Lecture 07 The diffusion term II

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非结构化网格

- 非正交
- 最小修正，mimimum correction
- 正交修正，orthogonal correction
- 超松弛，over-relaxed
- Cross-diffusion 项修正
- 梯度计算
- 非正交网格代数方程组
- 边界条件

非正交 non orthogonal

- 扩散项有限体积法回顾

$$\int_V \left(\nabla \cdot (\mathrm{D}\nabla\phi) + \mathrm{Q}\right) \mathrm{dV} = 0$$

$$\sum_f \int_f \mathrm{D}\nabla\phi \cdot \mathrm{d}\mathbf{S} + \mathrm{Q_p V_p} = 0$$

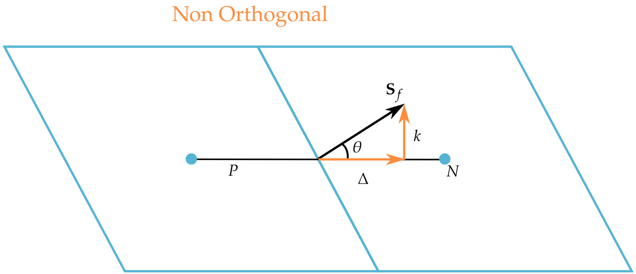
$$\sum_f \mathrm{D}(\nabla\phi)_f \cdot \mathbf{S}_f + \mathrm{Q_p V_p} = 0$$

非正交 non orthogonal

- 可将 $(\nabla\phi)_\mathrm{f} \cdot \mathbf{S}_\mathrm{f}$ 分为两个部分处理，目的是提高精度

$$(\nabla\phi)_\mathrm{f} \cdot \mathbf{S}_\mathrm{f} = \underbrace{(\nabla\phi)_\mathrm{f} \cdot \Delta}_{\text{正交贡献}} + \underbrace{(\nabla\phi)_\mathrm{f} \cdot \mathrm{k}}_{\text{非正交修正}}$$

- 分解



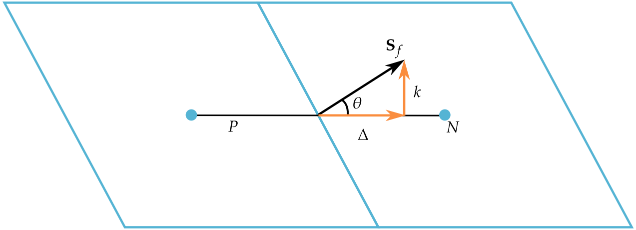
非正交 minimum correction

- 最小修正

$$(\nabla\phi)_f \cdot \mathbf{S}_f = \underbrace{(\nabla\phi)_f \cdot \Delta}_{\text{正交贡献}} + \underbrace{(\nabla\phi)_f \cdot \mathbf{k}}_{\text{非正交修正}}$$

- 分解

Minimum Correction



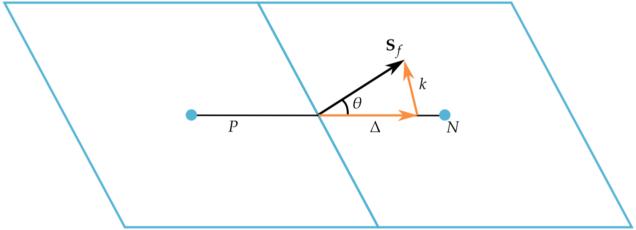
非正交 orthogonal correction

- 正交修正

$$(\nabla\phi)_f \cdot \mathbf{S}_f = \underbrace{(\nabla\phi)_f \cdot \Delta}_{\text{正交贡献}} + \underbrace{(\nabla\phi)_f \cdot \mathbf{k}}_{\text{非正交修正}}$$

- 分解

Orthogonal Correction



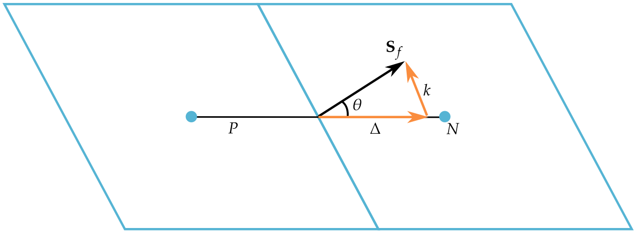
非正交 over-relaxed correction

- 超松弛修正

$$(\nabla\phi)_\mathrm{f} \cdot \mathbf{S}_\mathrm{f} = \underbrace{(\nabla\phi)_\mathrm{f} \cdot \Delta}_{\text{正交贡献}} + \underbrace{(\nabla\phi)_\mathrm{f} \cdot \mathrm{k}}_{\text{非正交修正}}$$

