Lecture 15 Convection Term

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对流项高分辨率格式 High Resolution Schemes

- 高阶格式，High Order Schemes
- 高分辨率格式，High Resolution Schemes
- 对流有界评价标准，Convection Boundedness Criterion
- Normalized Variable Formulation, NVF
- 总变差变小，Total Variation Diminishing
- 两种方法，Downwind Weighting Factor-DWF, Normalized Weighting Factor-NWF

对流项高分辨率格式 The Normalized Variable Formulation-NVF

- ▸ Leonard 最早提出，后续学者发扬。
- ▸ Gaskell, Lau 1988 简化提出了 Convection Boundedness Criterion。
- The normalized Variable Diagram-NVD 可以用来分析高阶和高分辨率格式

对流项高分辨率格式 The Normalized Variable Formulation-NVF

- Normalized Variable Formulation，归正变量公式
- 方法依赖 $\phi_U, \phi_P(\phi_C), \phi_D$

$$\tilde{\phi} = \frac{\phi - \phi_U}{\phi_D - \phi_U} \tag{1}$$

$$\phi_f = f(\phi_U, \phi_C, \phi_D) \tag{2}$$

- 所以可以简单看出

$$\tilde{\phi}_C = \frac{\phi_C - \phi_U}{\phi_D - \phi_U} \tag{3}$$

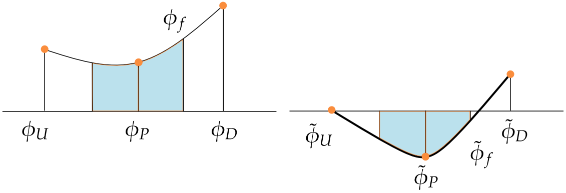
$$\tilde{\phi}_D = 1 \tag{4}$$

$$\tilde{\phi}_U = 0 \tag{5}$$

- 这里就是要思考一下，其实很简单

对流项高分辨率格式 The Normalized Variable Formulation-NVF

- 讨论一下几种情况
- 如果 $\tilde{\phi}_C$ 在 $[0,1]$ 区间
- 如果 $\tilde{\phi}_C < 0$
- 如果 $\tilde{\phi}_C > 0$



对流项高分辨率格式 The Normalized Variable Formulation-NVF

- 前面课程讲到的高阶格式

Scheme	插值公式	NVF
UD	$\phi_f = \phi_P$	$\tilde{\phi}_f = \tilde{\phi}_P$
LUD	$\phi_f = \frac{3}{2}\phi_P - \frac{1}{2}\phi_W$	$\tilde{\phi}_f = \frac{1}{2}\left(1 + \tilde{\phi}_P\right)$
CD	$\phi_f = \frac{\phi_E + \phi_P}{2}$	$\tilde{\phi}_f = \frac{3}{2}\tilde{\phi}_P$
QUICK	$\phi_f = \frac{6}{8}\phi_P + \frac{3}{8}\phi_E - \frac{1}{8}\phi_W$	$\tilde{\phi}_f = \frac{3}{8} + \frac{3}{4}\tilde{\phi}_P$
FROMM	$\phi_f = \phi_P + \frac{\phi_D - \phi_U}{4}$	$\tilde{\phi}_f = \tilde{\phi}_P + \frac{1}{4}$
DW	$\phi_f = \phi_D$	$\tilde{\phi}_f = 1$

对流项高分辨率格式 The Normalized Variable Formulation-NVF

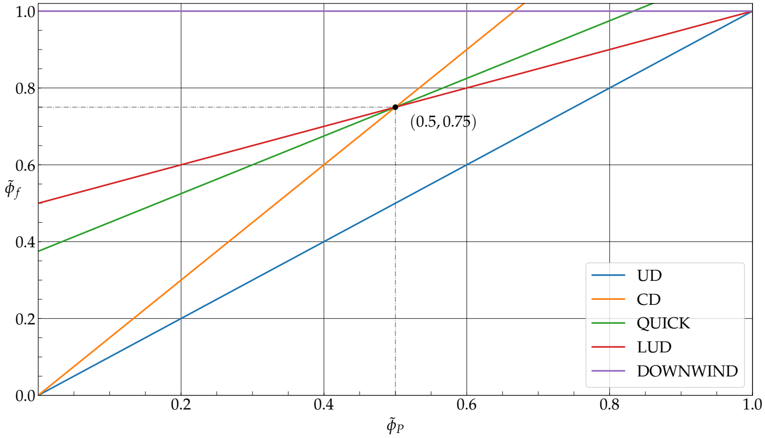
- 对于所有基于三个节点以内插值的高阶格式

$$\tilde{\phi}_f = a\tilde{\phi}_P + b \tag{6}$$

- 都可以写成上述关系，由此得到 $\tilde{\phi}_P, \tilde{\phi}_f$ 之间的关系图。
- 这个图又名 Normalized Variable Diagram-NVD。

