# Lecture 08 Gradient Computation

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# 梯度计算

- 梯度的作用
- 常见梯度项
- gradient

# 非结构化网格

- 格林高斯梯度，Green-Gauss Gradient
- 最小二乘, Least-Square Gradient
- 梯度插值

# 非正交 computing gradients in cartesian grides

- 一维

$$\left(\frac{\mathrm{d}\phi}{\mathrm{dx}}\right)_{\mathrm{e}} = \frac{\phi_{\mathrm{E}} - \phi_{\mathrm{P}}}{\mathrm{x_E} - \mathrm{x_P}}$$

- 二维

$$\left(\frac{\partial\phi}{\partial \mathrm{x}}\right)_{\mathrm{P}} = \frac{\phi_{\mathrm{E}} - \phi_{\mathrm{W}}}{\mathrm{x_E} - \mathrm{x_W}}$$
$$\left(\frac{\partial\phi}{\partial \mathrm{x}}\right)_{\mathrm{P}} = \frac{\phi_{\mathrm{N}} - \phi_{\mathrm{S}}}{\mathrm{x_N} - \mathrm{x_S}}$$

# 非正交 Green-Gauss Gradient

- 体心处梯度

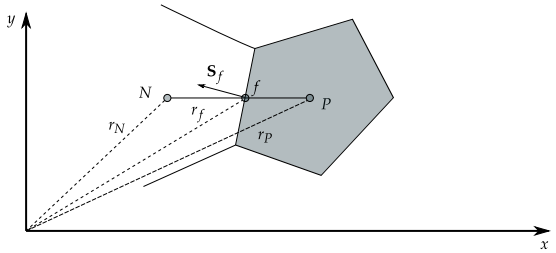
$$(\nabla\phi)_{\mathrm{P}} = \frac{1}{\mathrm{V_P}} \sum_{\mathrm{f}} \phi_{\mathrm{f}} \cdot \mathbf{S}_{\mathrm{f}}$$

- 两种方法计算 $\phi_{\mathrm{f}}$
- 1. `face-based`
- 2. `vertex-based`

# 非正交 face-based gradient computation

- 公式

$$\phi_{\mathrm{f}} = \gamma\phi_{\mathrm{P}} + (1-\gamma)\phi_{\mathrm{N}}$$



# 非正交 face-based gradient computation

- 其中 $\gamma$

$$\gamma = \frac{|\mathbf{r}_\mathrm{N} - \mathbf{r}_\mathrm{f}|}{|\mathbf{r}_\mathrm{N} - \mathbf{r}_\mathrm{P}|} = \frac{\mathrm{d}_\mathrm{Nf}}{\mathrm{d}_\mathrm{NP}}$$

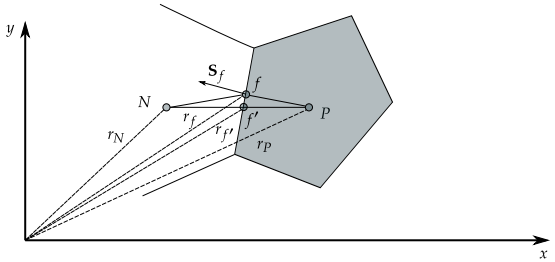
- 特殊情况

$$\phi_\mathrm{f} = \frac{\phi_\mathrm{P} + \phi_\mathrm{N}}{2}$$

- 二阶精度

# 非正交 face-based gradient computation

- skewness, $\phi_{f'}$




- 泰勒公式

$$\begin{aligned}\phi_{\mathrm{f}} &= \phi_{\mathrm{f}'} + \text{correction} \\ &= \phi_{\mathrm{f}'} + \nabla\phi_{\mathrm{f}'} \cdot (\mathbf{r}_{\mathrm{f}} - \mathbf{r}_{\mathrm{f}'})\end{aligned}$$

