

Question1:

Heat loss through a composite wall

A 3-m high and 5-m wide wall consists of long 32-m 22-m cross section horizontal bricks($k=0.72\text{W/m} \cdot ^\circ\text{C}$) separated by 3cm thick plaster layers($k=0.22\text{W/m}\cdot^\circ\text{C}$).

There are also 2cm thick plaster layers on each side of the brick and a 3-cm thick rigid foam ($k=0.026\text{W/m}\cdot^\circ\text{C}$) on the inner side of the wall. The indoor and the outdoor temperatures are 20°C and -10°C , and the convection heat transfer coefficients on the inner and outer sides are $h_1=10\text{W/m}^2\cdot^\circ\text{C}$ and $h_2=40\text{W/m}^2\cdot^\circ\text{C}$, respectively. Assuming one dimensional heat transfer and disregarding radiation, determine the rate of heat transfer through the wall.

Answer:

$$R_{1,\text{conv.}} = \frac{1}{h_1 * A_{1-\text{dimen.}}} = \frac{1}{10 \frac{\text{W}}{\text{m}^2\cdot^\circ\text{C}} * (0.015 + 0.22 + 0.015)\text{m} * 1\text{m}} = 0.4 \frac{^\circ\text{C}}{\text{W}}$$

$$R_{\text{Foam}} = \frac{L_{\text{Foam}}}{k_{\text{Foam}} * A_{1-\text{dimen}}} = \frac{0.03\text{m}}{0.026 \frac{\text{W}}{\text{m}\cdot^\circ\text{C}} * (0.015 + 0.22 + 0.015)\text{m} * 1\text{m}} \approx 4.615 \frac{^\circ\text{C}}{\text{W}}$$

$$\therefore R_{\text{plaster.up}} = R_{\text{plaster.down}} = \frac{L_{p.\text{up or dn}}}{k_p * A_{p.\text{up or dn}(1-\text{dimen})}} = \frac{0.32\text{m}}{0.22 \frac{\text{W}}{\text{m}\cdot^\circ\text{C}} * 0.015\text{m} * 1\text{m}} \approx 96.97 \frac{^\circ\text{C}}{\text{W}}$$

$$R_{\text{Brick}} = \frac{L_{\text{Brick}}}{k_{\text{Brick}} * A_{\text{Brick}}(1-\text{dimen})} = \frac{0.32m}{0.72 \frac{W}{m^2 \cdot ^\circ C} * 0.22m * 1m} \approx 2.02 \frac{^\circ C}{W}$$

$$\therefore \frac{1}{R_{\text{total-parallel}}} = \frac{1}{R_{\text{plaster.up}}} + \frac{1}{R_{\text{Brick}}} + \frac{1}{R_{\text{plaster.down}}} \approx \frac{1}{96.97 \frac{^\circ C}{W}} + \frac{1}{2.02 \frac{^\circ C}{W}} + \frac{1}{96.97 \frac{^\circ C}{W}} \approx 0.516 \frac{W}{^\circ C}$$

$$\text{i.e., } R_{\text{total-parallel}} = \frac{1}{0.516 \frac{W}{^\circ C}} \approx 1.94 \frac{^\circ C}{W}$$

$$R_{\text{plaster.left}} = R_{\text{plaster.right}} = \frac{L_{\text{p.lt or rt}}}{k_p * A_{\text{p.lt or rt}}(1-\text{dimen})} = \frac{0.22m}{0.022 \frac{W}{m^2 \cdot ^\circ C} * (0.015 + 0.22 + 0.015)m * 1m} = 0.363 \frac{^\circ C}{W}$$

$$R_{2,\text{conv.}} = \frac{1}{h_2 * A_{1-\text{dimen.}}} = \frac{1}{40 \frac{W}{m^2 \cdot ^\circ C} * (0.015 + 0.22 + 0.015)m * 1m} = 0.1 \frac{^\circ C}{W}$$

$$R_{\text{wall,total}(1-\text{dimen})} = R_{1,\text{conv.}} + R_{\text{Foam}} + R_{\text{plaster.left}} + R_{\text{total-parallel}} + R_{\text{plaster.right}} +$$

$$R_{2,\text{conv.}} \approx 0.4 \frac{^\circ C}{W} + 4.615 \frac{^\circ C}{W} + 0.363 \frac{^\circ C}{W} + 1.94 \frac{^\circ C}{W} + 0.363 \frac{^\circ C}{W} + 0.1 \frac{^\circ C}{W} = 7.781 \frac{^\circ C}{W}$$

The heat transfer rate is:

$$\dot{Q} = \frac{T_1 - T_\infty}{R_{\text{wall,total}}} \approx \frac{20^\circ C - (-10)^\circ C}{7.781 \frac{^\circ C}{W}} \approx 3.86W$$

Plus, we have already calculated the $R_{\text{wall,total}}$ while the thickness of brick in this composite wall is 16mm,

$$R_{\text{wall,total}(\text{thickness of the brick}=16mm)} \approx 6.81 \frac{^\circ C}{W}$$

In this condition, the heat transfer rate is:

$$\dot{Q}' = \frac{T_1 - T_\infty}{R_{\text{wall,total}(\text{thickness of the brick}=16mm)}} \approx \frac{20^\circ C - (-10)^\circ C}{6.81 \frac{^\circ C}{W}} \approx 4.41W$$

By comparing the two result, we can have this conclusion:

Simply double the thickness of a brick inside a composite wall doesn't significantly increase the thermal resistance of the whole wall, thus the rate of heat transfer

dosen't significantly decrease.

Question2:

A wood frame wall that is built around 38-mm 90-mm wood studs with a center-to-center distance of 400mm. The 90-mm-wide cavity between the studs is filled with urethane rigid foam insulation. The inside is finish with 13-mm gypsum wallboard and the outside with 13-mm plywood and 13-mm 200-mm wood bevel lapped siding. The insulated cavity constitutes 75 percent of the heat transmission area while the studs, plates, and sills constitute 21 percent. The headers constitute 4 percent of the area, and they can be treated as studs.

Find the two R_{Unit} values.

Answer:

	Wood	Insulation
Outside Air	0.03	0.03
Wood Bevel(13mm*200mm)	0.14	0.14
Plywood(13mm)	0.11	0.11
Urethane Rigid Foam Ins.(90mm)	No	$0.98 \times 90 / 25 = 3.528$
Wood Studs(90mm)	0.63	No
Gypsum Board(13mm)	0.079	0.079
Inside Surface	0.12	0.12

$$R'_{with\ wood} = (0.03 + 0.14 + 0.11 + 0.63 + 0.079 + 0.12) m^2 \frac{^{\circ}C}{W} = 1.109 \frac{m^2 \cdot ^{\circ}C}{W}$$

$$R'_{with\ insulation} = (0.03+0.14+0.11+3.528+0.079+0.12) = 4.007 \frac{m^2 \cdot ^\circ C}{W}$$