Q1: $\sum_{\text{all } x} P(X) = 1$ < Properity of P.D.F of a discrete r.v.)

Q2: Distinguish Discrete & Continuous r.v.

Q3: R.V.: Sample Space $\rightarrow$ Real Number

Q4: Find the P.D.F. (Dis.)

Q5: Find the C.D.F. (Dis.)

Q6 - Q13: Calculate $F_X(x)$ (Con.)

Q14 - Q21: $P(A|B) = \frac{P(A \cap B)}{P(B)}$ or $P(A|B)$

Q22 - Q23: Baye's Rule.

1. Determine the value of c so that each of the following functions can serve as a probability distribution of the discrete random variable X:

(a) $f(x) = c(x^2 + 4)$, for $x = 0, 1, 2, 3$

(b) $f(x) = c\binom{2}{x}\binom{3}{3-x}$, for $x = 0, 1, 2$

(a) $\sum_{\text{all } x} f(x) = 1$ ← key

$f(0) + f(1) + f(2) + f(3) = c[(0+4) + (1+4) + (4+4) + (9+4)]$

$= c[14 + \underbrace{4\times4}_{16}] = 30c = 1$

$\Rightarrow c = \frac{1}{30}$

(b) $f(0) + f(1) + f(2) = 1$

$= c\left[\binom{2}{0}\binom{3}{3} + \binom{2}{1}\binom{3}{2} + \binom{2}{2}\binom{3}{1}\right]$

$= c[1\times1 + 2\times3 + 1\times3] = c(1+6+3) = 10c$

$\Rightarrow c = \frac{1}{10}$

2. Classify the following random variables as discrete or continuous:

(a) X: the number of automobile accidents per year in Virginia

(b) Y: the length of time to play 18 holes of golf

(c) M: the amount of milk produced yearly by a particular cow

(d) N: the number of eggs laid each month by a hen

(e) P: the number of building permits issued each month in a certain city

(f) Q: the weight of grain produced per acre

key: { discrete variables: can be counted
continuous variables: can NOT be counted

(a) Discrete (b) Continuous (c) Continuous

(d) Discrete (e) Discrete (f) Continuous

3. Let W be a random variable giving the number of heads minus the number of tails in three tosses of a coin. List the elements of the sample space S for the three tosses of the coin and to each sample point assign a value of W.

$W = \#H - \#T$

	Sample Space	W		Sample Space	W
3H	(H, H, H)	3	1H	(H, T, T)	−1
2H	(H, H, T)	1		(T, H, T)	−1
	(H, T, H)	1		(T, T, H)	−1
	(T, H, H)	1	0H	(T, T, T)	−3

4. A shipment of 7 television sets contains 2 defective sets. A hotel makes a random purchase of 3 of the sets. If x is the number of defective sets purchased by the hotel, find the probability distribution of X. Express the results graphically as a probability histogram.

	X	P(X)
Max	2	$\binom{2}{2}\binom{5}{1}/\binom{7}{3} = \frac{1}{7}$
	1	$\binom{2}{1}\binom{5}{2}/\binom{7}{3} = \frac{4}{7}$
	0	$\binom{2}{0}\binom{5}{3}/\binom{7}{3} = \frac{2}{7}$

Hint $\longrightarrow$ $\binom{2}{2}\binom{5}{1}/\binom{7}{3}$

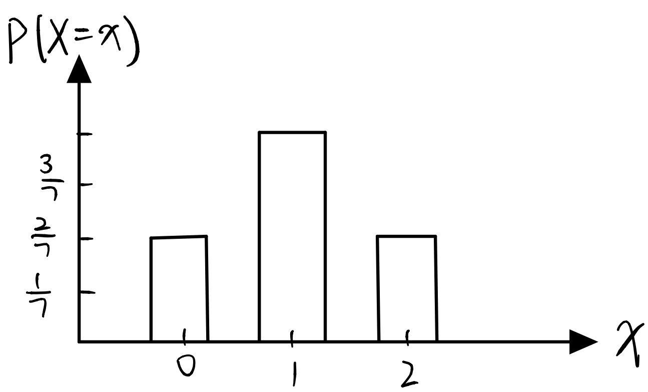
choose 2 bad of total 2 bad.