对流项高分辨率格式 Normalized Variable Diagram-NVD

- Normalized Variable Diagram-NVD 图



对流项高分辨率格式 The Convection Boundedness Criterion-CBC

- 在局部坐标系下，如果保持单调性

$$min(\phi_P, \phi_D) \leq \phi_f \leq max(\phi_P, \phi_D) \tag{7}$$

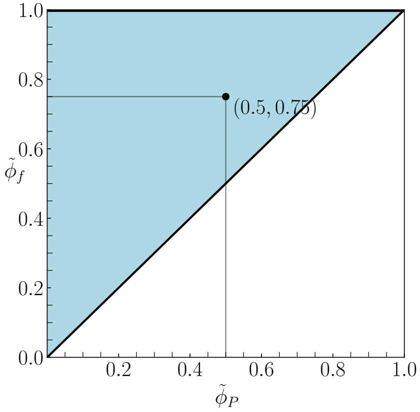
- 规则化 (Normalizing)

$$min(\tilde{\phi}_P, 1) \leq \tilde{\phi}_f \leq max(\tilde{\phi}_P, 1) \tag{8}$$

对流项高分辨率格式 The Convection Boundedness Criterion-CBC

- 公式

$$\tilde{\phi}_f = \begin{cases} f(\tilde{\phi}_P) & continouse \\ f(\tilde{\phi}_P) = 1 & \tilde{\phi}_P = 1 \\ f(\tilde{\phi}_P) \quad \tilde{\phi}_P < f(\tilde{\phi}_P) < 1 & 0 < \tilde{\phi}_P < 1 \\ f(\tilde{\phi}_P) = 0 & \tilde{\phi}_P = 0 \\ f(\tilde{\phi}_P) = \tilde{\phi}_P & \tilde{\phi}_P < 0 \; or \; \tilde{\phi}_P > 1 \end{cases} \tag{9}$$



- 思考下，$\tilde{\phi}_P > 1$ 和 $\tilde{\phi}_P < 0$ 这两种情况的物理意义

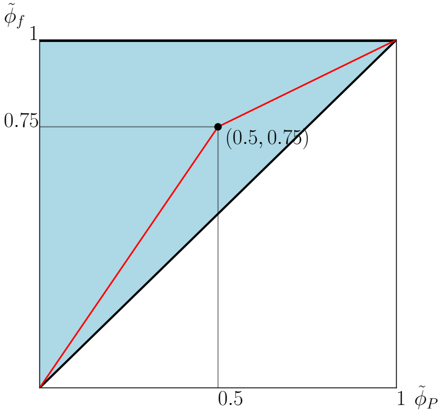
对流项高分辨率格式 High Resolution Schemes

- 构建高分辨率格式
- 高阶格式，对于 $\tilde{\phi}_P$ 在 $[0,1]$ 必须经过点 $(0,0),(1,1)$ 两个点
- 在 $\tilde{\phi}_P < 0, \tilde{\phi}_P > 1$ 非单调区间，采用**迎风格式**
- 许多高分辨率格式都是采用这种思路构建的

对流项高分辨率格式 High Resolution Schemes

- MINMOD

$$\tilde{\phi}_f = \begin{cases} \frac{3}{2}\tilde{\phi}_P & 0 \le \tilde{\phi}_P \le \frac{1}{2} \\ \frac{3}{2}\tilde{\phi}_P + \frac{1}{2} & \frac{1}{2} \le \tilde{\phi}_P \le 1 \\ \tilde{\phi}_P & \text{其他} \end{cases} \tag{10}$$

