

WEEK 3

2019年10月22日 18:19

QUESTIONS:

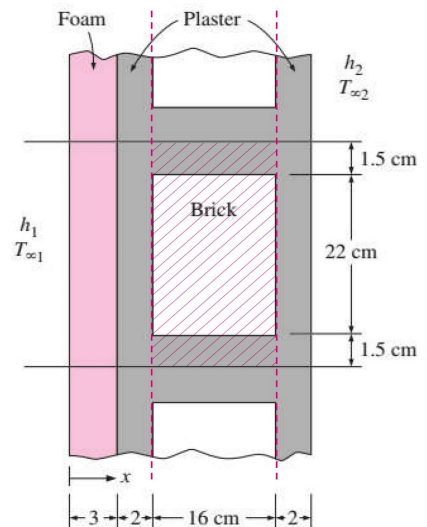
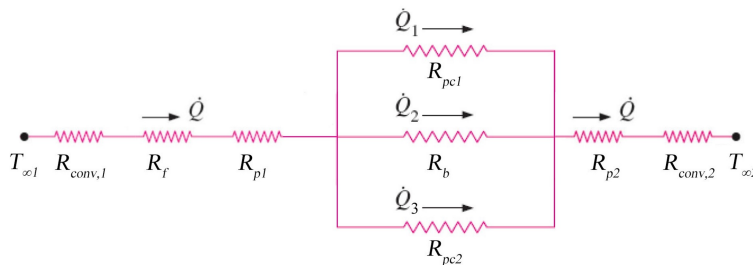
1. In this week's assignment you should first finish the composite wall question by finding the heat transfer rate, and then solve the same question while the thickness of the brick is increased to 32 cm and comment on the results
2. You should solve again the simplified wall calculation procedure replacing the glass fiber one with urethane rigid foam and while replacing the fiberboard with plywood and find the two R_{unit} values

ANSWERS:

1. 1) Heat loss through a composite wall

- A 3 m high and 5 m wide wall consists of long 16 cm 22 cm cross section horizontal bricks ($k = 0.72 \text{ W/m} \cdot ^\circ\text{C}$) separated by 3 cm thick plaster layers ($k = 0.22 \text{ W/m} \cdot ^\circ\text{C}$).
- There are also 2 cm thick plaster layers on each side of the brick and a 3-cm-thick rigid foam ($k = 0.026 \text{ W/m} \cdot ^\circ\text{C}$) on the inner side of the wall
- The indoor and the outdoor temperatures are 20°C and 10°C , and the convection heat transfer coefficients on the inner and the outer sides are $h_1 = 10 \text{ W/m}^2 \cdot ^\circ\text{C}$ and $h_2 = 40 \text{ W/m}^2 \cdot ^\circ\text{C}$, respectively. Assuming one-dimensional heat transfer and disregarding radiation, determine the rate of heat transfer through the wall.

If we regard an 1-metre-long symmetrical section around every brick as a unit of composite wall, we can first calculate the rate of heat transfer of a unit, then sum it up.



$$R_{conv,1} = \frac{1}{h_1 \cdot A_1} = \frac{1}{10 \cdot 0.25 \cdot 1} \approx 0.4 \text{ } ^\circ\text{C/W}$$

$$R_f = \frac{L_f}{k_f \cdot A_f} = \frac{0.03}{0.026 \cdot 0.25 \cdot 1} \approx 4.615 \text{ } ^\circ\text{C/W}$$

