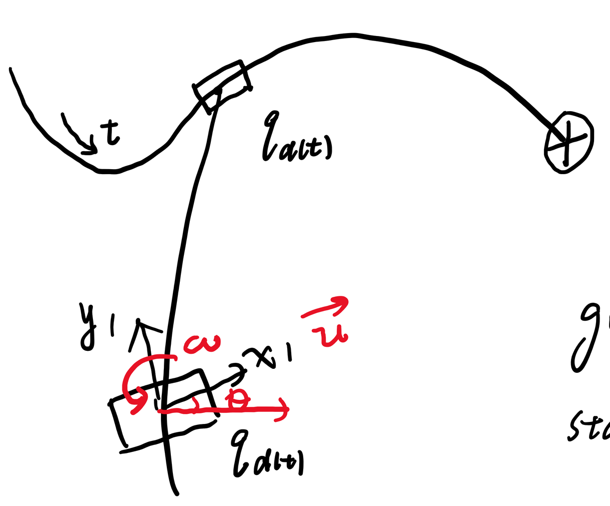
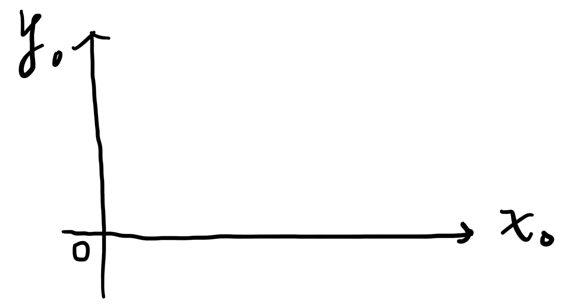
Class 1

2019年9月4日 星期三 11:43

Trajectory Tracking





$\begin{cases} o\,x_0y_0z_0 & \text{world} \\ o_1x_1y_1z_1 & \text{Robot} \end{cases}$

goal to follow the trajectory asymptotically

state vector:

$$q = \begin{bmatrix} x_1 \\ y_1 \\ \theta \end{bmatrix}$$

$$q_d = \begin{bmatrix} x_d \\ y_d \\ \theta_d \end{bmatrix}$$

Kinematics:

$$\dot{x} = u\cos\theta$$
$$\dot{y} = u\sin\theta$$
$$\dot{\theta} = \omega$$

given

$$\begin{cases} \dot{x}_d = u_d\cos\theta_d \\ \dot{y}_d = u_d\sin\theta_d \end{cases} \quad \tan\theta_d = \frac{\dot{y}_d}{\dot{x}_d} \Rightarrow \theta_d = \tan^{-1}\left(\frac{\dot{y}_d}{\dot{x}_d}\right) + k\pi, \quad k = 0, 1$$

"constrain"

$$\dot{\theta}_d = \omega_d$$

$$u_d^2 = \dot{x}_d^2 + \dot{y}_d^2$$

$$\left(\frac{d}{dt}(\tan^{-1}(x)) = \frac{1}{1+x^2}\right)$$

$$\omega_d = \frac{d}{dt}(\theta_d) = \frac{1}{1+\left(\frac{\dot{y}_d}{\dot{x}_d}\right)^2} \cdot \frac{\ddot{y}_d\dot{x}_d - \dot{y}_d\ddot{x}_d}{\dot{x}_d^2} = \frac{\ddot{y}_d\dot{x}_d - \dot{y}_d\ddot{x}_d}{\dot{x}_d^2 + \dot{y}_d^2}$$

Approach:

$$x \to x_d$$
$$y \to y_d$$
$$\theta \to \theta_d,$$

"Rotation"

$$e = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \end{bmatrix} \triangleq \begin{bmatrix} \cos\theta & \sin\theta & 0 \\ -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_d - x \\ y_d - y \\ \theta_d - \theta \end{bmatrix}$$

$$e_1^2 + e_2^2 = (x_d - x)^2 + (y_d - y)^2$$

$\boxed{\text{error dynamics}}$

$$\dot{e} = \begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \\ \dot{e}_3 \end{bmatrix}$$

derivative

$$\dot{e}_1 = \frac{d}{dt}[\cos\theta(x_d - x) + \sin\theta(y_d - y)]$$
$$= -\sin\theta(x_d - x)\dot{\theta} + \cos\theta(\dot{x}_d - \dot{x}) + \cos\theta(y_d - y)\dot{\theta} + \sin\theta(\dot{y}_d - \dot{y})$$
$$= u_d\cos(\underbrace{\theta - \theta_d}_{e_3}) - u - \underbrace{(x_d - x)\sin\theta\cdot\omega + (y_d - y)\cos\theta\cdot\omega}_{\omega\cdot e_2}$$
$$= u_d\cos\cdot e_3 - u + \omega\cdot e_2$$

Similarly

$$\dot{e}_2 = u_d\sin\cdot e_3 - \omega\cdot e_1$$
$$\dot{e}_3 = \omega_d - \omega$$

$e_3 \to 0$

$$v = u_d\cos e_3 - u_1$$
$$\omega = \omega_d - u_2$$

desire: $\begin{matrix} e_3 \to 0 \\ v \to u_d \end{matrix} \quad \begin{cases} u_1 \to 0 \\ u_2 \to 0 \end{cases}$

"desire 0"

$$\dot{e} = \begin{bmatrix} 0 & \omega_d & 0 \\ -\omega_d & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} e + \begin{bmatrix} 0 \\ \sin e_3 \\ 0 \end{bmatrix} u_d + \begin{bmatrix} 1 & -e_2 \\ 0 & e_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \cdot v$$

$e_3 \to 0$

$$= \begin{bmatrix} 0 & \omega_d & 0 \\ -\omega_d & 0 & u_d \\ 0 & 0 & 0 \end{bmatrix} e + \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

$$\boxed{\text{General: } \dot{e} = Ae + Bv, \quad u = ke}$$

$$u_1 = -k_1 e_1$$
$$u_2 = -k_2 e_2 - k_3 e_3$$

$$\dot{e} = A(t)e = \begin{bmatrix} -k_1 & \omega_{d(t)} & 0 \\ -\omega_d & 0 & v_{d(t)} \\ 0 & -k_2 & -k_3 \end{bmatrix}.$$

$$\dot{e} = A(t)e$$
$$\mathcal{L}\downarrow$$
$$sE - e[0] = AE$$

$$[sI - A]E = e[0]$$

$$E = [sI - A]^{-1}e[0]$$

$$p(\lambda) = \lambda(\lambda + k_1)(\lambda + k_3) + \omega_d^2(\lambda + k_3) + v_d k_2(\lambda + k_1)$$

$$k_1 = k_3 = 2\xi a$$

$$k_2 = \frac{a^2 - \omega_d^2}{v_d}$$