All possibilities to choose 3 of 7. (combination type)

choose 1 good of total 5 good

$$f(x) = \begin{cases} \frac{1}{7}, & X = 2 \\ \frac{4}{7}, & X = 1 \\ \frac{2}{7}, & X = 0 \end{cases}$$

In General: $\longrightarrow$ $P(X=x) = \dfrac{\binom{5}{3-x}\binom{2}{x}}{\binom{7}{3}}$, for $X = 0, 1, 2$



5. The probability distribution of X, the number of imperfections per 10 meters of a synthetic fabric in continuous rolls of uniform width, is given by

x	0	1	2	3	4
f(x)	0.41	0.37	0.16	0.05	0.01

Construct the cumulative distribution function of X.

Key : $F(x) = \sum_{X \leq x} f(x)$ for discrect variables.

X → random variable (r.v.); x → value

x	0	1	2	3	4
$f(x)$	0.41	0.37	0.16	0.05	0.01
$F(x)$	$f(0) = 0.41$	$f(0)+f(1) = 0.78$	$\sum_{i=0}^{2} f(i) = 0.94$	$\sum_{i=0}^{3} f(i) = 0.99$	$\sum_{i=0}^{4} f(i) = 1$

6. The shelf life, in days, for bottles of a certain prescribed medicines a random variable having the density function

$$f(x) = \begin{cases} \frac{20{,}000}{(x+100)^3} & x > 0, \\ 0 & \text{elsewhere} \end{cases}$$

Find the probability that a bottle of this medicine will have a shelf life of

(a) at least 200 days

(b) anywhere from 80 to 120 days

$$F(x) = \int_{-\infty}^{x} f(x) \cdot dx$$

$$= \int_{0}^{x} 2\times10^4 (x+100)^{-3} \cdot dx + \int_{-\infty}^{0} 0 \cdot dx$$

$$= 2\times10^4 \int_0^x (x+100)^{-3} \cdot dx$$

$$= 2\times10^4 \times \left[\frac{1}{-2}(x+100)^{-2}\right]_0^x$$

$$= 2\times\left(-\frac{1}{2}\right)\times10^4 \left[\frac{1}{(x+100)^2}\right]_0^x$$

$$= -10^4 \left[(x+100)^{-2} - 100^{-2}\right]$$

$$\left[(x+100)^{-2}\right]' = -2(x+100)^{-3}$$

$x \to -\infty$

$x+100 \to -\infty$

$(x+100)^2 \to +\infty$

$\frac{1}{(x+100)^2} \to 0$

(a) $P(X \geq 200) = 1 - F(200) = 1 + 10^4\left[300^{-2} - 100^{-2}\right] = \frac{1}{9}$

(b) $P(80 \leq X \leq 120) = F(120) - F(80)$

$$= -10^4\left[(220^{-2} - \cancel{100^{-2}}) - (180^{-2} - \cancel{100^{-2}})\right] = \cdots$$

7. The total number of hours, measured in units of 100 hours, that a family runs a vacuum cleaner over a period of one year in a continuous random variable X that has a density function

$$f(x) = \begin{cases} x & 0 < x < 1 \\ 2 - x & 1 \le x \le 2, \\ 0 & \text{elsewhere} \end{cases}$$

Find the probability that over a period of one year, a family runs their vacuum cleaner

(a) less than 120 hours

(b) between 50 and 100 hours

(a) $X < 1.2$

$$P(X<1.2) = \int_{-\infty}^{1.2} f(x)\cdot dx = \int_0^1 x \cdot dx + \int_1^{1.2} (2-x)\cdot dx$$

$$= \left[\tfrac{1}{2}x^2\right]_0^1 + \left[2x - \tfrac{1}{2}x^2\right]_1^{1.2}$$