# 非正交 face-based gradient computation

- 泰勒公式

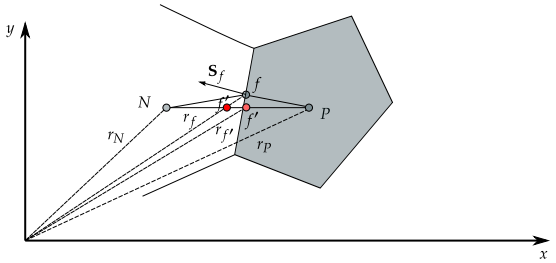
$$\begin{aligned}\phi_{\mathrm{f}} &= \gamma\{\phi_{\mathbf{P}} + (\nabla\phi)_{\mathbf{P}}(\mathbf{r_f} - \mathbf{r_P})\} + (\mathbf{1}-\gamma)\{\phi_{\mathbf{N}} + (\nabla\phi)_{\mathbf{N}} \cdot (\mathbf{r_f} - \mathbf{r_N})\} \\ &= \phi_{\mathrm{f}'} + \underbrace{\gamma(\nabla\phi)_{\mathrm{P}} \cdot (\mathbf{r}_{\mathrm{f}} - \mathbf{r}_{\mathrm{P}}) + (1-\gamma)(\nabla\phi)_{\mathrm{N}} \cdot (\mathbf{r}_{\mathrm{f}} - \mathbf{r}_{\mathrm{N}})}_{\text{correction}}\end{aligned}$$

# 计算方法 1face-based gradient computation

- 1. 计算 $\phi_{f'} = \gamma\phi_P + (1-\gamma)\phi_N$
- 2. 计算 $(\nabla\phi)_P = \frac{1}{V_P}\sum_f \phi_{f'} \cdot \mathbf{S}_f$
- 3. 计算更新的 $\phi_f = \phi_{f'} + \gamma(\nabla\phi)_P \cdot (\mathbf{r}_f - \mathbf{r}_P) + (1-\gamma)(\nabla\phi)_N \cdot (\mathbf{r}_f - \mathbf{r}_N)$
- 4. 更新 $(\nabla\phi)_P = \frac{1}{V_P}\sum_f \phi_f \cdot \mathbf{S}_f$
- 5. 重复 3

# 非正交 face-based gradient computation

- 另外两种方法



# 计算方法 2face-based gradient computation

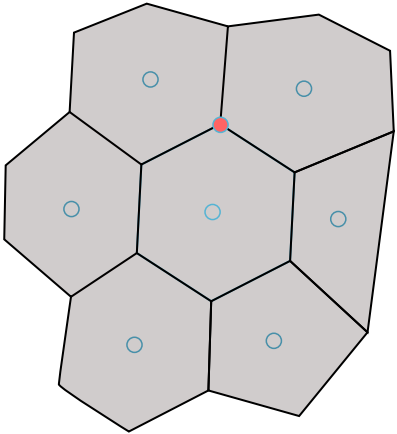
- 1. 计算 $\phi_{f'} = 0.5\phi_P + 0.5\phi_N$
- 2. 计算 $(\nabla\phi)_P = \frac{1}{V_P}\sum_f \phi_{f'} \cdot \mathbf{S}_f$
- 3. 计算更新的 $\phi_f = \phi_{f'} + 0.5(\nabla\phi)_P \cdot (\mathbf{r}_f - \mathbf{r}_P) + 0.5(\nabla\phi)_N \cdot (\mathbf{r}_f - \mathbf{r}_N)$
- 4. 更新 $(\nabla\phi)_P = \frac{1}{V_P}\sum_f \phi_f \cdot \mathbf{S}_f$
- 5. 重复 3

# 计算方法 3face-based gradient computation

- 1. 计算 $\phi_{f'} = \gamma\phi_P + (1-\gamma)\phi_N$
- 2. 计算 $(\nabla\phi)_P = \frac{1}{V_P}\sum_f \phi_{f'} \cdot \mathbf{S}_f$
- 3. 计算更新的 $\phi_f = \phi_{f'} + \gamma(\nabla\phi)_P \cdot (\mathbf{r}_f - \mathbf{r}_P) + (1-\gamma)(\nabla\phi)_N \cdot (\mathbf{r}_f - \mathbf{r}_N)$
- 4. 更新 $(\nabla\phi)_P = \frac{1}{V_P}\sum_f \phi_f \cdot \mathbf{S}_f$
- 5. 重复 3

# 非正交 node-based gradient computation

- 顶点值



# 非正交 node-based gradient computation

- 顶点值

$$\phi_{\mathrm{n}} = \frac{\sum_{\mathrm{i}=1}^{\mathrm{n}} \frac{\phi_{\mathrm{i}}}{\mathrm{d}_{\mathrm{i}}}}{\sum_{\mathrm{i}=1}^{\mathrm{n}} \frac{1}{\mathrm{d}_{\mathrm{i}}}}$$

- 面心值

$$\phi_{\mathrm{n}} = \frac{\phi_1 + \phi_2}{2}$$