# Autonomous Vehicle Planning and Control

Wu Ning

# Session 6

Vehicle Motion Planning/Local Planner

# Outline

## Motion planner

- Motion planner basic concept
- Functionality and Common method
- Fundamental Concepts and Key terminology 关键术语

## Stochastic Sampling Methods 随机采样方法

- RRT
- RRT*
- Other improve methods

## Lattice planner

- Frenet Coordinate
- Speed planning
- Trajectory planning

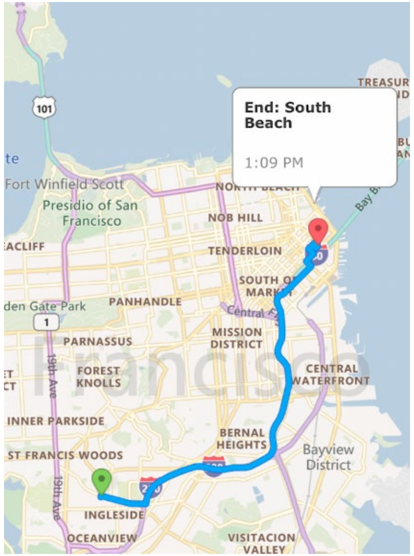
# Motion Planner

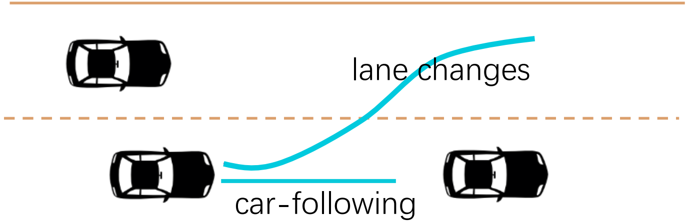
Part I: Main concept and key terminology

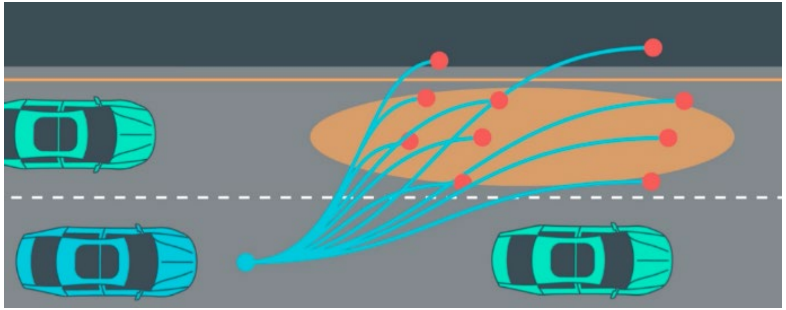
# Motion Planning Problem

Determine a sequence of actions to reach a specified goal

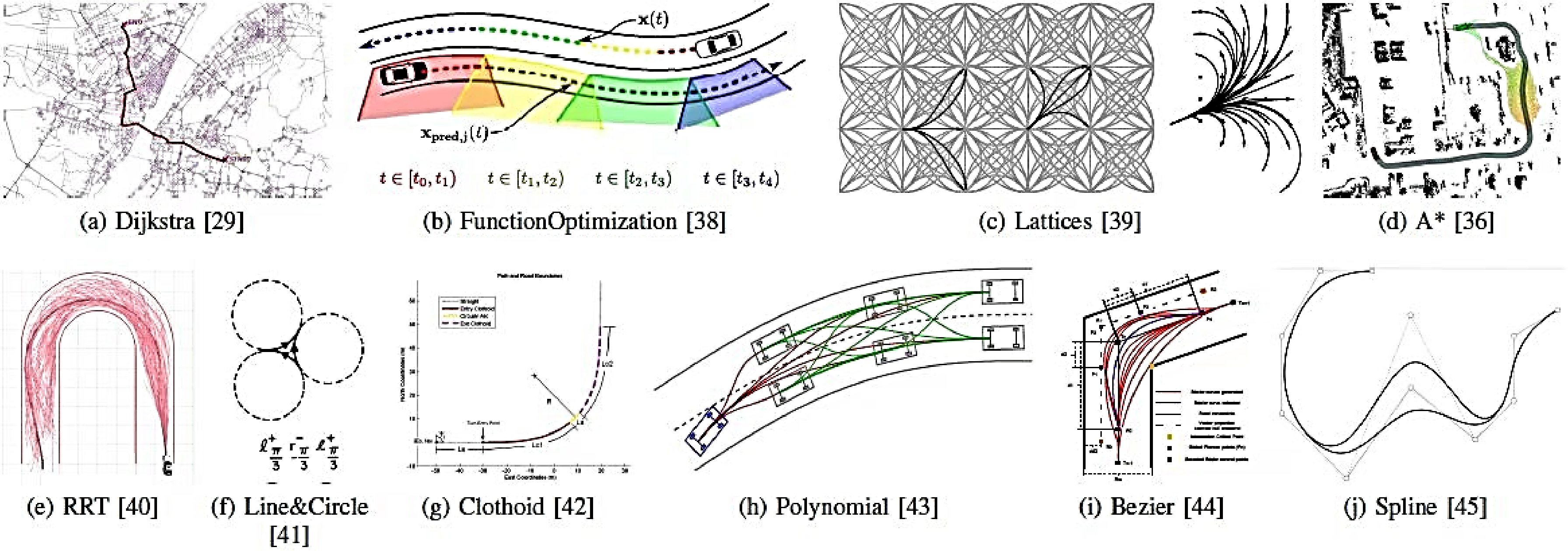
Mission/ Route Planning → Behavior Planner/ Decision making → Motion planning/ Local planner → Motion Control → Vehicle



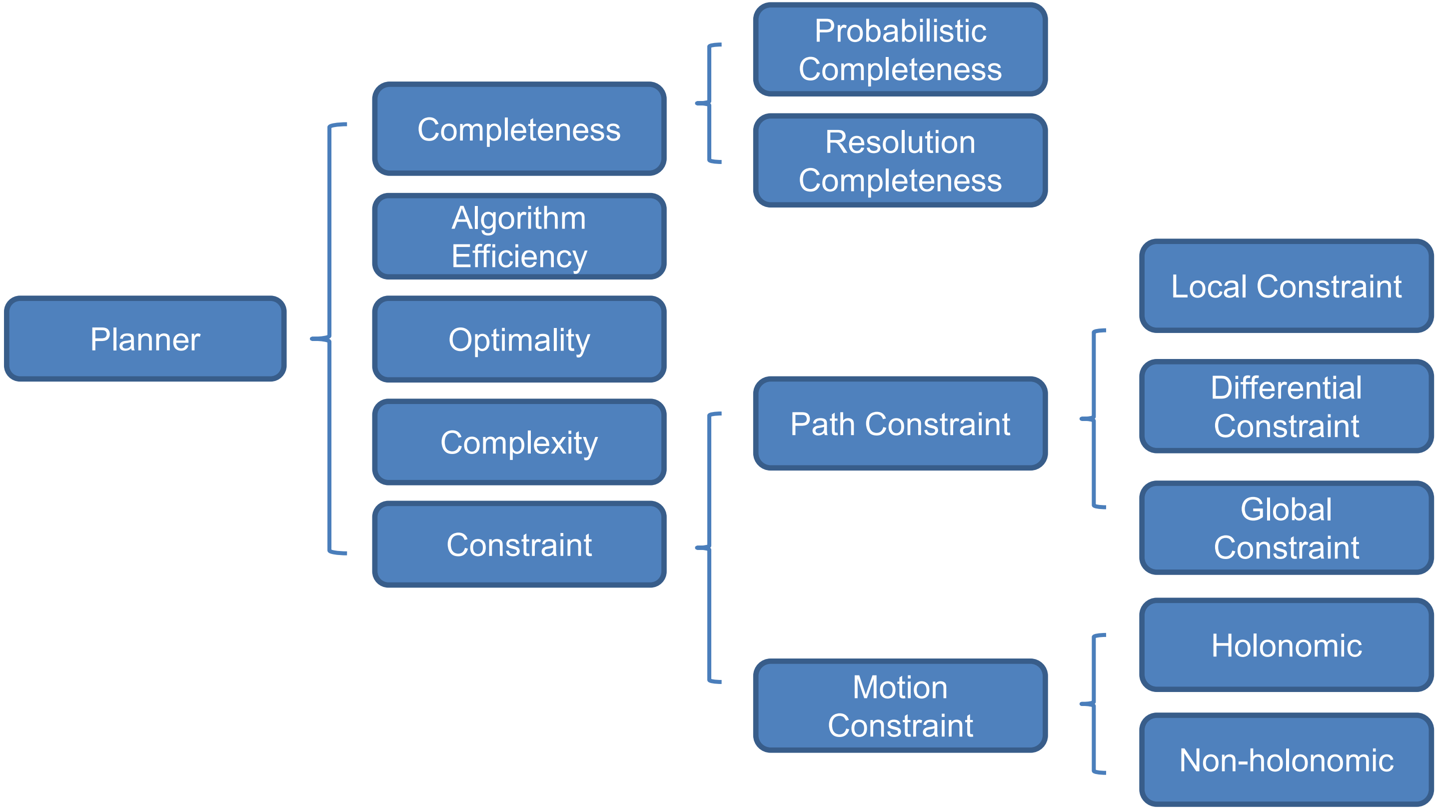



# The Common Motion Planning Methods




(a)Global path by Dijkastra. (b) Trajectory optimization considering a vehicle in the other lane. (c) Lattices and motion primitives. (d) Hybrid A* in DRAPA Junior. (e) RRT. (f) Optimal path to turn the vehicle around. (g) Planning a turn from Stanford. (h) Different motion states, planned with polynomial curves. (i) Evaluation of several Bezie curves. (j) Spline behaviour when a knot changes places

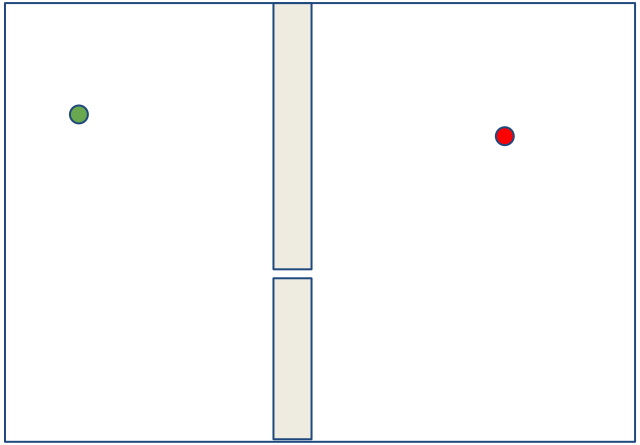
# Planner Properties



# Fundamental Concepts

**Completeness:**

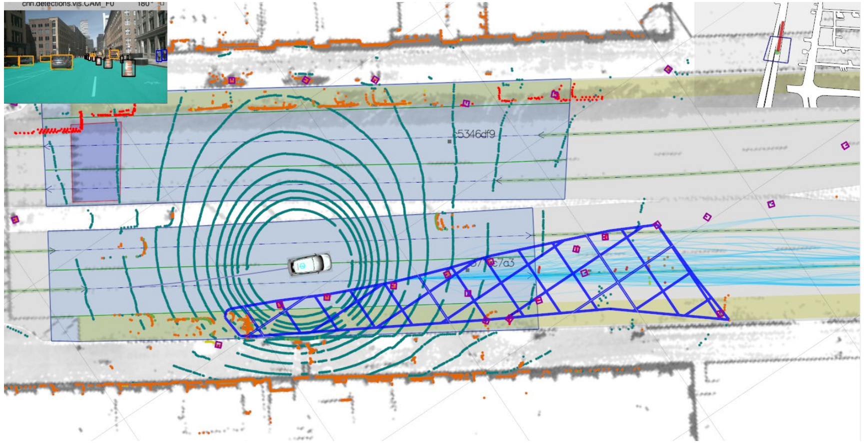
- Ability to find a solution if one exists,
- Narrow Gap Problem Examples.