Central part resistance:

$$R_{pc1} = R_{pc2} = \frac{L_{brick}}{k_p \cdot A_{pc1}} = \frac{0.16}{0.22 \cdot 0.015 \cdot 1} \approx 48.48 \text{ } ^\circ\text{C/W}$$

$$R_b = \frac{L_{brick}}{k_b \cdot A_b} = \frac{0.16}{0.72 \cdot 0.22 \cdot 1} \approx 1.01 \text{ } ^\circ\text{C/W}$$

$$\therefore \frac{1}{R_{total,central}} = \frac{1}{R_{pc1}} + \frac{1}{R_{pc2}} + \frac{1}{R_b} = \frac{1}{48.48} + \frac{1}{48.48} + \frac{1}{1.01} \approx 1.03 \text{ } ^\circ\text{C/W}$$

$$\therefore R_{total,central} = \frac{1}{1.03} = 0.97 \text{ } ^\circ\text{C/W}$$

$$R_{p1} = R_{p2} = \frac{L_{p1}}{k_p \cdot A_{p1}} = \frac{0.02}{0.22 \cdot 0.25 \cdot 1} \approx 0.36 \text{ } ^\circ\text{C/W}$$

$$R_{conv,2} = \frac{1}{h_2 \cdot A_2} = \frac{1}{40 \cdot 0.25 \cdot 1} \approx 0.1 \text{ } ^\circ\text{C/W}$$

$$R_{total} = R_{conv,1} + R_f + R_{p1} + R_{total,central} + R_{p2} + R_{conv,2} \\ = 0.4 + 4.615 + 0.36 + 0.97 + 0.36 + 0.1 = 6.805 \text{ } ^\circ\text{C/W}$$

$$\dot{Q}_{unit} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}} = \frac{20 - 10}{6.805} \approx 1.47 \text{ W}$$

$$\dot{Q} = \dot{Q}_{unit} \cdot 5 \cdot \frac{3}{0.25} = 1.47 \cdot 5 \cdot 12 = 88.2 \text{ W}$$

2) thickness of the brick is increased to 32 cm

Only R_b , R_{pc1} and R_{pc2} are changed in this case:

$$R'_{pc1} = R'_{pc2} = \frac{L'_{brick}}{k_p \cdot A_{pc1}} = \frac{0.32}{0.22 \cdot 0.015 \cdot 1} \approx 96.97 \text{ } ^\circ\text{C/W}$$

$$R_b = \frac{L_{brick}}{k_b \cdot A_b} = \frac{0.32}{0.72 \cdot 0.22 \cdot 1} \approx 2.02 \text{ } ^\circ\text{C/W}$$

$$\frac{1}{R'_{total,central}} = \frac{1}{R'_{pc1}} + \frac{1}{R'_{pc2}} + \frac{1}{R'_b} = \frac{1}{96.97} + \frac{1}{96.97} + \frac{1}{2.02} \approx 0.52 \text{ } ^\circ\text{C/W}$$

$$R'_{total,central} = \frac{1}{0.52} \approx 1.92 \text{ } ^\circ\text{C/W}$$

$$R'_{total} = R_{conv,1} + R_f + R_{p1} + R'_{total,central} + R_{p2} + R_{conv,2} \\ = 0.4 + 4.615 + 0.36 + 1.92 + 0.36 + 0.1 = 7.755 \text{ } ^\circ\text{C/W}$$

$$\dot{Q}' = \dot{Q}'_{unit} \cdot 5 \cdot \frac{3}{0.25} = \frac{T_{\infty 1} - T_{\infty 2}}{R'_{total}} \cdot 5 \cdot 12 = \frac{20 - 10}{7.755} \cdot 60 \approx 77.37 \text{ W} < \dot{Q}$$

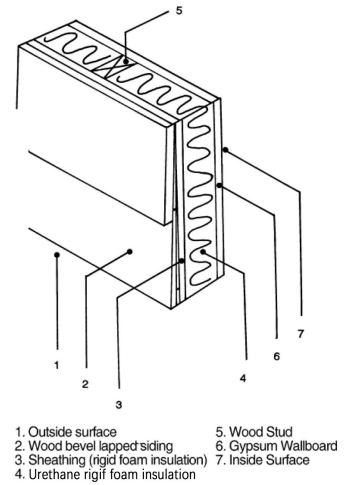
- We can see when thickness of the brick is increased, the total resistance of wall is increased, so the rate of heat transfer is decreased, but not significantly.

