

$$\begin{aligned}
 (3) \int \frac{\sqrt{1+2\arctan x}}{1+x^2} dx &= \int \sqrt{1+2\arctan x} d(\arctan x) \\
 &= \int \sqrt{1+2u} du (u = \arctan x) \\
 &= \frac{1}{2} \int \sqrt{1+2u} d(1+2u) = \frac{1}{3} (1+2u)^{\frac{3}{2}} + C \\
 &= \frac{1}{3} (1+2\arctan x)^{\frac{3}{2}} + C.
 \end{aligned}$$

$$\begin{aligned}
 (4) \quad &\text{令 } x = a \sin t \left(-\frac{\pi}{2} \leq t \leq \frac{\pi}{2} \right), dx = a \cos t dt \\
 \int \sqrt{a^2 - x^2} dx &= \int \sqrt{a^2(1 - \sin^2 t)} \cdot a \cos t dt = a^2 \int \cos^2 t dt \\
 &= a^2 \int \frac{1 + \cos 2t}{2} dt = \frac{a^2}{2} \left(t + \frac{\sin 2t}{2} \right) + C \\
 &= \frac{a^2}{2} \arcsin \frac{x}{a} + \frac{x}{2} \sqrt{a^2 - x^2} + C.
 \end{aligned}$$