

The rate of heat transfer through the composite wall analyzed in class is:

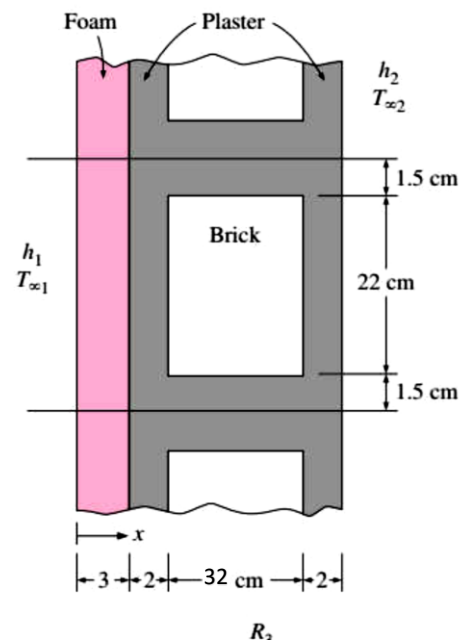
$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{tot}}} = \frac{20 - (-10) \text{ }^{\circ}\text{C}}{6,81 \text{ }^{\circ}\text{C/W}} = 4,417 \text{ W}$$

A 3 m high and 5 m wide wall consists of long 32 cm 22 cm cross section horizontal bricks ($k = 0.72 \text{ W/m} \cdot ^{\circ}\text{C}$) separated by 3 cm thick plaster layers ($k = 0.22 \text{ W/m} \cdot ^{\circ}\text{C}$).

There are also 2 cm thick plaster layers on each side of the brick and a 3-cm-thick rigid foam ($k = 0.026 \text{ W/m} \cdot ^{\circ}\text{C}$) on the inner side of the wall.

The indoor and the outdoor temperatures are 20°C and 10°C , and the convection heat transfer coefficients on the inner and the outer sides are $h_1 = 10 \text{ W/m}^2 \cdot ^{\circ}\text{C}$ and $h_2 = 25 \text{ W/m}^2 \cdot ^{\circ}\text{C}$, respectively.

Assuming one-dimensional heat transfer and disregarding radiation, determine the rate of heat transfer through the wall.



Solving:

$$R_i = \frac{1}{h_i A} = \frac{1}{10 \cdot 0,25} = 0,4 \frac{^{\circ}\text{C}}{\text{W}} \quad \text{Inner resistance}$$

$$R_f = \frac{L_f}{k_f A} = \frac{0,3}{0,026 \cdot 0,25} = 4,615 \frac{^{\circ}\text{C}}{\text{W}} \quad \text{Foam resistance}$$

$$R_{pc1} = R_{pc2} = \frac{L_{pc1}}{k_p A_{pc1}} = \frac{0,02}{0,22 \cdot 0,25} = 0,36 \frac{^{\circ}\text{C}}{\text{W}} \quad \text{Plaster resistance}$$

$$R_{pb1} = R_{pb2} = \frac{L_{pb1}}{k_p A_{pb1}} = \frac{0,32}{0,22 \cdot 0,015} = 96,97 \frac{^{\circ}\text{C}}{\text{W}} \quad \text{Plaster resistance in parallel}$$

$$R_b = \frac{L_b}{k_b A} = \frac{0,32}{0,72 \cdot 0,22} = 2,02 \frac{^{\circ}\text{C}}{\text{W}} \quad \text{Brick resistance in parallel}$$

$$\frac{1}{R_{\text{tot}}} = \frac{1}{R_{pc1}} + \frac{1}{R_b} + \frac{1}{R_{pc2}} = \frac{1}{96,97} + \frac{1}{2,02} + \frac{1}{96,97} = 0,516 \frac{\text{W}}{^{\circ}\text{C}}$$

$$R_{\text{tot parallel}} = \frac{1}{0,516} = 1,938 \frac{^{\circ}\text{C}}{\text{W}} \quad \text{Total resistance in parallel}$$

$$R_o = \frac{1}{h_2 A} = \frac{1}{50 \cdot 0,25} = 0,1 \frac{^{\circ}\text{C}}{\text{W}} \quad \text{Outer resistance}$$

$$R_{\text{tot}} = 0.4 + 4.615 + 0.36 + 0.36 + 1.938 + 0.1 = 7,773 \frac{^{\circ}\text{C}}{\text{W}} \quad \text{Total resistance}$$

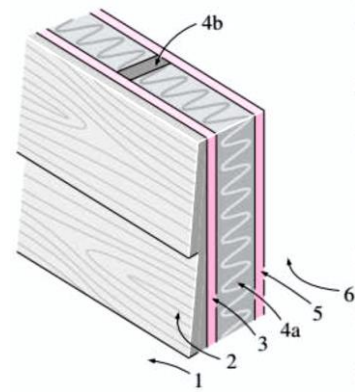
$$\dot{Q} = \frac{T_{\infty 1} - T_{\infty 2}}{R_{\text{tot}}} = \frac{20 - (-10) \text{ } ^{\circ}\text{C}}{7,773 \text{ } ^{\circ}\text{C/W}} = 3,86 \text{ W} \quad \text{Heat transfer through the composite wall}$$