2D space ($\mathbb{R}^2$), point robot

无人驾驶汽车在建筑周围导航

Self-driving car navigating around construction

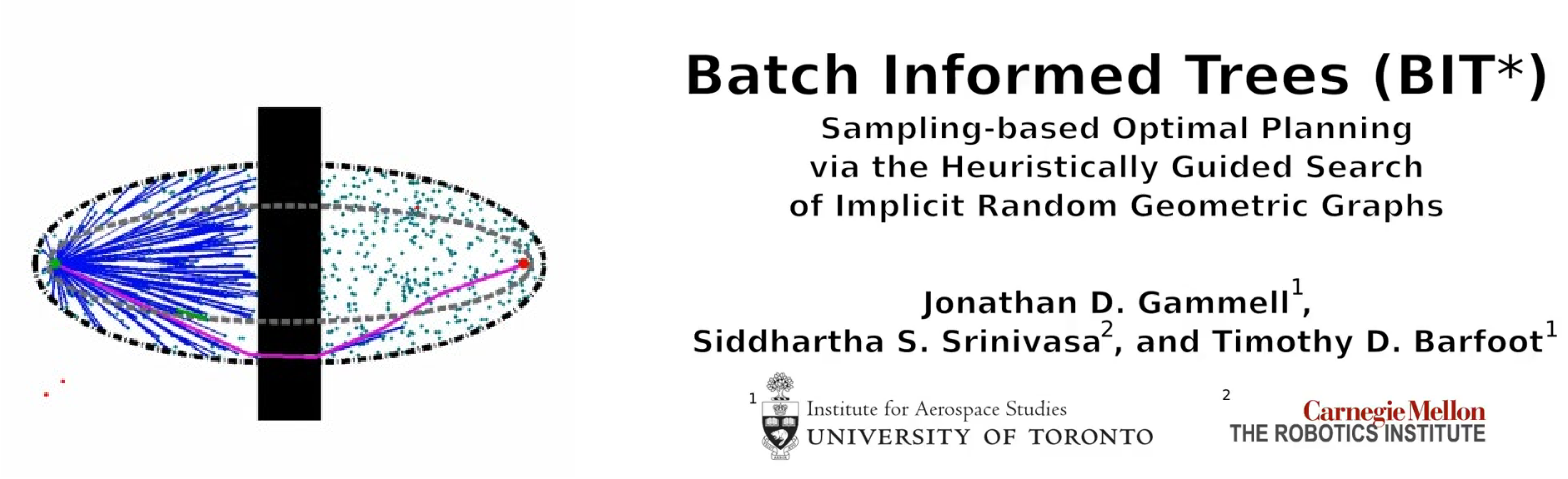


# Fundamental Concepts

**Probabilistic Completeness:**

- Probability of finding a solution (when one exists) increases as computational time spent on the problem increases

找到解决方案的可能性(如果存在的话)随着花费在问题上的计算时间的增加而增加




[Batch Informed Trees (BIT*) @ UTIAS / CMU (ICRA 2015)]

# Fundamental Concepts

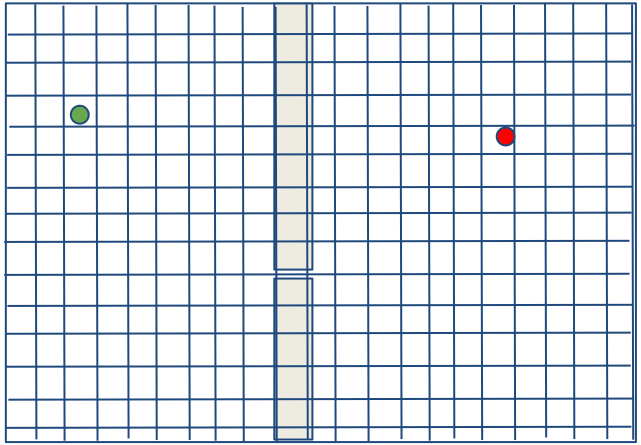
**Resolution Completeness:**

- Ability to find a solution if one exists AND using fine enough resolution in discretization of the state and/or control space

如果存在解决方案，能够找到解决方案，并在状态和/或控制空间的离散化中使用足够精细的分辨率

Narrow Gap Problem Examples:

2D space ($\mathbb{R}^2$), point robot



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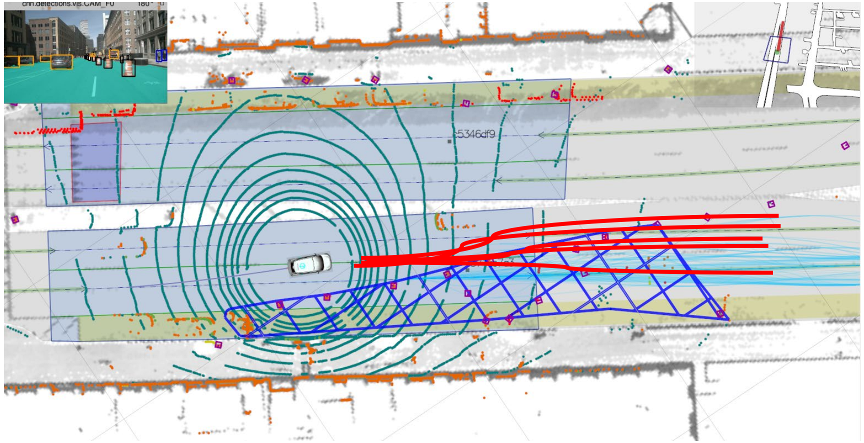


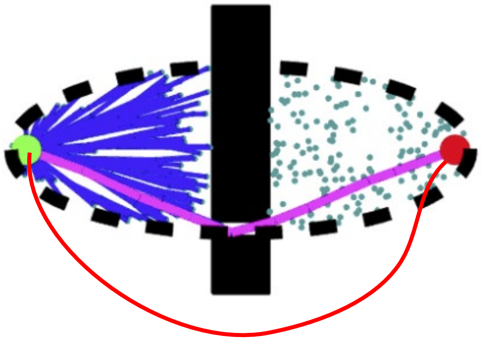
# Fundamental Concepts

**Algorithmic efficiency:** 算法如何求解与输入数据大小成比例的时间标度,

- How an algorithm solve time scales proportionally with respect to size of the input data,
- Big $O$ Notation, $O(n)$ takes time proportional to number of elements.
  花费的时间与元素数量成正比

**Optimality:**

- Optimal is able to find the lowest cost solution of all possible options, optimal能够找到所有可能选项的最低成本解决方案,
- Suboptimal of a lower cost solution exists,
  存在较低成本的次优解决方案,
- Asymptotically optimal if guaranteed to converge to the optimal solution given increasing,
  渐近最优如果保证收敛到给定增加的最优解,
  computational time spent on the problem.
  花在这个问题上的计算时间。



# Fundamental Concepts

**Complexity**

- Space dimensionality
  - configuration space is a $\mathbb{R}^3$ for rigid body. But for the multi-bodies track, bicycle mode is not accurate enough to capture all the constraints.
- Geometric complexity
  - How bounding box and bounding box interact；
  - How to detect a path between polygons and interact with another obstacles;
  - ...

# Fundamental Concepts

There are three types of **Path Constraints**:

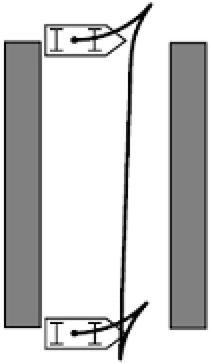
- **Local Constraints**
  - Avoid collision with static obstacles
- **Differential constraints**
  - Bounded curvature, limited steering angle for vehicle
- **Global constraints**
  - Find the shortest path by A*

# Fundamental Concepts

**Holonomic vs non-holonomic motion constraints**

- Holonomic if # of controllable DOF = # of total DOF

Cars are non-holonomic since control throttle and steering (2 DOF), but move in $SE(2)$ $(x, y, \theta)$



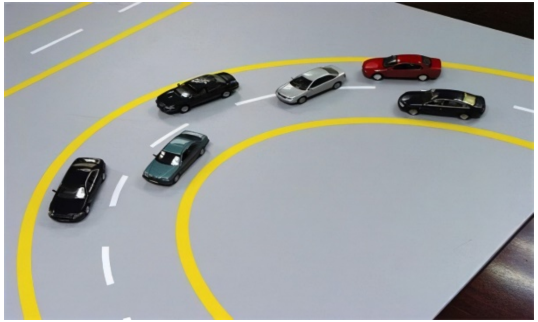
# Local Planning

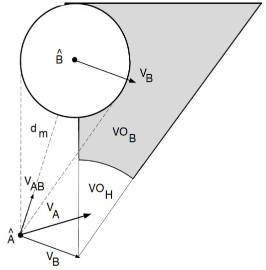
Find higher precision “good” (near optimal w.r.t. cost function) path to execute.

**Where to search?**

**Configuration Space Parameterization Options**

- Workspace (direct physical environment, traditional)
- Control Space (e.g. velocity space, see link)
    - Only saves effort in simple problems
- Belief Space (POMDP, more later in Session 7)



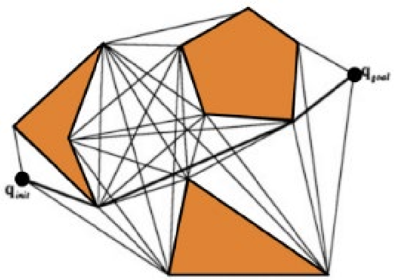



Velocity Obstacle (control space)

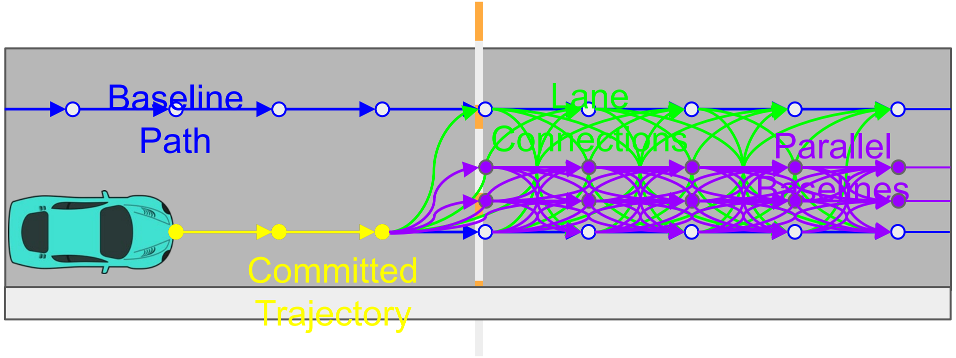
# Local Planning

**How to search?**

- **Combinatorial methods (exact complete solution, e.g. visibility graph - link)**
  - Rarely exists to find optimal solution to complex problems
- **Sampling-based methods**
  - 确定性的 Deterministic (resolution complete), e.g. uniform grid or road structure graph, repeatable
  - Stochastic (probabilistic completeness), e.g. random sampling 随机的
- May need some **smoothing/post-process** to improve quality of solution




Visibility Graph (combinatorial)



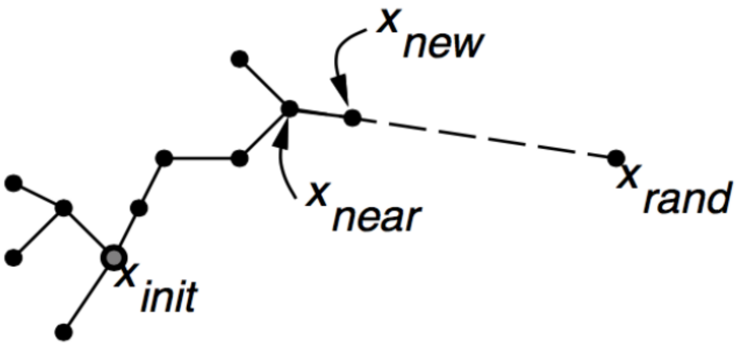

road "structural graph" (deterministic sampling)

# Motion Planner

## Part II Stochastic Sampling Methods

# Stochastic Sampling Methods: Rapidly-exploring Random Trees

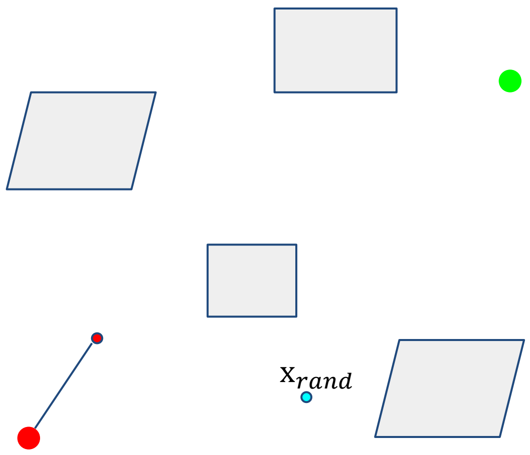
Build up a tree from start to goal through generating “next states” in the tree by executing random controls.



