1. Suppose X is uniformly distributed on the interval from 1 to 5, or $X \sim U(1,5)$. Determine the conditional probability $P(X > 2.5 \mid X \leq 4)$. (Hint: Use the formula of a conditional probability)

$$X \sim U(1,5) \quad \text{or} \quad f(x) = \begin{cases} \frac{1}{4}, & 1 \leq x \leq 5 \\ 0, & \text{elsewhere} \end{cases}$$

$$P(X > 2.5 \mid X \leq 4)$$

$$= \frac{P(X>2.5, X\leq 4)}{P(X\leq 4)} = \frac{P(2.5 < X \leq 4)}{P(X \leq 4)} = \frac{\int_{2.5}^{4} \frac{1}{4}\cdot dx}{\int_{1}^{4} \underbrace{\frac{1}{4}}_{f(x)} \cdot dx} = \frac{\frac{1}{4}(4-2.5)}{\frac{1}{4}\times(4-1)} = \frac{1.5}{3} = \frac{1}{2}$$

2. The daily amount of coffee, in liters, dispensed by a machine located in an airport lobby is a random variable X having a continuous uniform distribution from 7 to 10. Find the probability that on a given day the amount of coffee dispensed by this machine will be

(a) at most 8.8 liters;

(b) more than 7.4 liters but less than 9.5 liters;

(c) at least 8.5 liters.

$$f(x) = \begin{cases} \frac{1}{10-7} = \frac{1}{3}, & 7 \leq x \leq 10 \\ 0, & \text{elsewhere} \end{cases}$$

(a) $P(X \leq 8.8) = \int_{-\infty}^{8.8} f(x)\cdot dx = \int_{7}^{8.8} \frac{1}{3}\cdot dx = \frac{1}{3}\times(8.8-7) = 0.6$

(b) $P(7.4 < X < 9.5) = \int_{7.4}^{9.5} \frac{1}{3}\cdot dx = \frac{1}{3}\times(9.5-7.4) = 0.7$

(c) $P(X \geq 8.5) = \int_{8.5}^{+\infty} f(x)\cdot dx = \int_{8.5}^{10} \frac{1}{3}\cdot dx = \frac{1}{3}\times(10-8.5) = 0.5$

3. Let $X \sim U(a,b)$. Show that $V(X) = \frac{1}{12}(b-a)^2$

$$X \sim U(a,b) \iff f(x) = \begin{cases} \frac{1}{b-a}, & a \leq x \leq b \\ 0, & \text{elsewhere} \end{cases}$$

Recall : $V(X) = E(X^2) - [E(X)]^2$

$E(X^2)$ Recall : $E[h(x)] = \int_{-\infty}^{+\infty} h(x) \cdot f(x) \cdot dx$, $h(x) = x^2$

$$E(X^2) = \int_{-\infty}^{+\infty} x^2 \cdot f(x) \cdot dx = \int_a^b x^2 \cdot \frac{1}{b-a} \cdot dx = \frac{1}{b-a} \cdot \int_a^b x^2 \cdot dx = \frac{1}{b-a} \cdot \left[\frac{1}{3}x^3\right]_a^b$$

$$= \frac{1}{b-a} \cdot \frac{1}{3}(b^3 - a^3) = \frac{1}{3}(b^2 + ab + a^2)$$

$$E(X) = \int_{-\infty}^{+\infty} x \cdot f(x) \cdot dx = \int_a^b x \cdot \frac{1}{b-a} \cdot dx = \frac{1}{b-a} \cdot \int_a^b x \cdot dx = \frac{1}{b-a} \cdot \left[\frac{1}{2}x^2\right]_{x=a}^b$$

$$= \frac{1}{2}(b+a)$$

$$V(X) = E(X^2) - [E(X)]^2 = \frac{1}{3}(b^2 + ab + a^2) - \frac{1}{4}(b+a)^2$$

$$= b^2\left(\frac{1}{3} - \frac{1}{4}\right) + ab\left(\frac{1}{3} - \frac{1}{4} \times 2\right) + a^2\left(\frac{1}{3} - \frac{1}{4}\right) = \frac{1}{12}b^2 - \frac{1}{6}ab + \frac{1}{12}a^2$$

$$= \frac{1}{12}(b-a)^2$$

4. A bus arrives every 10 minutes at a bus stop. It is assumed that the waiting time for a particular individual is a random variable with a (continuous) uniform distribution. What is the probability that an individual waits

(a) more than 7 minutes?