Conclusion:

Duplicating the thickness of the brick (from 16 cm to 32 cm), the total parallel resistance increases only to $1 \frac{^{\circ}\text{C}}{\text{W}}$ (from $6.81 \frac{^{\circ}\text{C}}{\text{W}}$ to $7.77 \frac{^{\circ}\text{C}}{\text{W}}$) and, consequently, the heat transfer through the wall decreases of 0,8 W (from 4.417 W to 3.86 W).

As a consequence, to reduce the heat transfer, the better solution is increasing the thickness of the foam (because it has the biggest resistance); increasing the thickness of the brick doesn't produce a significant effect in order to reduce the heat transfer.

Determine the overall unit thermal resistance (the R_{value}) of a wood frame wall that is built around 38-mm 90-mm wood studs with a center-to-center distance of 400 mm. The 90-mm-wide cavity between the studs is filled with urethane rigid insulation. The inside is finished with 13-mm gypsum wallboard and the outside with 13-mm plywood and 13-mm 200-mm wood bevel lapped siding.



Unit thermal resistance (the R -value) of common components used in buildings

Component	R -Value		Component	R -Value	
	$\text{m}^2 \cdot ^\circ\text{C}/\text{W}$	$\text{ft}^2 \cdot \text{h} \cdot ^\circ\text{F}/\text{Btu}$		$\text{m}^2 \cdot ^\circ\text{C}/\text{W}$	$\text{ft}^2 \cdot \text{h} \cdot ^\circ\text{F}/\text{Btu}$
Outside surface (winter)	0.030	0.17	Wood stud, nominal 2 in \times 6 in (5.5 in or 140 mm wide)	0.98	5.56
Outside surface (summer)	0.044	0.25	Clay tile, 100 mm (4 in)	0.18	1.01
Inside surface, still air	0.12	0.68	Acoustic tile	0.32	1.79
Plane air space, vertical, ordinary surfaces ($\epsilon_{\text{eff}} = 0.82$):			Asphalt shingle roofing	0.077	0.44
13 mm ($\frac{1}{2}$ in)	0.16	0.90	Building paper	0.011	0.06
20 mm ($\frac{3}{4}$ in)	0.17	0.94	Concrete block, 100 mm (4 in):		
40 mm (1.5 in)	0.16	0.90	Lightweight	0.27	1.51
90 mm (3.5 in)	0.16	0.91	Heavyweight	0.13	0.71
Insulation, 25 mm (1 in)			Plaster or gypsum board, 13 mm ($\frac{1}{2}$ in)	0.079	0.45
Glass fiber	0.70	4.00	Wood fiberboard, 13 mm ($\frac{1}{2}$ in)	0.23	1.31
Mineral fiber batt	0.66	3.73	Plywood, 13 mm ($\frac{1}{2}$ in)	0.11	0.62
Urethane rigid foam	0.98	5.56	Concrete, 200 mm (8 in)		
Stucco, 25 mm (1 in)	0.037	0.21	Lightweight	1.17	6.67
Face brick, 100 mm (4 in)	0.075	0.43	Heavyweight	0.12	0.67
Common brick, 100 mm (4 in)	0.12	0.79	Cement mortar, 13 mm (1/2 in)	0.018	0.10
Steel siding	0.00	0.00	Wood bevel lapped siding, 13 mm \times 200 mm (1/2 in \times 8 in)	0.14	0.81
Slag, 13 mm ($\frac{1}{2}$ in)	0.067	0.38			
Wood, 25 mm (1 in)	0.22	1.25			
Wood stud, nominal 2 in \times 4 in (3.5 in or 90 mm wide)	0.63	3.58			

	$\frac{\text{m}^2 \cdot ^\circ\text{C}}{\text{W}}$	$\frac{\text{m}^2 \cdot ^\circ\text{C}}{\text{W}}$
	Insulation	Wood Studs
1. Outside air surface	0.03	0.3
2. Wood bevel (13*200) mm	0.14	0.14
3. Plywood (13 mm)	0.11	0.11
4.a Urethane rigid foam	$0.98 \times \frac{90}{25} = 3.528$	/
4.b Wood Studs (90 mm)	/	0.63
5. Gypsum Wallboard (13 mm)	0.079	0.079
6. Inside air surface	0.12	0.12

The R_{unit} values are respectively:

$R'_{\text{with insulation}} =$	$4,007 \frac{\text{m}^2 \cdot ^\circ\text{C}}{\text{W}}$	
$R'_{\text{with wood studs}} =$		$1,379 \frac{\text{m}^2 \cdot ^\circ\text{C}}{\text{W}}$