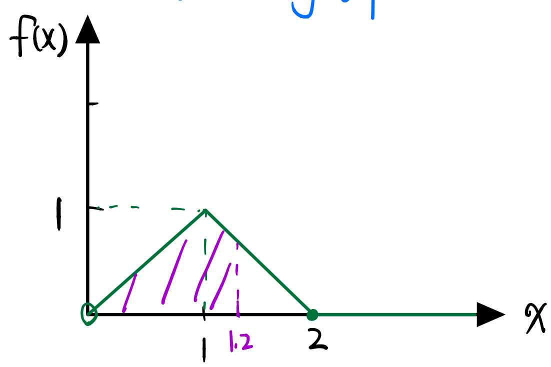
$$= \left(\tfrac{1}{2}\times 1 - \tfrac{1}{2}\times 0\right) + \left[\left(2\times 1.2 - \tfrac{1}{2}\times 1.2^2\right) - \left(2\times 1 - \tfrac{1}{2}\times 1\right)\right]$$

$$= 0.68$$

(b) $$P(0.5 < X < 1) = \int_{0.5}^{1} x \cdot dx = \left[\tfrac{1}{2}x^2\right]_{\frac{1}{2}}^{1}$$

$$= \left(\tfrac{1}{2}\times 1 - \tfrac{1}{2}\times \tfrac{1}{4}\right) = \tfrac{1}{2} - \tfrac{1}{8} = \tfrac{3}{8}$$

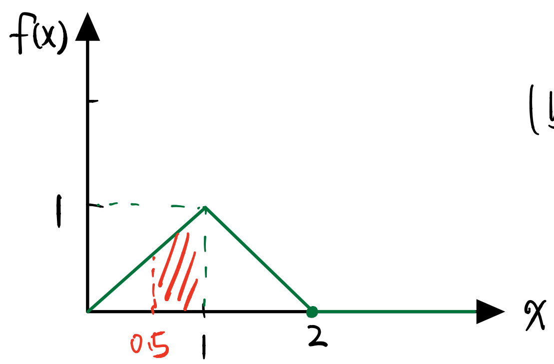
If we use graph:



(a) $P(X<1.2) = 1 - \frac{1}{2}\times(0.8\times 0.8)$

$= 1 - 0.32 = 0.68$

(Area of purple)



(b) $P(0.5 < X < 1) = \frac{1}{2}\times 1\times 1 - \frac{1}{2}\times 0.5\times 0.5$

$= \frac{1}{2} - \frac{1}{8} = \frac{3}{8}$

(Area of red)

8. A continuous random variable X that can assume values between $x = 1$ and $x = 3$ has a density function given by

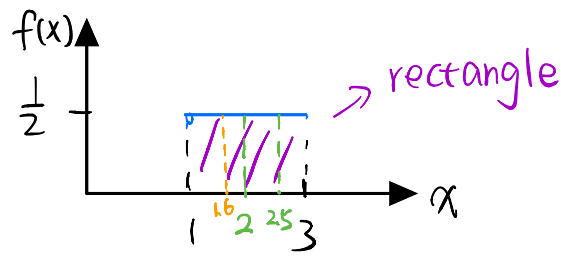
$$f(x) = \frac{1}{2} \quad (8\text{-}13)$$

(a) Show that the area under the curve is equal to 1.

(b) Find $P(2 < X < 2.5)$

(c) Find $P(X \le 1.6)$

(a) Use Graph:



Area under the curve $= \frac{1}{2} \times (3-1) = 1$

Use Integration:

$$S = \int_1^3 f(x) \cdot dx = \int_1^3 \frac{1}{2} \cdot dx = \frac{1}{2} \int_1^3 dx = \frac{1}{2} \times (3-1) = 1$$

(b) $P(2 < X < 2.5) = (2.5 - 2) \times \frac{1}{2} = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$

(c) $P(X \le 1.6) = (1.6 - 1) \times \frac{1}{2} = 0.6 \times \frac{1}{2} = 0.3$

9. A continuous random variable X that can have values between $x = 2$ and $x = 5$ has a density function given by

$$f(x) = \frac{2(1+x)}{27}$$

Find

(a) $P(X < 4)$

(b) $P(3 \le X < 4)$

$$F(x) = \begin{cases} 0, & x \le 2 \\ \int_2^x \frac{2}{27}(1+x) \cdot dx = \frac{1}{27}(x-2)(x+4), & 2 < X < 5 \\ 1, & x \ge 5 \end{cases}$$

$$\int_2^x \frac{2}{27}(1+x) \cdot dx = \frac{2}{27}\left[x + \frac{1}{2}x^2\right]_2^x = \frac{2}{27}\left[\frac{1}{2}x^2 + x - 4\right] = \frac{1}{27}(x^2 + 2x - 8) = \frac{1}{27}(x-2)(x+4)$$

