Week 1 • Problem Set 2 • Acyclic closures and minimal models

Quick Problem:

Let $R = k[x, y/(x^3 + y^3)$.

Write out the complex, up to homological degree 3, underlying the following acyclic closure.

$$R\langle\, e_1, e_2, f \mid \partial(e_1) = x^2,\ \partial(e_2) = y^2,\ \partial(f) = xe_1 + ye_2, \rangle$$

That is, write the bases of the free R-modules and their images under the differentials.
Furthermore, what is the algebra (R-module) being resolved?

Problems:

1. Begin calculating an acyclic closure of the following canonical surjection.

$$R = k[[x, y]]/(x^2, xy) \longrightarrow S = k = R/(x, y)$$

Try to construct up through at least some of the degree 3 variables needed.

Be careful – you should kill cycles that give a minimal generating set for each homology – you may initially guess too many generating cycles, hence dependent in the homology, and need to throw some out.

2. Let k be a field with characteristic 2. (For example, take $k = \mathbb{Z}/2\mathbb{Z}$).

To illustrate why Cartan and Tate used *divided power* variables rather than *polynomial* variables, let us compute an example.

$$\text{Let } R = k[[x, y]]/(f) \text{ with } f = xy, \text{ and set } S = R/(x).$$

(a) Find the acyclic closure of S over R.

Note: By a theorem of Tate that we will see in Lecture 3 that one need only adjoin variables up to degree two as this is an embedded c.i.

(b) Compute the first two steps of the minimal model for S over R.

(c) (Fun challenge) Find the minimal model for S over R. If short on time just try to write it down, you don't need to fully justify.