Control Systems 2: MATLAB Laboratory Course

Lab one: Flight Control

Faculty of
Electrical Engineering and Information Technology

General Comment

The computer-aided laboratory course is designed in three parts as a complement to the Control Systems 2 lectures. Within each part of the laboratory, we try to make a bridge between the principals of control theory provided in the lectures, and its engineering application by studying real models of different plants. It is expected that after completing the laboratory course, the students become capable to analyze the linear models' behavior of real plants, in both time and frequency domains, and also design simple controllers to meet the control objectives.

The present manuscript, is the first part of the computer-aided lab, and considers the basic concepts of multi-input multi-output (MIMO) control systems. Fundamental control concepts such as stability, controllability, observability, etc. are to be practiced with a simplified model of an aircraft to see how an aircraft basically behaves, and how it can be controlled.

Chapter 1

Introduction

This section presents a short introduction about how an aircraft flies and how it can be controlled. In references, more detailed information is provided, that interested readers can refer to. Ignoring take-off and landing, we focus on the main part of the flight which is called cruise. While cruising, the aircraft is flying in a straight line at a constant speed. We investigate the ways we can control the aircraft while it is cruising. During the flight, there are four main forces acting on the plane, **weight**, **lift**, **drag** and **thrust**.

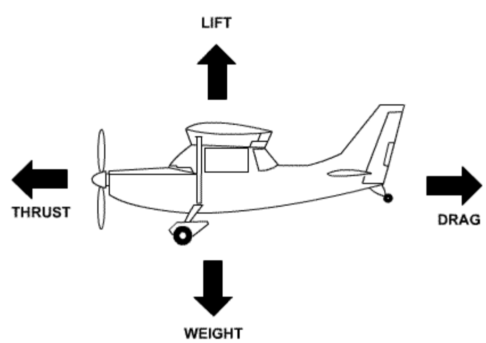

Figure 1.1: Acting forces on aircraft (*photo:www.learntoflylv.com*)

Lift is a force that pulls the plane up and balances the plane's **weight** force, mostly by the wings. A plane's wing has a special shape, called an airfoil, which forces air to flow over the top surface of the wing quicker than the bottom surface, resulting an upward force acting on the wing. As we see later on, the **elevators** which are mounted horizontally at the rear of the fuselage, move downward to increase the lift force when the aircraft takes-off.

Drag is the force that opposes the motion of the plane through the air, due to the friction between aircraft and air. Therefore, to keep a plane moving forward at a constant speed, another force is needed to overcome the drag force.

Thrust is the force produced by the plane's engines. This force pulls the plane forward through the air and overcomes the drag force produced by the air friction. Therefore, the aircraft accelerates if thrust force overcomes drag, and decelerates if engines produce less force in comparison with drag. It also ascends if lift force is greater that weight, and descends vice versa. A plane is composed of a large number of different components. Main components involve wings, fuselage, cockpit, engine, cargo bay, and landing gears. There are in adition control components of which the most important ones are stabilizers, ailerons, rudder, elevators, and flaps. What we are focused on, are mainly the **ailerons** and **rudder** which are mounted at the back of the wings and rear of the fuselage, respectively. The figure below, shows schematically the different components of a typical aircraft.

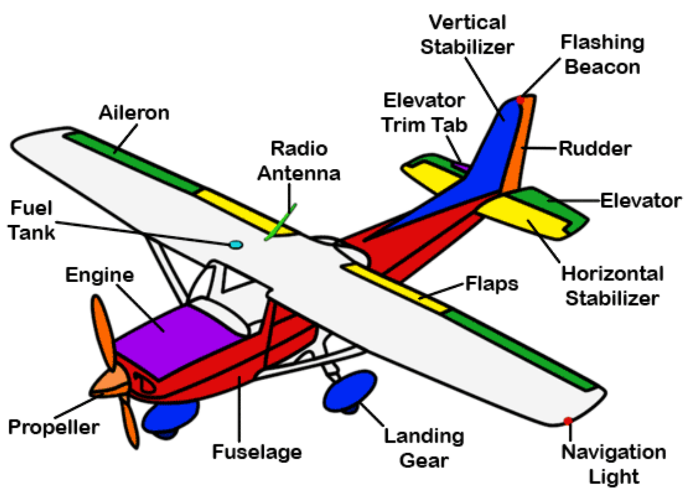

Figure 1.2: Aircraft components (*photo:www.tailspin.com*)