Verify $\frac{1}{27} \times (5-2)(5+4) = \frac{1}{27} \times 3 \times 9 = 1$

$\Rightarrow \lim_{x \to -\infty} F(x) = 0, \quad \lim_{x \to +\infty} F(x) = 1$

(a) $P(X<4) = F(4) = \frac{1}{27} \times (4-2) \times (4+4)$

$= \frac{1}{27} \times 2 \times 8 = \frac{16}{27}$

(b) $P(3 \leq X < 4) = F(4) - F(3) = \frac{16}{27} - \frac{1}{27} \times (3-2)(3+4)$

$= \frac{1}{27}(16-7) = \frac{9}{27} = \frac{1}{3}$

10. The proportion of people who respond to a certain mail-order solicitation is a continuous random variable X that has density function

$$f(x) = \begin{cases} \frac{2(x+2)}{5} & 0 < x < 1, \\ 0 & \text{elsewhere} \end{cases}$$

(a) Show that $P(0 < X < 1) = 1$

(b) Find the probability that more than 1/4 but fewer than 1/2 of the people contacted will respond to this type of solicitation.

(a) $P(0<X<1) = \int_0^1 f(x) \cdot dx = \int_0^1 \frac{2}{5}(x+2) \cdot dx = \frac{2}{5} \int_0^1 (x+2) \cdot dx$

$= \frac{2}{5} \left[\frac{1}{2}x^2 + 2x\right]_0^1 = \frac{2}{5}\left[\left(\frac{1}{2}+2\right) - 0\right]$

$= \frac{2}{5} \times \frac{5}{2} = 1$

(b) $P\left(\frac{1}{4} < X < \frac{1}{2}\right) = \int_{\frac{1}{4}}^{\frac{1}{2}} \frac{2}{5}(x+2) \cdot dx = \frac{2}{5}\left[\frac{1}{2}x^2 + 2x\right]_{\frac{1}{4}}^{\frac{1}{2}}$

$= \frac{2}{5}\left[\left(\frac{1}{2} \times \frac{1}{4} + 2 \times \frac{1}{2}\right) - \left(\frac{1}{2} \times \frac{1}{16} + 2 \times \frac{1}{4}\right)\right]$

$= \frac{2}{5}\left[\left(\frac{1}{8}+1\right) - \left(\frac{1}{32} + \frac{1}{2}\right)\right] = 0.2375$

11. An important factor in solid missile fuel is the particle size distribution. Significant problems occur if the particle sizes are too large. From production data in the past, it has been determined that the particle size (in micrometers) distribution is characterized by

$$f(x) = \begin{cases} 3x^{-4} & x > 1, \\ 0 & \text{elsewhere} \end{cases}$$

(a) Verify that this is a valid density function

(b) Find $F(x)$

(c) What is the probability that a random particle from the manufactured fuel exceeds 4 micrometers?

(a) Need to show $\int_{-\infty}^{+\infty} f(x) \cdot dx = 1$

$[x^{-3}]' = -3 \cdot x^{-4}$

$[-x^{-3}]' = 3 \cdot x^{-4}$

$$\int_{-\infty}^{+\infty} f(x) \cdot dx = \int_1^{+\infty} 3x^{-4} \cdot dx = [-x^{-3}]_1^{+\infty}$$

$x \to +\infty$

$$= (0) - (\underbrace{-1^{-3}}_{-1}) = 1$$

$x^{-3} \to 0$

(b)
$$F(x) = \begin{cases} 0, & x \le 1 \\ \int_1^x 3x^{-4} \cdot dx = [-x^{-3}]_1^x = -\frac{1}{x^3} - (-1) = -\frac{1}{x^3} + 1, & x > 1 \end{cases}$$

(c) $P(X > 4) = 1 - F(4) = 1 - \left(-\frac{1}{4^3} + 1\right) = \frac{1}{4^3} = \frac{1}{64}$

12. Based on extensive testing, it is determined by the manufacturer of a washing machine that the time Y (in years) before a major repair is required is characterized by the probability density function

$$f(y) = \begin{cases} \frac{1}{4} e^{\frac{-y}{4}} & y > 0 \\ 0 & \text{elsewhere} \end{cases}$$

(a) Critics would certainly consider the product a bargain if it is unlikely to require a major repair before the sixth year. Comment on this by determining $P(Y > 6)$.

