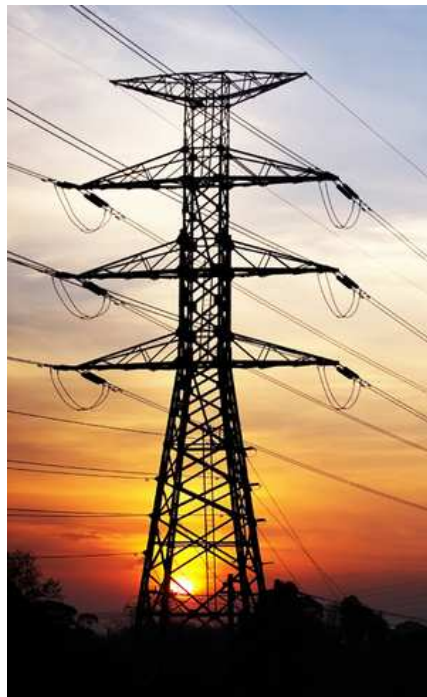


# CONTROL SYSTEMS 2: MATLAB LABORATORY COURSE

## LAB TWO: ELECTRIC POWER NETWORK CONTROL



FACULTY OF  
ELECTRICAL ENGINEERING AND INFORMATION TECHNOLOGY

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## General Comment

The computer-aided laboratory course is designed in three parts as a complement to the Control Systems 2 lectures. Within each part of the laboratory, we try to make a bridge between the principles of control theory provided in the lectures, and its engineering application by studying real models in different domains. It is expected that after completing this laboratory course, the students be able to analyze the linear models' behavior of real plants, in both time and frequency domains, and also design MIMO control laws to meet some specified control objectives.

The present manuscript, is the second part of the computer-aided laboratory, and considers the concept of control design for multi-input multi-output (MIMO) control systems. Feed-forward control design (pre-filtering), modal synthesis, decoupled and decentralized control design are the main topics to be considered in this laboratory. A real model of an electric power network is utilized to apply the design methods.



# Chapter 1

## Introduction

Electric power networks are among those most important and developed areas of engineering. Electric power networks are MIMO systems which are geographically distributed and are considered as large-scale systems. Electric power networks are expected to evermore grow in size and complexity in the future due to the fact that every household is becoming potentially also a producer of electric energy for example by means of solar and wind. Figure 1.1 shows the dispersion of the synchronous transmission power networks across Europe.

Imagine only one hour of your life without electricity to understand how important is the power networks' control, its performance and reliability. In this chapter we present a short introduction about electric power networks and specifically about voltage control in distributed transformers. In the references, more detailed information is provided where interested readers are referred to. The overall objective in an electric power system is to maintain the balance between the electric power produced by the generators and the power consumed by the loads, including the network losses, at all time instants. If this balance is not maintained, it will lead to frequency deviations. If the deviations are too large, then it will have serious impacts on the system operation; and even causes blackouts.

In an electric power network, the delivered electricity must regard certain quality criteria, such as; **voltage magnitude**, **frequency**, and **wave shape**. Voltage control in power networks considers the regulation of voltage at the infeed nodes of the power plants to keep track of predetermined values. The desired values of voltage depends on many factors such as the capacity of the individual power plants, transmission lines, consumption rate, and price of the electricity. Usually, in a power network, the power plants are located far from each other and are potentially operated by different companies. Therefore a centralized control approach is typically not feasible, and traditionally decentralized control mechanisms are employed.

From the control perspective, the power networks are coupled multi-input multi-output control systems, in which one specific output is affected by more than one input. In terms of control performance, centralized control laws are the most efficient methods to control such a coupled system, however, as we already discussed,

