

## LOCAL PATH PLANNING AND MOTION CONTROL FOR AGV IN POSITIONING

Arata Takahashi    Takero Hongo    Yoshiki Ninomiya  
Gunji Sugimoto

Toyota Central Research & Development Labs., Inc.  
Nagakute-cho, Aich-gun, Aich-prefecture, 480-11  
Japan

**Abstract** — Local-path-planning-method has been proposed for positioning autonomous guided vehicles (AGVs). This method minimizes the cost function to constrain AGV's sudden acceleration change (jerk). The method allows AGVs to make the velocity and the curvature of the local path continuous, so that AGVs can smoothly travel along the path. The method was implemented on the CPU of an experimental vehicle for positioning. It was shown that the experimental vehicle could plan the local path in real time and travel along the path.

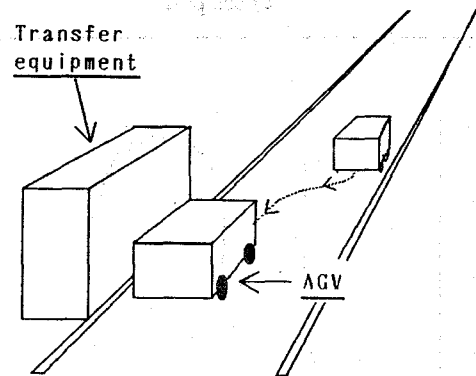


Figure 1: positioning of AGV

### Introduction

#### Problem

Recently, a new AGV system requiring no guide wire cables embedded in the floor has been under development. For applications of this new AGV system, there are many problems to be solved. One of the problems is positioning, because it is difficult for AGVs to stop at the predetermined position. Therefore it is necessary to establish a method of positioning and stopping AGVs precisely at the required position without guide wire cables in front of the workstation (Figure 1).

In this paper, a new local-path-planning-method is proposed. Using this method, an AGV generates a local path to guide itself to the required position. Experiments shows that the proposed method is useful in positioning.

#### Requirements

The local-path-planning-method generates functions representing the trajectory of the AGV between its start position and the destination. In positioning, the AGV must take the following steps as the start position and destination of the local path cannot be predetermined.

The AGV first measures the relative distance between the required destination and the AGV's current position using some instruments on the traveling AGV.

Based on the measured data of the relative distance, the AGV plans the local path in real time to guide itself to the destination.

The AGV then calculates the desired steering and rotating speeds of wheels to travel along the planned local path.

In positioning, the local path must be planned to satisfy the following requirements:

First, the executing time of the local-path-planning-method must be short enough to control the AGV in real time.

Second, the curvature and velocity of the planned local path must be time-continuous. Therefore, the local-path-planning-method using lines and arcs [1] is not optimal for the positioning. This method bring about the discontinuous change of the curvature and the velocity at the conjunction points between lines and arcs, inducing trajectory errors.

Third, the planning method must generate the local path which is optimized even if there is a velocity change, because in positioning the AGV must decelerate the speed and then stop. Therefore, the methods using interpolation functions (i.e. spline functions [2][3], clothoid functions[4] [5]) are not optimal in positioning. This is because these methods make the curvature of the local path continuous only at a constant velocity.

To satisfy the above requirements, we propose a new local-path-planning- method based on the following two guidelines:

One of the guidelines is to plan both of the position and velocity of the local path coordinately for considering the AGV's velocity changing.

Another guideline is to constrain the AGV's sudden acceleration change in order to travel along the local path smoothly.

To realize the above concepts, we applied the acceleration change (jerk) as a cost function.

## Theory

Defining the cost function of jerk, the local path is obtained as functions constraining the sudden acceleration change.

The cost function  $C$  is defined as the time integral of the square of jerk when the AGV moves between the two positions within the time interval  $T$ .

$$C = \frac{1}{2} \int_0^T L dt$$

, where the performance index  $L$  is

$$L = \left(\frac{d^3x}{dt^3}\right)^2 + \left(\frac{d^3y}{dt^3}\right)^2.$$

Here,  $x$  and  $y$  indicate the AGV's position components on the  $x-y$  coordinate system fixed on the ground (Figure 2).