# Stochastic Sampling Methods: RRT

**Algorithm 1:** RRT Algorithm

**Input:** $\mathcal{M}$,$x_{init}$,$x_{goal}$
**Result:** A path $\Gamma$ from $x_{init}$ to $x_{goal}$
$\mathcal{T}$.init();
**for** $i = 1$ *to* $n$ **do**
  $x_{rand} \leftarrow Sample(\mathcal{M})$ ;
  $x_{near} \leftarrow Near(x_{rand}, \mathcal{T})$;
  $x_{new} \leftarrow Steer(x_{rand}, x_{near}, StepSize)$;
  $E_i \leftarrow Edge(x_{new}, x_{near})$;
  **if** *CollisionFree*($\mathcal{M}$, $E_i$) **then**
    $\mathcal{T}.addNode(x_{new})$;
    $\mathcal{T}.addEdge(E_i)$;
  **if** $x_{new} = x_{goal}$ **then**
    Success();




Sample a node $X_{rand}$ in the free space

# Stochastic Sampling Methods: RRT

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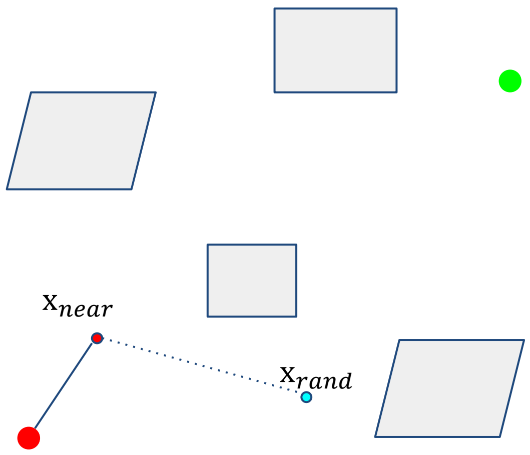


Find the nearest node $X_{near}$ in current tree

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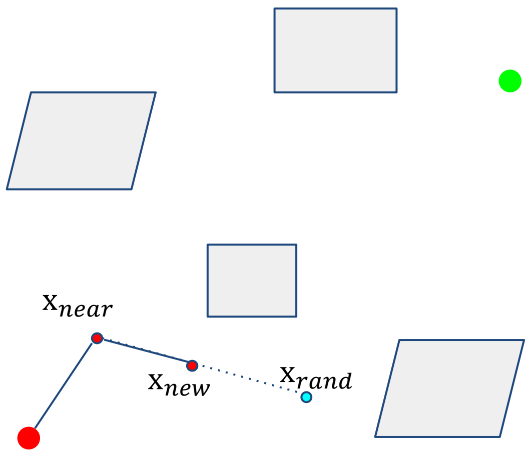


Grow a new node $X_{new}$ and path $E_i$ from $X_{near}$

# Stochastic Sampling Methods: RRT

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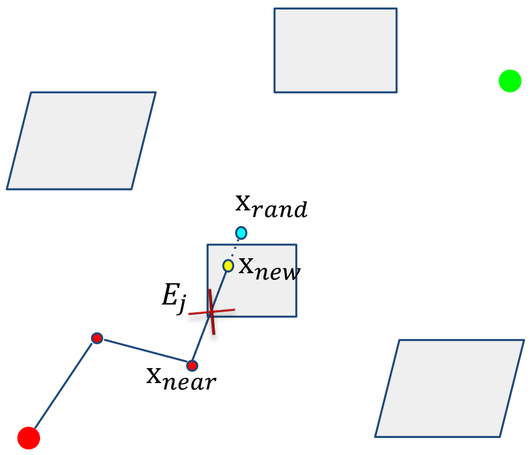


Do not grow if collision

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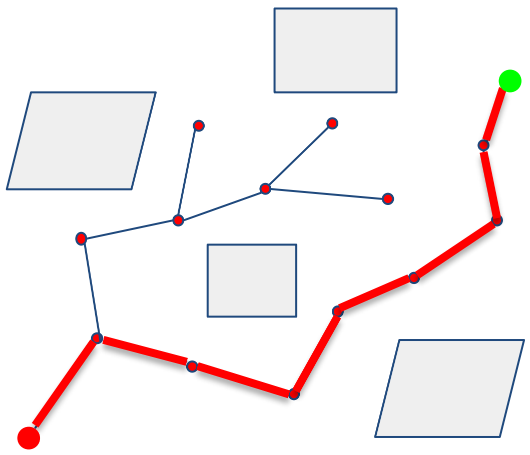


Repeat sampling for n times until the tree reaches the goal or goal region

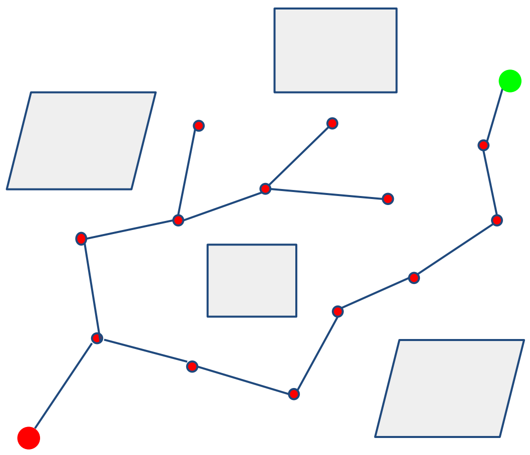
# Stochastic Sampling Methods: RRT

**Pros**

- Easy to implement
- Aims to find a path from the start to the goal
- More target-oriented than PRM

**Cons**

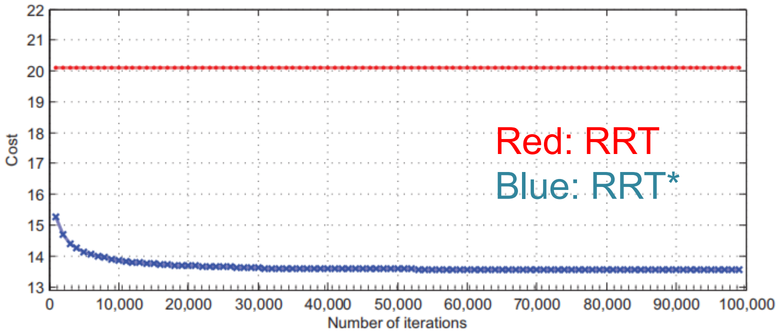
- Not optimal solution
- Not efficient (leave room for improvement)
- Sample in the whole space



# Stochastic Sampling Methods: RRT*

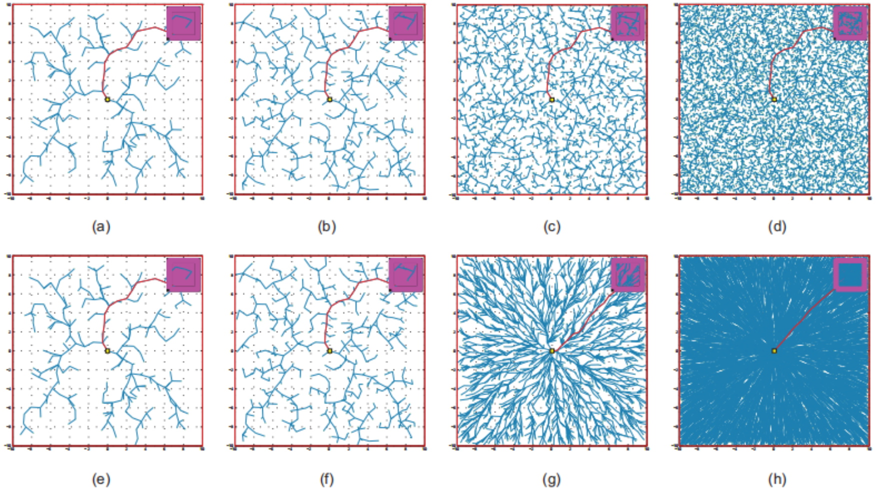
- An improvement of RRT
- Asymptotically optimal (under conditions)

渐近最优(在条件下)



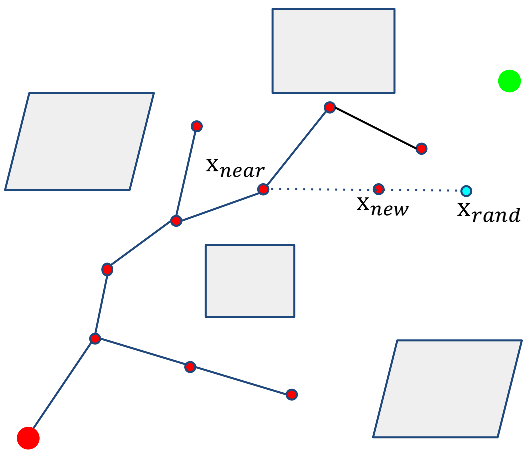

RRT:

RRT*:



# Stochastic Sampling Methods: RRT*

Consider $N$ nearing nodes

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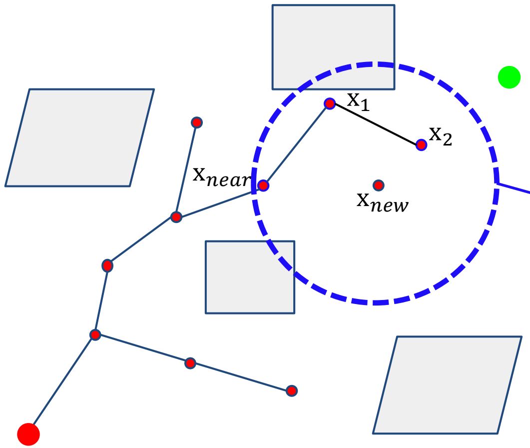


