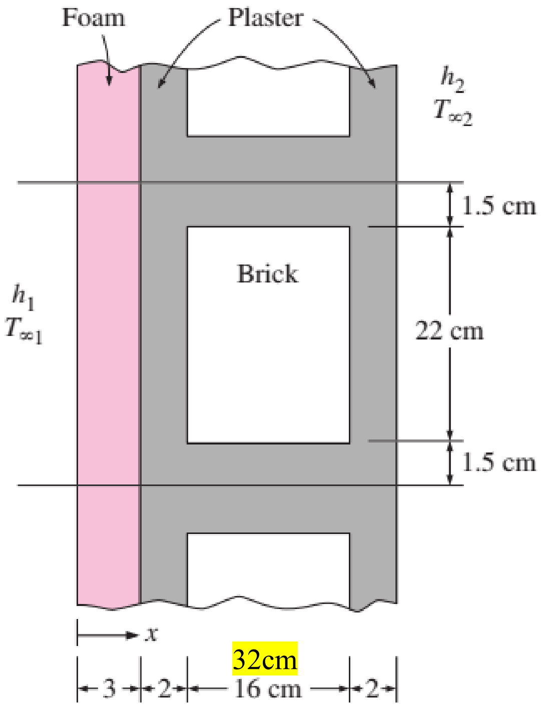
QUESTION 1



A 3 m high and 5 m wide wall consists of long (case 1: 16cm, case 2: 32 cm), 22 cm cross section horizontal bricks (k =0.72 W/m · °C) separated by 3 cm thick plaster layers (k =0.22 W/m · °C).

There are also 2 cm thick plaster layers on each side of the brick and a 3-cm-thick rigid foam (k= 0.026 W/m · °C) on the inner side of the wall

The indoor and the outdoor temperatures are 20°C and -10°C, and the convection heat transfer coefficients on the inner and the outer sides are h1=10 W/m2·°C and h2 =40 W/m2·°C, respectively.

Assuming one-dimensional heat transfer and disregarding radiation, determine the rate of heat transfer through the wall.

ANSWER

We consider the wall as a repetition of a-1m-long-row-of-brick units: It's dimension will be 1m in length and h_{unit} in height.

$$h_{unit} = 22 + 1.5 \times 2 = 25cm = 0.25m$$
$$A = h_{unit} \times 1m = 0.25m^2$$

In this single unit, the heat transfer can be illustrated by this diagram:

$$T_{\infty1} \rightarrow R_{conv1} \rightarrow R_{foam} \rightarrow R_{plaster1} \rightarrow R_{parallel} \begin{cases} R_{plaster_p} \\ R_{brick_p} \\ R_{plaster_p} \end{cases} \rightarrow R_{plaster2} \rightarrow R_{conv2} \rightarrow T_{\infty2}$$

$$R_{conv1} = \frac{1}{h_1 * A} = \frac{1}{10 * 0.25} = 0.4\frac{℃}{W}$$
$$R_{conv2} = \frac{1}{h_2 * A} = \frac{1}{40 * 0.25} = 0.1\frac{℃}{W}$$
$$R_{foam} = \frac{L_{foam}}{K_{foam} * A} = \frac{0.03}{0.026 * 0.25} = 4.615℃\frac{℃}{W}$$

$$R_{plaster1} = R_{plaster2} = \frac{L_{plaster1}}{K_{plaster} * A} = \frac{0.02}{0.22 * 0.25} = 0.363℃\frac{℃}{W}$$

To calculate for brick thickness 16cm:

$$R_{plaster_P} = \frac{L_{plaster_p}}{K_{plaster} * A} = \frac{0.16}{0.22*(0.015*1)} = 48.48℃\frac{℃}{W}$$

$$R_{brick_p} = \frac{L_{brick_p}}{K_{brick} * A} = \frac{0.16}{0.72 * (0.22 * 1)} = 1.01\frac{℃}{W}$$

$$\frac{1}{R_{parallel_{16}}} = \frac{1}{R_{brick_p}} + \frac{1}{R_{plaster_p}} + \frac{1}{R_{p_{c_2}}} = \frac{1}{1.01} + 2 * \left(\frac{1}{48.48}\right)$$