The problem is to deduce the local path  $(x(t), y(t))$  minimizing the cost function  $C$ .

For any functions  $x$  and  $y$  which are sufficiently differential-able in the interval  $0 \leq t \leq T$ , the cost function  $C$  is analytically assumed to be an extremum when  $x(t)$  and  $y(t)$  are the solution of the following Euler equations. (The terms depending on the two position components,  $x$  and  $y$  can be uncoupled.)

$$\frac{\partial L}{\partial x} - \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{x}} \right) \cdots + (-1)^n \frac{d^n}{dt^n} \frac{\partial L}{\partial (x^{(n)})} = 0$$

$$\frac{\partial L}{\partial y} - \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{y}} \right) \cdots + (-1)^n \frac{d^n}{dt^n} \frac{\partial L}{\partial (y^{(n)})} = 0$$

From these equations and the performance index  $L$ , the following equations are deduced.

$$\frac{d^3}{dt^3} \frac{\partial (\ddot{x}^2)}{\partial (\ddot{x})} = 0$$

$$\frac{d^3}{dt^3} \frac{\partial (\ddot{y}^2)}{\partial (\ddot{y})} = 0$$

Then

$$\frac{d^6x}{dt^6} = 0,$$

$$\frac{d^6y}{dt^6} = 0.$$

From the above differential equations, the following equations are derived.

$$\left. \begin{aligned} x(t) &= a_0 + a_1t + a_2t^2 + a_3t^3 + a_4t^4 + a_5t^5 \\ y(t) &= b_0 + b_1t + b_2t^2 + b_3t^3 + b_4t^4 + b_5t^5 \end{aligned} \right\}$$

Resulting solutions  $(x(t), y(t))$  that minimize the time integral of the square jerk are represented by fifth order polynomials

In other words, the AGV can constrain the sudden acceleration change using the local path represented by fifth polynomials.

Additionally, the first derivatives and second derivatives of the fifth polynomials are continuous, so that the requirements of the continuous velocity and curvature are satisfied.

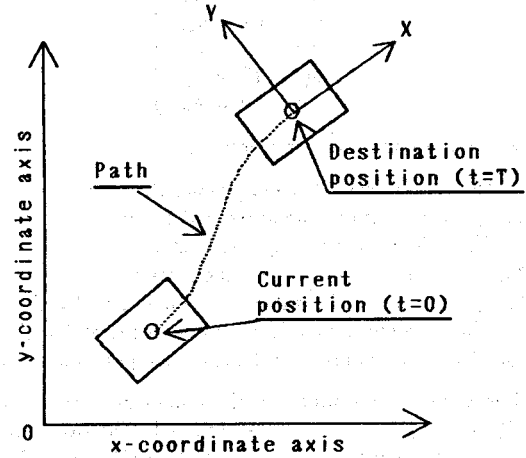


Figure 2: Coordinate system

## Application to AGV

The method of positioning AGVs using the fifth polynomials is to be described. In other words, it will be explained that AGVs need the following three functions for positioning.

1. Generate the local path with the fifth polynomials.
2. Calculate for traveling along the path.
3. Restrain the trajectory error.

### Local Path Generation

The local path is generated on the assumption that we can get the data of AGV state, i.e., the velocity and acceleration of the AGV and the relative position between the AGV's current position and the destination position.

Defining the origin of the  $X-Y$  coordinate system at the destination position, we fix the  $X$  direction of the  $X-Y$  coordinate system in parallel with the destination direction. The time  $t$  is defined to be zero at the start position of the local path. (Essentially these definitions have no influence on the process of the local path planning.)

