$$\arcsin[\sin x]$$
$$= \arcsin[-\sin(x-\pi)]$$
$$= -(x-\pi)$$

$$(3)\int \frac{\sqrt{1+2\arctan x}}{1+x^2}\mathrm{d}x = \int \sqrt{1+2\arctan x}\,\mathrm{d}(\arctan x)$$

$$= \int \sqrt{1+2u}\,\mathrm{d}u\,(u = \arctan x)$$

$$= \frac{1}{2}\int \sqrt{1+2u}\,\mathrm{d}(1+2u) = \frac{1}{3}(1+2u)^{\frac{3}{2}} + C$$

$$= \frac{1}{3}(1+2\arctan x)^{\frac{3}{2}} + C.$$

(4) 令 $x = a\sin t\left(-\frac{\pi}{2} \leqslant t \leqslant \frac{\pi}{2}\right)$，$\mathrm{d}x = a\cos t\mathrm{d}t$

$$\int \sqrt{a^2 - x^2}\,\mathrm{d}x = \int \sqrt{a^2(1-\sin^2 t)} \cdot a\cos t\mathrm{d}t = a^2\int \cos^2 t\mathrm{d}t$$

$$= a^2\int \frac{1+\cos 2t}{2}\mathrm{d}t = \frac{a^2}{2}\left(t + \frac{\sin 2t}{2}\right) + C$$

$$= \frac{a^2}{2}\arcsin \frac{x}{a} + \frac{x}{2}\sqrt{a^2 - x^2} + C.$$

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