Lecture 05 The Finite Volume Mesh

汪　洋

wangyangstayfoolish@gmail.com

July 2022

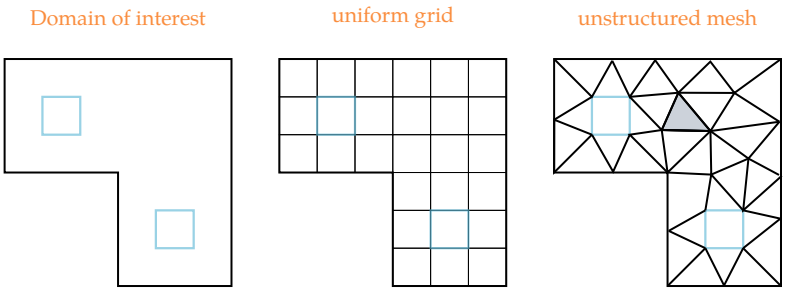
简介

- 有限体积法的关键是对几何的划分
- 通过一系列面将连续的区域分割成不重叠的单元，离散
- 通过边界面来确定物理边界
- 获得不同组件 (点、面、单元等) 的拓扑信息 (topology)
- 本章就是针对有限体积网格所需的几何和拓扑要求

几何离散 Domain Discretization

- 结构化网格 structured mesh/grid

- 非结构化网格 unstructured mesh/grid



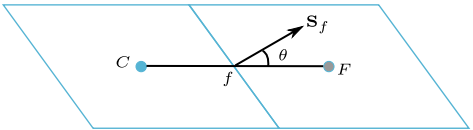
有限体积法网格 The Finite Volume Mesh

- 梯度计算

$$\overline{\nabla\phi}_{\mathrm{C}} = \frac{1}{\mathrm{V_C}}\int_{\mathrm{V_C}} \nabla\phi \mathrm{dV} = \frac{1}{\mathrm{V_C}}\int_{\partial \mathrm{V_C}} \phi \mathrm{d}\mathbf{S}$$

- 非正交

Non Orthogonal



有限体积法网格 The Finite Volume Mesh

- 梯度计算

$$\overline{\nabla\phi}_C = \frac{1}{V_C}\int_{V_C} \nabla\phi dV = \frac{1}{V_C}\int_{\partial V_C} \phi d\mathbf{S}$$

- 梯度离散

$$\overline{\nabla\phi}_C = \frac{1}{V_C}\sum_f \int_f \phi d\mathbf{S} = \frac{1}{V_C}\sum_f \overline{\phi}_f \mathbf{S}_f$$

- 面心值 $\overline{\phi}_f$

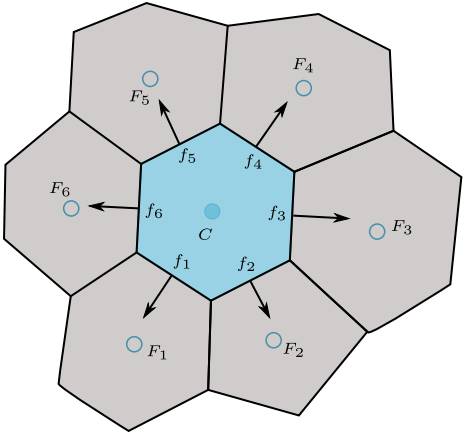
$$\overline{\phi}_f = g_F\phi_F + g_C\phi_C$$

$$g_F = \frac{V_C}{V_C + V_F}$$

$$g_C = \frac{V_F}{V_C + V_F}$$

有限体积法网格 The Finite Volume Mesh

- 例题 1



有限体积法网格 The Finite Volume Mesh

- 几何数据

index	$\mathbf{S}_f$	Volume	Field1	Field2
C	-	37.8	1	6
1	(-2.4 -3.24)	-	1	10
2	(2.4 -3.48	-	1	9
3	(4.1 -6.7)	-	1	5
4	(2.2 3.7)	-	1	3
5	(-2.64 2.9)	-	1	4
6	(-3.66 6.82)	-	1	8