Planning the local path is to determine the coefficients of the fifth polynomials,  $a_0 \sim a_5, b_0 \sim b_5$ , by solving the following six simultaneous equations satisfying boundary conditions.

$$\left. \begin{aligned} x(t=0) &= X_s \\ \dot{x}(t=0) &= V_{xs} \\ \ddot{x}(t=0) &= A_{xs} \\ x(t=T) &= X_e \\ \dot{x}(t=T) &= V_{xe} \\ \ddot{x}(t=T) &= A_{xe} \end{aligned} \right\}$$

$$\left. \begin{aligned} y(t=0) &= Y_s \\ \dot{y}(t=0) &= V_{ys} \\ \ddot{y}(t=0) &= A_{ys} \\ y(t=T) &= Y_e \\ \dot{y}(t=T) &= V_{ye} \\ \ddot{y}(t=T) &= A_{ye} \end{aligned} \right\}$$

where the boundary conditions are

$$\left. \begin{aligned} T &: \text{the time interval traveling along the path} \\ X_s, Y_s &: \text{the position components at } t=0 \\ V_{xs}, V_{ys} &: \text{the velocity components at } t=0 \\ A_{xs}, A_{ys} &: \text{the acceleration components at } t=0 \\ X_e, Y_e &: \text{the required position components at } t=T \\ V_{xe}, V_{ye} &: \text{the required velocity components at } t=T \\ A_{xe}, A_{ye} &: \text{the required acceleration components at } t=T. \end{aligned} \right\}$$

Based on the assumptions, the boundary conditions  $X_s, Y_s, X_e, Y_e, V_{xs}, V_{ys}, A_{xs}, A_{ys}$  can be obtained. The values of  $V_{xe}, V_{ye}, A_{xe}, A_{ye}$  are predetermined. (When the AGV stops,  $X_e = Y_e = 0, V_{xe} = V_{ye} = 0, A_{xe} = A_{ye} = 0$ .)

Following the coefficients of the fifth polynomials,  $a_0 \sim a_5, b_0 \sim b_5$  can be easily determined by a calculation of the boundary conditions and parameter  $T$ .

Parameter  $T$  represents the time required for AGV to travel along the local path. When parameter  $T$  is small, the magnitude of acceleration of the local path tends to be large (Figure 3(a)). When parameter  $T$  is large, the magnitude of acceleration of the local path tends to be small (Figure 3(b)). Namely, the maximum value of acceleration is the function of parameter  $T$ .

It will be difficult for AGV to travel along the local path, when the magnitude of acceleration exceeds. Therefore, parameter  $T$  is set so that the maximum value of acceleration is below the predetermined value. Parameter  $T$  can be adjusted by repeating calculations a few times.

The main position is that the AGV generates the local path from fifth polynomials taking the following steps:

1. Define the boundary conditions.
2. Adjust parameter  $T$ .
3. Calculate the coefficients of the fifth polynomials.

### Calculation for traveling

The calculation for traveling along the generated local path is to be discussed.

Various types of AGV's driving and steering need their own calculation method. A calculation method for a vehicle having two independent driving wheels on both sides is shown. This type of vehicle requires the desired rotating speeds of driving wheels.

First, using the first and second deviations of the planned local path, the desired velocity vector,  $\mathbf{v} = (\dot{x}(t), \dot{y}(t))$ , and the desired acceleration vector,  $\mathbf{a} = (\ddot{x}(t), \ddot{y}(t))$ , can be calculated at any time  $t$ .

Next, the desired velocity  $v_s$  for the AGV can be calculated by the following equation:

$$v_s = \sqrt{\dot{x}^2 + \dot{y}^2}.$$

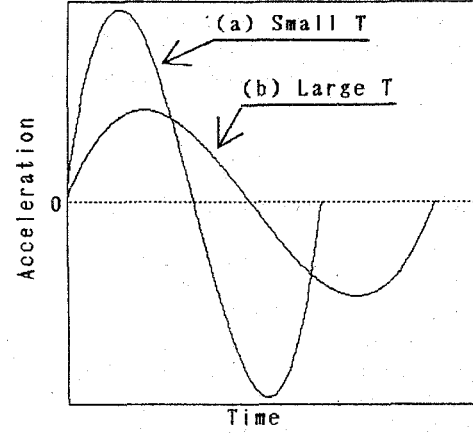


Figure 3: Adjusting parameter  $T$

Also, the desired curvature  $\kappa$  is calculated by the following equation:

$$\kappa = \frac{a_d}{v_s^2}$$

where

$$a_d = \left| \mathbf{a} - \frac{\mathbf{v}}{|\mathbf{v}|} \left( \mathbf{a} \cdot \frac{\mathbf{v}}{|\mathbf{v}|} \right) \right| : \text{the AGV's sideways acceleration,}$$

$$\mathbf{v} = (\dot{x}, \dot{y}) : \text{the AGV's velocity vector,}$$

$$\mathbf{a} = (\ddot{x}, \ddot{y}) : \text{the AGV's acceleration vector.}$$

$\kappa$  and  $a_d$  are transformed into the desired rotating speeds of wheels  $v_r, v_l$  (Figure 4).