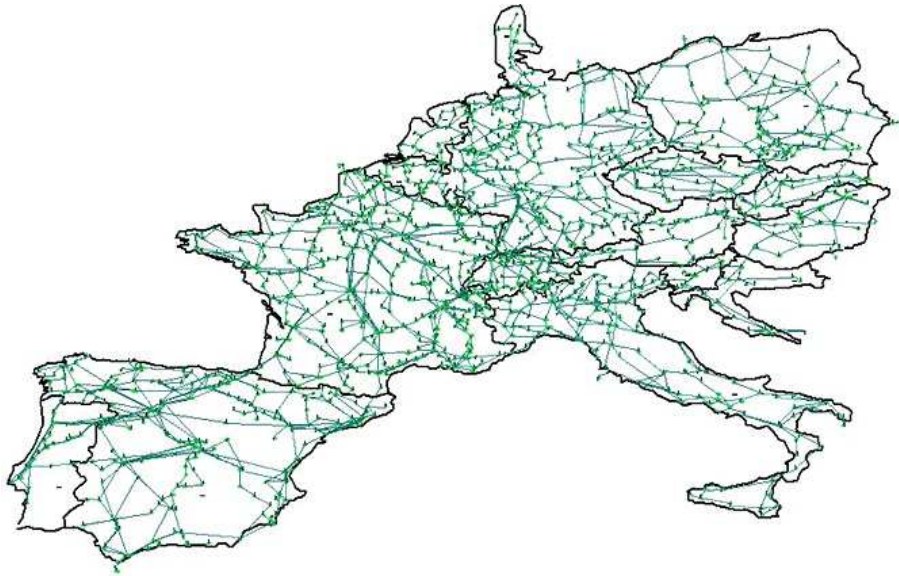


Figure 1.1: Europe transmission grid (*Source: UCTE, internet*)

the distance between power plants, their different operating companies, and also the scale of the power networks make it infeasible to employ centralized approaches. Decentralized control is a suitable approach to control such distributed and large-scale networks. In decentralized voltage control, we aim to design local controls for each individual power plant such that the terminal voltage be as close as possible to the predetermined set values. Modeling the power networks as graphs, each individual plant's state vector is considered as a node and the edges represent the couplings between the nodes. The aim is to design decentralized control for each power plant to regulate the voltage output, which are measured by sensors placed at specified nodes.

# Chapter 2

## Exercises

In this section, the linearized model of an electric power network is provided, and afterwards the tasks are followed. We will investigate how power networks can be controlled by the rules of decentralized MIMO control synthesis.

### 2.1 Linearized Model of an Electric Power Network

We consider the following model describing the linearized state space presentation of an electric power network which includes three different power plants  $\mathcal{P}_i$  each with three states, and the plants are located far from each other. The states of each individual power plant are the voltages of the infeed nodes. These voltages are influenced by some factors such as; terminal voltages of synchronous machines, impedances of transmission lines, and turn ratio of transformers. However, the voltages at infeed nodes must not fluctuate dramatically and must track the set values, therefore voltage control is inevitable. The main problem is that the factors which affect the infeed voltages at every power plant are not local. Within each power plant  $\mathcal{P}_i$ , for  $i \in \{1, 2, 3\}$ , the local dynamics of the state vector  $x_i \in \mathbb{R}^{3 \times 1}$ , which are voltages of infeed nodes, can be stated as follows:

$$\dot{x}_i = A_i x_i + \sum_{j \neq i} A_{ij} x_j + B_i u_i \quad (2.1)$$

$$y_i = C_i x_i \quad (2.2)$$

where,  $A_i \in \mathbb{R}^{3 \times 3}$  is the local dynamic matrix, and  $A_{ij} \in \mathbb{R}^{3 \times 3}$  represents the couplings between the states of different power plants. As it can be seen from eq. (2.1), we aim to have a local voltage regulator  $u_i \in \mathbb{R}$  for each plant, which determines the set values of the terminal voltages, depending on the plant's operation status. There are also voltage sensors connected to certain nodes to measure the output voltage  $y_i \in \mathbb{R}$  produced by the infeed nodes.

