

Question 1: Finalizing the composite wall question by finding the heat transfer rate?

- What we concluded in the class was the following:

$$- R_{total} = R_{conv 1} + R_{foam} + R_{left.plaster} + R_{paralle.layers} + R_{right.plaster} + R_{conv 2}$$

$$\rightarrow R_{total} = 0.4 + 4.61 + 0.36 + 0.97 + 0.36 + 0.1$$

$$\rightarrow R_{total} \approx 6.8 \frac{C^{\circ}}{W}$$

- Now we can calculate the partial heat transfer rate as below:

$$- \dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$

$$\rightarrow \dot{Q} = \frac{20 - (-10)}{6.8}$$

$$\rightarrow \dot{Q} \approx \boxed{4.41 \text{ W}}$$

The calculated amount represents the partial heat transfer of the wall for the area of 0.25 m². Then the total heat transfer through this wall will be:

$$- \dot{Q} = 4.41 \times \frac{3 \times 5}{0.25}$$

$$\rightarrow \dot{Q} \approx \boxed{264.6 \text{ W}}$$

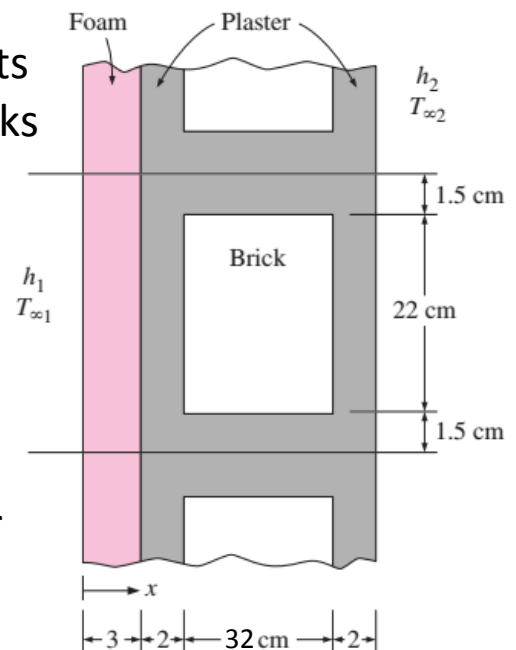


Question 2: A 3 m high and 5 m wide wall consists of long 32 cm 22 cm cross section horizontal bricks ($k = 0.72 \text{ W/m} \cdot ^\circ\text{C}$) separated by 3 cm thick plaster layers ($k = 0.22 \text{ W/m} \cdot ^\circ\text{C}$).

There are also 2 cm thick plaster layers on each side of the brick and a 3-cm-thick rigid foam ($k = 0.026 \text{ W/m} \cdot ^\circ\text{C}$) on the inner side of the wall.

The indoor and the outdoor temperatures are 20°C and -10°C , and the convection heat transfer coefficients on the inner and the outer sides are $h_1 = 10 \text{ W/m}^2 \cdot ^\circ\text{C}$ and $h_2 = 40 \text{ W/m}^2 \cdot ^\circ\text{C}$,

respectively. Assuming one-dimensional heat transfer and disregarding radiation, determine the rate of heat transfer through the wall.



$$\begin{aligned}
 - R_{conv 1} &= \frac{1}{h_1 \times A} = \frac{1}{10 \times [(0.015 + 0.22 + 0.015) \times 1]} = 0.4 \frac{^\circ\text{C}}{\text{W}} \\
 - R_{left.plaster} &= \frac{L_{plaster}}{k_{plaster} \times A} = \frac{1}{0.22 \times [(0.015 + 0.22 + 0.015) \times 1]} = 0.36 \frac{^\circ\text{C}}{\text{W}} \\
 - R_{foam} &= \frac{L_{foam}}{k_{foam} \times A} = \frac{0.03}{0.026 \times [(0.015 + 0.22 + 0.015) \times 1]} = 4.61 \frac{^\circ\text{C}}{\text{W}} \\
 - \frac{1}{R_{parallel.layers}} &= \frac{1}{R_{plaster.up}} + \frac{1}{R_{brick}} + \frac{1}{R_{plaster.down}} \\
 \rightarrow \frac{1}{R_{parallel.layers}} &= \frac{1}{\frac{L_{plaster.up}}{k_{plaster} \times A}} + \frac{1}{\frac{L_{brick}}{k_{brick} \times A}} + \frac{1}{\frac{L_{plaster.down}}{k_{plaster} \times A}}
 \end{aligned}$$