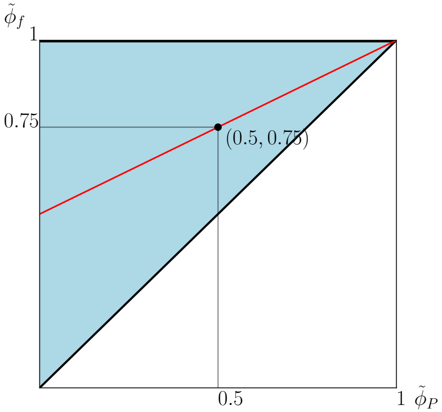


- 对照一下 OpenFOAM ▸ MINMOD

对流项高分辨率格式 High Resolution Schemes

- Bounded CD

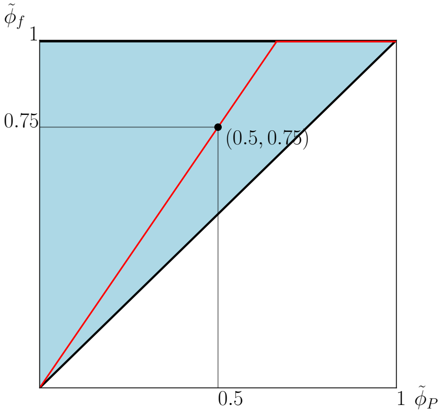
$$\tilde{\phi}_f = \begin{cases} \frac{1}{2}\tilde{\phi}_P + \frac{1}{2} & 0 \leq \tilde{\phi}_P \leq 1 \\ \tilde{\phi}_P & \text{其他} \end{cases} \tag{11}$$



对流项高分辨率格式 High Resolution Schemes

- OSHER

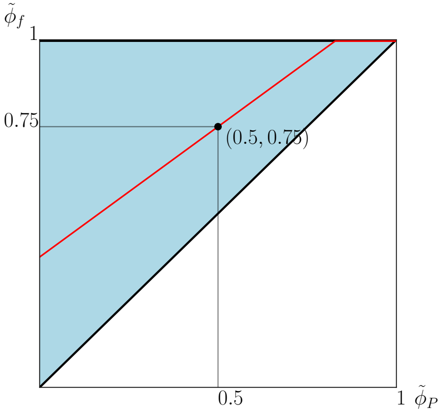
$$\tilde{\phi}_f = \begin{cases} \frac{3}{2}\tilde{\phi}_P & 0 \leq \tilde{\phi}_P \leq \frac{2}{3} \\ 1 & \frac{2}{3} \leq \tilde{\phi}_P \leq 1 \\ \tilde{\phi}_P & \text{其他} \end{cases} \tag{12}$$



对流项高分辨率格式 High Resolution Schemes

- SMART

$$\tilde{\phi}_f = \begin{cases} \frac{3}{4}\tilde{\phi}_P + \frac{3}{8} & 0 \le \tilde{\phi}_P \le \frac{5}{6} \\ 1 & \frac{5}{6} \le \tilde{\phi}_P \le 1 \\ \tilde{\phi}_P & \text{其他} \end{cases} \quad (13)$$

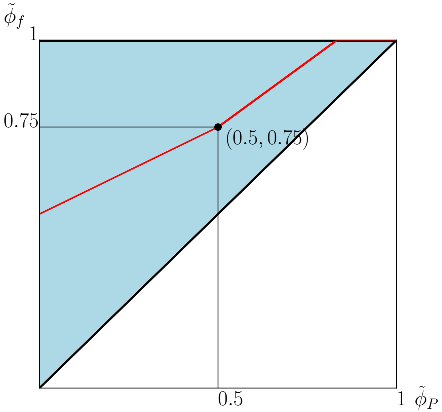


- 对照一下 OpenFOAM ▸ QUICK

对流项高分辨率格式 High Resolution Schemes

- STOTIC

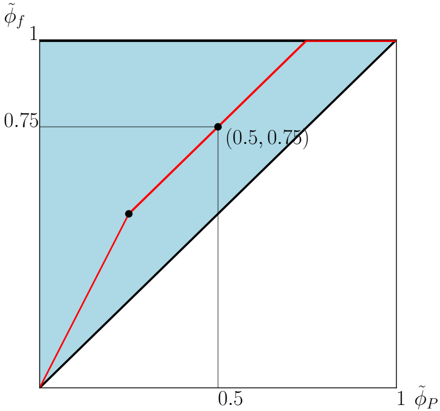
$$\tilde{\phi}_f = \begin{cases} \frac{1}{2}\tilde{\phi}_P + \frac{1}{2} & 0 \le \tilde{\phi}_P \le \frac{1}{2} \\ \frac{3}{4}\tilde{\phi}_P + \frac{3}{8} & \frac{1}{2} \le \tilde{\phi}_P \le \frac{5}{6} \\ 1 & \frac{5}{6} \le \tilde{\phi}_P \le 1 \\ \tilde{\phi}_P & \text{其他} \end{cases} \tag{14}$$