The aggregate state vector  $x = (x_1 \ x_2 \ x_3)^\top$  has the following dynamics, which  $x_i$  represents the state vector of each power plant, for  $i \in \{1, 2, 3\}$ :

$$\dot{x} = Ax + Bu \quad (2.3)$$

$$y = Cx \quad (2.4)$$

where,  $x \in \mathbb{R}^{9 \times 1}$  is the aggregate state vector including the node voltages of the power network, with the given initial value  $x(0) = x_0$ ,  $y \in \mathbb{R}^{3 \times 1}$  represents the node voltage measurements by the sensors, and  $u \in \mathbb{R}^{3 \times 1}$  is the vector of control input, which is accompanied by the input matrix  $B$  of appropriate dimension. Unless otherwise stated, the matrix  $A$  is:

$$A = \begin{pmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{pmatrix} = \quad (2.5)$$

$$\begin{pmatrix} -2.445 & -0.16 & 0 & -0.162 & 0 & 0 & -0.129 & 0 & 0 \\ 2.557 & 0 & 0 & -0.007 & 0 & 0 & -0.006 & 0 & 0 \\ 3.06 & 0 & -2 & 0.984 & 0 & 0 & 0.785 & 0 & 0 \\ -0.162 & 0 & 0 & -2.546 & -0.16 & 0 & -0.111 & 0 & 0 \\ -0.007 & 0 & 0 & 2.552 & 0 & 0 & -0.005 & 0 & 0 \\ 0.984 & 0 & 0 & 3.672 & 0 & -2 & 0.673 & 0 & 0 \\ -0.129 & 0 & 0 & -0.111 & 0 & 0 & -2.289 & -0.16 & 0 \\ -0.006 & 0 & 0 & -0.005 & 0 & 0 & 2.564 & 0 & 0 \\ 0.785 & 0 & 0 & 0.673 & 0 & 0 & 2.116 & 0 & -2 \end{pmatrix}$$



and the matrices,  $B$ , and  $C$  are:

$$B = \begin{pmatrix} B_1 & \{0\}_{3 \times 1} & \{0\}_{3 \times 1} \\ \{0\}_{3 \times 1} & B_2 & \{0\}_{3 \times 1} \\ \{0\}_{3 \times 1} & \{0\}_{3 \times 1} & B_3 \end{pmatrix} = \begin{pmatrix} 0.9 & 0 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0.9 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0.9 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix} \quad (2.6)$$

$$C = \begin{pmatrix} C_1 & \{0\}_{1 \times 3} & \{0\}_{1 \times 3} \\ \{0\}_{1 \times 3} & C_2 & \{0\}_{1 \times 3} \\ \{0\}_{1 \times 3} & \{0\}_{1 \times 3} & C_3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (2.7)$$

As it can be seen from matrix  $A$ , the plants are coupled through the off-diagonal blocks  $A_{ij}, i \neq j$ . Matrix  $B$  shows that the control input affects only two out of the three states of each power plant, as the third, sixth, and ninth rows are zero. The sensors are also measuring the voltages of those uncontrolled nodes. The overall objective is to design decentralized control for the power plants such that the node voltages reach their desired values as fast as possible and without overshoot.

## 2.2 Tasks

### 2.2.1 Pre-filter design

Suppose that the node voltages of the different power plants are controlled with the following control laws

$$u_i(t) = -K_i x_i(t) + L_i r_i(t), \quad i \in \{1, 2, 3\} \quad (2.8)$$

where  $K_i$  is the state feedback gain,  $L_i$  is the pre-filter gain, and  $r_i$  is the desired voltage trajectory to be tracked. The desired closed loop system should have the following eigenvalues:

$$\lambda_K = (-3 \quad -4.5 \quad -2 \quad -3 \quad -2.7 \quad -1.1 \quad -1 \quad -0.2 \quad -0.8)^\top$$

1. Assume for now that there exists a control law resulting in the desired eigenvalues of the closed loop system. Does a pre-filter  $L_i$  for perfect stationary tracking exist? If yes, provide the pre-filter design. (Hint: In order to design the pre-filter, choose the appropriate coefficients for the characteristic polynomial  $\det(sI - A + BK)$ .)

Plot the stationary step response of the controlled power network to see how your pre-filter design makes the output track the desired trajectory.