Most aircrafts have four main control systems: **Elevators** which are used to make a plane ascend or descend by moving upward and downward, respectively. They are mounted horizontally on the sides of the vertical stabilizer, and they move in the same direction. **Ailerons** which help the plane to roll left or right, **rudder** which makes the aircraft turns left or right, and **throttles** which control the speed of the plane. Focusing on aileron and rudder, the former one is mounted on the tips of the wings and changes the lift force by moving up and down. By moving one aileron up and the one mounted on the other wing down, one wing will generate more lift than the other, which causes the plane to turn around the roll axis. The rudder is located on the vertical tail of the plane and steers the plane to left or right. The vertical tail on an aircraft does not normally have a curved shape, so does not generally produce lift force, while the rudder stays in middle. By moving the rudder to one direction, the tail is curved and start producing lift. Since, lift forces are always acting perpendicular to the wing or tail that generates it, so the lift produced by the motion of rudder act horizontally, which makes the plane to rotate left or right, in the horizontal plane.

Moving the rudder to the left side, will generate lift force to the right, which will move the nose of the aircraft to the left side, and vice versa. Rudders are often slower at turning an aircraft than the ailerons, but they can turn the aircraft without rolling it and are employed for small adjustments during takeoff, landing and cruising. Sometimes pilots use both the rudder and the ailerons together while turning in order to produce a smoother flight. The pilot controls ailerons and rudder by **yoke** and **rudder pedals**.

The orientation of an aircraft can effectively be captured by measuring three critical flight dynamics parameters, so called **angles of rotation**, and they are known as **roll**, **pitch** and **yaw** angles, also known as **Euler angles**. These angles represent the rotation of an aircraft around its **center of Mass**, and all the actuators discussed earlier are exerting appropriate forces to correctly tune these three essential parameters at every segment of the flight. The figure below, illustrates these three angles around the center of mass of a plane.

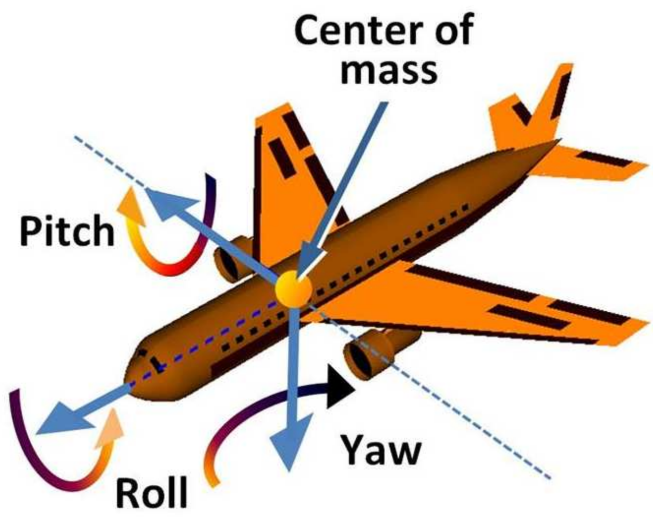

Figure 1.3: Angles of rotation (*photo:www.wikimedia.com*)

There is another important variable to be watched out during the flight so called **sideslip angle**, which relates the rotation of the aircraft centerline (roll axis) from the relative wind. As it can be seen in the following figure, sideslip angle is the displacement of the aircraft centerline from the relative airflow, rather than from a reference axis. This angle plays an important role in stability of the aircraft.

The ailerons are in charge of controlling **bank angle** at which the aircraft is inclined

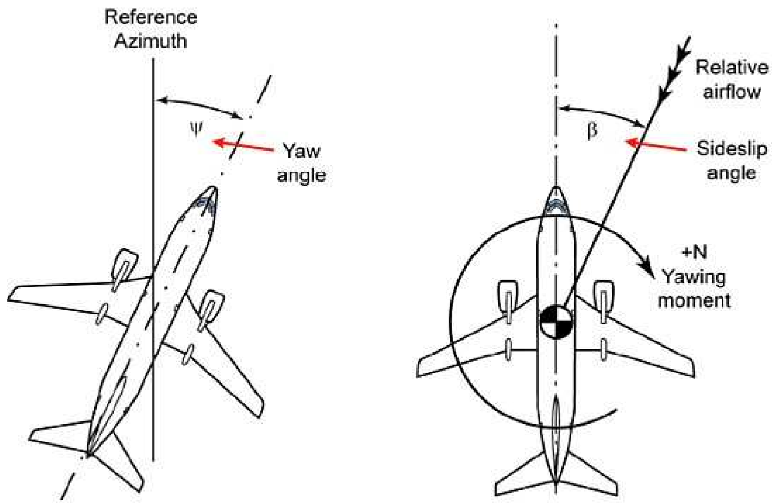

Figure 1.4: Sideslip angle (*photo:www.theairlinepilot.com*)

about. In case of having non-zero bank angle, the horizontal component of the lift force acting on the wings causes centripetal acceleration to make the turning possible. As already discussed, rudder's movement changes the yaw angle around the vertical axis and facilitates the rotation of aircraft in horizontal plane. In practice, the pilot uses both ailerons and rudder control inputs for a turn of aircraft. A rudder alone can also turn a fixed-wing aircraft but much more slowly than if the ailerons collaborate. Using both together induces coordinated turns, in which the roll axis is in line with the arc of the turn, preventing slipping and skidding. Using rudders solely can cause a fast and unexpected spin which can be dangerous at low altitudes.