- 图片

Over-relaxed Correction



非正交 over-relaxed correction

- 梯度计算

$$\overline{\nabla\phi}_P = \frac{1}{V_P}\int_{V_C}\nabla\phi \mathrm{d}V = \frac{1}{V_P}\int_{\partial V_P}\phi \mathrm{d}\mathbf{S}$$

- 梯度离散

$$\overline{\nabla\phi}_P = \frac{1}{V_P}\sum_f\int_f\phi \mathrm{d}\mathbf{S} = \frac{1}{V_P}\sum_f\phi_f\mathbf{S}_f$$

- 面心值 $\nabla\phi_f$

$$\nabla\phi_f = g_N\nabla\phi_N + g_P\nabla\phi_P$$

$$g_N = \frac{V_P}{V_P + V_N}$$
$$g_P = \frac{V_N}{V_P + V_N}$$

非正交 Algebraic equation for Non-orthogonal Meshes

- 公式推导

$$\sum_{\mathrm{f}} (\mathrm{D}\nabla\phi) \cdot \mathbf{S}_{\mathrm{f}} = \sum_{\mathrm{f}} \Big((\mathrm{D}\nabla\phi) \cdot (\Delta + \mathbf{k}) \Big)$$

$$\sum_{\mathrm{f}} \Big((\mathrm{D}\nabla\phi) \cdot (\Delta + \mathbf{k}) \Big)$$

$$\underbrace{\sum_{\mathrm{f}} \Big(\mathrm{D}\nabla\phi\Big) \cdot \Delta}_{\text{orthogonal part}} + \underbrace{\sum_{\mathrm{f}} \Big(\mathrm{D}\nabla\phi\Big) \cdot \mathrm{k}}_{\text{non}-\text{orthogonal part}}$$

$$\sum_{\mathrm{f}} \Big(\mathrm{D}\frac{\phi_{\mathrm{N}} - \phi_{\mathrm{P}}}{\mathrm{PN}}\Big) |\Delta| + \sum_{\mathrm{f}} \Big(\mathrm{D}\nabla\phi\Big) \cdot \mathbf{k}$$

非正交 Algebraic equation for Non-orthogonal Meshes

- 公式推导

$$a_P \phi_p + \sum_f a_N \phi_N = b_P$$

- 其中

$$a_N = \frac{D}{PN} |\Delta|_f$$

$$a_P = \sum_f \frac{D}{PN} |\Delta|_f$$

$$b_P = S_P V_P + \sum_f \left(D \nabla \phi \right) \cdot \mathbf{k}$$

非正交 Algebraic equation for Non-orthogonal Meshes

- 例题

非正交 Boundary conditions for non-orthogonal grids

- Dirichlet boundary condition

$$(\nabla\phi)_{\mathrm{b}} = \frac{\phi_{\mathrm{b}} - \phi_{\mathrm{P}}}{\mathrm{PN}}$$

非正交 Boundary conditions for non-orthogonal grids

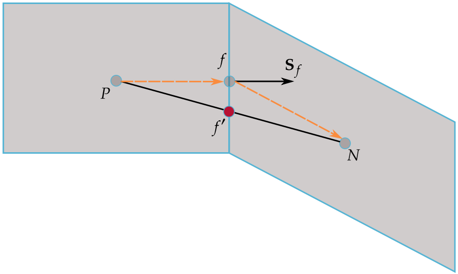
- Neumann boundary condition

$$(\mathrm{D}\nabla\phi)_{\mathrm{b}} \cdot \mathbf{S}_{\mathrm{f}} = \mathrm{q}_{\mathrm{b}}|\mathrm{S}_{\mathrm{f}}|$$

奇异 skewness

- skewness

$$\phi_{\mathrm{f}} = \phi_{\mathrm{f'}} + (\nabla\phi)_{\mathrm{f'}} \cdot \mathbf{d}_{\mathrm{f'f}}$$



低松弛 under relaxation

- 代数方程组

$$a_P \phi_P + \sum_f a_N \phi_N = b_P$$

$$\phi_P = \frac{b_P - \sum_f a_N \phi_N}{a_P}$$

低松弛 under relaxation

- 显式

$$\phi_P = \phi_P^{old} + \alpha\left(\frac{b_P - \sum_f a_N\phi_N}{a_P} - \phi_P^{old}\right)$$

- 隐式

$$\frac{a_P}{\alpha}\phi_P + \sum_f a_N\phi_N = b_P + \frac{(1-\alpha)a_P}{\alpha}\phi_P^{old}$$

低松弛 under relaxation

- 显式, `fields p 0.3`
- 隐式, `equations U 0.9`