2. Now suppose that due to some limitations on the capacity of transmission lines, the feedback gain  $K$  is constrained in some of its elements as follows:

$$K = \begin{pmatrix} k_{11} & 0 & 0.5 & 0 & k_{15} & -2 & k_{17} & 0 & k_{19} \\ 1 & k_{22} & 1.5 & k_{24} & -1 & k_{26} & 1.7 & k_{28} & -1 \\ k_{31} & 0 & k_{33} & k_{34} & 0 & k_{36} & 0 & k_{38} & k_{39} \end{pmatrix}$$

and  $k_{11} > -2.5$ , and  $k_{36} > -0.5$ . Consider the same desired closed-loop behavior. Does a pre-filter  $L_i$  for perfect stationary tracking exist considering the constraints on feedback gain  $K$ ? If yes, provide the pre-filter design.

## 2.2.2 Disturbance Decoupling Control

Due to a sudden change in the load on the third node of the first power plant, the LTI MIMO model defined in (2.1)-(2.2) is perturbed. The mentioned load causes changes in the states dynamics, which we model it as follows:

$$\dot{x} = Ax(t) + Bu(t) + Nd(t) \quad (2.9)$$

$$y(t) = Cx(t) \quad (2.10)$$

where,  $N = (0 \ 0 \ -1.2 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)^\top$  is the disturbance matrix and  $d(t) \in \mathbb{R}$  is the disturbance input.

1. Design a state feedback control  $u = -Kx(t)$  for the power network such that for all times  $t > 0$ , the change in the load of the first power plant ( $d(t)$ ) is not observable for the sensors which measure the voltage of the nodes. (Hint: The invariant zero is approximately equal to one of the system eigenvalues, and in this task they can be considered as equal, for the sake of simplicity.)
2. Check if the unknown load is observable in the sensor measurements, after employing the designed control law.

### 2.2.3 Dynamic Decoupling Control

If a sub-system is dynamically decoupled from the other subsystems, then changes in the set-point of one plant leads to a response solely in that plant and all other modes remain constant. As a result of dynamic decoupling, the MIMO system can be analyzed as a set of decoupled SISO systems. In this task, we would like to design a dynamic decoupling control scheme for the described power network.

Assume the  $A$ ,  $B$ , and  $C$  matrices as in (2.3)-(2.4).

1. Check the existence of a stable decoupling control for the given power network, with the given  $A$ ,  $B$ , and  $C$  matrices.
2. So far, the states 3, 6, and 9 were measured by the voltage sensors, as in eq. (2.7). Now, a new set of states are measured which are supposed to be three different states than 3, 6, and 9, such that the relative grades  $\delta_i = 1$  for each power plant  $i \in \{1, 2, 3\}$ . Calculate the decoupling control with the appropriate choice of  $C$  matrix such that the following relations hold between the individual voltage outputs and input control: (Hint: It is possible that more than one set of appropriate state measurements exist!)

$$\frac{y_1(s)}{w_i(s)} = \frac{10}{s + 10}$$

$$\frac{y_2(s)}{w_j(s)} = \frac{5}{s + 7}$$

$$\frac{y_3(s)}{w_k(s)} = \frac{30}{s + 50}$$

where,  $i, j, k$  are either 1, 2, or 3, depending on the sensor placement ( $C$  matrix).

### 2.2.4 Decentralized Control

In this task, the decentralized control design of the electric power network is considered. By decentralization, we aim to control the power plants locally, by considering the strongest coupling between each pair of input-output. Assuming the original model of the power network:

1. We aim to design a decentralized PI control of the following form for each individual power plant:

$$U_i^{\text{PI}}(s) = g_i \left( 1 + \frac{1}{sT_i} \right), \quad i \in \{1, 2, 3\} \quad (2.11)$$

where,  $g_i$  is the proportional gain, and  $T_i$  is the time constant (integration time) of the PI control law. Due to the change of the season from autumn to winter, the electricity consumption has grown, which demands more electricity generation. This leads us to change the operation point of the power plant to have the cross-over frequency  $\omega_c = 1$  rad/s, and the desired phase margin  $\frac{\pi}{3}$ . Design the decentralized PI control as given in eq. (2.11) by pairing the appropriate inputs and outputs. Plot the Bode diagram of the loop gain to see if the decentralized control fulfills the margin requirements. (Hint: RGA analysis helps to find the best I/O pairing)

## References

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