$$\left. \begin{aligned} v_r &= v_s(1 - \kappa w/2) \\ v_l &= v_s(1 + \kappa w/2) \end{aligned} \right\} \text{, if the turning direction is clockwise,}$$

$$\left. \begin{aligned} v_r &= v_s(1 + \kappa w/2) \\ v_l &= v_s(1 - \kappa w/2) \end{aligned} \right\} \text{, otherwise,}$$

where

$$\begin{aligned} v_r &: \text{the desired rotating speed of the right driving wheel,} \\ v_l &: \text{the desired rotating speed of the left driving wheel,} \\ w &: \text{the AGV's tread.} \end{aligned}$$

It was verified that the calculation method for the other types of AGV can also be deduced.

The point is that the desired rotating speeds of wheels can be calculated through the following steps:

1. Calculate the velocity  $v_s$  and the curvature  $\kappa$  using the generated local path  $(x(t), y(t))$ .
2. Transform the velocity  $v_s$  and the curvature  $\kappa$  into the the desired rotating speeds of wheels.

## Restraining Trajectory Error

The method to restrain the trajectory error is to be described. The trajectory error is the distance between the AGV and the planned local path.

In practice, the trajectory errors are caused from slip between the driving wheels and the floor and also from the concave and convex of the floor. Therefore, it is necessary to decrease the trajectory error in order to position the AGV at the destination precisely. When the trajectory error is measurable, the AGV can restrain the error.

Therefore, we planned to restrain the trajectory error by the following method in real time when AGV was traveling (Figure 5).

1. Calculate the trajectory error  $e$  between the AGV's desired position and direction  $(x, y, \theta)$  and the measured AGV's current position  $(x_s, y_s)$ .

$$e = (x_s - x) \cos \theta + (y_s - y) \sin \theta,$$

$$\theta = \arctan(\dot{x}/\dot{y}).$$

2. If  $|e| >$  the set value, then re-generate the local path using the latest measured data. If  $|e| <$  the set value, then continue the traveling calculation without restraining the trajectory error.

If the sensor on board the AGV is used to measure the displacement between the current AGV's position and the destination position, the measurement accuracy is tend to be improved as the AGV approaches the destination. Therefore, the re-generation of the local path can also compensate the measuring error.

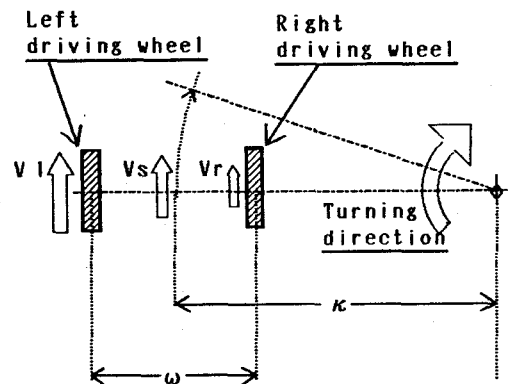


Figure 4: Calculation for traveling

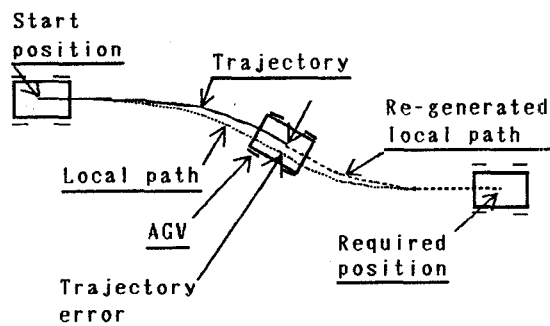


Figure 5: Restraining the trajectory error

## Experiments and Discussion

Using an experimental vehicle (Figure 6), we verified that the proposed local-path-planning-method can be used practically.