(b) between 2 and 7 minutes?

Maximum wait time = 10 min   Minimum wait time = 0 min

X = waiting time for a individual

$$X \sim U(0, 10) \text{ or } f(x) = \begin{cases} \frac{1}{10}, & 0 \le X \le 10 \\ 0, & \text{elsewhere} \end{cases}$$

(a) $P(X > 7) = \int_7^{+\infty} f(x) \cdot dx = \int_7^{10} \frac{1}{10} \cdot dx = \frac{1}{10} \cdot (10 - 7) = \frac{3}{10}$

(b) $P(2 \le X \le 7) = \int_2^7 f(x) \cdot dx = \int_2^7 \frac{1}{10} \cdot dx = \frac{1}{10} \times (7-2) = \frac{5}{10}$

5. The life, in years, of a certain type of electrical switch has an exponential distribution with an average life $\beta = 2$. (Notice that $\beta = \frac{1}{\lambda}$, where λ is the parameter used in the lecture, so in this exercise we have $\lambda = \frac{1}{2}$.) If 100 of these switches are installed in different systems, what is the probability that at most 30 fail during the first year? (Hint: Calculate first the probability that a randomly selected switch fails during the first year)

For a randomly selected switch : $E(X) = \frac{1}{\lambda} = 2$

X = life time of this switch. $X \sim Exp(\lambda = \frac{1}{2})$

$P(X < 1) = 1 - e^{-\frac{1}{2} \times 1} = 0.3935$

Recall : $P(X \le x) = 1 - e^{-\lambda x}$ if $X \sim Exp(\lambda)$

But we have $n=100$ switches (= experiment), ← independent & identical

outcome: fail or not fail in the first year.

P(fail) is constant.

Y = # switches fail during the first year in $n=100$ switches

$Y \sim B(n=100, p=0.3935)$ But n is not big, p is not small

$\approx N(\mu=39.35, \sigma=\sqrt{100\times 0.3935\times 0.6065}=4.885)$ → Normal Approximation to Binomial

$$Z=\frac{Y-\mu}{\sigma}\sim N(0,1)$$

$$P(Y\leq 30)=P(Y\leq 30.5)=P\left(\frac{Y-\mu}{\sigma}\leq\frac{30.5-39.35}{4.885}\right)=P(Z\leq -1.81)=0.0351$$

continuity correction.

6. Computer response time is an important application of the exponential distribution. Suppose that a study of a certain computer system reveals that the response time, in seconds, has an exponential distribution with a mean of 3 seconds.

(a) What is the probability that response time exceeds 5 seconds?

(b) What is the probability that response time exceeds 10 seconds?

$\frac{1}{\lambda}=3 \Rightarrow \lambda=\frac{1}{3}$ X = response time. $X\sim Exp(\lambda=\frac{1}{3})$

(a) $P(X>5)=e^{-\frac{1}{3}\times 5}=e^{-\frac{5}{3}}=0.1889$ Recall: $P(X>x)=e^{-\lambda x}$ if $X\sim Exp(\lambda)$

(b) $P(X>10)=e^{-\frac{1}{3}\times 10}=e^{-\frac{10}{3}}=0.0357$

7. The number of automobiles that arrive at a certain intersection per minute has a Poisson distribution with a mean of 5. Interest centers around the time that elapses before 10 automobiles appear at the intersection.

(a) What is the probability that more than 10 automobiles appear at the intersection during any given minute of time?

(b) What is the probability that more than 2 minutes elapse before 10 cars arrive?

(c) What is the probability that more than 1 minute elapses between two arrivals?

(d) What is the mean number of minutes that elapse between two car arrivals?

(e) What is the mean number of minutes that elapse between three car arrivals?

(a) X = # automobiles appear during any given minute.