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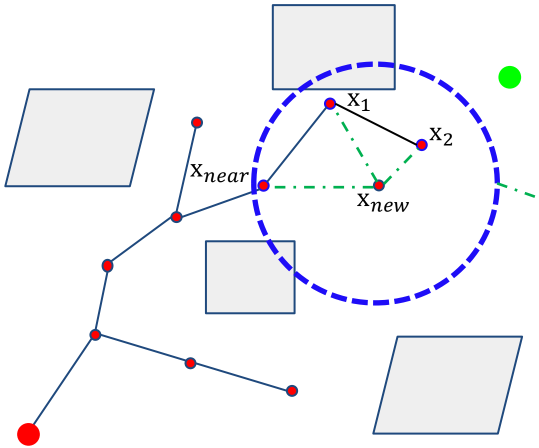
# Stochastic Sampling Methods: RRT*

Consider history cost instead of only local information

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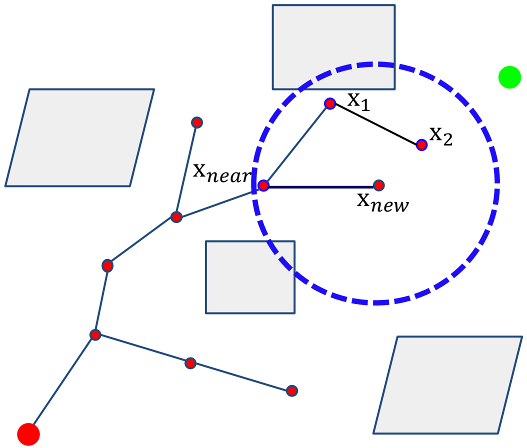


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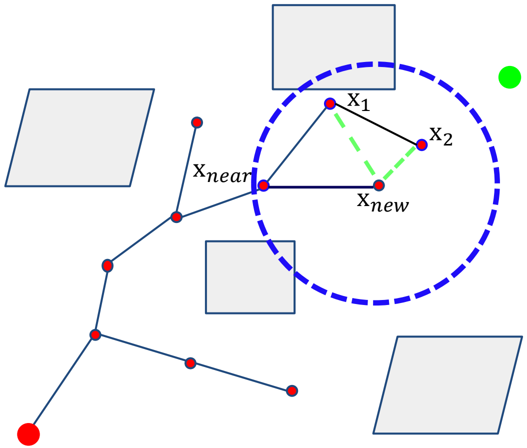
# Stochastic Sampling Methods: RRT*

Rewire to improve local optimality

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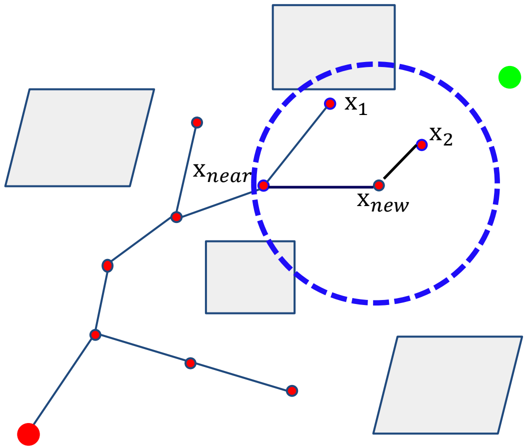


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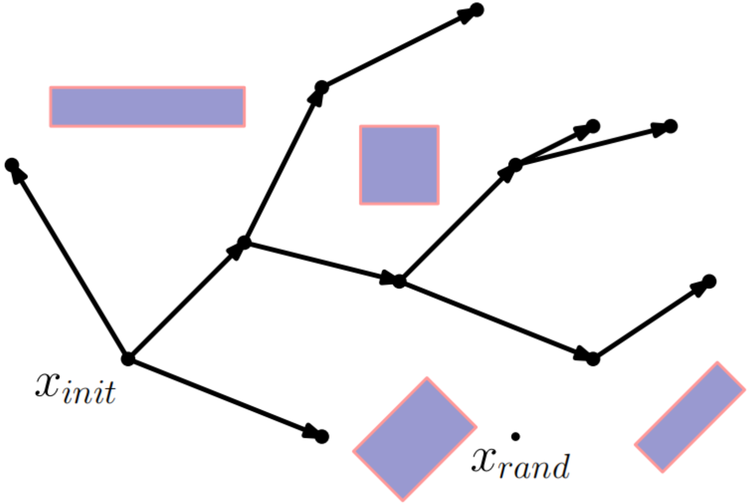
# Stochastic Sampling Methods: RRT*

Generate a random point $x_{rand}$

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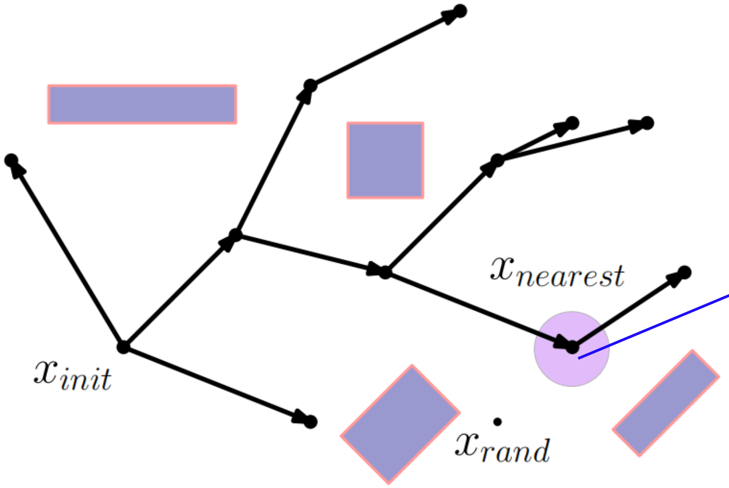
    $x_{min} \leftarrow ChooseParent(X_{near}, x_{near}, x_{new})$ ;
    $\mathcal{T}.addNodEdge(x_{min}, x_{new})$;
    $\mathcal{T}.rewire()$;

# Stochastic Sampling Methods: RRT*

Find the nearest nodes $x_{nearest}$

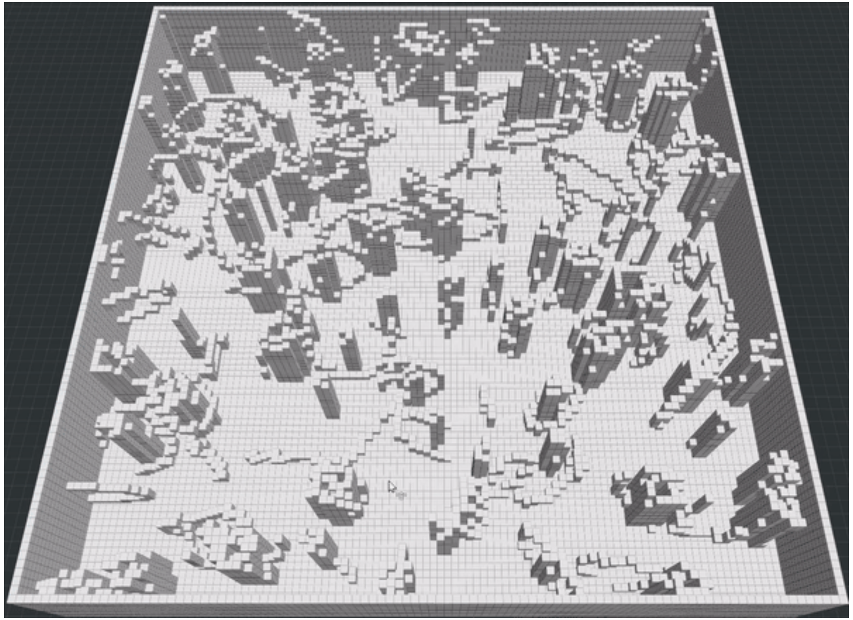



**Algorithm 2:** RRT* Algorithm

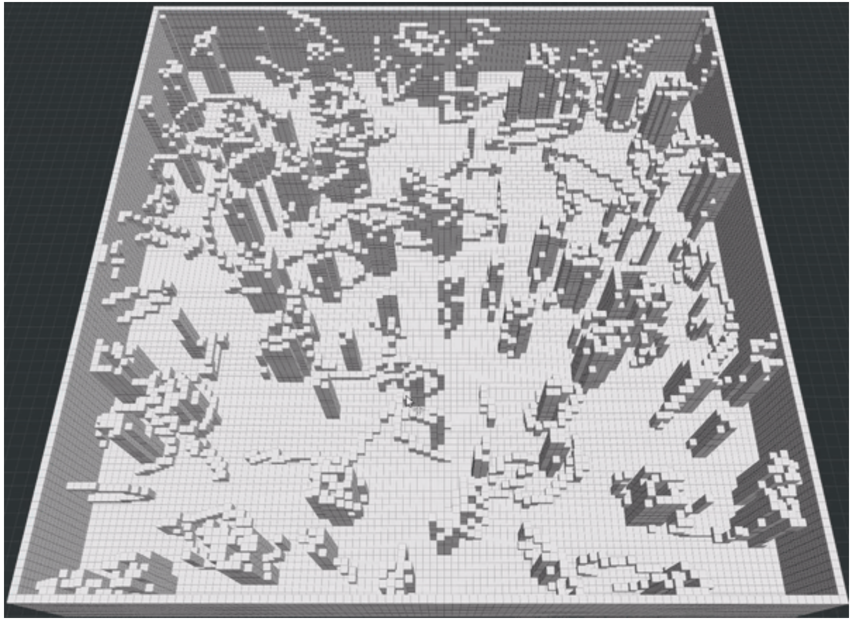
**Input:** $\mathcal{M}$,$x_{init}$,$x_{goal}$
**Result:** A path $\Gamma$ from $x_{init}$ to $x_{goal}$
$\mathcal{T}$.init();
**for** $i = 1$ *to* $n$ **do**
    $x_{rand} \leftarrow Sample(\mathcal{M})$ ;
    $x_{near} \leftarrow Near(x_{rand}, \mathcal{T})$;
    $x_{new} \leftarrow Steer(x_{rand}, x_{near}, StepSize)$;
    **if** *CollisionFree*($x_{new}$) **then**
        $X_{near} \leftarrow NearC(\mathcal{T}, x_{new})$;
        $x_{min} \leftarrow ChooseParent(X_{near}, x_{near}, x_{new})$ ;
        $\mathcal{T}.addNodEdge(x_{min}, x_{new})$;
        $\mathcal{T}.rewire()$;

# Stochastic Sampling Methods: RRT*

RRT



RRT*



https://github.com/ZJU-FAST-Lab/sampling-based-path-finding

# Stochastic Sampling Methods: Obstacle avoidance

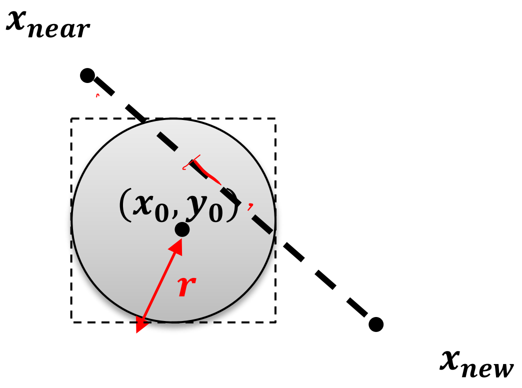
**Obstacle avoidance check:**

**To simplify the check, we used circle or polygon (rectangle) to represent the objects**

- For the circle object, we can check the obstacle easily by

$$x_0 - r - \varepsilon < x_{new,x} < x_0 + r + \varepsilon$$

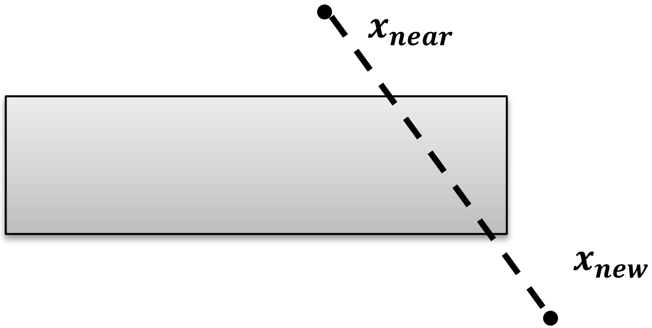
$$y_0 - r - \varepsilon < x_{new,y} < y_0 + r + \varepsilon$$



# Stochastic Sampling Methods: Obstacle avoidance

**Obstacle avoidance check: (polygon: rectangle)**

- There are two steps for collision checking for polygon
- If $x_{near}$ and $x_{new}$ are in the same side of obstacle;
  - If they are at the same side, there will be no interaction with the obstacle;
- If $x_{near}$ and $x_{new}$ are not at the same side, there will be two situations:
  - $x_{new}$ is inside the polygon, and there must be an intersection between the connection of $x_{near}$ and $x_{new}$
  - If both $x_{near}$ and $x_{new}$ are at the outside of the polygon, we will need to use the connect line to check the collision.