The experimental vehicle had two independent driving wheels on both sides of the body.

The maximum acceleration of the vehicle was limited within  $0.3m/s^2$  and the resolution of the velocity was  $0.01m/s$ .

The coupled follower wheel was coaxially installed on the outside of the driving wheels for measuring the current position of the vehicle. The position and direction data of the vehicle can be calculated by integrating the rotation of the wheels.

The program of the proposed positioning method was written in C language. Then it was executed by the CPU68000 (clock cycle 12.5MHz) on the experimental vehicle. The entire flow of the positioning is shown in Figure 7.

First, the executing time was measured to verify that the CPU executes the proposed method in real time. The result indicated that the calculation each of the planning local path and the desired rotating speeds of wheels was executed within 70msec. Considering the response of the actuators, these execution time was proper for the control. Because of the short execution time, the CPU was able to plan the local path several times in real time to decrease the trajectory error.

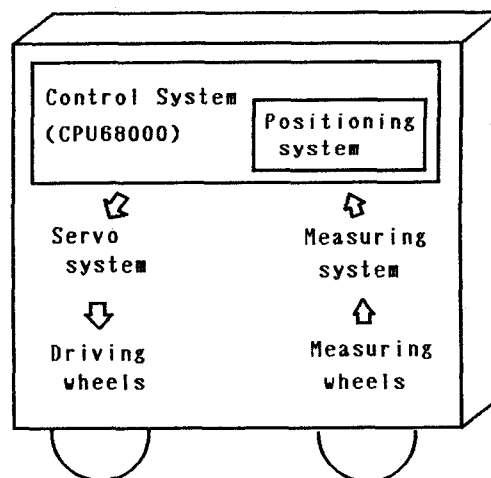


Figure 6: Experimental vehicle

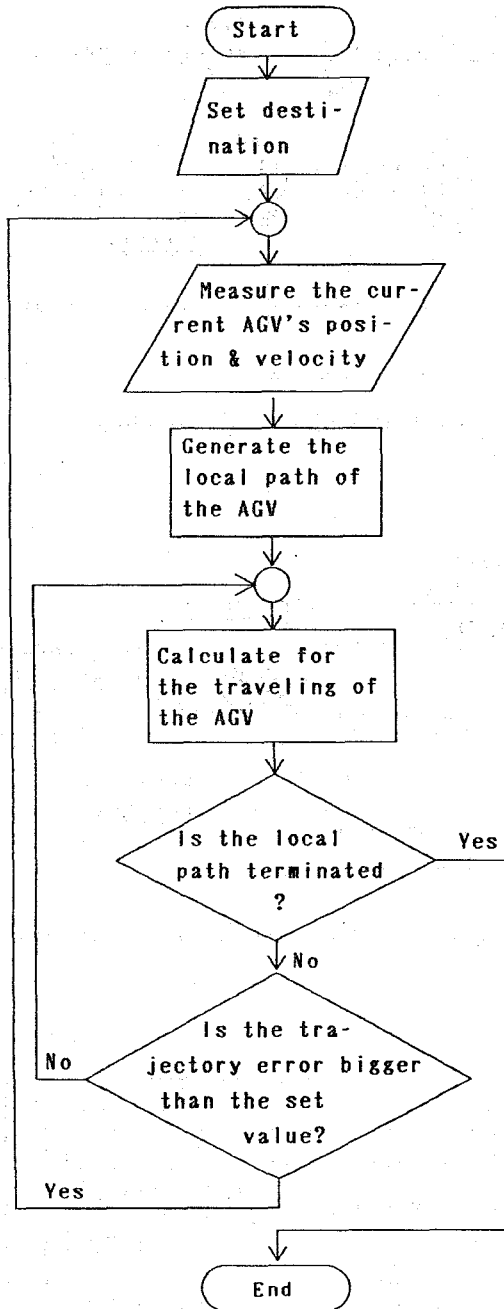


Figure 7: Flow chart of the positioning

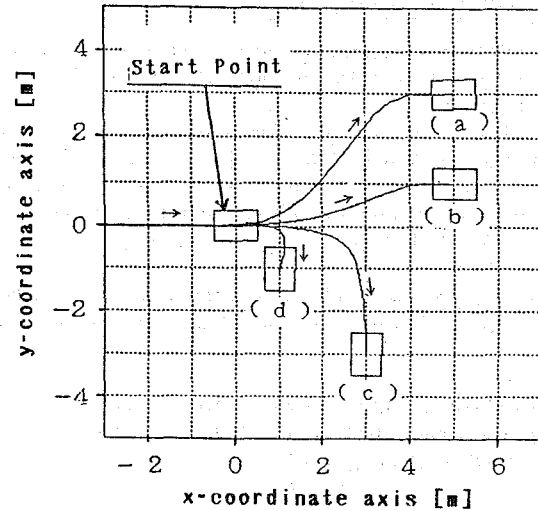


Figure 8: Experimented trajectory

Local path planning and motion control started at the Start Point with 0.5m/s initial velocity.

Next, two patterns of the AGV's movements were tested for smooth traveling using the proposed method. One of the tested patterns was the S-shaped movement pattern (Figure 8(a)(b)) and the other is the quadrantal movement pattern (Figure 8(c)(d)). The destination position was set when AGV was running at 0.5m/s, then AGV's CPU planned the local path in real time. As a result, the proposed method was verified to be useful for the smooth positioning.

Additionally, when the maximum acceleration of the planned local path was set under  $0.3m/s^2$ , the experimental maximum acceleration was really limited under  $0.3m/s^2$ . This means that the generated local path was practicable from the view point of the performance of actuators.

