1.2

2. For the function $y = g(x)$ graphed in Figure 1.14, find each of the following limits or explain why it does not exist.
(a) $\lim_{x\to 1} g(x)$, (b) $\lim_{x\to 2} g(x)$, (c) $\lim_{x\to 3} g(x)$

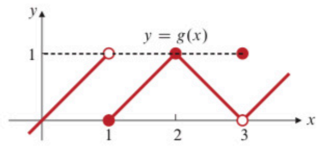

Figure 1.14

(a) $\lim_{x\to 1} g(x)$

$\lim_{x\to 1^-} g(x) = 1 \qquad \lim_{x\to 1^+} g(x) = 0$

$\because \lim_{x\to 1^-} g(x) \neq \lim_{x\to 1^+} g(x)$

$\therefore$ Not exist

(b) $\lim_{x\to 2} g(x)$

$\lim_{x\to 2^-} g(x) = 1 \qquad \lim_{x\to 2^+} g(x) = 1$

$\therefore \lim_{x\to 2^-} g(x) = \lim_{x\to 2^+} g(x) = 1$

$\therefore \lim_{x\to 2} g(x) = 1$

(c) $\lim_{x\to 3} g(x)$

$\lim_{x\to 3^+} g(x) = 0 \qquad \lim_{x\to 3^-} g(x) = 0$

$\therefore \lim_{x\to 3^+} g(x) = \lim_{x\to 3^-} g(x) = 0$

$\therefore \lim_{x\to 3} g(x) = 0.$

In Exercises 3–6, find the indicated one-sided limit of the function g whose graph is given in Figure 1.14.

3. $\lim_{x\to 1-} g(x)$ 4. $\lim_{x\to 1+} g(x)$
5. $\lim_{x\to 3+} g(x)$ 6. $\lim_{x\to 3-} g(x)$

3. $\lim_{x\to 1^-} g(x) = 1$ 4. $\lim_{x\to 1^+} g(x) = 0$

5. $\lim_{x\to 3^+} g(x) = 0$ 6. $\lim_{x\to 3^-} g(x) = 0.$

21. $\lim_{x\to 0} \dfrac{|x-2|}{x-2}$ 22. $\lim_{x\to 2} \dfrac{|x-2|}{x-2}$

21. $\lim_{x\to 0} \dfrac{|x-2|}{x-2}$

$\lim_{x\to 0^+} \dfrac{|x-2|}{x-2} = \lim_{x\to 0^+} \dfrac{-(x-2)}{x-2} = -1$

$\lim_{x\to 0^-} \dfrac{|x-2|}{x-2} = \lim_{x\to 0^-} \dfrac{-(x-2)}{x-2} = -1$

$\therefore \lim_{x\to 0^+} \dfrac{|x-2|}{x-2} = \lim_{x\to 0^-} \dfrac{|x-2|}{x-2} = -1$

$\therefore \lim_{x\to 0} \dfrac{|x-2|}{x-2} = -1$

22. $\lim_{x\to 2} \dfrac{|x-2|}{x-2}$

$\lim_{x\to 2^+} \dfrac{|x-2|}{x-2} = \lim_{x\to 2^+} \dfrac{x-2}{x-2} = 1$

$\lim_{x\to 2^-} \dfrac{|x-2|}{x-2} = \lim_{x\to 2^-} \dfrac{-(x-2)}{x-2} = -1$

$\because \lim_{x\to 2^+} \dfrac{|x-2|}{x-2} \neq \lim_{x\to 2^-} \dfrac{|x-2|}{x-2}$

$\therefore$ Not exist.

Solutions are Created by Yulin

25. $\lim_{t\to 0} \dfrac{t}{\sqrt{4+t}-\sqrt{4-t}}$ 26. $\lim_{x\to 1} \dfrac{x^2-1}{\sqrt{x+3}-2}$

27. $\lim_{t\to 0} \dfrac{t^2+3t}{(t+2)^2-(t-2)^2}$ 28. $\lim_{s\to 0} \dfrac{(s+1)^2-(s-1)^2}{s}$

25. $\lim_{t\to 0} \dfrac{t}{\sqrt{4+t}-\sqrt{4-t}}$ we don't want such complex denominator.