(b) What is the probability that a major repair occurs in the first year?

Since y represents time, so $y \geq 0$

$\boxed{[e^{-\frac{y}{4}}]' = e^{-\frac{y}{4}} \cdot (-\frac{1}{4})}$

$$F(y) = \begin{cases} \int_0^y \frac{1}{4} e^{-\frac{y}{4}} \cdot dy = [-e^{-\frac{y}{4}}]_0^y = -e^{-\frac{y}{4}} - (-e^0) = -e^{-\frac{y}{4}} + 1, & y > 0 \\ 0, & \text{elsewhere} \end{cases}$$

(a) $P(Y > 6) = 1 - F(6) = 1 - \left[-e^{-\frac{6}{4}} + 1\right] = e^{-\frac{3}{2}}$

(b) $P(Y < 1) = P(Y \le 1) = F(1) = -e^{-\frac{1}{4}} + 1$

13. Suppose a certain type of small data processing firm is so specialized that some have difficulty making a profit in their first year of operation. The probability density function that characterizes the proportion Y that make a profit is given by

$$f(y) = \begin{cases} k\,y^4(1-y)^3 & 0 \le y \le 1, \\ 0 & \text{elsewhere} \end{cases}$$

(a) What is the value of k that renders the above a valid density function?

(b) Find the probability that at most 50 % of the firms make a profit in the first year?

(c) Find the probability that at most 80 % of the firms make a profit in the first year?

(a) $\int_{-\infty}^{+\infty} f(x)\cdot dx = 1$

$(1-y)^3 = (1-y)(1-2y+y^2) = 1-2y+y^2-y+2y^2-y^3 = -y^3+3y^2-3y+1$

$$\int_{-\infty}^{+\infty} f(x)\cdot dx = \int_0^1 k\cdot y^4(1-y)^3\cdot dx = k\int_0^1 y^4(-y^3+3y^2-3y+1)\cdot dy$$

$$= k\int_0^1(-y^7+3y^6-3y^5+y^4)\cdot dy = k\cdot\left[-\frac{1}{8}\cdot y^8+\frac{3}{7}y^7-\frac{3}{6}y^6+\frac{1}{5}y^5\right]_0^1$$

$$= k\left[\left(-\frac{1}{8}+\frac{3}{7}-\frac{1}{2}+\frac{1}{5}\right)-0\right]$$

$$= k\cdot\frac{1}{280} = 1 \Rightarrow k = 280$$

$$F(x) = \begin{cases} 0, \; y<0 \longrightarrow \text{Actually, } y \text{ won't smaller than } 0. \\ \int_0^y \frac{1}{280}\cdot(-y^7+3y^6-3y^5+y^4)\cdot dy = \frac{1}{280}\cdot\left[-\frac{1}{8}y^8+\frac{3}{7}y^7-\frac{1}{2}y^6+\frac{1}{5}y^5\right]_0^y \\ \quad = \frac{1}{280}\cdot\left(-\frac{1}{8}y^8+\frac{3}{7}y^7-\frac{1}{2}y^6+\frac{1}{5}y^5\right) \\ \quad = -35y^8+120y^7-140y^6+56y^5, \quad 0\le y\le 1 \\ 1, \; y \ge 1 \longrightarrow \text{Actually, } y \text{ won't larger than } 0. \end{cases}$$

(b) $P(X<0.5) = F(0.5) = -35\left(\frac{1}{2}\right)^8+120\left(\frac{1}{2}\right)^7-140\left(\frac{1}{2}\right)^6+56\left(\frac{1}{2}\right)^5 =$

(c) $P(X<0.8) = F(0.8) = -35(0.8)^8+120(0.8)^7-140(0.8)^6+56(0.8)^5$

$=$

14. A class in advanced physics is composed of 10 juniors, 30 seniors, and 10 graduate students. The final grades show that 3 of the juniors, 10 of the seniors, and 5 of the graduate students received an A for the course. If a student is chosen at random from this class and is found to have earned an A, what is the probability that he or she is a senior?

