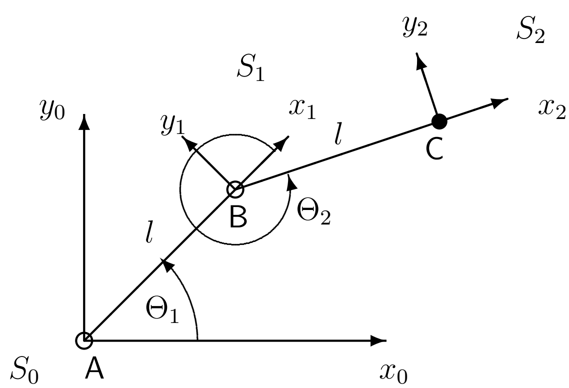
Lehrstuhl für Steuerungs- und Regelungstechnik / Lehrstuhl für Informationstechnische Regelung	Einführung in die Roboterregelung (ERR)
Technische Universität München	Kurzlösung zur 4. Übung

Aufgabe 1:

1. Aufgabe

1.1



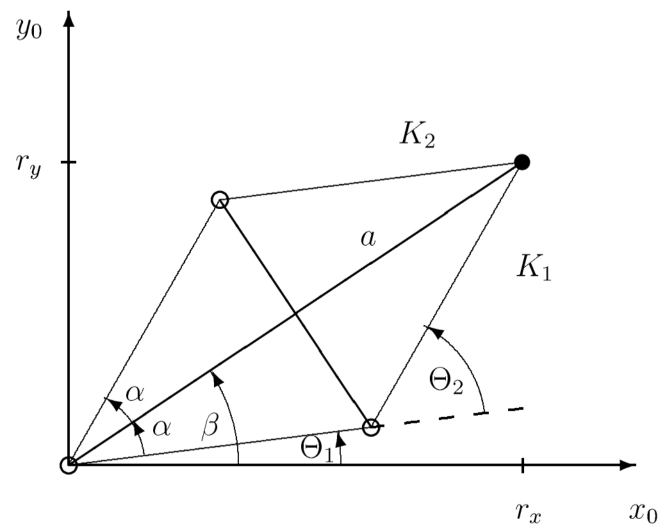
1.2

	d_i	a_i	α_i
$i=1$	0	l	0
$i=2$	0	l	0

$$A_i = \left[\begin{array}{ccc|c} c\Theta_i & -s\Theta_i & 0 & lc\Theta_i \\ s\Theta_i & c\Theta_i & 0 & ls\Theta_i \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

$$\begin{aligned} {}^0T_2 &= A_1 \cdot A_2 \\ &= \left[\begin{array}{ccc|c} c(\Theta_1+\Theta_2) & -s(\Theta_1+\Theta_2) & 0 & lc\Theta_1 + lc(\Theta_1+\Theta_2) \\ s(\Theta_1+\Theta_2) & c(\Theta_1+\Theta_2) & 0 & ls\Theta_1 + ls(\Theta_1+\Theta_2) \\ 0 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 1 \end{array}\right] \end{aligned}$$

1.3



$$
\begin{aligned}
K_1: \quad \Theta_1 &= \beta - \alpha & \qquad K_2: \quad \Theta_1 &= \beta + \alpha \\
\Theta_2 &= 2\alpha & \Theta_2 &= -2\alpha
\end{aligned}
$$

$$a = \sqrt{r_x^2 + r_y^2}$$

$$\alpha = \arccos \frac{a}{2l} \qquad , \qquad 0 \leq \alpha \leq \frac{\pi}{2}$$

$$\beta = \operatorname{atan2}(r_y, r_x)$$

$$
\begin{aligned}
a = \sqrt{r_x^2 + r_y^2} > 2l \quad &: \quad \text{keine Lösung} \\
a = 2l \quad &: \quad \text{eine Lösung} \\
0 < a < 2l \quad &: \quad \text{zwei Lösungen } (K_1, K_2) \\
a = 0 \quad &: \quad \text{unendlich viele Lösungen (Arm gefaltet)} \\
& \quad\;\; \Theta_1 \text{ beliebig} \\
& \quad\;\; \Theta_2 \; = \pi
\end{aligned}
$$

