不确定性量化导论

1 数值差商

对于微分方程 $\begin{cases} \frac{dy}{dx} = f(x,y) \\ y(a) = y_0 \end{cases}$ 向前差商 $\frac{y(x_{n+1}-y(x_n))}{\Delta x} \approx y'(x_n)$

2 Runge-Kutta 方法（泰勒展开）

$y'(x) = f(x,y), \quad y''(x) = f_x(x,y) + f_y(x,y)f(x,y), \cdots$ 截断 $T = O(\Delta x^p)$，带入可得

$$y^{n+1} = y^n + \cdots$$

如

- $p = 1, \quad y^{n+1} = y^n + \Delta x f(x_n, y^n)$ 即为欧拉方法;
- $p = 2, \quad y^{n+1} = y^n + \Delta x f(x_n, y^n) + \frac{\Delta x^2}{2} \left[f_x(x,y) + f_y(x,y)f(x,y) \right]$

2.1 3 阶 Strong Stability Preserving 方法

3 PDE 的有限差分方法

$$u_t(x,t) = a u_x(x,t), \quad x \in [0, 2\pi]$$
$$u(x,0) = f(x)$$

边界是周期的：

$$\frac{d^p f}{dx^p}(0) = \frac{d^p f}{dx^p}(2\pi)$$