对流项高分辨率格式 High Resolution Schemes

- MUSCL

$$
\tilde{\phi}_f = \begin{cases} 2\tilde{\phi}_P & 0 \le \tilde{\phi}_P \le \frac{1}{4} \\ \tilde{\phi}_P + \frac{3}{4} & \frac{1}{4} \le \tilde{\phi}_P \le \frac{3}{4} \\ 1 & \frac{3}{4} \le \tilde{\phi}_P \le 1 \\ \tilde{\phi}_P & \text{其他} \end{cases} \tag{15}
$$

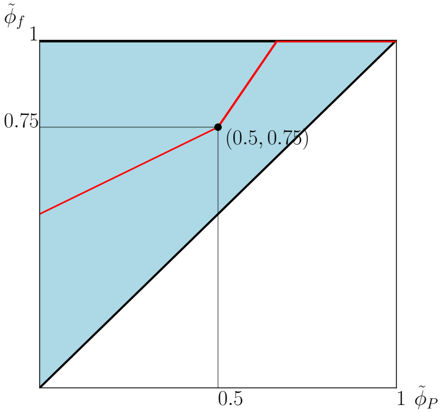


- 对照一下 OpenFOAM ▸ MUSCL

对流项高分辨率格式 High Resolution Schemes

- SUPERBEE

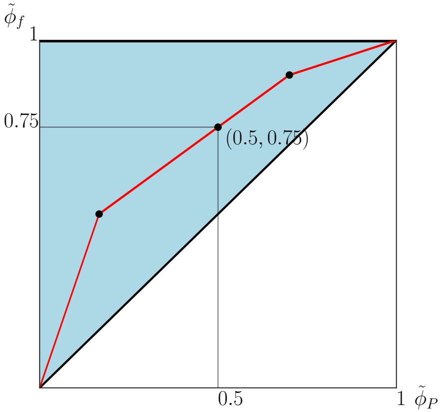
$$\tilde{\phi}_f = \begin{cases} \frac{1}{2}\tilde{\phi}_P + \frac{1}{2} & 0 \le \tilde{\phi}_P \le \frac{1}{2} \\ \frac{2}{3}\tilde{\phi}_P & \frac{1}{2} \le \tilde{\phi}_P \le \frac{2}{3} \\ 1 & \frac{2}{3} \le \tilde{\phi}_P \le 1 \\ \tilde{\phi}_P & \text{其他} \end{cases} \tag{16}$$



对流项高分辨率格式 High Resolution Schemes

- Modified SMART

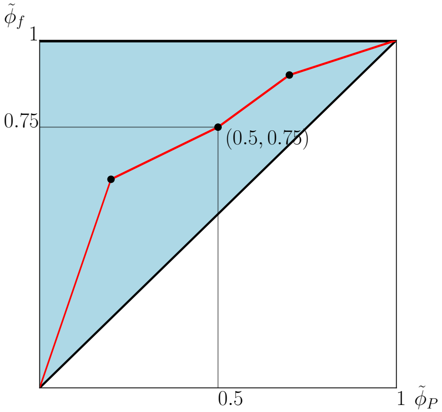
$$\tilde{\phi}_f = \begin{cases} 3\tilde{\phi}_P & 0 \le \tilde{\phi}_P \le \frac{1}{6} \\ \frac{3}{4}\tilde{\phi}_P + \frac{3}{8} & \frac{1}{6} \le \tilde{\phi}_P \le \frac{7}{10} \\ \frac{1}{3}\tilde{\phi}_P + \frac{2}{3} & \frac{7}{10} \le \tilde{\phi}_P \le 1 \\ \tilde{\phi}_P & \text{其他} \end{cases} \tag{17}$$