1.4

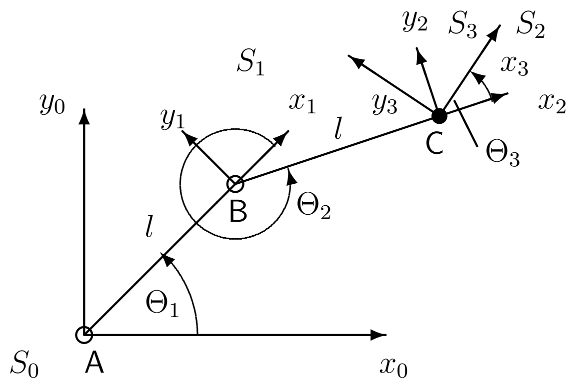
$$J = \begin{bmatrix} \dfrac{\partial r_x}{\partial \Theta_1} & \dfrac{\partial r_x}{\partial \Theta_2} \\ \dfrac{\partial r_y}{\partial \Theta_1} & \dfrac{\partial r_y}{\partial \Theta_2} \end{bmatrix} = \begin{bmatrix} -ls\Theta_1 - ls(\Theta_1 + \Theta_2) & -ls(\Theta_1 + \Theta_2) \\ lc\Theta_1 + lc(\Theta_1 + \Theta_2) & lc(\Theta_1 + \Theta_2) \end{bmatrix}$$

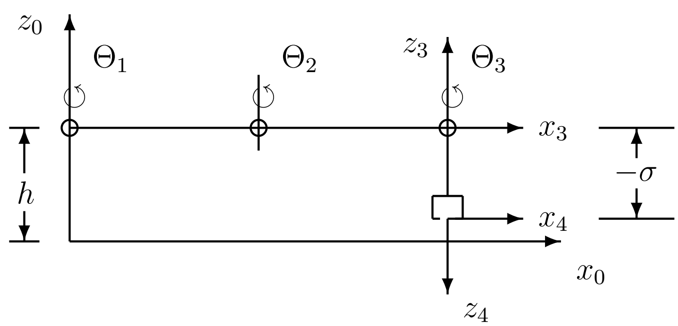
$$\det J = l^2 s\Theta_2$$

Jacobi-Matrix wird singulär für $\Theta_2 = 0$ (Arm gestreckt) und $\Theta_2 = \pi$ (Arm gefaltet).

Aufgabe 2:

2.1





2.2

	Θ_1	d_i	a_i	α_i
$i=1$	Θ_1	h	l	0
$i=2$	Θ_2	0	l	0
$i=3$	Θ_3	0	0	0
$i=4$	0	σ	0	π

$$
\begin{aligned}
{}^0T_4 &= A_1' \cdot A_2 \cdot A_3 \cdot A_4 \\
&= \left[\begin{array}{ccc|c}
c(\Theta_1+\Theta_2) & -s(\Theta_1+\Theta_2) & 0 & lc\Theta_1 + lc(\Theta_1+\Theta_2) \\
s(\Theta_1+\Theta_2) & c(\Theta_1+\Theta_2) & 0 & ls\Theta_1 + ls(\Theta_1+\Theta_2) \\
0 & 0 & 1 & h \\ \hline
0 & 0 & 0 & 1
\end{array}\right] \\
&\cdot \left[\begin{array}{ccc|c}
c\Theta_3 & -s\Theta_3 & 0 & 0 \\
s\Theta_3 & c\Theta_3 & 0 & 0 \\
0 & 0 & 1 & 0 \\ \hline
0 & 0 & 0 & 1
\end{array}\right] \cdot \left[\begin{array}{ccc|c}
1 & 0 & 0 & 0 \\
0 & -1 & 0 & 0 \\
0 & 0 & -1 & \sigma \\ \hline
0 & 0 & 0 & 1
\end{array}\right] \\
&= \left[\begin{array}{ccc|c}
c(\Theta_1+\Theta_2+\Theta_3) & s(\Theta_1+\Theta_2+\Theta_3) & 0 & lc\Theta_1 + lc(\Theta_1+\Theta_2) \\
s(\Theta_1+\Theta_2+\Theta_3) & -c(\Theta_1+\Theta_2+\Theta_3) & 0 & ls\Theta_1 + ls(\Theta_1+\Theta_2) \\
0 & 0 & -1 & h+\sigma \\ \hline
0 & 0 & 0 & 1
\end{array}\right]
\end{aligned}
$$

