Multivariable Control Systems

Laboratory Course:

Lab 1: Aircraft Flight Control

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Outline

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  - ✓ Flight control
- Control toolbox
- Matlab® control toolbox
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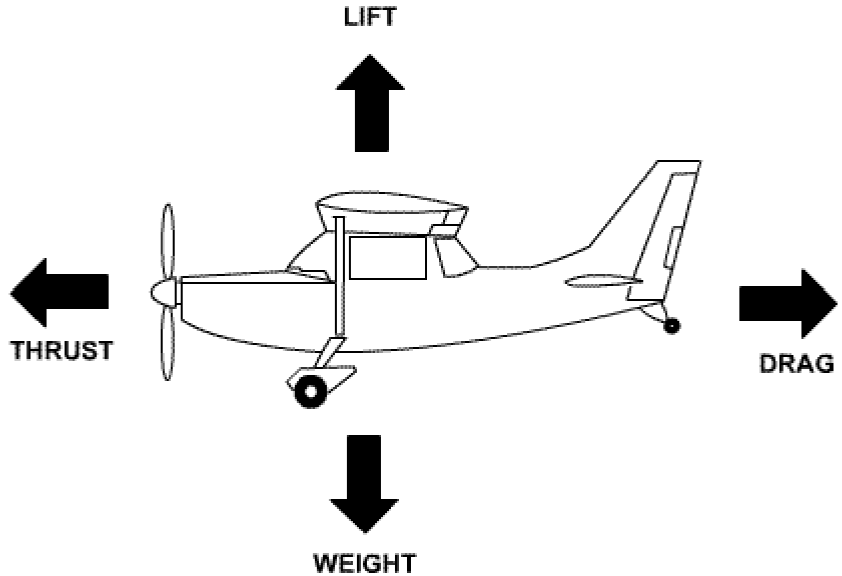
About Laboratory Course

Objectives of the course

- ✓ From theory towards application
- ✓ How to model and control real systems
- ✓ Simple tools to evaluate the behavior of real control systems
- ✓ How to use Matlab® for computer-aided control design
- ✓ Interpret the obtained data from Matlab®