对流项高分辨率格式 High Resolution Schemes

- Modified STOIC

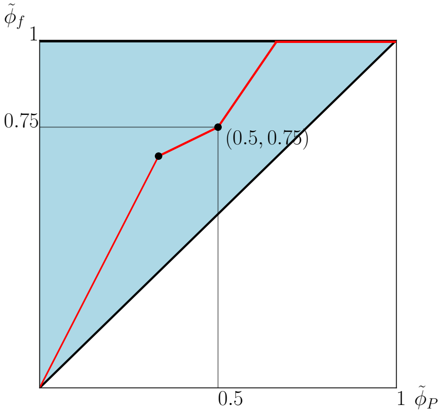
$$\tilde{\phi}_f = \begin{cases} 3\tilde{\phi}_P & 0 \le \tilde{\phi}_P \le \frac{1}{5} \\ \frac{1}{2}\tilde{\phi}_P + \frac{1}{2} & \frac{1}{5} \le \tilde{\phi}_P \le \frac{1}{2} \\ \frac{3}{4}\tilde{\phi}_P + \frac{3}{8} & \frac{1}{2} \le \tilde{\phi}_P \le \frac{7}{10} \\ \frac{1}{3}\tilde{\phi}_P + \frac{2}{3} & \frac{7}{10} \le \tilde{\phi}_P \le 1 \\ \tilde{\phi}_P & \text{其他} \end{cases} \tag{18}$$



对流项高分辨率格式 High Resolution Schemes

- Modified SUPERBEE

$$\tilde{\phi}_f = \begin{cases} 2\tilde{\phi}_P & 0 \le \tilde{\phi}_P \le \frac{1}{3} \\ \frac{1}{2}\tilde{\phi}_P + \frac{1}{2} & \frac{1}{3} \le \tilde{\phi}_P \le \frac{1}{2} \\ \frac{3}{2}\tilde{\phi}_P & \frac{1}{2} \le \tilde{\phi}_P \le \frac{2}{3} \\ 1 & \frac{2}{3} \le \tilde{\phi}_P \le 1 \\ \tilde{\phi}_P & \text{其他} \end{cases} \tag{19}$$



对流项高分辨率格式 The NVF-TVD Relation

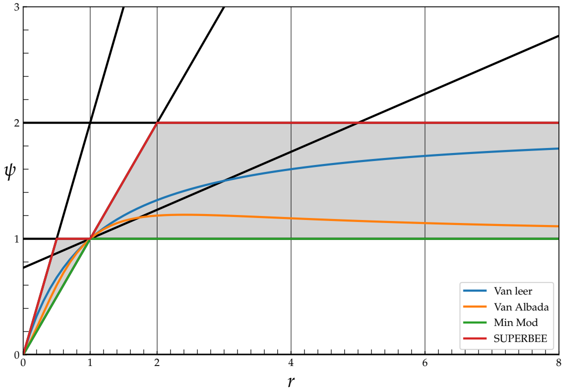
- TVD 复习

$$\phi_f = \phi_P + \frac{1}{2}\psi(r)(\phi_D - \phi_P)$$

$$\psi = \frac{\phi_P - \phi_U}{\phi_D - \phi_P}$$

$$r = \frac{\phi_P - \phi_U}{\phi_D - \phi_P}$$

- 各种格式 TVD



对流项高分辨率格式 The NVF-TVD Relation

- NVF-TVD 关系

$$
\begin{aligned}
r_f = \frac{\phi_P - \phi_U}{\phi_D - \phi_P} &= \frac{(\phi_P - \phi_U)/(\phi_D - \phi_U)}{(\phi_D - \phi_U + \phi_U - \phi_P)/(\phi_D - \phi_U)} \\
&= \frac{\tilde{\phi}_P}{1 - \tilde{\phi}_P}
\end{aligned}
$$

- 所以

$$
\tilde{\phi}_P = \frac{r_f}{1 + r_f} \tag{20}
$$

对流项高分辨率格式 HR Schemes in Unstrctured Grids

- 非结构化网格的 r 采用 Darwish and Moukalled 2003 的推荐公式

$$r = \frac{2(\nabla\phi)_P \cdot \mathbf{r}_{PA}}{\phi_A - \phi_P} - 1 \tag{21}$$

- 因为上面公式是个广义形式，所以改成上下游的写法

$$r = \frac{2(\nabla\phi)_P \cdot \mathbf{r}_{PA}}{\phi_D - \phi_U} - 1 \tag{22}$$

- 对流通量的 TVD 表达式变为

$$\boxed{\phi_i = \phi_U + \frac{\psi(r)}{2}(\phi_D - \phi_U)} \tag{23}$$

源项处理 Treatment of source terms

- 源项处理

$$\int_V S dV = \overline{S} \Delta V \tag{24}$$

- 延迟修正，**Deferred Correction-DC**