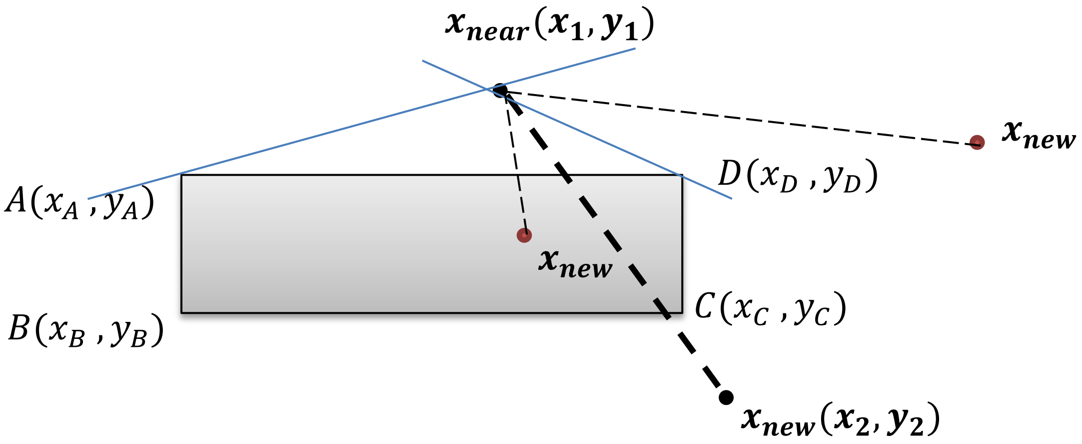
# Stochastic Sampling Methods: Obstacle avoidance

**Obstacle avoidance check:**

• If both $x_{near}$ and $x_{new}$ are at the outside of the polygon, we will need to use the connect line to check the collision.

$$k_{x_{near}x_{new}} < k_{Dx_{near}} \ \&\& \ k_{x_{near}x_{new}} > k_{Ax_{near}}$$

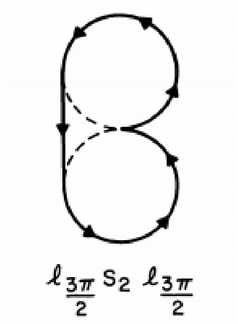
where k is the slope of line

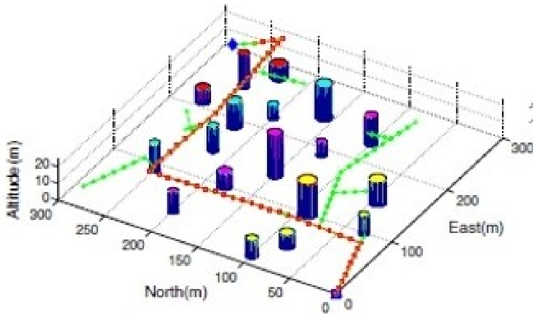


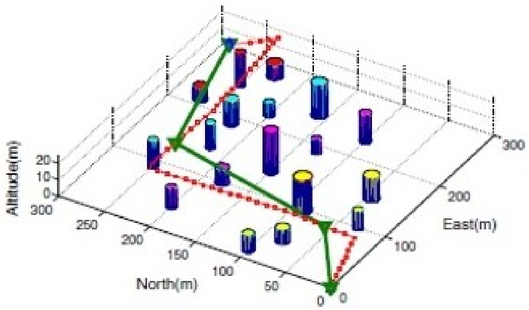
# Stochastic Sampling Methods: Other improvement

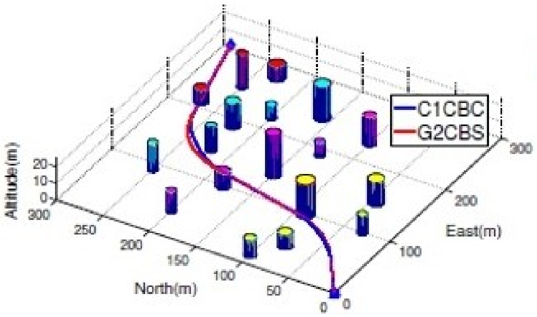
Improvements may come through focus on any of those steps individually, e.g.:

- Connect with edges of desirable properties
  - ✓ Dubins for shortest length w/fixed turn radius
  - ✓ Reeds-Shepp for forward/backward w/fixed turn radius
  - ✓ Spline, clothoid, Bezier for continuous curvature



# Stochastic Sampling Methods: $\mathrm{RRT}^*$ Improvement

**Bias Sampling**

- Sample biasing toward the goal

**Sample Rejection**

- Reject samples that don't meet some threshold until you reach the number of samples you need

**Tree Pruning**

- Prune the non-promising sub trees to reduce neighbor query cost.

**Graph Sparsify**

- Reject samples by resolution. Introduce near optimality.

**Delay Collision Check**

- Sort the neighbours by potential cost-to-come values. Check collisions in order and stop once a collision-free edge is found.

**Anytime RRT**

- Store the collision-checking results for ChooseParent and Rewire.

[Informed RRT*: Optimal Sampling-based Path Planning Focused via Direct Sampling of an Admissible Ellipsoidal Heuristic]

# Motion Planner

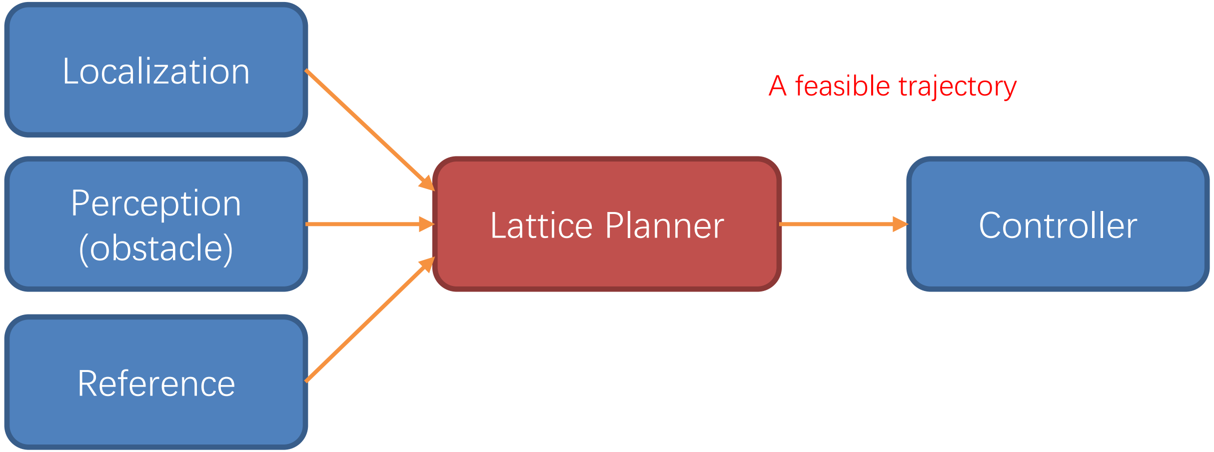
## Part III Lattice Planner

# Lattice Planner

The Lattice Planner algorithm is a local motion planner,

- Sample based motion planner;
- Plan in Frenet coordinate;
- The output is a smooth, safe and collision-free local trajectory that satisfies the vehicle's kinematics and speed constraints which is directly feed into controller.

输出是一个平滑、安全和无碰撞的局部轨迹，满足车辆的运动学和速度约束，并直接馈入控制器。



# Lattice Planner

## The basic process of Lattice planner:

**Transform to Frenet coordinate, and calculate the look ahead**

↓

**Sample the path**
(based on time, target speed and lateral displacement of the reference path) 参考路径的横向位移

↓

**Construct lateral and longitudinal displacement planning function $s(t)$ and $d(s)$**

↓

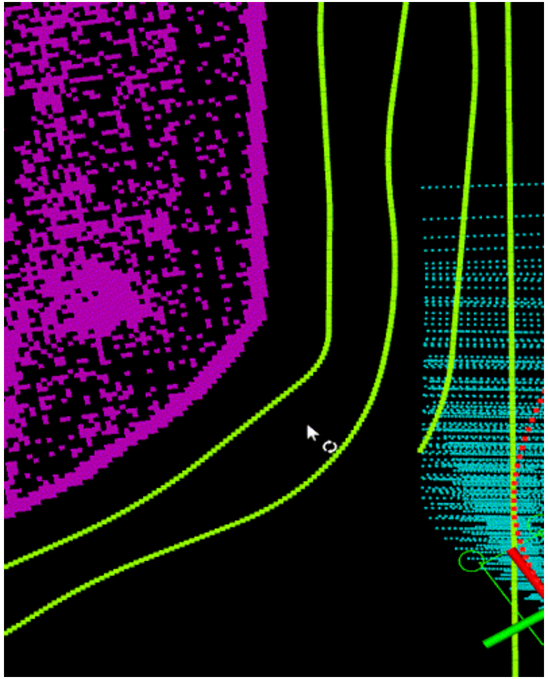
**Calculate the reference paths in Frenet coordinate by sampling time $t$**

↓

**Transform the paths back to global Cartesian coordinate**

↓

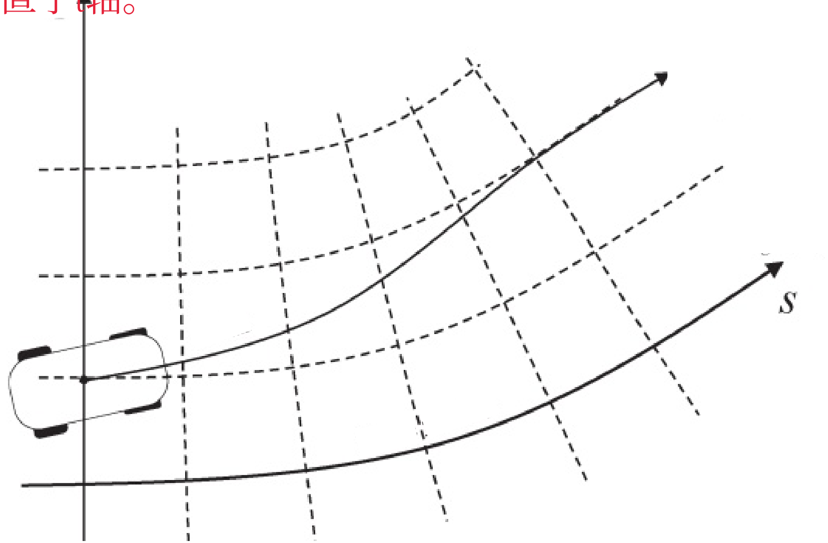
**Trajectory scoring**
(based on the cost and constraints, such as collision cost and vehicle dynamics constraints)

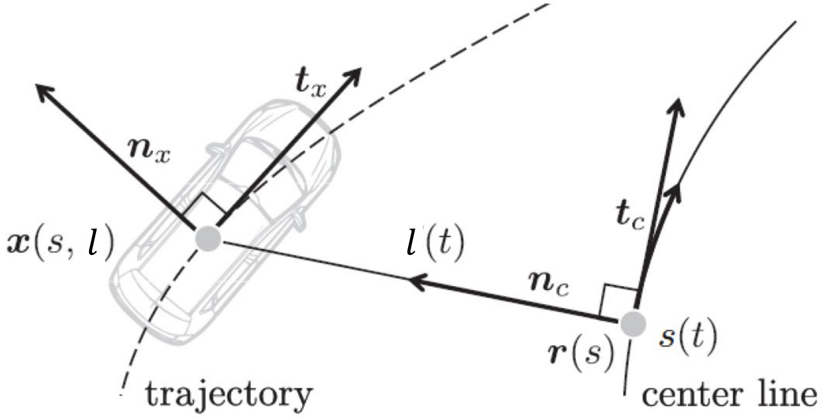


# Frenet coordinate and Cartesian coordinate transformation

Frenet coordinate is a frame on the reference line, is a moving frame. Its origin *r* is the nearest point from the vehicle to the path. The *t*-axis is along the tangiential direction at *r* while *n*-axis is pependicular to *t*-axis.