有限体积法网格 The Finite Volume Mesh

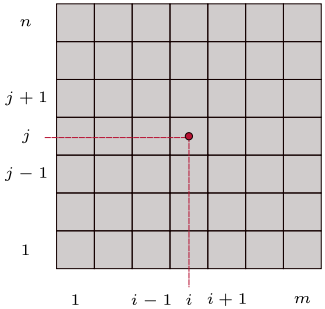
- 1 求解

$$\overline{\nabla \phi}_{\mathrm{C}} =$$

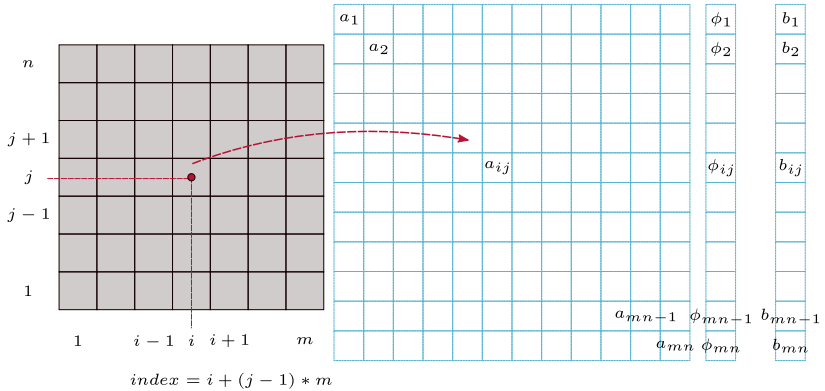
- 2 求解

$$\overline{\nabla \phi}_{\mathrm{C}} =$$

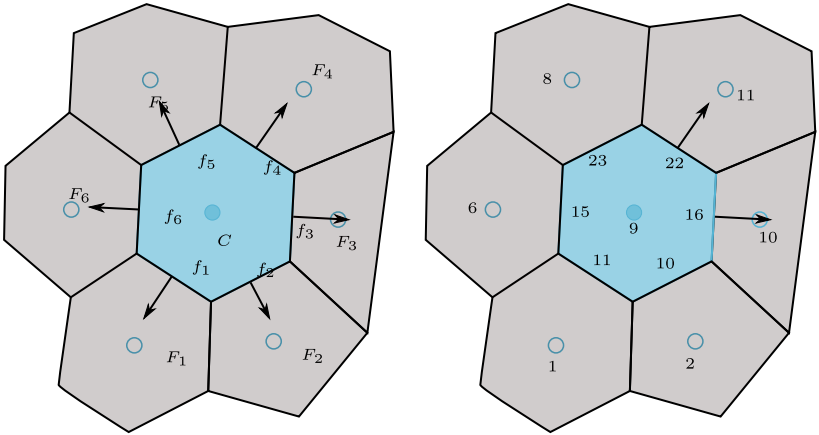
结构网格 structured mesh/grid



结构网格 structured mesh/grid



非结构化网格 unstructured mesh/grid



非结构化网格 unstructured mesh/grid

- 9 单元的梯度值

$$\overline{\nabla\phi}_9 = \frac{1}{V_9}\left(\overline{\phi}_{16}\mathbf{S}_{16} + \overline{\phi}_{22}\mathbf{S}_{22} - \overline{\phi}_{23}\mathbf{S}_{23} - \overline{\phi}_{15}\mathbf{S}_{15} - \overline{\phi}_{11}\mathbf{S}_{11} - \overline{\phi}_{10}\mathbf{S}_{10}\right)$$

非结构化网格 unstructured mesh/grid

- 例题 2

几何量 Geometric Quantities

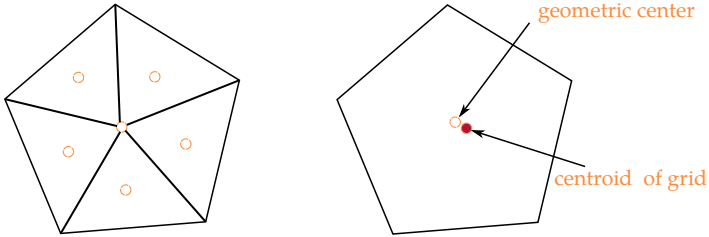
- 三维网格 Tetrahedron, Hexahedron, Prism, Polyhedron
- 二维网格 Quadrilateral, Triangle, Pentagon

几何量 Geometric Quantities

- 面积和面心
- 体积和体心
- 奇异 skewness

几何量 Geometric Quantities

- 面积和面心



几何量 Geometric Quantities

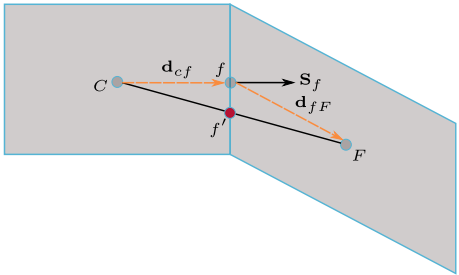
- 面积和面心, 几何中心

$$\mathbf{x}_G = \frac{1}{k}\sum_{i=1}^{k}\mathbf{x}_i$$

$$\mathbf{x}_{CE} = \frac{\sum_{i=1}^{k}\mathbf{x}_i * S_i}{\mathbf{S}}$$

几何量 Geometric Quantities

- 奇异 skewness



几何量 Geometric Quantities

- 奇异性 $\overline{ff'}$