$X\sim P(\mu=5)$

$P(X>10)=1-P(X\leq 10)=1-0.9863=0.0137$

(b) X = # automobiles appear per 2 mins

$X\sim P(\mu=10)$

$P(X<10)=P(X\leq 9)=0.4579$

(c) Y = time betwen 2 car arrivals

$Y \sim Exp(\lambda = 5)$ $\quad E(Y) = \frac{1}{\lambda} = \frac{1}{5}$

$P(Y > 1) = e^{-5 \cdot 1} = e^{-5} = 0.0067$

(d) $E(Y) = \frac{1}{\lambda} = \frac{1}{5}$ (mins)

(e) Y: time between 3 car arrival

$E(Y) = \frac{1}{5} + \frac{1}{5} = \frac{2}{5}$

average time between 2 cars.

8. Let $X \sim Exp(\lambda)$. Show that $V(X) = \left(\frac{1}{\lambda}\right)^2$

$X \sim Exp(\lambda)$ $\quad f(x) = \begin{cases} \lambda \cdot e^{-\lambda x}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$

$V(X) = E(X^2) - [E(X)]^2$

$$E(X^2) = \int_{-\infty}^{+\infty} x^2 \cdot f(x) \cdot dx = \int_0^{+\infty} x^2 \cdot \lambda \cdot e^{-\lambda x} \cdot dx$$

$$= \int_0^{+\infty} x^2 \cdot (-e^{-\lambda x})' \cdot dx = \left[x^2 \cdot (-e^{-\lambda x})\right]_0^{+\infty} - \int_0^{+\infty} 2x \cdot (-e^{-\lambda x}) \cdot dx$$

$$= 0 + \int_0^{+\infty} 2x \left(-\frac{1}{\lambda} \cdot e^{-\lambda x}\right)' \cdot dx$$

$$= \left[2x \cdot -\frac{1}{\lambda} e^{-\lambda x}\right]_0^{+\infty} - \int_0^{+\infty} 2 \cdot \left(-\frac{1}{\lambda} \cdot e^{-\lambda x}\right) dx$$

$$= 0 + \frac{2}{\lambda} \int_0^{+\infty} e^{-\lambda x} \cdot dx$$

$$= \frac{2}{\lambda} \cdot \left[-\frac{1}{\lambda} \cdot e^{-\lambda x}\right]_0^{+\infty} = \frac{2}{\lambda}\left[0 - \left(-\frac{1}{\lambda} \cdot e^0\right)\right] = \frac{2}{\lambda^2}$$

Recall:

$$\int_a^b f(x) \cdot g'(x) \cdot dx = f(x) \cdot g(x) \Big|_{x=a}^{b} - \int_a^b f'(x) \cdot g(x) \cdot dx$$

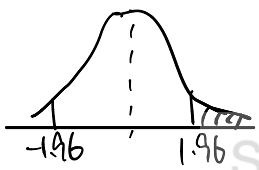
9. Look-up exercise (table A3): Given the standard normal distribution $\mathcal{N}(0,1)$, find the area under the curve that lies

(a) to the left of $z = -1.39$;
(b) to the right of $z = 1.96$;
(c) between $z = -2.16$ and $z = -0.65$;
(d) to the left of $z = 1.43$;
(e) to the right of $z = -0.89$;
(f) between $z = -0.48$ and $z = 1.74$.

$X \sim N(0,1)$

(a) $P(X \leq -1.39) = 0.0823$

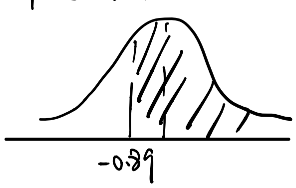
(b) $P(X \geqslant 1.96) = \frac{1}{2} - P(X \leq -1.96) = 0.5 - 0.0250 = 0.0250$



(c) $P(-2.16 \leq X \leq -0.65)$
$= P(X \leq -0.65) - P(X \leq -2.16) = 0.2578 - 0.0154 = 0.2424$

(d) $P(X \leq 1.43) = 0.9236$

(e) $P(X \geqslant -0.89) = 1 - P(X \leq -0.89) = 1 - 0.1867 = 0.8133$



(f) $P(-0.48 \leq X \leq 1.74) = P(X \leq 1.74) - P(X \leq -0.48)$
$= 0.9591 - 0.3156 = 0.6435$.