Lab one: Aircraft flight control

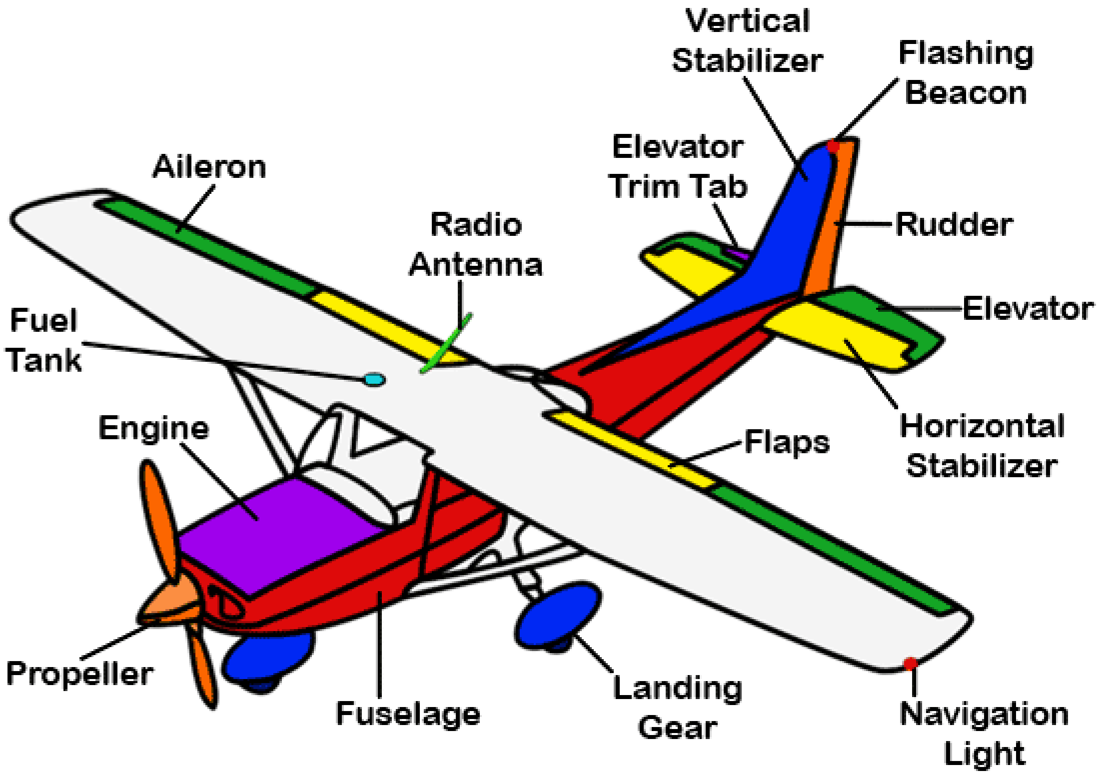
Flight dynamics



- Main forces acting on the aircraft
- Take-off/Landing highly nonlinear.
- Focus on cruising
- Shape of the wings is the main source of lift force
- Engines provides thrust
- Drag is the effect of air friction

Lab one: Aircraft flight control

Aircraft components



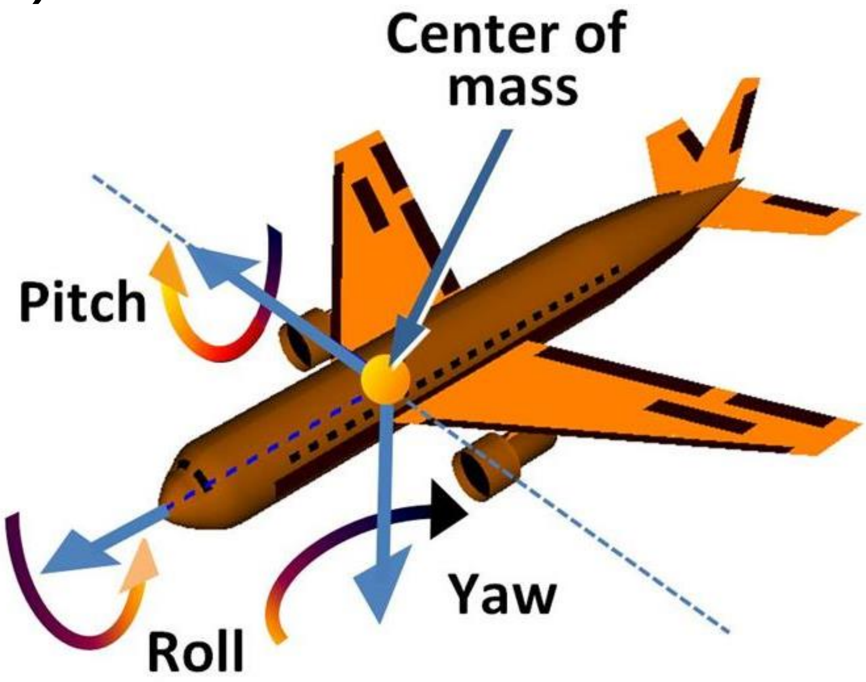
Lab one: Aircraft flight control

Flight control

Four main control systems

- Ailerons: Roll (bank angle) control
- Rudder: Yaw control
- Elevators: Pitch control
- Throttles: Speed control

➢ We are focused on the first two control systems



Control toolbox

Theoretical overview*

- Poles and zeros:

  Pole polynomial is obtained by calculating the least common denominator of all sub-determinants of $G(s)$

  Zeros are the roots of $\det G(s) = 0$, for square transfer matrices

- Singular values:

  S.V. of a real matrix A are defined as the square root of the eigenvalues of $A^{\mathrm{T}}A$. For a linear stable MIMO system we have

$$\underline{\sigma}(G(j\omega)) \leq \frac{|Y(j\omega)|}{|U(j\omega)|} \leq \overline{\sigma}(G(j\omega))$$

  Where, $\underline{\sigma}(G(j\omega))$ and $\overline{\sigma}(G(j\omega))$ are the smallest and largest singular values of $G(j\omega)$, and $\frac{|Y(j\omega)|}{|U(j\omega)|}$ is the gain of the system.