To control the pitch angle, the elevators mounted horizontally at the rear of the tail are used. The pitch angle determines the rotation of the aircraft about its center of gravity or in the other words imposes the aircraft's nose movement. The elevators create a downward force which balances the nose down torque due to the aircraft's center of mass being located in front of the center of lift. As already stated, the elevators work in pairs and unlike the ailerons, they move in the same direction, both up or down.

To measure these three essential angles, gyroscopes are employed. In general, three different packages of gyros are used in an aircraft, **heading indicator**, **attitude indicator**, and **turn indicator**. Two primary characteristics of gyro which makes it an indispensable sensor in aviation are **rigidity in space**, and **precision**. The spinning rotor inside a gyro maintains a constant attitude in space as long as no external force is applied. The stability of gyro increases with increasing the mass and speed of the rotor, therefore, the gyros in an aircraft are constructed from heavy materials and are designed to spin rapidly. For instance, the attitude and heading indicators are designed to rotate with around 15000 and 10000 rpm, respectively.

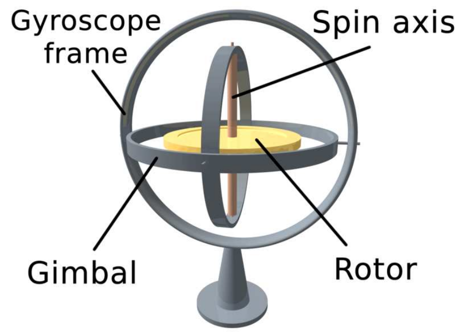

Figure 1.5: Gyroscope (*photo:www.wikipedia.com*)

The other characteristic corresponds the tilting of the gyro axis by applying forces. Then rotor tilts in the same direction as the force is applied, however, the same forces results in a move in a different direction as though the force had been applied to a point 90° around the rim in the direction of rotation. This turning movement or precession, places the rotor in a new plane of rotation parallel to the applied force. To have more knowledge about how the gyros work inside an aircraft, refer to references.

Chapter 2

Exercises

In this section, a simplified model of an aircraft, which is essentially a multivariable control system, is provided, and some tasks are followed afterwards. We will investigate the interactions between different inputs and outputs in the model which is linearized about steady state values of the dynamical system's states.

2.1 Linearized Model of an Aircraft

A well-known linearized state space model of an aircraft during the steady state cruise with speed cruise 0.8 Mach and altitude 40,000 ft. is as follows:

$$\dot{x} = Ax + Bu \tag{2.1}$$

where, $x = (\beta \ \omega \ \theta \ \phi)^{\mathsf{T}}$ is the state vector includes the four subsequent states: sideslip angle (β), angle of roll or bank angle (ϕ), angular rate of roll (θ), and angular rate of yaw (ω), with the initial value $x_0 = [0]_{4\times 1}$. The vector of control input is also represented by u, which is accompanied by the input matrix B of appropriate dimension. The system matrix A is given as follows:

$$A = \begin{pmatrix} -0.0558 & -0.9968 & 0.0802 & 0.0415 \\ 0.5980 & -0.1150 & -0.0318 & 0 \\ -3.0500 & 0.3880 & -0.4650 & 0 \\ 0 & 0.0805 & 1.0000 & 0 \end{pmatrix}$$

2.2 Preparatory Tasks

1. How the given model of the aircraft behaves without any control input (i.e. open-loop behavior). Plot the eigenvalues of the A matrix in the s-plane. Is the system stable in open-loop? Assuming the zero initial conditions for all

the states, plot the open-loop response. (Hint: Given the initial conditions, the open-loop response can be obtained by solving the differential equation $\dot{x} = Ax$.)

2. Determine the minimum number of control inputs the pilot needs to employ in order to control the aircraft in an arbitrary state, according to the dynamics matrix A. Is your answer in accordance with the discussions in the introduction? (hint: check the controllability of the system, and in addition check how strong the states are coupled to see how a change in one state affects the other states).