2. Simplified wall calculation

- Determine the overall unit thermal resistance (the R-value) and the overall heat transfer coefficient (the U-factor) of a wood frame wall that is built around 38-mm 90-mm wood studs with a center-to-center distance of 400 mm. The 90-mm-wide cavity between the studs is filled with urethane rigid foam insulation. The inside is finished with 13-mm gypsum wallboard and the outside with 13-mm plywood and 13-mm 200-mm wood bevel lapped siding.
- The insulated cavity constitutes 75 percent of the heat transmission area while the studs, plates, and sills constitute 21 percent. The headers constitute 4 percent of the area, and they can be treated as studs.
- Also, determine the rate of heat loss through the walls of a house whose perimeter is 50 m and wall height is 2.5 m in Las Vegas, Nevada, whose winter design temperature is -2 °C. Take the indoor design temperature to be 22 °C and assume 20 percent of the wall area is occupied by glazing.

Unit thermal resistance (the *R*-value) of common components used in buildings

Component	<i>R</i> -Value		Component	<i>R</i> -Value	
	m ² · °C/W	ft ² · h · °F/Btu		m ² · °C/W	ft ² · h · °F/Btu
Outside surface (winter)	0.030	0.17	Wood stud, nominal 2 in × 6 in (5.5 in or 140 mm wide)	0.98	5.56
Outside surface (summer)	0.044	0.25	Clay tile, 100 mm (4 in)	0.18	1.01
Inside surface, still air	0.12	0.68	Acoustic tile	0.32	1.79
Plane air space, vertical, ordinary surfaces (<i>e_{eff}</i> = 0.82):			Asphalt shingle roofing	0.077	0.44
13 mm ($\frac{1}{2}$ in)	0.16	0.90	Building paper	0.011	0.06
20 mm ($\frac{3}{4}$ in)	0.17	0.94	Concrete block, 100 mm (4 in):		
40 mm (1.5 in)	0.16	0.90	Lightweight	0.27	1.51
90 mm (3.5 in)	0.16	0.91	Heavyweight	0.13	0.71
Insulation, 25 mm (1 in)			Plaster or gypsum board, 13 mm ($\frac{1}{2}$ in)	0.079	0.45
Glass fiber	0.70	4.00	Wood fiberboard, 13 mm ($\frac{1}{2}$ in)	0.23	1.31
Mineral fiber batt	0.66	3.73	Plywood, 13 mm ($\frac{1}{2}$ in)	0.11	0.62
Urethane rigid foam	0.98	5.56	Concrete, 200 mm (8 in)		
Stucco, 25 mm (1 in)	0.037	0.21	Lightweight	1.17	6.67
Face brick, 100 mm (4 in)	0.075	0.43	Heavyweight	0.12	0.67
Common brick, 100 mm (4 in)	0.12	0.79	Cement mortar, 13 mm (1/2 in)	0.018	0.10
Steel siding	0.00	0.00	Wood bevel lapped siding, 13 mm × 200 mm (1/2 in × 8 in)	0.14	0.81
Slag, 13 mm ($\frac{1}{2}$ in)	0.067	0.38			
Wood, 25 mm (1 in)	0.22	1.25			
Wood stud, nominal 2 in × 4 in (3.5 in or 90 mm wide)	0.63	3.58			



	Wood studs	Insulation
Outside air	0.03	0.03
Wood bevel lapped siding	0.14	0.14
Plywood (13mm)	0.11	0.11
Urethane rigid foam insulation (90mm)	/	0.98*90/25=3.528
Wood studs (38-mm 90-mm)	0.63	/
Gypsum wallboard (13mm)	0.079	0.079
Inside surface	0.12	0.12

$$R_{withwood} = 0.03 + 0.14 + 0.11 + 0.63 + 0.079 + 0.12 = 1.109^{\circ}C/W$$

$$R_{withinsulation} = 0.03 + 0.14 + 0.11 + 3.528 + 0.079 + 0.12 = 4.007^{\circ}C/W$$