*RS2 script, section 2.1.5, subsections: 2.1.5.1-2.1.5.3

Control toolbox

Theoretical overview

- I/O pairings and RGA:

  A measure of how strong the pairings are in a MIMO system is given by the RGA of the transfer matrix $G(s)$

$$\mathrm{RGA}\big(G(j\omega)\big) \coloneqq G(j\omega).* \left[G^{-1}(j\omega)\right]^{\mathrm{T}}$$

  This gives a measure of how the inputs affect the outputs in a certain frequency. A zero in the RGA matrix shows that the corresponding I/O are decoupled. The bigger the RGA element is, the stronger the coupling will be.

  - RGA analysis is a fundamental tool for decentralized control design.

Control toolbox

Theoretical overview

- Observability/Controllability:

  In the state space representation of a system

$$\dot{x} = \boldsymbol{A}x + \boldsymbol{B}u$$
$$y = \boldsymbol{C}x + \boldsymbol{D}w$$

➢ The system (A, B) is *Controllable* iff for every eigenvalue λ_i of A, $\operatorname{rank}\,(\lambda_i\boldsymbol{I} - \boldsymbol{A}, \boldsymbol{B}) = n$

➢ The system (A, C) is *Observable* iff for every eigenvalue λ_i of A, $\operatorname{rank}\begin{pmatrix}\lambda_i\boldsymbol{I} - \boldsymbol{A} \\ \boldsymbol{C}\end{pmatrix} = n$

➢ The pair (A, B) is *Stabilizable* iff all non-controllable λ_i's are asymptotically stable

➢ The pair (A, C) is *Detectable* iff all unstable λ_i's are observable

Matlab® control toolbox

Getting started with Matlab®

- Type `help` <*function name*> to get help from Matlab®

<u>Useful functions and commands</u>:

1. Creation and conversion of LTI models:
   - ✓ `ss(A,B,C,D)`: creates a state space model
   - ✓ `tf(num,denum)`: creates a transfer function model
   - ✓ `minreal(sys)`: cancels the pole/zero pairs
   - ✓ `ssdata(sys)`: extracts data from a LTI model
   - ✓ `c2d(sys,`τ_s`,meth)/d2c(sys,meth)`: converts continuous model to discrete, and vice versa

Matlab® control toolbox

Useful functions and commands:

2. Poles, zeros, singular values, eigenvalues:

- ✓ `pole(sys)`: returns the poles of the model
- ✓ `zero(sys)`: returns the zeros of the model
- ✓ `pzmap(sys)`: plots the pole/zero map of the model
- ✓ `S=svd(A)`: returns the vector S with the elements as the singular values of A
- ✓ `sigma(sys)`: returns a singular value plot of the frequency response
- ✓ `[V,D]=eig(A)`: returns the diagonal matrix D with the eigenvalues, and a full matrix V of eigenvectors

Matlab® control toolbox

Useful functions and commands:

3. Time/frequency responses:

- ✓ `step(sys)`: plots the step response of the model
- ✓ `impulse(sys)`: plots the impulse response
- ✓ `lsim(sys,U,T)`: plots the time domain response of the system w.r.t. the input signal `U` and time vector `T`
- ✓ `bode(sys)`: plots the model's frequency response
- ✓ `margin(sys)`: returns the gain and phase margins
- ✓ `nyquist(sys)`: plots the Nyquist frequency response

Matlab® control toolbox

Useful functions and commands:

4. Controllability/Observability:

- ✓ `ctrb(sys)/ctrb(A,B)`: returns the controllability matrix, either by providing A and B matrices, or by having a state space model
- ✓ `obsv(sys)/obsv(A,C)`: returns the observability matrix, either by providing A and C matrices, or by having a state space model
- ✓ `rank(A)`: returns the rank of the A matrix by computing the number of linearly independent rows or columns of A

References

Robert F. Stengel
Flight Dynamics.
In: *Princeton University Press, November 2004, ISBN: 0-691-11407-2.*

Malcolm J. Abzug, Eugene Larrabee
Airplane Stability and Control.
In: *Cambridge University Press, 2002, ISBN: 0-521-80992-4.*

Sigurd Skogestad, Ian Postlethwaite
Multivariable Feedback Control, Analysis and Design.
In: *John Wiley & Sons, 2nd Edition, August 2001.*

Daniel P. Raymer
Aircraft Design: A conceptual Approach.
In: American Institute of Aeronautics and Astronautics, *4th Edition, 2002.*