3. Suppose that, due to modeling uncertainties, the matrices A, B and C are largely unknown except for the zeros entries of the matrices i.e.

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & 0 \\ a_{31} & a_{32} & a_{33} & 0 \\ 0 & a_{42} & a_{43} & 0 \end{pmatrix}, B = \begin{pmatrix} b_{11} & 0 \\ b_{21} & b_{22} \\ b_{31} & b_{32} \\ 0 & 0 \end{pmatrix}, C = \begin{pmatrix} 0 & c_{12} & 0 & 0 \\ 0 & 0 & 0 & c_{24} \end{pmatrix}$$

Is the model structurally controllable/observable? Exist there parameter combinations such that the system becomes controllable for $b_{11} = b_{22} = 0$?

4. Suppose that the pilot desires to have the output information about only one of the states at a certain time, e.g. the roll angle ϕ. Construct the appropriate output matrix C for that specific output, and discuss its observability. Repeat the task for the other states as well, when they are the only states to be measured. What can you say about the observability in case two of the aircraft's states need to be measured together, e.g. ϕ and ω? Check the observability for every pair of the given four states.

2.3 Main Tasks

1. Suppose that a sudden side wind starts blowing while the aircraft is steadily cruising. The disturbance can be considered as an input to the control system. Let's model the input by step function. How the aircraft would response to that disturbance input without applying any control input? Discuss the behavior during the transient and steady state responses. (Hint: Plot the step response of the open loop system and check the behavior over transient and steady-state responses.)

2. Suppose that the pilot needs to change the direction of the aircraft in the horizontal plane (yaw angle), only by applying the control input to the rudder.

The input matrix is

$$B_1 = (0.00729 \ \ -0.475 \ \ 0.153 \ \ 0)^{\mathsf{T}}$$

Does the aircraft remain stable, after applying the control input to reach the desired rate of yaw angle?

3. How the control input in task 2 affects the bank angle output ϕ? (Hint: Derive the transfer function which relates the changes in rudder position to the bank angle and check the closed-loop behavior.)

4. Now assume that, the pilot is using the ailerons as control input to reach the desired bank angle. The corresponding input signal is as follows

$$B_2 = (0 \ \ 0.00775 \ \ 0.143 \ \ 0)^{\mathsf{T}}$$

Does the aircraft remain stable, after applying the control input to reach the desired bank angle? Does this control input also affect ω? How fast ω changes when the ailerons are the only control input? Plot the closed-loop responses for both outputs ϕ and ω.

5. Considering ϕ and ω as the two controlled outputs, how they are coupled to the control inputs in the tasks 2 and 4? (Hint: Compute the RGA of the aircraft's transfer function, and discuss the resulting matrix.)

6. As discussed in introduction, the pilot is able to turn the aircraft around roll and yaw axes by rudder, ailerons, or both together. To confirm this, investigate the aircraft's controllability in three different cases of control inputs: only rudder, only ailerons, both together. The input matrices are B_1 for rudder, B_2 for ailerons, and $B = [B_1, B_2]$ for applying both inputs together.

7. Now the pilot is using both rudder and ailerons to control the aircraft's orientation, (ϕ and ω). The input matrix is as already defined, $B = [B_1, B_2]$. Find an appropriate Lyapunov function to show that the aircraft can be stabilized by the aforementioned inputs and outputs.

8. Flight is coming to the end, and the pilot is going to turn the aircraft while he is also decreasing the altitude. The state matrix A during the landing is subject to some changes compared to the steady state cruising, as follows:

$$A = \begin{pmatrix} -0.0428 & -0.9863 & 0.0844 & 0.0391 \\ 0.6083 & -0.1726 & -0.0417 & -0.0097 \\ -2.6894 & 1.0542 & -0.2743 & -0.0243 \\ -0.6040 & -2.8825 & 0.3116 & -0.1760 \end{pmatrix}$$

Is the aircraft stable? Is the aircraft controllable by applying single control inputs rudder and aileron with the input matrices B_1 and B_2, respectively? Is the aircraft controllable when applying both inputs with the input matrix $B = [B_1, B_2]$?

References

[1] S. Skogestad, I. Postlethwaite, '*Multivariable Feedback Control-Analysis and Design*', 2nd Edition, John Wiley & Sons, 2001.

[2] O. H. Bosgra, H. Kwakernaak, G. Meinsma, '*Design Methods for Control Systems*', Notes for a course of the Dutch Institute of Systems and Control, 2001-2002.

[3] R. F. Stengel, '*Flight Dynamics*', Princeton University Press, November 2004, ISBN: 0-691-11407-2.