Bay's Rule

	(J) juniors	(S) seniors	(G) graduate
Total	10	30	10
# A	3	10	5

$$P(x \mid A) = \frac{P(A \mid x) \cdot P(x)}{P(A)} \qquad x = J, S, G$$

$$P(A) = \frac{3+10+5}{10+30+10} = \frac{18}{50} = \frac{9}{25}$$

$$P(A \mid S) = \frac{10}{30} = \frac{1}{3} \qquad P(S) = \frac{30}{10+30+10} = \frac{3}{5}$$

$$P(S \mid A) = \frac{P(A \mid S) \cdot P(S)}{P(A)} = \frac{\frac{1}{3} \times \frac{3}{5}}{\frac{9}{25}} = \frac{5}{9}$$

$$= \frac{P(A \cap S)}{P(A)} = \frac{\frac{10}{50}}{\frac{9}{25}} = \frac{10}{18} = \frac{5}{9}$$

15. A random sample of 200 adults are classified below by sex and their level of education attained.

Education	Male	Female
Elementary	38	45
Secondary	28	50
College	22	17

If a person is picked at random from this group, find the probability that

(a) the person is a male, given that the person has a secondary education.

(b) the person does not have a college degree, given that the person is female.

	Ele	Sec	Col	Sum (of row)
Male	38	28	22	88

Female	45	50	17	112
Total	83 +	78 +	39 =	200

(a) $P(\text{male} \mid \text{Secondary})$

$$= \frac{28}{78} = \frac{14}{39}.$$

(b) $P(Col^c \mid \text{Female}) = \frac{45+50}{112} = \frac{95}{112}$

16. In an experiment to study the relationship of hypertension and smoking habits, the following data are collected for 180 individuals:

	Nonsmokers	Moderate Smokers	Heavy Smokers
H	21	36	30
NH	48	26	19

Here H and NH stand for Hypertension and Nonhypertension, respectively. If one of these individuals is selected at random, find the probability that the person is

(a) experiencing hypertension, given that the person is a heavy smoker.

(b) a nonsmoker, given that the person is experiencing no hypertension.

	NS	MS	HS	Sum of row
H	21	36	30	87
NH	48	26	19	93
sum of column	69	62	49	180

Heavy Smokers.

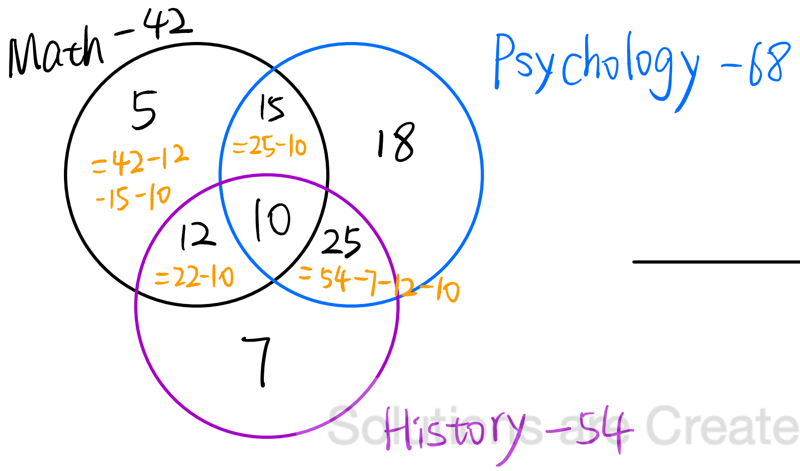
(a) $P(H \mid HS) = \frac{30}{49}$

(b) $P(NS \mid NH) = \frac{48}{93}$

17. In the senior year of a high school graduating class of 100 students, 42 studied mathematics, 68 studied psychology, 54 studied history, 22 studied both mathematics and history, 25 studied both mathematics and psychology, 7 studied history but neither mathematics nor psychology, 10 studied all three subjects, and 8 did not take any of the three. Randomly select a student from the class and find the probabilities of the following events.