Frenet坐标是参考线上的一个坐标系，是一个运动坐标系。它的原点是从车辆到路径的最近点。t轴沿切线方向，而n轴垂直于t轴。



# Frenet coordinate and Cartesian coordinate transformation

**Frenet coordinate:** $[s, \dot{s}, \ddot{s}, l, \dot{l}, \ddot{l}, l', l'']$

$s$: longitudinal axis (T- axis) of Frenet coordinate

$\dot{s} = \frac{ds}{dt}$: differentiation of longitudinal axis w.r.t. to time,

i.e. speed 纵轴相对于时间的微分，即速度

$\dot{s} = \frac{d\dot{s}}{dt}$: longitudinal acceleration

$l$: lateral axis of Frenet coordinate

$\dot{l} = \frac{dl}{dt}$: lateral speed

$\dot{l} = \frac{d\dot{l}}{dt}$: lateral acceleration

横轴相对于纵轴的微分

$l'$: differentiation of lateral axis w.r.t. to longitudinal axis

$l''$: 2nd derivative of lateral axis w.r.t. to longitudinal axis

**Cartesian coordinate:** $[\vec{x}, v_x, a_x, \theta_x, \kappa_x]$

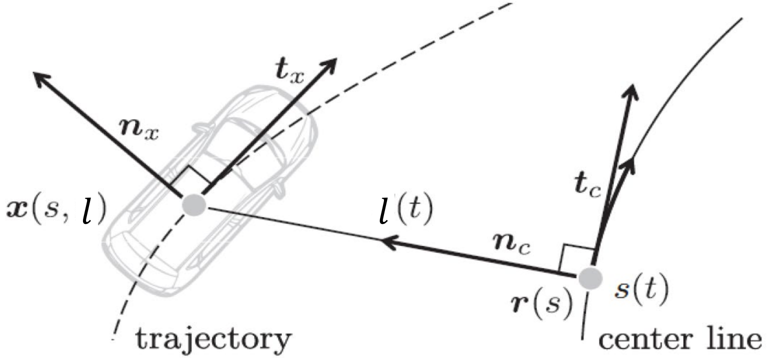
$\vec{x}$: a position vector in Cartesian coordinate

$v_x = \left\|\dot{x}\right\|_2$: speed in Cartesian coordinate

$a_x = \frac{v_x}{dt}$: acceleration in Cartesian coordinate

$\theta_x$: heading in Cartesian coordinate

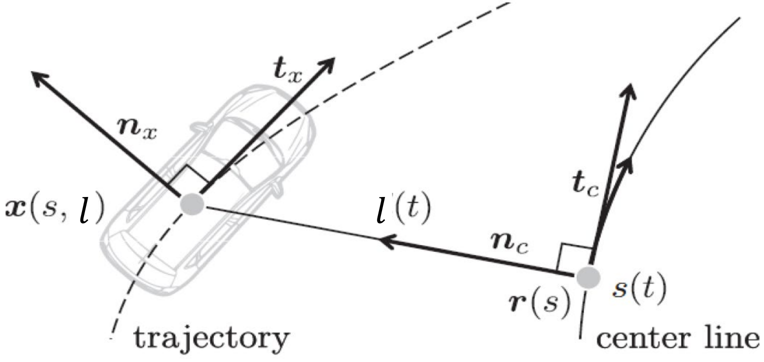
$\kappa_x = \frac{d\theta_x}{ds}$: curvature



# Frenet coordinate and Cartesian coordinate transformation

Cartesian to Frenet

$$
\begin{cases}
s = s_r \\
\dot{s} = \dfrac{v_x cos(\theta_x - \theta_r)}{1 - k_r l} \\
\ddot{s} = \dfrac{a_x cos(\theta_x - \theta_r) - \dot{s}^2 \left[ l' \left( k_x \dfrac{1 - k_r l}{cos(\theta_x - \theta_r)} - k_r \right) - (k_r' l + k_r l') \right]}{1 - k_r l} \\
l = sign\big((x_x - x_r)cos(\theta_r) - (y_x - y_r)sin(\theta_r)\big)\sqrt{(x_x - x_r)^2 + (y_x - y_r)^2} \\
l' = (1 - k_r l)tan(\theta_x - \theta_r) \\
l'' = -(k_r' l + k_r l')tan(\theta_x - \theta_r) + \dfrac{1 - k_r l}{cos^2(\theta_x - \theta_r)} \left( \dfrac{1 - k_r l}{cos(\theta_x - \theta_r)} k_x - k_r \right)
\end{cases}
$$



# Frenet coordinate and Cartesian coordinate transformation

Frenet to Cartesian

$$
\begin{cases}
x_x = x_r - l\sin(\theta_r) \\
y_x = y_r + l\cos(\theta_r) \\
\theta_x = \arctan\left(\dfrac{l'}{1 - k_r l}\right) + \theta_r \\
v_x = \sqrt{[\dot{s}(1 - k_r l)]^2 + (\dot{s} l')^2} \\
a_x = \ddot{s}\dfrac{1 - k_r l}{\cos(\theta_x - \theta_r)} + \dfrac{\dot{s}^2}{\cos(\theta_x - \theta_r)}\left[l'\left(k_x \dfrac{1 - k_r l}{\cos(\theta_x - \theta_r)} - k_r\right) - (k_r' l + k_r l')\right] \\
k_x = \left(\left(l'' + (k_r' l + k_r l')\tan(\theta_x - \theta_r)\right)\dfrac{\cos^2(\theta_x - \theta_r)}{1 - k_r l} + k_r\right)\dfrac{\cos(\theta_x - \theta_r)}{1 - k_r l}
\end{cases}
$$


$$
\begin{cases}
x_x = x_r - l\sin(\theta_r) \\
y_x = y_r + l\cos(\theta_r) \\
\theta_x = \arctan\left(\dfrac{l'}{1 - k_r l}\right) + \theta_r \\
v_x = \sqrt{[\dot{s}(1 - k_r l)]^2 + (\dot{s} l')^2} \\
a_x = \ddot{s}\dfrac{1 - k_r l}{\cos(\theta_x - \theta_r)} + \dfrac{\dot{s}^2}{\cos(\theta_x - \theta_r)}\left[l'\left(k_x \dfrac{1 - k_r l}{\cos(\theta_x - \theta_r)} - k_r\right) - (k_r' l + k_r l')\right] \\
k_x = \left(\left(l'' + (k_r' l + k_r l')\tan(\theta_x - \theta_r)\right)\dfrac{\cos^2(\theta_x - \theta_r)}{1 - k_r l} + k_r\right)\dfrac{\cos(\theta_x - \theta_r)}{1 - k_r l}
\end{cases}
$$

# Lattice planner sampling

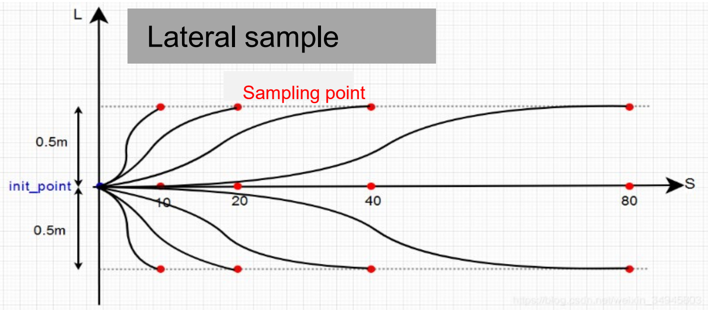
Lattice planner sampling includes:

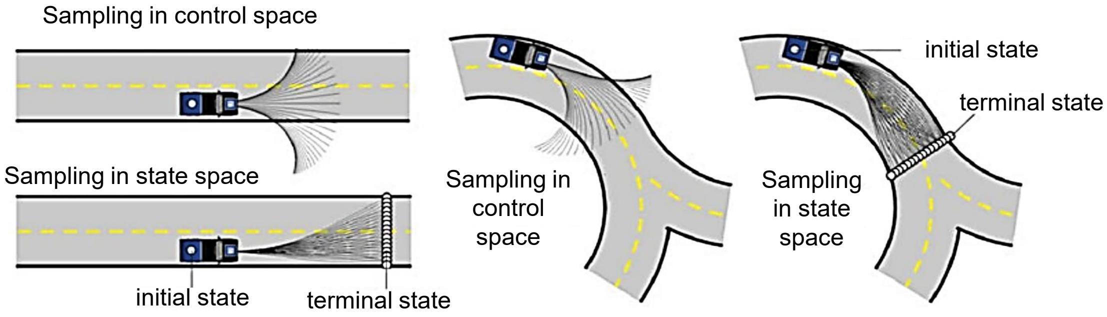
- Lateral sampling
- Longitudinal sampling
- Time sampling

状态空间:源于环境的空间

- State space: Those arising from the environment
- Control space: Those arising from vehicle mobility

控制空间:由车辆机动性产生的空间




State Space Sampling of Feasible Motions for High-Performance Mobile Robot Navigation in Complex Environments, Thomas M. Howard, Colin J. Green, and Alonzo Kelly

# Lattice planner: speed planning

基于采样使用多项式公式化横向规划函数 l(s）和纵向规划函数 s(t)

- Formulate lateral planning function $l(s)$ and longitudinal planning function $s(t)$ using polynomials based on the sampling.
- Typically, 4th or 5th order polynomials are used to ensure the smoothness of the path.

