Lecture 19 Multigrid Method

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MG 算法

- 多重网格思路就是，波长短的误差在细网格容易衰减，但是波长长的网格却不行，但是如果用多套网格即可达到此目的。

$$\mathbf{A} \cdot \mathbf{x} = \mathbf{b} \tag{1}$$

$$\mathbf{A} \cdot \mathbf{y} = \mathbf{b} - \mathbf{r} \tag{2}$$

$$\mathbf{e} = \mathbf{x} - \mathbf{y} \tag{3}$$

$$\mathbf{A} \cdot \mathbf{e} = \mathbf{r} \tag{4}$$

MG 算法 An outline of a multigrid

- 1. fine grid iterations
- 2. Restriction
- 3. Prologation
- 4. Correction and final iterations

MG 算法 An outline of a multigrid

- 1. fine grid iterations

$$\mathbf{A}^h \cdot \mathbf{x} = \mathbf{b} \tag{5}$$

$$\mathbf{r}^h = \mathbf{b} - \mathbf{A}^h \cdot \mathbf{y}^h \tag{6}$$

$$\mathbf{e}^h = \mathbf{x} - \mathbf{y}^h \tag{7}$$

$$\mathbf{A}^h \cdot \mathbf{e}^h = \mathbf{r}^h \tag{8}$$

MG 算法 An outline of a multigrid

- 2. Restriction
- 选取粗化的间距 $ch, c > 1$

$$\mathbf{A}^{ch} \cdot \mathbf{e}^{ch} = \mathbf{r}^{ch} \tag{9}$$

MG 算法 Operator splitting

- 3. Prologation 延拓
- 将第二步计算的误差 $\mathbf{e}^{ch}$ 插值回来 $\mathbf{e}'$

MG 算法 Operator splitting

- 4. Correction and final iterations
- 然后将第三步计算结果反代回来

$$\mathbf{y}^{improved} = \mathbf{y}^{h} + \mathbf{e}'^{h} \tag{10}$$