

Question 1:

Heat loss through a composite wall

A 3-m high and 5-m wide wall consists of long 32-cm 22-cm cross section horizontal bricks ($k = 0.72 \text{ W/m} \cdot ^\circ\text{C}$) separated by 3 cm thick plaster layers ($k = 0.22 \text{ W/m} \cdot ^\circ\text{C}$).

There are also 2 cm thick plaster layers on each side of the brick and a 3-cm-thick rigid foam ($k = 0.026 \text{ W/m} \cdot ^\circ\text{C}$) on the inner side of the wall. The indoor and the outdoor temperatures are 20°C and -10°C , and the convection heat transfer coefficients on the inner and the outer sides are $h_1 = 10 \text{ W/m}^2 \cdot ^\circ\text{C}$ and $h_2 = 40 \text{ W/m}^2 \cdot ^\circ\text{C}$, respectively. Assuming one-dimensional heat transfer and disregarding radiation, determine the rate of heat transfer through the wall.

Answer:

$$R_{1,conv.} = \frac{1}{h_1 * A_{1-dimen.}} = \frac{1}{10 \frac{W}{m^2 \cdot ^\circ C} * (0.015 + 0.22 + 0.015)m * 1m} = 0.4 \frac{^\circ C}{W}$$

$$R_{Foam} = \frac{L_{Foam}}{k_{Foam} * A_{1-dimen.}} = \frac{0.03m}{0.026 \frac{W}{m \cdot ^\circ C} * (0.015 + 0.22 + 0.015)m * 1m} \approx 4.615 \frac{^\circ C}{W}$$

$$\therefore R_{Plaster.up} = R_{Plaster.down} = \frac{L_{P.up \text{ or } dn}}{k_P * A_{P.up \text{ or } dn(1-dimen)}} = \frac{0.032m}{0.22 \frac{W}{m \cdot ^\circ C} * 0.015m * 1m} \approx 96.97 \frac{^\circ C}{W}$$

$$R_{Brick} = \frac{L_{Brick}}{k_{Brick} * A_{Brick(1-dimen)}} = \frac{0.32m}{0.72 \frac{W}{m \cdot ^\circ C} * 0.22m * 1m} \approx 2.02 \frac{^\circ C}{W}$$

$$\therefore \frac{1}{R_{total-parallel}} = \frac{1}{R_{Plaster.up}} + \frac{1}{R_{Brick}} + \frac{1}{R_{Plaster.down}} \approx \frac{1}{96.97 \frac{^\circ C}{W}} + \frac{1}{2.02 \frac{^\circ C}{W}} + \frac{1}{96.97 \frac{^\circ C}{W}} \approx 0.516 \frac{W}{^\circ C}$$

$$i.e., R_{total-parallel} = \frac{1}{0.516 \frac{W}{^\circ C}} \approx 1.94 \frac{^\circ C}{W}$$

$$R_{Plaster.left} = R_{Plaster.right} = \frac{L_{P.lt \text{ or } rt}}{k_P * A_{P.lt \text{ or } rt(1-dimen)}} = \frac{0.02m}{0.022 \frac{W}{m \cdot ^\circ C} * (0.015 + 0.22 + 0.015)m * 1m} = 0.363 \frac{^\circ C}{W}$$

$$R_{2,conv.} = \frac{1}{h_2 * A_{1-dimen.}} = \frac{1}{40 \frac{W}{m^2 \cdot ^\circ C} * (0.015 + 0.22 + 0.015)m * 1m} = 0.1 \frac{^\circ C}{W}$$

$$R_{wall,total(1-dimen)} = R_{1,conv.} + R_{Foam} + R_{Plaster.left} + R_{total-parallel} + R_{Plaster.right} + R_{2,conv.}$$

$$\approx 0.4 \frac{^\circ C}{W} + 4.615 \frac{^\circ C}{W} + 0.363 \frac{^\circ C}{W} + 1.94 \frac{^\circ C}{W} + 0.363 \frac{^\circ C}{W} + 0.1 \frac{^\circ C}{W} = 7.781 \frac{^\circ C}{W}$$

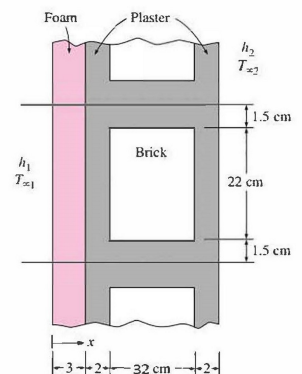
The heat transfer rate is:

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{wall,total}} \approx \frac{20^\circ\text{C} - (-10)^\circ\text{C}}{7.781 \frac{^\circ C}{W}} \approx 3.86W$$

Plus, we have already calculated the $R_{wall,total}$ while the thickness of brick in this composite wall is 16 mm,

$$R_{wall,total(Thickness \text{ of the brick}=16mm)} \approx 6.81 \frac{^\circ C}{W}$$

In this condition, the heat transfer rate is:



$$\dot{Q}' = \frac{T_1 - T_\infty}{R_{wall, total}(\text{Thickness of the brick}=16\text{mm})} \approx \frac{20^\circ\text{C} - (-10)^\circ\text{C}}{6.81 \frac{^\circ\text{C}}{\text{W}}} \approx 4.41\text{W}$$

By comparing the two results, we can have this conclusion:

Simply double the thickness of a brick inside a composite wall doesn't significantly increase the thermal resistance of the whole wall, thus the rate of heat transfer doesn't significantly decrease.

Qusetion 2:

A wood frame wall that is built around 38-mm 90-mm wood studs with a center-to-center distance of 400 mm. The 90-mm-wide cavity between the studs is filled with urethane rigid foam insulation. The inside is finished with 13-mm gypsum wallboard and the outside with 13 mm plywood and 13-mm 200-mm wood bevel lapped siding. The insulated cavity constitutes 75 percent of the heat transmission area while the studs, plates, and sills constitute 21 percent. The headers constitute 4 percent of the area, and they can be treated as studs.

Find the two R_{unit} values.

Answer:

| | Wood | Insulation |
|--------------------------------|-------|------------------------------|
| Outside Air | 0.03 | 0.03 |
| Wood Bevel(13mm*200mm) | 0.14 | 0.14 |
| Plywood(13mm) | 0.11 | 0.11 |
| Urethane Rigid Foam Ins.(90mm) | No | $0.98 \cdot 90 / 25 = 3.528$ |
| Wood Studs(90mm) | 0.63 | No |
| Gypsum Board(13mm) | 0.079 | 0.079 |
| Inside Surface | 0.12 | 0.12 |

$$R'_{with\ wood} = (0.03 + 0.14 + 0.11 + 0.63 + 0.079 + 0.12)m^2 \frac{^\circ\text{C}}{\text{W}} = 1.109 \frac{m^2 \cdot ^\circ\text{C}}{\text{W}}$$

$$R'_{with\ insulation} = (0.03 + 0.14 + 0.11 + 3.528 + 0.079 + 0.12) = 4.007 \frac{m^2 \cdot ^\circ\text{C}}{\text{W}}$$

