Ergänzung zu Kapitel V 5.2

ERR

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Ermittlung der Translationskomponente für die geometrische Jakobimatrix

$$^0\dot{r}_E =^0 \dot{r}_{n-1} +^0 \dot{r}_{n-1,n} +^0 \omega_{n-1} \times^0 r_{n-1,n}$$

Sukzessives Einsetzen von (*) und (**)

$$
\begin{aligned}
^0\dot{r}_E =& ^0\dot{r}_{n-1} +^0 \dot{r}_{n-1,n} +^0 \omega_{n-1} \times^0 r_{n-1,n} \\
&\qquad \rightarrow^0 \dot{r}_{n-1} =^0 \dot{r}_{n-2} +^0 \omega_{n-2} \times^0 r_{n-2,n-1} +^0 \dot{r}_{n-2,n-1} \\
&\qquad \rightarrow^0 \omega_{n-1} =^0 \omega_{n-2} +^0 \omega_{n-2,n-1} \\
=& ^0\dot{r}_{n-2} +^0 r_{n-2,n-1} +^0 \omega_{n-2} \times^0 r_{n-2,n-1} +^0 \dot{r}_{n-1,n} + \left(^0\omega_{n-2} -^0 \omega_{n-2,n-1}\right) \times^0 r_{n-1,n} \\
&\qquad \rightarrow^0 \dot{r}_{n-2} =^0 \dot{r}_{n-3} +^0 \omega_{n-3} \times^0 r_{n-3,n-2} +^0 \dot{r}_{n-3,n-2} \\
&\qquad \rightarrow^0 \omega_{n-2} =^0 \omega_{n-3} +^0 \omega_{n-3,n-2} \\
=& ^0\dot{r}_{n-3} +^0 \dot{r}_{n-3,n-2} +^0 \omega_{n-3} \times^0 r_{n-3,n-2} +^0 \dot{r}_{n-2,n-1} + \left(^0\omega_{n-3} +^0 \omega_{n-3,n-2}\right) \times^0 r_{n-2,n-1} +^0 \dot{r}_{n-1,n} +^0 \omega_{n-2} + \\
& + \left(^0\omega_{n-3,n-2} +^0 \omega_{n-2,n-1}\right) \times^0 r_{n-1,n} \\
=& \sum_{n=1}^{N} \left[^0\dot{r}_{n-1,n}\right] +^0 \omega_{n-2,n-1} \times^0 r_{n-1,n} +^0 \omega_{n-3,n-2} \times \left(^0 r_{n-2,n-1} +^0 r_{n-1,n}\right) + \dots \\
=& \sum_{n=1}^{N} \left[^0\dot{r}_{n-1,n} +^0 \omega_{n-2,n-1} \times \left(\sum_{i=1}^{N} {}^0 r_{i-1,i}\right)\right] \\
=& \sum_{n=1}^{N} \left[^0\dot{r}_{n-1,n} +^0 \omega_{n-2,n-1} \times^0 r_{n-1,n}\right]
\end{aligned}
$$

Anwendung für rotatorisches Gelenk

$$
\begin{aligned}
^0\dot{r}_E =& \sum_{n=1}^{N} {}^0\omega_{n-1,n} \times^0 r_{n-1,n} +^0 \omega_{n-2,n-1} \times^0 r_{n-1,n} \\
=& \sum_{n=1}^{N} {}^0\omega_{n-1,n} \times^0 r_{n-1,n} \\
=& ^0\omega_{n-1,n} \times^0 r_{n-1,n} +^0 \omega_{n-2,n-1} \times^0 r_{n-1,n} +^0 \omega_{n-2,n-1} \times^0 r_{n-2,n-1} +^0 \omega_{n-3,n-2} \times^0 r_{n-2,n} + \\
& +^0 \omega_{n-3,n-2} \times^0 r_{n-3,n-2} +^0 \omega_{n-4,n-3} \times^0 r_{n-3,n}
\end{aligned}
$$

$$^0\dot{r}_E =^0\dot{r}_{n-1} +^0 \dot{r}_{n-1,n} +^0 \omega_{n-1} \times^0 r_{n-1,n}$$

Einsetzen für rotatorisches Gelenk:

$$
\begin{aligned}
{}^0\dot{r}_E &= {}^0\dot{r}_{n-1} + {}^0\omega_{n-1,n} \times {}^0 r_{n-1,n} + {}^0\omega_{n-1} \times {}^0 r_{n-1,n} \\
&= {}^0\dot{r}_{n-1} + {}^0\omega_n \times {}^0 r_{n-1,n} \\
&\qquad \rightarrow {}^0\dot{r}_{n-1} = {}^0\dot{r}_{n-2} + {}^0\omega_{n-1} \times {}^0 r_{n-2,n-1} \\
&\qquad \rightarrow {}^0\omega_n = {}^0\omega_{n-1} + {}^0\omega_{n-1,n} \\
&= {}^0\dot{r}_{n-2} + {}^0\omega_{n-1} \times {}^0 r_{n-2,n-1} + \left({}^0\omega_{n-1} + {}^0\omega_{n-1,n}\right) \times {}^0 r_{n-1,n} \\
&\qquad \rightarrow {}^0\dot{r}_{n-2} = {}^0\dot{r}_{n-3} + {}^0\omega_{n-2} \times {}^0 r_{n-3,n-2} \\
&\qquad \rightarrow {}^0\omega_{n-1} = {}^0\omega_{n-2} + {}^0\omega_{n-2,n-1} \\
&= {}^0\dot{r}_{n-3} + {}^0\omega_{n-2} \times {}^0 r_{n-3,n-2} + \left({}^0\omega_{n-2} + {}^0\omega_{n-2,n-1}\right) \times {}^0 r_{n-2,n-1} + \left({}^0\omega_{n-2} + {}^0\omega_{n-2,n-1} + {}^0\omega_{n-1,n}\right) \times {}^0 r_{n-1,n} \\
&= {}^0\omega_{n-1,n} \times {}^0 r_{n-1,n} + {}^0\omega_{n-2,n-1} \times \left({}^0 r_{n-1,n} + {}^0 r_{n-2,n-1}\right) + \dots \\
&= \sum_{n=1}^{N} {}^0\omega_{n-1,n} \times \left(\sum_{i=n}^{N} {}^0 r_{i-1,i}\right) \\
&= \sum_{n=1}^{N} {}^0\omega_{n-1,n} \times {}^0 r_{n-1,n}
\end{aligned}
$$