10. Given the standard normal distribution, find the value of k such that

(a) $P(Z > k) = 0.2946$
(b) $P(Z < k) = 0.0427$;
(c) $P(-0.93 < Z < k) = 0.7235$.

(a) $P(Z > k) = 0.2946$
$\Rightarrow P(Z \leq k) = 1 - 0.2946 = 0.7054 \xrightarrow{\text{Table}} k = 0.54$

(b) $P(Z<k) = 0.0429 \xrightarrow{\text{Table}} k \approx -1.72$

(c) $P(-0.93 < Z < k) = P(Z \le k) - P(Z \le -0.93)$

$= P(Z \le k) - 0.1762 = 0.7235$

$\Rightarrow P(Z \le k) = 0.8997$

$\xrightarrow{\text{Table}} k = 1.28$

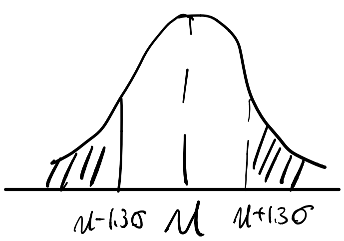
11. If a set of observations is normally distributed, what percent of these differ from the mean by

(a) more than $1.3\,\sigma$?

(b) less than $0.52\,\sigma$?

$X \sim N(\mu, \sigma)$ $\quad P(X < \mu - 1.3\sigma) + P(X > \mu + 1.3\sigma)$

(a)



$= 1 - P(\mu - 1.3\sigma \le X \le \mu + 1.3\sigma)$

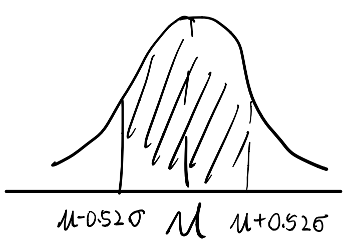
$= 1 - P\left(-1.3 \le \frac{X-\mu}{\sigma} \le 1.3\right)$

$= 1 - P(-1.3 \le Z \le 1.3)$

$= 1 - [P(Z \le 1.3) - P(Z \le -1.3)]$

$= 1 - (0.9032 - 0.0968) = 0.1936$

(b)



$P(\mu - 0.52\sigma < X < \mu + 0.52\sigma)$

$= P\left(-0.52 < \frac{X-\mu}{\sigma} < 0.52\right)$

$= P(-0.52 < Z < 0.52)$

$= P(Z \le 0.52) - P(Z \le -0.52)$

$= 0.6985 - 0.3015 = 0.3970$

12. The IQs of 600 applicants to a certain college are approximately normally distributed with a mean of 115 and a standard deviation of 12. If the college requires an IQ of at least 95, how many of these students will be rejected on this basis of IQ, regardless of their other qualifications? Note that IQs are recorded to the nearest integers.

$\mu = 115$, $\sigma = 12$ $\quad X$ = IQs of 600 applicants

$X \sim N(\mu = 115, \sigma = 12)$

$$P(X < 94.5) = P\left(\frac{X-\mu}{\sigma} < \frac{94.5 - 115}{12}\right) = P(Z < -1.71)$$

$$= 0.0436$$

13. The weights of a large number of miniature poodles are approximately normally distributed with a mean of 8 kilograms and a standard deviation of 0.9 kilograms. Assuming that measurements are recorded to the nearest tenth of a kilogram, find the fraction of these poodles with recorded weights

(a) over 9.5 kilograms;

(b) of at most 8.6 kilograms;

(c) between 7.3 and 9.1 kilograms inclusive.

(d) What is the probability that the combined weight of five poodles exceeds 42 kilograms?

(e) The weights of people who own poodles, is also normally distributed, with a mean of 85 kilograms and a standard deviation of 11.3 kilograms. What is the probability that the combined weight of 2 poodles and 2 owners is between 182 and 192 kilograms?