(a) A person enrolled in psychology takes all three subjects.

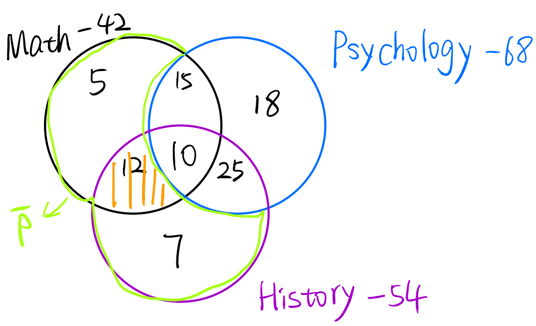
(b) A person not taking psychology is taking both history and mathematics.



$\longrightarrow$ Venn-diagram

(a) $P(M \cap P \cap H \mid P) = \dfrac{10}{68}$

(b) $P(H \cap M \mid P^c) = \dfrac{12}{100-68} = \dfrac{12}{32} = \dfrac{3}{8}$



18. In USA Today, the results of a survey involving the use of sleepware while traveling were listed as follows:

(a) What is the probability that a traveler is a female who sleeps nude?

(b) What is the probability that a traveler is male?

(c) Assuming the traveler is male, what is the probability that he sleeps in pyjamas?

(d) What is the probability that a traveler is male if the traveler sleeps in pyjamas or a T-shirt?

	Male	Female	Total
Underwear	0.220	0.024	0.224
Nightgown	0.002	0.180	0.182
Nothing	0.160	0.018	0.178
Pyjamas	0.102	0.073	0.175
T-shirt	0.046	0.088	0.134
Other	0.084	0.003	0.087

Female

(a) $P(F \cap \text{Nude}) = P(F \cap \text{Nothing}) = 0.018$

(b) $P(M) = 0.220 + 0.002 + 0.160 + 0.102 + 0.046 + 0.084 = 0.614$

(c) $P(\text{Pyjamas} \mid M) = \dfrac{P(P \cap M)}{P(M)} = \dfrac{0.102}{0.614} \approx 0.166$

(d) $P(M \mid \text{Pyj} \cup \text{T-shirt})$

$$= \frac{P(M \cap [\text{Pyj} \cup \text{T-shirt}])}{P(\text{Pyj} \cup \text{T-shirt})} = \frac{0.102 + 0.046}{0.175 + 0.134} \approx 0.479$$

19. The probability that an automobile being filled with gasoline also needs an oil change is 0.25; the probability that it needs a new oil filter is 0.40; the probability that both the oil and the filter need changing is 0.14.

(a) If the oil has to be changed, what is the probability that a new oil filter is needed?

(b) If a new oil filter is needed, what is the probability that the oil has to be changed?

$P(\text{oil charge}) = 0.25$ $\quad P(\text{new oil filter}) = 0.4$

$= P(A)$ $\quad = P(B)$ $\quad P(A \cap B) = 0.14$

(a) $P(\text{new oil filter} \mid \text{oil charge}) = P(B \mid A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{0.14}{0.25}$

$= 0.56$

(b) $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.14}{0.4} = 0.35$

20. The probability that a vehicle entering the Luray Caverns has Canadian licence plates is 0.12; the probability that it is a camper is 0.28; the probability that it is a camper with Canadian licence plate is 0.09. What is the probability that

(a) a camper entering the Luray Caverns has Canadian licence plates?

(b) a vehicle with Canadian licence plates entering the Luray Caverns is a Camper?

(c) a vehicle entering the Luray Caverns does not have Canadian plates or is not a camper?