2.3 $$\left.\begin{array}{lcl}\Theta_1 &=& f_1(r_x, r_y)\\ \Theta_2 &=& f_2(r_x, r_y)\end{array}\right\}\text{ siehe 1.3}$$

$$\begin{array}{lcl}\Theta_3 &=& \operatorname{atan2}(n_y, n_x) - \Theta_1 - \Theta_2\\ \sigma &=& r_z - h\end{array}$$

Aufgabe 3:

$$\underline{w} = [r_x,\, r_z]^T, \quad \underline{q} = [x_F,\, \Theta_1,\, \Theta_2]^T$$

3.1

$$^0T_3(\underline{q}) = {}^0T_1(x_F) \cdot {}^1T_3(\Theta_1,\, \Theta_2)$$

$$= \left[\begin{array}{ccc|c} c(\Theta_1+\Theta_2) & -s(\Theta_1+\Theta_2) & 0 & lc\Theta_1 + lc(\Theta_1+\Theta_2) + x_F \\ 0 & 0 & -1 & 0 \\ s(\Theta_1+\Theta_2) & c(\Theta_1+\Theta_2) & 0 & ls\Theta_1 + ls(\Theta_1+\Theta_2) \\ \hline 0 & 0 & 0 & 1 \end{array}\right]$$

3.2

$$J(\underline{q}) = \begin{bmatrix} 1 & -ls\Theta_1 - ls(\Theta_1+\Theta_2) & -ls(\Theta_1+\Theta_2) \\ 0 & lc\Theta_1 + lc(\Theta_1+\Theta_2) & lc(\Theta_1+\Theta_2) \end{bmatrix}$$

$J(\underline{q})$: liegende Matrix $\longrightarrow$ Lösungsvielfalt

3.3 $J^+ = J^T \cdot (J \cdot J^T)^{-1}$

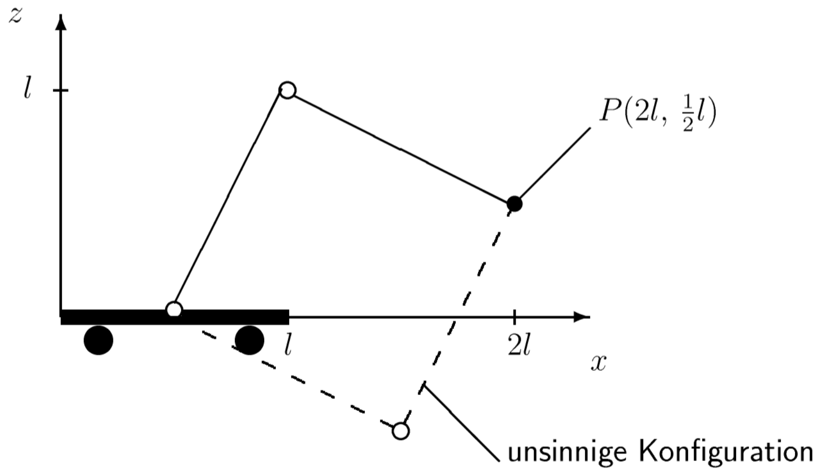
$$J^+ = \frac{1}{3}\begin{bmatrix} 2 & 1 \\ -1 & 1 \\ 1 & 2 \end{bmatrix}$$

3.4 $\mu(\underline{q}) = \sqrt{\det[J(\underline{q}) \cdot J^T(\underline{q})]}$

Hier gilt für den Manipulator $\mu(\underline{q}) = |\det J|$.

Mit 1.4. folgt $\mu(\underline{q}) = |l^2 s\Theta_2| \longrightarrow \max$ für $\Theta_2 = \pm 90^0$.

$$\Theta_2 = -90^0 \text{ (aus Skizze)} \qquad \Theta_1 \approx 66^0 \qquad x_F \approx 0{,}7 \cdot l$$



3.5 $\underline{w} = f(\underline{q})$, mit $\underline{w} = \begin{bmatrix} r_x \\ r_z \end{bmatrix}$

$$\underline{0} = g(\underline{q}) = \Theta_1 + \Theta_2 - 90^0$$

$$J^* = \begin{bmatrix} 1 & -ls\Theta_1 - ls(\Theta_1 + \Theta_2) & -ls(\Theta_1 + \Theta_2) \\ 0 & lc\Theta_1 + lc(\Theta_1 + \Theta_2) & lc(\Theta_1 + \Theta_2) \\ 0 & 1 & 1 \end{bmatrix}$$