X = weights of miniature poodles. $X \sim N(\mu = 8, \sigma = 0.9)$

(a) $P(X > 9.55) = P\left(\frac{X-\mu}{\sigma} > \frac{9.55 - 8}{0.9}\right) =$

$(9.45, 9.55) \to 9.5$

(b) $P(X < 8.65) = P\left(\frac{X-\mu}{\sigma} < \frac{8.65-8}{0.9}\right) = P(Z < 0.72) = 0.7642$

$(8.55, 8.65) \to 8.6$

(c) $P(7.25 < X < 9.15) = P\left(\frac{7.25-8}{0.9} < \frac{X-\mu}{\sigma} < \frac{9.15-8}{0.9}\right)$

$= P(-0.83 < Z < 1.28) = 0.8997 - 0.2033 = 0.6964$

(e) X_1, X_2 : weight of poodles

Y_1, Y_2 : weight of owners

$X_1, X_2 \sim N(\mu = 8, \sigma = 0.9)$ $\quad Y_1, Y_2 \sim N(\mu = 85, \sigma = 11.3)$

$L = X_1 + X_2 + Y_1 + Y_2 \sim N(\mu = 8+8+85+85, \sigma = \sqrt{0.9^2+0.9^2+11.3^2+11.3^2})$

$N(\mu = 186, \sigma = 16.03)$

$P(182 \le L \le 192) = P\left(\frac{182-186}{16.03} \le \frac{L-\mu}{\sigma} \le \frac{192-186}{16.03}\right)$

$= P(-0.25 \le Z \le 0.37) = 0.6443 - 0.4013 = 0.2430.$ (?)

14. The heights of 1000 students are normally distributed with a mean of 174.5 cm and a standard deviation of 6.9 cm. Assuming that the heights are recorded to the nearest half-centimeter, how many of these students would you expect to have heights

(a) less than 160.0 cm?

(b) between 171.5 and 182.0 cm?

(c) exactly 175.0 cm?

(d) What is the probability that the average height of 3 students is between 172.0 and 175.0 cm?

X = height of student $X \sim N(\mu = 174.5, \sigma = 6.9)$

(a) $P(X < 159.75) = P\left(\frac{X-\mu}{\sigma} < \frac{159.75-174.5}{6.9}\right) = P(Z < -2.14)$

$(159.75, 160.25) \to 160$.

$= 0.0162$

$0.0162 \times 1000 \approx 16$

$(171.25, 171.75)$ $(181.75, 182.25)$

(b) $P(171.25 < X < 182.25) = P\left(\frac{171.25-174.5}{6.9} < \frac{X-\mu}{\sigma} < \frac{182.25-174.5}{6.9}\right)$

$= P(-0.47 < Z < 1.12) = 0.8686 - 0.3192 = 0.5496.$

$1000 \times 0.5494 \approx 550$

(c) $P(174.75 < X < 175.25) = P\left(\frac{174.75-174.5}{6.9} < \frac{X-\mu}{\sigma} < \frac{175.25-174.5}{6.9}\right)$

$= P(0.04 < Z < 0.11) = 0.5438 - 0.5160 = 0.0278$

$1000 \times 0.0278 \approx 28$ $\sqrt{\sigma^2+\sigma^2+\sigma^2} = \sqrt{3\sigma^2}$ $\sqrt{3}\cdot\sigma$

(d) X_1, X_2, X_3 : heights of students

$L = X_1 + X_2 + X_3 \sim N(3\mu, \sqrt{3}\sigma)$

$V(\frac{1}{3}L) = \frac{1}{9}\cdot V(L) = \frac{1}{9}\times 3\sigma^2 = \frac{1}{3}\sigma^2$

$\frac{1}{3} L \sim N(\mu, \frac{\sigma}{\sqrt{3}}) = N(174.5, 3.984)$
$= A$

$$P(172.0 < A < 175.0) = P\left(\frac{172-174.5}{3.984} < \frac{A-\mu_A}{\sigma_A} < \frac{175-174.5}{3.984}\right)$$
$$= P(-0.63 < Z < 0.13) = 0.5517 - 0.2643 = 0.2874.$$

15. Let $X \sim B(n, p)$ with $n = 15$ and $p = 0.4$. Calculate $P(1 \leq X \leq 4)$

(a) exactly (so using the binomial distribution (table A1));

(b) using the normal approximation.