"Canadian": Canadian licence plate

$P(\text{Canadian}) = 0.12 \qquad P(\text{Camper}) = 0.28$

$P(\text{Canadian} \cap \text{Camper}) = 0.09$

(a) $$P(\text{Canadian} \mid \text{Camper}) = \frac{P(\text{Canadian} \cap \text{Camper})}{P(\text{Camper})}$$

$$= \frac{0.09}{0.28} = 0.321$$

(b) $$P(\text{Camper} \mid \text{Canadian}) = \frac{P(\text{Canadian} \cap \text{Camper})}{P(\text{Canadian})}$$

$$= \frac{0.09}{0.12} = 0.75$$

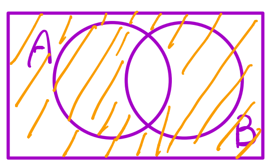
(c) $$P(\text{Canadian}^c \cup \text{Camper}^c)$$

$$= P([\text{Canadian} \cap \text{Camper}]^c)$$

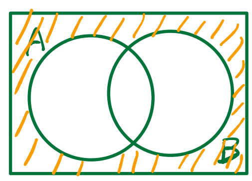
$$= 1 - P(\text{Canadian} \cap \text{Camper})$$

$$= 1 - 0.09 = 0.91$$

$A^c \cup B^c = (A \cap B)^c$



$A^c \cap B^c = [A \cup B]^c$



21. A fair coin is flipped twice. What is the probability that both flips land on heads given that

a the first flip lands on heads

b at least one flips lands on heads

The result of two filps are independent, meaning that they won't affect each other.

H : Head T: Tail

(a) $P(HH \mid \text{first is Head}) = P(\text{sencond is Head}) = 0.5$

Way 2 : $\{\underbrace{HH}_{2H}, \underbrace{HT, TH}_{1H}, \underbrace{TT}_{0H}\}$

$$P(HH \mid \text{first is Head}) = \frac{P(HH \cap \text{first is Head})}{P(\text{first is Head})} = \frac{1}{2}$$

(b) $$P(HH \mid \#H \geqslant 1) = \frac{P(HH \cap \#H \geqslant 1)}{P(\#H \geqslant 1)} = \frac{1}{3}$$

22. Every morning during college weeks, a period of 6 weeks, a student rolls a die. If the outcome is a six, then he will study; otherwise he goes back to sleep. During examination week, rather than rolling a die, our student flips a coin and he will study if he flips heads. If our student is studying one morning, what is the probability that it is examination week?

College Week (6 weeks) $\longrightarrow$ 6 : Study
(CW) 1–5 : sleep.

Examination Week (1 week) $\longrightarrow$ Head : Study
(EW) Tail : sleep.

$$P(CW) = \frac{6}{6+1} = \frac{6}{7} \ ; \ P(EW) = \frac{1}{7}$$

$$P(Study|CW) = \frac{1}{6} \ ; \ P(Sleep|CW) = \frac{5}{6}$$

$$P(Study|EW) = \frac{1}{2} \ ; \ P(Sleep|EW) = \frac{1}{2}$$

$$P(EW|Study) = \frac{P(EW \cap Study)}{P(Study)}$$

$$= \frac{P(Study|EW) \cdot P(EW)}{P(Study|EW) \cdot P(EW) + P(Study|CW) \cdot P(CW)}$$

$$= \frac{\frac{1}{2} \times \frac{1}{7}}{\frac{1}{2} \times \frac{1}{7} + \frac{1}{6} \times \frac{6}{7}} = \frac{\frac{1}{14}}{\frac{1}{14} + \frac{1}{7}} = \frac{1}{3}$$

23. A truth serum has the property that 90 % of the guilty suspects are properly judged while, of course, 10 % of the guilty suspects are improperly found innocent. On the other hand, innocent suspects are misjudged 1 % of the time. If the suspect was selected from a group of suspects of which only 5 % have ever committed a crime, and the serum indicates that he is guilty, what is the probability that he is innocent?

G : Guilty in truth FG : Found Guilty

$$P(FG|G) = 0.9 \ ; \ P(FG|G^c) = 0.01$$

$$P(G) = 0.05 \ , \ P(G^c) = 0.95$$

$$P(G^c|FG) = \frac{P(FG|G^c) \cdot P(G^c)}{P(FG)} = \frac{P(FG|G^c) \cdot P(G^c)}{P(FG|G^c) \cdot P(G^c) + P(FG|G) \cdot P(G)}$$

$$= \frac{0.01 \times 0.95}{0.01 \times 0.95 + 0.9 \times 0.05} \approx 0.174$$