$\lim_{t\to 0} \dfrac{t}{\sqrt{4+t}-\sqrt{4-t}} = \lim_{t\to 0} \dfrac{t\,(\sqrt{4+t}+\sqrt{4-t})}{(\sqrt{4+t}-\sqrt{4-t})(\sqrt{4+t}+\sqrt{4-t})}$

Recall: $a^2-b^2 = (a-b)(a+b)$ OR $a-b = (\sqrt{a}-\sqrt{b})(\sqrt{a}+\sqrt{b})$

$= \lim_{t\to 0} \dfrac{t\,(\sqrt{4+t}+\sqrt{4-t})}{(4+t)-(4-t)} = \lim_{t\to 0} \dfrac{\cancel{t}\,(\sqrt{4+t}+\sqrt{4-t})}{2\cancel{t}}$

$= \dfrac{\sqrt{4+0}+\sqrt{4-0}}{2} = 2$

28. $\lim_{s\to 0} \dfrac{(s+1)^2-(s-1)^2}{s} = \lim_{s\to 0} \dfrac{\overbrace{[(s+1)-(s-1)]}^{\cancel{s}+1-\cancel{s}+1}\overbrace{[(s+1)+(s-1)]}^{s+\cancel{1}+s-\cancel{1}}}{s}$

$= \lim_{s\to 0} \dfrac{2\cdot 2s}{s} = 4$

34. $\lim_{x\to 2}\left(\frac{1}{x-2}-\frac{1}{x^2-4}\right)$

36. $\lim_{x\to 0}\frac{|3x-1|-|3x+1|}{x}$

34. $\lim_{x\to 2}\left(\frac{1}{x-2}-\frac{1}{x^2-4}\right)$

$= \lim_{x\to 2}\left(\frac{x+2}{(x-2)(x+2)}-\frac{1}{(x-2)(x+2)}\right)$

$= \lim_{x\to 2}\frac{x+2-1}{(x-2)(x+2)} = \lim_{x\to 2}\frac{x+1}{(x-2)(x+2)}$

$\lim_{x\to 2^+}\frac{\overset{\to 3}{x+1}}{\underset{\to 0^+}{(x-2)}\underset{\to 4}{(x+2)}} = +\infty \neq \lim_{x\to 2^-}\frac{\overset{\to 3}{x+1}}{\underset{\to 0^-}{(x-2)}\underset{\to 4}{(x+2)}} = -\infty$

Not exist

36. $\lim_{x\to 0}\frac{\overset{\to|-1|}{|3x-1|}-\overset{\to|1|}{|3x+1|}}{x}$

$= \lim_{x\to 0}\frac{(1-3x)-(3x+1)}{x} = \lim_{x\to 0}\frac{-6x}{x} = -6$

Solutions are Created by Yulin

53. $\lim_{x\to 0}\sqrt{x^3-x}$ 54. $\lim_{x\to 0-}\sqrt{x^3-x}$

55. $\lim_{x\to 0+}\sqrt{x^3-x}$ 56. $\lim_{x\to 0+}\sqrt{x^2-x^4}$

53. $\lim_{x\to 0}\sqrt{x^3-x} = \lim_{x\to 0}\sqrt{x(x^2-1)} = \lim_{x\to 0}\sqrt{x(x-1)(x+1)}$

$\lim_{x\to 0^-}\sqrt{\underset{\to 0^-}{x}\underset{\to -1}{(x-1)}\underset{\to 1}{(x+1)}} = \sqrt{0^+} = 0$

$\neq$ Not exist

$\lim_{x\to 0^+}\sqrt{\underset{\to 0^+}{x}\underset{\to -1}{(x-1)}\underset{\to 1}{(x+1)}} = \sqrt{0^-}$

54. $\lim_{x\to 0^-}\sqrt{x^3-x}$ Refer to (53.). It's 0.

