

Bar resolution \cong Priddy resn

2

Let R be std gdd k -algebra

The bar resn of k is:

$$B(R, R, k)_n \stackrel{\text{def}}{=} R \otimes_k B(R)_n \otimes_k k$$

$$= R \otimes_k \underbrace{\bar{R} \otimes_k \dots \otimes_k \bar{R}}_n \otimes_k k$$

where

$$\bar{R} = R_+ = \bigoplus_{n \geq 0} R_i = m$$

$$\partial(r \otimes r_1 \otimes \dots \otimes r_n \otimes 1)$$

$$= r r_1 \otimes r_2 \otimes \dots \otimes r_n - r \otimes r_1 r_2 \otimes \dots \otimes r_n + \dots + (-1)^i r \otimes r_1 \otimes \dots \otimes r_i r_{i+1} \otimes \dots \otimes r_n \pm \dots$$

this part of ∂
is called d

this part of ∂
is called b

$[b^2 \neq 0$, so $B(R)$ not a cx,
but satisfies Maurer-Cartan eqn, curved alg...]

Thm: $B = B(R, R, k) \xrightarrow{\sim} k$ is an R -free resn of k

It is very non-min! $R \otimes_k \underbrace{\bar{R}^{\otimes i}}_{\text{huge } k\text{-v.sp.}} = \text{huge free } R\text{-mod}$

but universal (same form any R).

(any M , $B(R, R, M) \xrightarrow{\sim} M$)

Separate B out by graded pieces:

Note 1st:

$$R = \frac{T(V)}{(W)} = \underbrace{k}_{R_0} \oplus \underbrace{V}_{R_1} \oplus \underbrace{\frac{V^{\otimes 2}}{W}}_{R_2} \oplus \underbrace{\frac{V^{\otimes 3}}{V \otimes W + W \otimes V}}_{R_3} \oplus \dots$$

delete in \bar{R}

cont'd

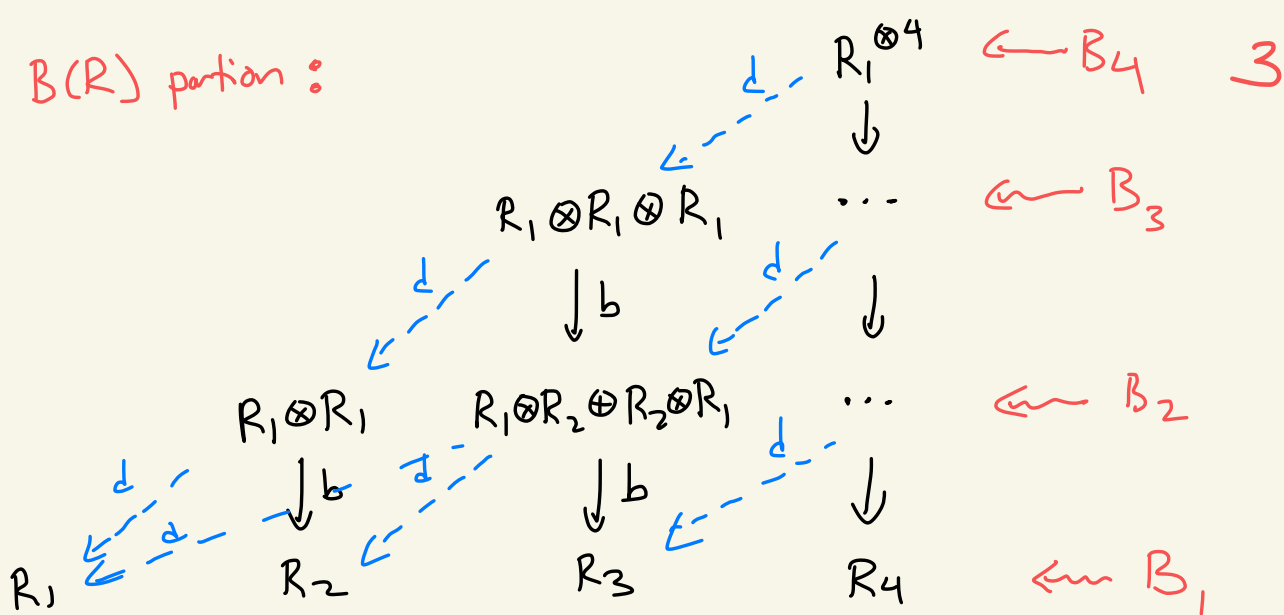
$$B = R \otimes_k R$$

$B(R)$ portion:

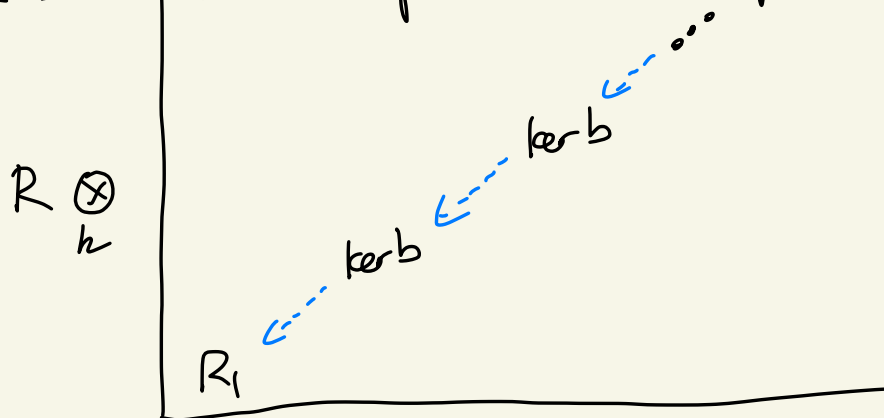
$$B = R \otimes_k R$$

b maps: 0,1's

d maps:
min'l
(ind = m...)



The kernel of each top vertical b map forms a subcx:



In column 2:

$$\ker(R_1 \otimes_k R_1 \xrightarrow{b} R_2) = \ker(V \otimes_k V \rightarrow \frac{V \otimes V}{W}) = W = (R_2^!)^*$$

In general:

$$\ker(\underbrace{R_1 \otimes \dots \otimes R_1}_n \rightarrow \bigoplus_{\substack{i+j+2 \\ =n}} \underbrace{R_1 \otimes \dots \otimes R_2}_{i} \otimes \dots \otimes \underbrace{R_1}_{j}) = \bigcap_{\substack{i+j+2 \\ =n}} \underbrace{V \otimes \dots \otimes W}_{i} \otimes \dots \otimes \underbrace{V}_{j} = (R_n^!)^*$$

Inherited diff'l: just d

$$= \alpha \cdot (x_{i_1} \otimes x_{i_1}^*) = 2(\alpha) \text{ in } P(R)!$$

$$d(\alpha) = d(1 \otimes x_{i_1} \otimes x_{i_2} \otimes \dots \otimes x_{i_n}) = 1 \cdot x_{i_1} \otimes x_{i_2} \otimes \dots \otimes x_{i_n}$$

\uparrow
basis
elt

$$\xrightarrow{\text{basis elt}} x_{i_1}^* x_{i_2}^* \dots x_{i_n}^* \in (R_n^!)^*$$

$$\text{if basis elt} \quad x_{i_2}^* \dots x_{i_n}^* \in (R_{n-1}^!)^*$$

So, $P(R) \hookrightarrow B = B(R, R, k)$ [Both recs of k , but $P(R)$ min'l]