通常，四阶或五阶多项式用于确保路径的平滑度。

In stop and go, or spacing control ($5^{th}$ order)

$$s(t) = c_1 t^5 + c_2 t^4 + c_3 t^3 + c_4 t^2 + c_5 t + c_6$$

$$v(t) = 5c_1 t^4 + 4c_2 t^3 + 3c_3 t^2 + 2c_4 t + c_5$$

$$a(t) = 20c_1 t^3 + 12c_2 t^2 + 6c_3 t + 2c_4$$

In cruse control ($4^{th}$ order)

$$s(t) = b_1 t^4 + b_2 t^3 + b_3 t^2 + b_4 t + b_5$$

$$v(t) = 4b_1 t^3 + 3b_2 t^2 + 2b_3 t + b_4$$

$$a(t) = 12b_1 t^2 + 6b_2 t + 2b_3$$

# Lattice planner: speed planning

纵向拟合多项式解；

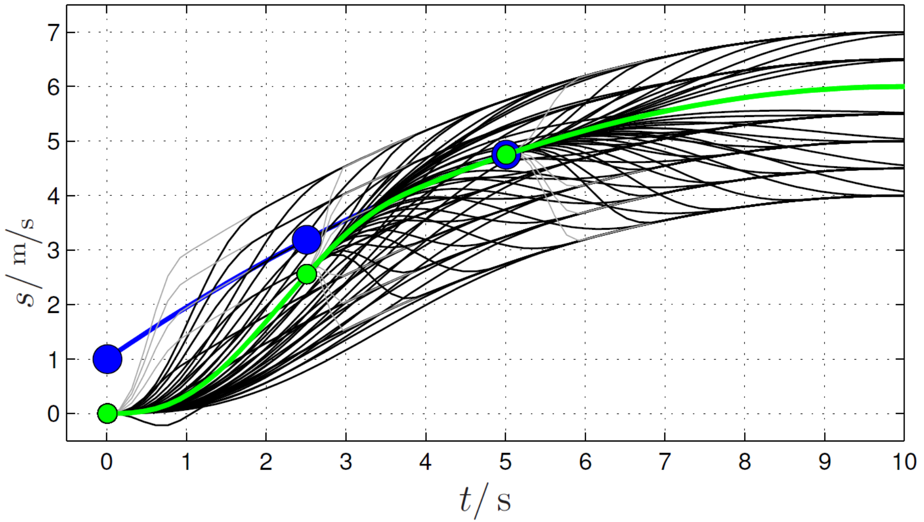
## Longitudinal fitting polynomial solution:

In stop and go, or spacing control (5th order example)

$$s(t) = c_1 t^5 + c_2 t^4 + c_3 t^3 + c_4 t^2 + c_5 t + c_6$$

$$v(t) = 5c_1 t^4 + 4c_2 t^3 + 3c_3 t^2 + 2c_4 t + c_5$$

$$a(t) = 20c_1 t^3 + 12c_2 t^2 + 6c_3 t + 2c_4$$




## Constraint functions:

$$s(t_0) = c_6 = s_0$$

$$v(t_0) = c_5 = ts_0$$

$$a(t_0) = 2c_4 = tts_0$$

$$s(t_1) = c_1 t_1^5 + c_2 t_1^4 + c_3 t_1^3 + c_4 t_1^2 + c_5 t_1 + c_6 = s_1$$

$$v(t_1) = 5c_1 t_1^4 + 4c_2 t_1^3 + 3c_3 t_1^2 + 2c_4 t_1 + c_5 = ts_1$$

$$a(t_1) = 20c_1 t_1^3 + 12c_2 t_1^2 + 6c_3 t_1 + 2c_4 = tts_1$$

Solving the equations to get coefficients

# Lattice planner: speed planning

## Longitudinal fitting polynomial solution:

In cruise control (4th order example)

$$s(t) = b_1 t^4 + b_2 t^3 + b_3 t^2 + b_4 t + b_5$$

$$v(t) = 4b_1 t^3 + 3b_2 t^2 + 2b_3 t + b_4$$

$$a(t) = 12b_1 t^2 + 6b_2 t + 2b_3$$

## Constraint functions:

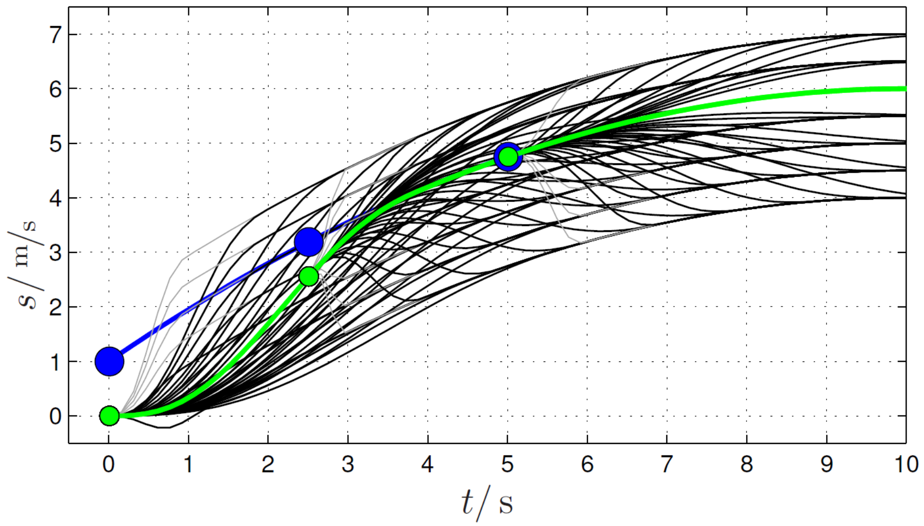
$$s(t_0) = b_5 = s_0$$

$$v(t_0) = b_4 = ts_0$$

$$a(t_0) = 2b_3 = tts_0$$

$$v(t_1) = 4b_1 t_1^3 + 3b_2 t_1^2 + 2b_3 t_1 + b_4 = ts_1$$

$$a(t_1) = 12b_1 t_1^2 + 6b_2 t_1 + 2b_3 = tts_1$$




Solving the equations to get coefficients

# Lattice planner: lateral trajectory planning

Lateral fitting polynomial solution

$$d(s) = k_1 s^5 + k_2 s^4 + k_3 s^3 + k_4 s^2 + k_5 s + k_6$$

$$d_v(t) = 5k_1 s^4 + 4k_2 s^3 + 3k_3 s^2 + 2k_4 s + k_5$$

$$d_a(t) = 20k_1 s^3 + 12k_2 s^2 + 6k_3 s + 2k_4$$

Constraint function

$$d(s_0) = k_1 s_0^5 + k_2 s_0^4 + k_3 s_0^3 + k_4 s_0^2 + k_5 s_0 + k_6 = d_0$$

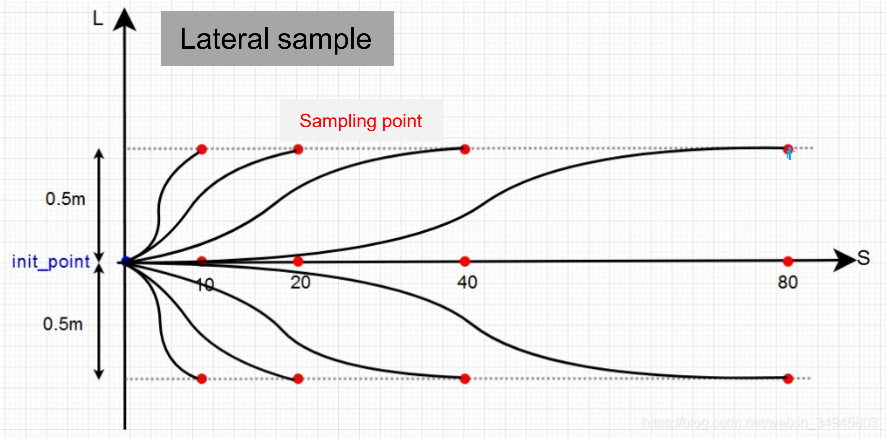
$$d_v(s_0) = 5k_1 s_0^4 + 4k_2 s_0^3 + 3k_3 s_0^2 + 2k_4 s_0 + k_5 = sd_0$$

$$d_a(s_0) = 20k_1 s_0^3 + 12k_2 s_0^2 + 6k_3 s_0 + 2k_4 = ssd_0$$

$$d(s_1) = k_1 s_1^5 + k_2 s_1^4 + k_3 s_1^3 + k_4 s_1^2 + k_5 s_1 + k_6 = d_1$$

$$d_v(s_1) = 5k_1 s_1^4 + 4k_2 s_1^3 + 3k_3 s_1^2 + 2k_4 s_1 + k_5 = sd_1$$

$$d_a(s_1) = 20k_1 s_1^3 + 12k_2 s_1^2 + 6k_3 s_1 + 2k_4 = ssd_1$$




Constraint variables:

$d_0$: initial lateral displacement

$sd_0$: initial lateral speed

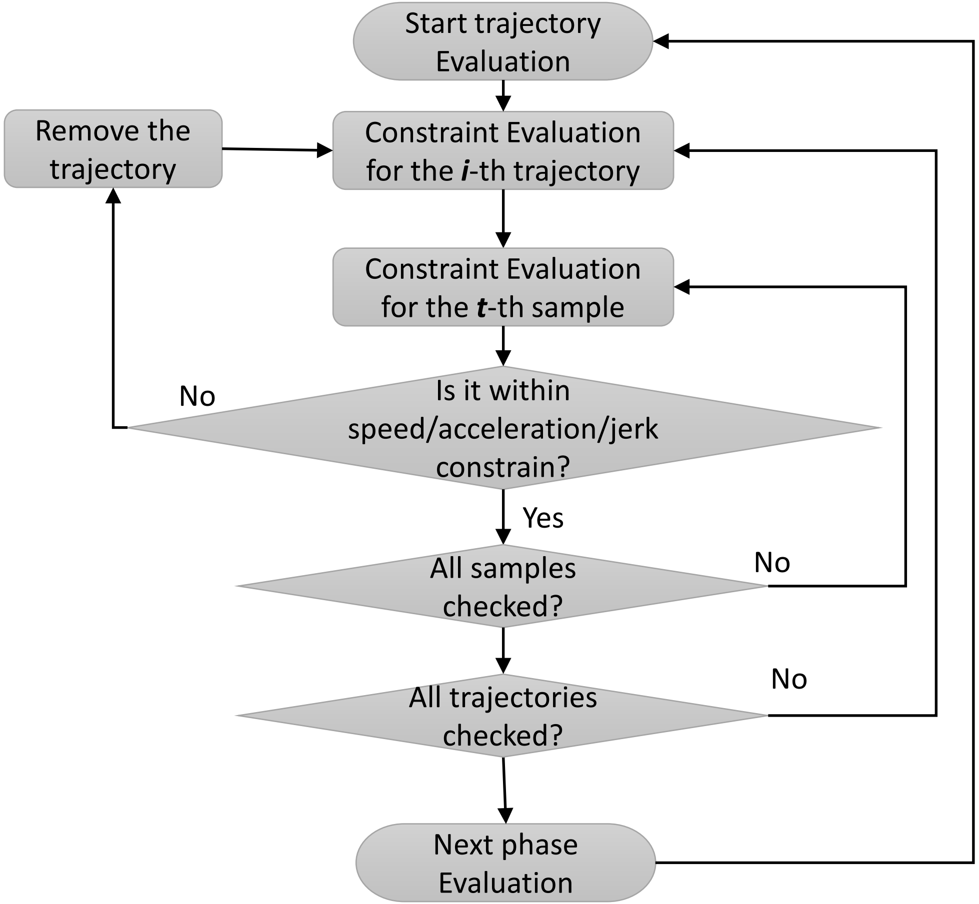
$ssd_0$: initial lateral acceleration

$d_1$: sampled lateral displacement

$sd_1$: sampled lateral speed

$ssd_1$: sampled lateral acceleration

# Lattice planner: Constraint evaluation




For every trajectory generated, it needs to be evaluated to check if it violates any constraint and remove it if it does.

# Lattice planner: trajectory cost function

**Objective**: to choose a feasible path that is nearest to the static reference path, and at the same time, avoid large speed change to ensure comfortability and stay away from obstacles.