$$\rightarrow R_{parallel_{16}} = 0.97℃/W$$

$$R_{total_{16}} = R_{conv1} + R_{foam} + R_{plaster1} + R_{parallel_{16}} + R_{plaster2} + R_{conv2}$$
$$= 0.4 + 4.615 + 0.363 + 0.97 + 0.363 + 0.1 = 6.81℃/W$$

$$\dot{Q}_{unit_{16}} = \frac{\Delta T}{R_{total_{16}}} = \frac{20 - (-10)}{6.81} = 4.41W$$
$$\dot{Q}_{wall_{16}} = \sum \dot{Q}_{unit_{16}} = \dot{Q}_{unit_{16}} * \frac{A_{wall}}{A} = 4.41 * \frac{3*5}{0.25} = 264.60W \ (1)$$

To calculate for brick thickness 32cm:

$$R_{plaster_P} = \frac{L_{plaster_p}}{K_{plaster} * A} = \frac{0.32}{0.22*(0.015*1)} = 96.96℃\frac{℃}{W}$$

$$R_{brick_p} = \frac{L_{brick_p}}{K_{brick} * A} = \frac{0.32}{0.72*(0.22*1)} = 2.02\frac{℃}{W}$$

$$\frac{1}{R_{parallel_{32}}} = \frac{1}{R_{brick_p}} + \frac{1}{R_{plaster_p}} + \frac{1}{R_{p_{c_2}}} = \frac{1}{2.02} + 2 * \left(\frac{1}{96.96}\right)$$

$$\rightarrow R_{parallel_{32}} = 1.94℃/W$$

$$(It's\ clear\ that\ R_{parallel_{32}} = 2 * R_{parallel_{16}})$$

$$R_{total_{16}} = R_{conv1} + R_{foam} + R_{plaster1} + R_{parallel_{32}} + R_{plaster2} + R_{conv2}$$
$$= 0.4 + 4.615 + 0.363 + 1.94 + 0.363 + 0.1 = 7.78℃/W$$

$$\dot{Q}_{unit_{32}} = \frac{\Delta T}{R_{total_{32}}} = \frac{20 - (-10)}{7.78} = 3.86W$$
$$\dot{Q}_{wall_{32}} = \sum \dot{Q}_{unit_{32}} = \dot{Q}_{unit_{32}} * \frac{A_{wall}}{A} = 4.41 * \frac{3 * 5}{0.25} = 231.36W \ (2)$$

From (1) and (2), it can be concluded that to increase the thickness of brick help reduce the heat transfer rate of the whole wall to some extent. In other words, this help improve the thermal resistance of the wall, but not significantly.

QUESTION 2

Determine the overall unit thermal resistance (the R-value) and the overall heat transfer coefficient (the U-factor) of a wood frame wall that is built around 38-mm 90-mm wood studs with a center-to-center distance of 400 mm. The 90-mm-wide cavity between the studs is filled with urethane rigid foam insulation. The inside is finished with 13-mm gypsum wallboard and the outside with 13 mm plywood and 13-mm 200-mm wood bevel lapped siding. The insulated cavity constitutes 75 percent of the heat transmission area while the studs, plates, and sills constitute 21 percent. The headers constitute 4 percent of the area, and they can be treated as studs.

ANSWER

	with Wood	with Insulation
Outside Air	0.03	0.03
Wood Bevel Lapped Siding (13mm * 200mm)	0.14	0.14
Plywood (13mm)	0.11	0.11
Urethane Rigid Foam Insulation (90mm)	-	0.98*90/25=3.528
Wood Studs (90mm)	0.63	-
Gypsum board (13mm)	0.079	0.079
Inside Surface	0.12	0.12

$$R_{withwood} = 0.03 + 0.14 + 0.11 + 0.63 + 0.079 + 0.12 = 1.109\ m^2.\text{°C}/W$$

$$R_{withinsulation} = 0.03 + 0.14 + 0.11 + 3.528 + 0.079 + 0.12 = 4.007\ m^2.\text{°C}/W$$

[to be continue with the calculation of R_{unit}]