To verify the practicability of the proposed method, the positioning accuracy was repeatedly measured with the S-shaped moving pattern, setting the desired position 5m forward and 1m sideward at 0.5m/s velocity. In this experiment, the local path was re-generated a few times to constrain the trajectory error when the error exceeds 10mm. The result was that the position accuracy was within  $\pm 10mm$  and the directional accuracy was within  $\pm 3deg$  when the AGV stopped. The position accuracy is good enough for practical use, but the directional accuracy is not. This directional error is thought to be caused by the unbalanced performance of the servo systems at low speeds.

In comparison with conventional methods, the advantage of the proposed method is that the AGV does not need a higher performance CPU, because the desired rotating speeds of wheels change smoothly and slowly. It was experimentally verified that the proposed method is better than the method using lines and arcs for path components in positioning. Experimental comparison of the proposed method with the method using the interpolation function (spline or clothoid) will be made in the future.

## Conclusion

Local-path-planning-method has been proposed for positioning AGV's. This method minimizes the cost function to constrain AGV's sudden acceleration change (jerk). The method enables AGVs to make the velocity and the curvature of the local path continuous so that AGV can smoothly travel along the path. The method was implemented on the CPU of an experimental vehicle for positioning. It was shown that the experimental vehicle could plan the local path in real time and travel along the path.

The proposed local-path-planning-method can be used not only for positioning, but also for various purposes. Further research will be carried out in the future for the application of this method to a smooth traveling between required two positions without stop.

## Reference

- [1] Hongo,T., Arakawa,H., Sugimoto,G., Tange,K. and Ymamoto,Y., "An Automatic Guidance System of a Self-Controlled Vehicle — The Command System and the Control Algorithm —", *IEEE Trans. on Industrial Electronics*, IE-34, 1, pp5-10, 1985.
- [2] Komoriya,K., Tanie,K., Tate,A. and Ohno,T, "Trajectory Control of a Wheel-Type Mobil Robots Using Spline Curve" , *Proceedings of the 24th symposium of Society of Instrument and Control Engineers of Japan*, pp343-344, 1985.
- [3] Nelson,W.L. and Cox,I.J., "Local Path Control for Autonomous Vehicle" , *IEEE*, pp1504-1511, 1988.
- [4] Kanayama,Y. and Miyake,N., "Trajectory Generation for Mobile Robots", *Preprint of the third international symposium of robotics research*, pp16-23, 1985.
- [5] Kanayama,Y. and Hartman,B.I., "Smooth Local Path Planning for Autonomous Vehicles", *Proceeding of the 6th symposium of Robotics Society of Japan*, pp89-96, 1988.