目的:选择最接近静态参考路径的可行路径，同时避免较大的速度变化，以保证舒适性并远离障碍物。

$$J = k_{\text{longi}} * J_{longi} + k_{comfort} * J_{comfort} + k_{collision} * J_{collision}$$

where

- $k_{longi}$: weight on longitudinal objective cost
- $J_{\text{longi}}$: tracking cost, considering speed error, distance to travel.
- $k_{comfort}$: weight on comfort cost
- $J_{comfort}$: comfort cost, considering longitudinal jerk
- $k_{collision}$: weight on collision cost.
- $J_{collision}$: cost of collision to the near objects.

# Lattice planner: trajectory cost function

**Longitudinal Objective achievement cost** : to choose a feasible path that is nearest to the static reference path.

- Input: $Lon\_trajectory, planning\_target, reference\_s\_dot$
- $J_{speed} = \frac{\sum_{t=0}^{length} t^2 \cdot |V_{ref_t} - V_{evaluation_t}|}{\sum_{t=0}^{length} t^2}$
- $J_{dist} = \frac{1}{1+dist}$
- $J_{\text{longi}} = \frac{W_{speed}Cost_{speed}+W_{dist}Cost_{dist}}{W_{speed}+W_{dist}}$

# Lattice planner: trajectory cost function

**Comfort Objective**: to choose a feasible path that is having less *jerk*

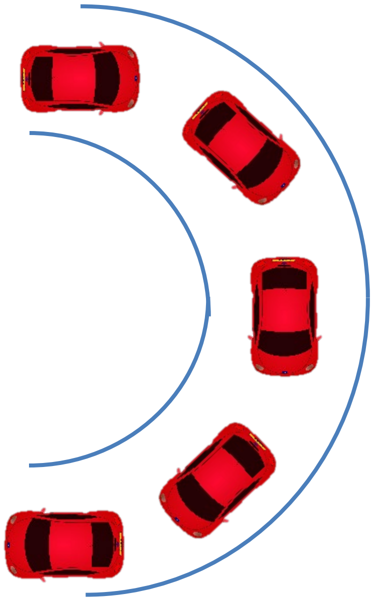
$$J_{comfort} = \frac{\sum_{t=0}^{length}(jerk_t)^2}{1 + \sum_{t=0}^{length}|jerk_t|}$$

$$J_{comfort} = \max(jerk)$$

# Lattice planner: trajectory cost function

**Centripetal Objective**: to choose a feasible path that is having less *less centripetal accel jerk*.
向心目标:选择一条具有较小向心加速度冲击的可行路径。

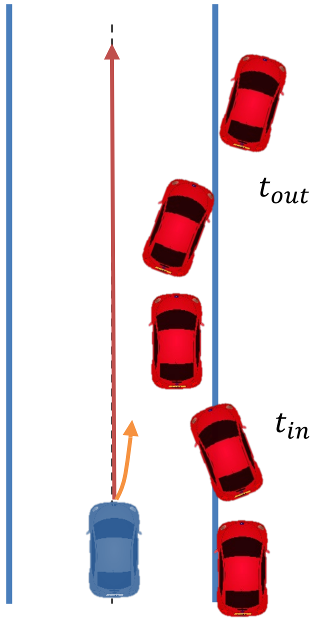
- $a_{centr\,t} = \frac{v_t^2}{R_t} = v_t^2\ k_t$
- $J_{CentriAcc} = \frac{\sum_{t=0}^{length} a_{centri_t}^2}{\sum_{t=0}^{length} |a_{centri_t}|}$

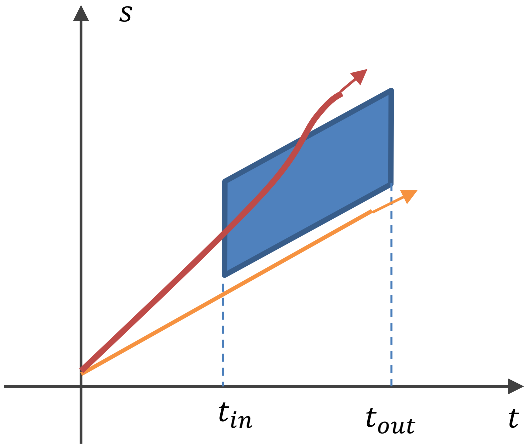


# Lattice planner: trajectory cost function

碰撞目标:选择一条距离障碍物最远的路径

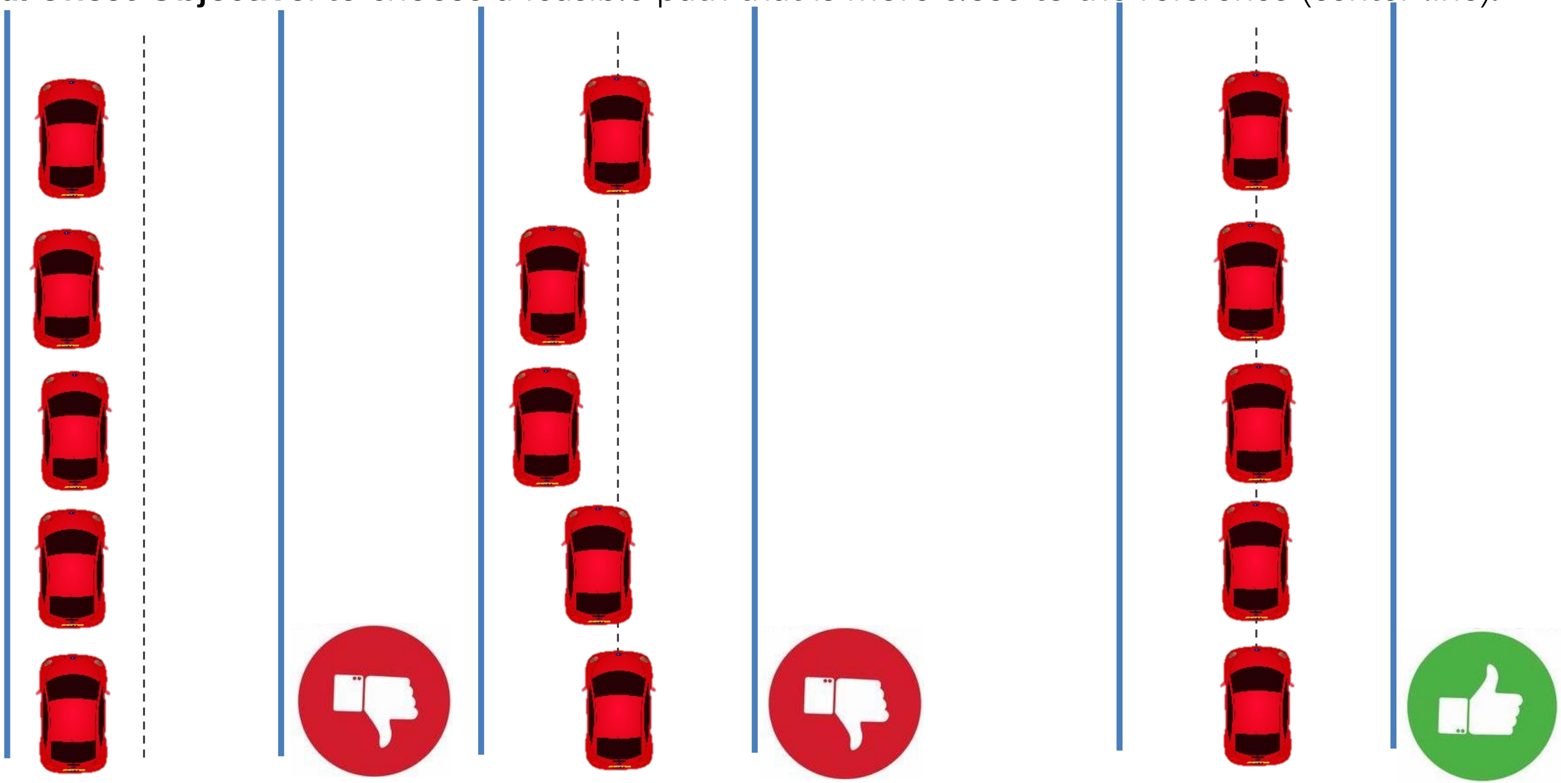
**Collision Objective**: to choose a path that is furthest from obstacles



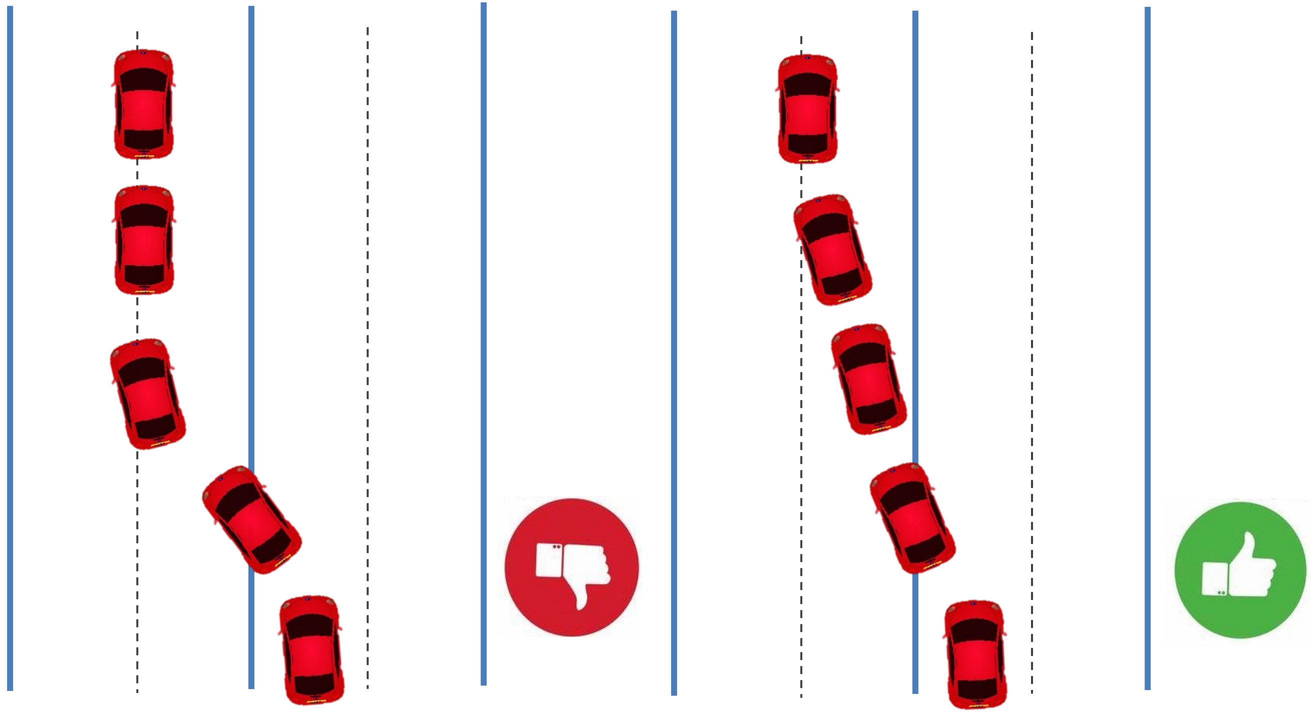
# Lattice planner: trajectory cost function

**Lateral offset Objective**: to choose a feasible path that is more close to the reference (center line).



# Lattice planner: trajectory cost function

**Lateral Acceleration Objective**: to choose a feasible path that is having smoother lane change



# Summary

- Motion planner
  - Motion planner basic concept
  - Functionality and Common method
  - Fundamental Concepts and Key terminology
- Stochastic Sampling Methods
  - RRT
  - RRT*
  - Other improve methods
- Lattice planner
  - Frenet Coordinate
  - Speed planning
  - Trajectory planning

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# Thanks for Listening !