55. $\lim_{x\to 0^+}\sqrt{x^3-x}$ Refer to (53). Not exist

1.3

19. $\lim_{x\to 2+}\frac{x}{(2-x)^3}$ 22. $\lim_{x\to 1-}\frac{1}{|x-1|}$

19. $\lim_{x\to 2^+}\frac{\overset{\to 2^+}{x}}{(2-x)^3} = -\infty$

$\to (0^-)^3 \to 0^-$

22. $\lim_{x\to 1^-}\frac{1}{|x-1|} = +\infty$

$\to |0^-| \to 0^+$

28. $\lim_{x\to\infty}\left(\frac{x^2}{x+1}-\frac{x^2}{x-1}\right)$

28. $\lim_{x\to\infty}\left(\frac{x^2}{x+1}-\frac{x^2}{x-1}\right)$

$= \lim_{x\to\infty}\frac{x^2(x-1)}{(x+1)(x-1)}-\frac{x^2(x+1)}{(x+1)(x-1)}$

$= \lim_{x\to\infty}\frac{x^2[x-1-(x+1)]}{(x+1)(x-1)}$ $= x^2(\cancel{x}-1-\cancel{x}-1)$

$= \lim_{x\to\infty}\frac{-2x^2}{x^2-1}$ $\to$ 2 ways

$= \lim_{x\to\infty}\frac{-2x^2}{x^2-1}$

Way 1: $\lim_{x\to\infty}\frac{-2}{1-\underset{\to 0}{\frac{1}{x^2}}} = \frac{-2}{1-0} = -2$

Way 2: $\lim_{x\to\infty}\frac{-2(x^2-1)-2}{x^2-1} = \lim_{x\to\infty} -2 - \boxed{\frac{2}{\underset{\to\infty}{(x^2-1)}}} = -2$

$\to 0$

1.4

Exercises 1–3 refer to the function g defined on $[-2, 2]$, whose graph is shown in Figure 1.33.

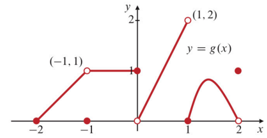

Figure 1.33

1. State whether g is (a) continuous, (b) left continuous, (c) right continuous, and (d) discontinuous at each of the points −2, −1, 0, 1, and 2.

2. At what points in its domain does g have a removable discontinuity, and how should g be redefined at each of those points so as to be continuous there?

1. (a) Continuous: $x=-2$

$\lim_{x\to -2} g(x) = g(-2) = 0$

(c) right continuous

$\lim_{x\to 1^+} g(x) = g(1) = 0$

(b) left continuous: $x=0$

$\lim_{x\to 0^-} g(x) = g(0) = 1$

(d) discontinuous: $x=-1, 0, 1, 2.$

ex. $\lim_{x\to -1} g(x) = 1 \neq g(-1) = 0.$

2. Recall: Removable discontinuities:

$x=c$ will be continuous if $f(c)$ is redefined.

($\lim_{x\to c} f(x) = L$ exists)

At $x=-1$, $x=2.$

Redefine $g(-1)=1$, $g(2)=0.$

33. If an even function f is right continuous at $x=0$, show that it is continuous at $x=0$.

Even fun. $f(x) = f(-x)$

Right con. at $x=0$: $f(0) = \lim_{x\to 0^+} f(x)$

$\because$ Even fun.

$\therefore f(x) = f(-x)$

$\therefore \lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} f(-x) \underset{\substack{x\to 0^+\\ -t\to 0^+\\ t\to 0^-}}{\overset{t=-x}{=\!=}} \boxed{\lim_{t\to 0^-} f(t)} = \lim_{x\to 0^-} f(x)$

$\therefore f(0) = \lim_{x\to 0^+} f(x) = \lim_{x\to 0^-} f(x) = \lim_{x\to 0} f(x)$

$\therefore$ It's continuous at $x=0$.