$$\rightarrow \frac{1}{R_{parallel.layers}} = \frac{1}{\frac{0.32}{0.22 \times 0.015}} + \frac{1}{\frac{0.32}{0.72 \times 0.22}} + \frac{1}{\frac{0.32}{0.22 \times 0.015}}$$

$$\rightarrow \frac{1}{R_{parallel.layers}} = \frac{1}{96.97} + \frac{1}{2.02} + \frac{1}{96.97} = 0.01 + 0.495 + 0.01 = 0.515$$

$$\rightarrow R_{parallel.layers} \approx 1.94 \frac{C^{\circ}}{W}$$

$$- R_{conv 2} = \frac{1}{h_2 \times A} = \frac{1}{40 \times [(0.015 + 0.22 + 0.015) \times 1]} = 0.1 \frac{C^{\circ}}{W}$$

- Now we can calculate the total R:

$$- R_{total} = R_{conv 1} + R_{foam} + R_{left.plaster} + R_{parallel.layers} + R_{right.plaster} + R_{conv 2}$$

$$- R_{total} = 0.4 + 4.61 + 0.36 + 1.94 + 0.36 + 0.1$$

$$- R_{total} \approx 7.8 \frac{C^{\circ}}{W}$$

- Now we can calculate the heat transfer rate as below:

$$- \dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{total}}$$

$$\rightarrow \dot{Q} = \frac{20 - (-10)}{7.8}$$

$$\rightarrow \dot{Q} \approx \boxed{3.85 \text{ W}}$$

The calculated amount represents the partial heat transfer of the wall for the area of 0.25 m². Then the total heat transfer through this wall will be:

$$\dot{Q} = 3.85 \times \frac{3 \times 5}{0.25}$$

$$\rightarrow \dot{Q} \approx 231 \text{ W}$$

- Although the thickness of brick has been doubled in the previous example, the effect on the total value of heat transfer has not been that significant. The reason is due to low value of brick's thermal conductivity ($k = 0.72 \text{ W/m} \cdot ^\circ\text{C}$) which results in a decimal number for its corresponding resistance value.

Question 3: Determine the overall unit thermal resistance (the R -value) and the overall heat transfer coefficient (the U -factor) of a wood frame wall that is built around 38-mm 90-mm wood studs with a center-to-center distance of 400 mm. The 90-mm-wide cavity between the studs is filled with glass urethane rigid foam insulation. The inside is finished with 13-mm gypsum wallboard and the outside with 13 mm plywood and 13-mm 200-mm wood bevel lapped siding.

- First thing we have to do is to calculate $R_{total} (R')$, while imagining every material within the cavity as a single layer.

- Second thing is to find the Unit Thermal Resistance from the corresponding table and also to proportionally calculate the ones which exceeds the standard unit dimensions.



	Wood Studs	Urethane Rigid Ins.
Outside Air	0.03	0.03
Wood Bevel	0.14	0.14
Plywood (13 mm)	0.11	0.11
Urethane Rigid Foam	-	$0.98 \times (\frac{90}{25}) = 3.528$
Wood Studs	0.63	-
Gypsum Board	0.079	0.079
Inside Surface	0.12	0.12

$$R'_{wood} = 0.03 + 0.14 + 0.11 + 0.63 + 0.079 + 0.12 = 1.109 \, m^2 \frac{C^{\circ}}{W}$$

$$R'_{urethane} = 0.03 + 0.14 + 0.11 + 3.528 + 0.079 + 0.12 = 4.007 \, m^2 \frac{C^{\circ}}{W}$$