(a) $P(1 \leq X \leq 4) = P(X \leq 4) - P(X \leq 0) = 0.2173 - 0.0005 = 0.2168$

(b) $X \sim B(n=15, p=0.4) \approx N(\mu = 15 \times 0.4 = 6, \sigma = \sqrt{15 \times 0.4 \times 0.6} \approx 1.897)$

continuity correction

$$P(1 \leq X \leq 4) = P(0.5 \leq X \leq 4.5) = P\left(\frac{0.5-6}{1.897} \leq \frac{X-\mu}{\sigma} \leq \frac{4.5-6}{1.897}\right)$$
$$= P(-2.90 \leq Z \leq -0.79) = 0.2148 - 0.0019 = 0.2129.$$

16. A coin is tossed 400 times. Find the probability of obtaining

(a) between 185 and 210 heads inclusive;

(b) exactly 205 heads;

(c) less than 176 or more than 227 heads.

X = # heads (success) in $n=400$ toss (experiment)

$P(\text{"S"}) = 0.5 \quad X \sim B(n=400, p=0.5)$

$X \sim B(n=400, p=0.5)$
$\approx N(\mu = 400 \times 0.5 = 200, \sigma = \sqrt{400 \times 0.5 \times 0.5} = 10)$

(a) $P(185 \le X \le 210) = P(184.5 \le X \le 210.5)$

$= P\left(\frac{184.5-200}{10} \le \frac{X-\mu}{\sigma} \le \frac{210.5-200}{10}\right) = P(-1.55 \le Z \le 1.05)$

$= 0.8531 - 0.0606 = 0.7925$

(b) $P(X=205) = P(204.5 < X < 205.5)$

$= P\left(\frac{204.5-200}{10} < Z < \frac{205.5-200}{10}\right) = P(0.45 < Z < 0.55)$

$= 0.7088 - 0.6736 = 0.0352$

(c) $P(X<176 \cup X>227) = P(X \le 175) + 1 - P(X \le 227)$

continuity correction $= P(X \le 175.5) + 1 - P(X \le 227.5)$

$= 1 + P\left(Z \le \frac{175.5-200}{10}\right) - P\left(Z \le \frac{227.5-200}{10}\right)$

$= 1 + P(Z \le -2.45) - P(Z \le 2.75)$

$= 1 + 0.0071 - 0.9970 = 0.0101$

17. A pair of dice is rolled 180 times. What is the probability that a total of 7 occurs

(a) at least 25 times?

(b) between 33 and 41 times inclusive?

(c) exactly 30 times?

1,6
2,5
3,4
4,3
5,2
6,1

$P(\text{"get total of 7"}) = P(\text{"S"}) = \frac{6}{36} = \frac{1}{6}.$

$X = \#$ "get total of 7" in $n=180$ trials.

$X \sim B\left(n=180, p=\frac{1}{6}\right) \approx N\left(\mu=30, \sigma=\sqrt{180 \times \frac{1}{6} \times \frac{5}{6}} = 5\right)$

(a) $P(X \ge 25) = P(X \ge 24.5) = P\left(Z \ge \frac{24.5-30}{5}\right) = P(Z \ge -1.1)$

continuity correction.

$= 1 - P(Z \le -1.1) = 1 - 0.1357 = 0.8643$

(b) $P(33 \le X \le 41) = P(32.5 \le X \le 41.5)$

$= P\left(\frac{32.5-30}{5} \le Z \le \frac{41.5-30}{5}\right) = P(0.5 \le Z \le 2.3)$

$$= 0.9893 - 0.6915 = 0.2978$$

(c) $P(X=30) = P(29.5 \le X \le 30.5) = P\left(\frac{29.5-30}{5} \le Z \le \frac{30.5-30}{5}\right)$

$= P(-0.1 \le Z \le 0.1) = 0.5398 - 0.4602 = 0.0796$

18. The probability that a patient recovers from a delicate heart operation is 0.9. Of the next 100 patients having this operation, what is the probability that

(a) between 84 and 95 inclusive survive?

(b) fewer than 86 survive?

X = # patients recover from the operation.

