Lecture 10 Convection Term

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对流项计算 Introduction

▶ 对流项的研究超过 30 年

▶ 对流项内容相当多,包含两章内容(11, 12)

▶ 本章主要讲解基本对流项格式,从 first order upwind 开始

▶ 下一章讲解高分辨格式 (High resolution schemes)

▶ 数值色散、数值扩散和数值稳定性,numerical dispersion, numerical diffusion, numerical stability

复习 Recap

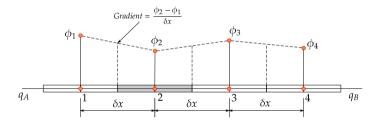
▶ 一维对流扩散方程

▶ 中心差分格式

▶ 迎风格式格式

守恒性 conservative

conservative



有界性 Boundedness

► Boundedness

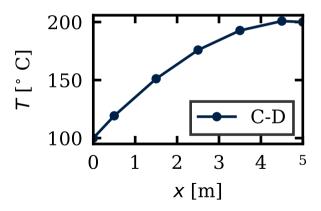
▶ 对角占优

$$\frac{\sum |a_{nb}|}{a_p}$$

▶ wiggles 现象

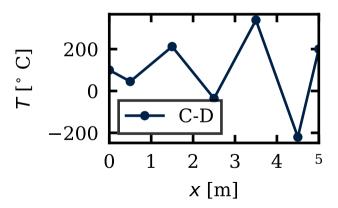
有界性 Boundedness

► Boundedness



有界性 Boundedness

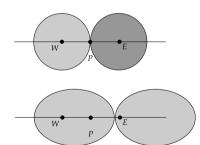
► Boundedness



输运性 transportiveness

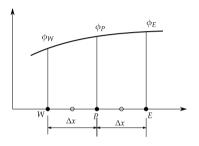
Peclet

$$Pe = \frac{\rho u}{\Gamma/\delta x}$$



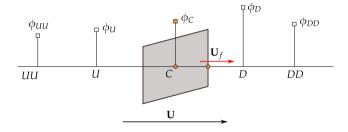
精度 accuracy

▶ taylor series



$$\phi(x + \Delta x) = \phi(x) + \frac{d\phi}{dx} \Delta x + \frac{d^2\phi}{dx^2} \frac{\Delta^2 x}{2} + \cdots$$

对流项计算 High order upwind schemes



对流项计算 High order upwind schemes

- ▶ 二阶迎风格式 second order upwind scheme
- ▶ 阶段误差

$$\phi(x) = k_0 + k_1(x - x_C)$$

▶ e 处的插值格式

$$\phi(x) = \phi_C + \frac{\phi_C - \phi_U}{x_C - x_U}(x - x_C)$$

▶ 对于均匀网格

$$\phi_e = \frac{3}{2}\phi_C - \frac{1}{2}\phi_U$$

对流项计算 QUICK scheme

► QUICK 格式

$$\phi_e = \frac{3}{8}\phi_E + \frac{6}{8}\phi_P - \frac{1}{8}\phi_W$$