$X \sim B(n=100, P=0.9)$

$\approx N(\mu = 100 \times 0.9 = 90, \sigma = \sqrt{100 \times 0.9 \times 0.1} = 3)$

continuity correction.

(a) $P(84 \le X \le 95) = P(83.5 \le X \le 95.5)$

$= P\left(\frac{83.5-90}{3} \le Z \le \frac{95.5-90}{3}\right) = P(-2.17 \le Z \le 1.83)$

$= 0.9664 - 0.0150 = 0.9514$

continuity correction.

(b) $P(X<86) = P(X \le 85) = P(X \le 85.5) = P\left(Z \le \frac{85.5-90}{3}\right)$

$= P(Z \le -1.5) = 0.0668$

19. A pharmaceutical company knows that approximately 5% of its birth-control pills have an ingredient that is below the minimum strength, thus rendering the pill ineffective. What is the probability that fewer than 10 in a sample of 200 pills will be effective? Calculate this probability using

(a) the Poisson approximation of the binomial;

(b) the normal approximation of the binomial.

X = # ineffective pill in n=200 pills

$X \sim B(n=200, P=0.05)$

(a) $X \sim \mathbb{P}(10)$

$P(X<10) = P(X \leq 9) = 0.4579$

(b) $X \sim N(\mu = 10, \sigma = \sqrt{200 \times 0.05 \times 0.95} = 3.082$

continuity correction

$P(X<10) = P(X \leq 9) \doteq P(X \leq 9.5) = P(Z \leq \frac{9.5-10}{3.082} = -0.16)$

$= 0.4364$

20. The serum cholesterol level in 14-year-old boys has approximately a normal distribution with mean 170 and standard deviation 30.

(a) Find the probability that the serum cholesterol level of a randomly selected 14-year-old boy exceeds 230.

(b) In a middle school there are 300 14-year-old boys. Find the probability that at least 8 of them have a serum cholesterol level that exceeds 230.

X = serum cholesterol level in 14-year-old boys

$X \sim N(\mu = 170, \sigma = 30)$

(a) $P(X > 230) = P(Z > \frac{230-170}{30} = 2)$

$= 1 - P(Z \leq 2) = 1 - 0.9772 = 0.0228$

(b) Y = # boys whose serum cholesterol level > 230

$P(\text{"S"}) = 0.0228 \qquad n = 300$

Can Not Find in Table

$Y \sim B(n=300, P=0.0228) \approx \mathbb{P}(6.84)$

Use Normal Approximation : $\approx N(6.84, 2.585 = \sqrt{300 \times 0.0228 \times 0.9772})$

continuity correction

$P(Y \geq 8) \doteq P(Y \geq 7.5) = P(Z \geq \frac{7.5-6.84}{2.585}) = P(Z \geq 0.26)$

$= 1 - P(Z \leq 0.26) = 1 - 0.6026 = 0.3974$

21. A random sample of twenty light bulbs is selected. These bulbs appear to have life times of

1.2, ~~0.9~~, ~~0.0~~, 1.0, 1.3, ~~0.8~~, ~~0.0~~, 1.0, 0.9, ~~0.5~~,
1.1, ~~0.7~~, ~~0.0~~, 1.5, 1.1, 0.9, ~~0.8~~, 1.0, 0.9 and ~~0.1~~

years respectively. Calculate (a) the sample mean $\overline{x}$, (b) the sample median, (c) the sample mode, (d) the sample variance s^2 (please look up the formula), (e) the sample standard deviation s and (f) the sample range.

(a) $\bar{X} = \frac{1}{20}(1.2+0.9+\cdots+0.1) = \frac{1}{20}\times 15.7 = 0.785$

(b) Median: $\frac{1}{2}(0.9+0.9) = 0.9$

(c) Mode: 0.9

(d) $s^2 = \frac{1}{20-1}\cdot\sum_{i=1}^{20}(X_i-\bar{X})^2 = \frac{1}{19}\left((1.2-0.785)^2+\cdots+(0.1-0.785)^2\right) = 0.204$

(e) $s = \sqrt{s^2} = 0.451$

(f) range $= 1.5 - 0.0 = 1.5$