Xi'an Jiaotong-Liverpool University
西交利物浦大学

UNIVERSITY OF LIVERPOOL

PhD Viva · Aug 2025

Robust Cardiac Feature Monitoring based on Millimeter-Wave Radar

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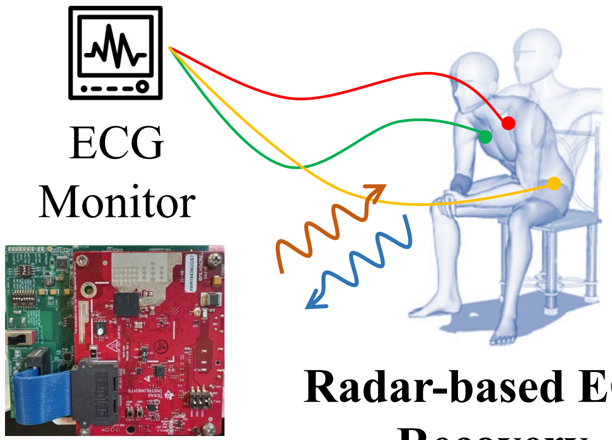
Co-Supervisor: Prof. Yi Huang

Prof. Steven Guan

Dr. Yutao Yue

Contents

Research Background – Radar-Based Sensing



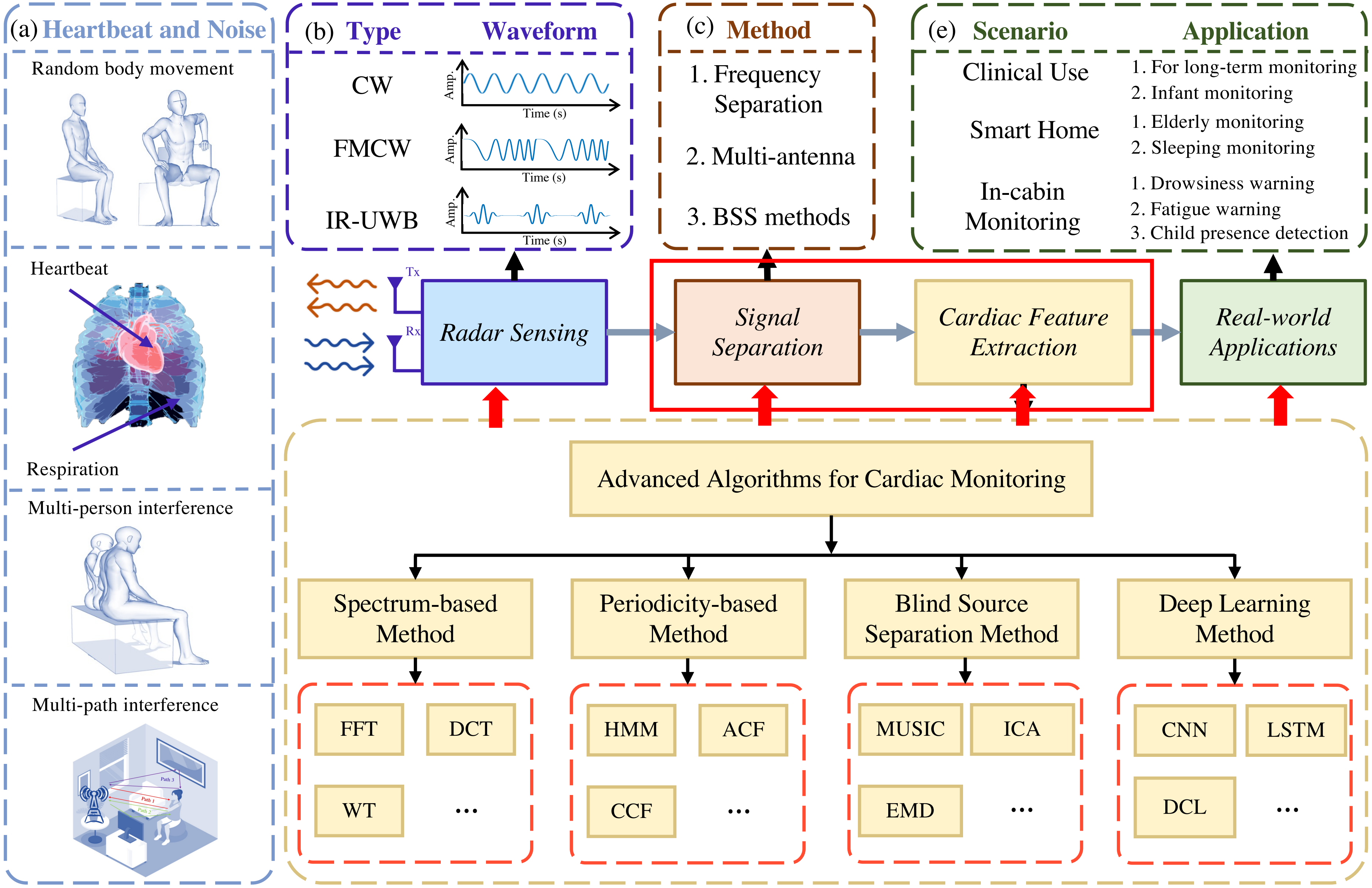
Why we need contactless cardiac features monitoring?

1. Suitable for long-term monitoring or special patients (e.g., burn patients)
2. Suitable for in-cabin monitoring or smart home (e.g., healthcare scenarios)

Why we use radar as the contactless sensor?

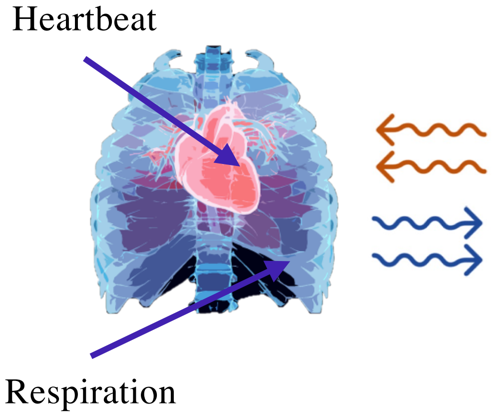
Sensors	Cameras	Acoustic Sensors	Wi-Fi Routers	Radars
Pros	1. SOTA. CV algorithms or DL frameworks	1. Directly monitor the heart sound	1. No need for extra devices 2. Unobtrusive	1. Unobtrusive 2. Contactless 3. Robustness
Cons	1. Privacy issue 2. Rely on high quality image.	1. Low accuracy 2. Vulnerable to acoustic noise	1. Low accuracy 2. Complex EM environment	1. Require special signal processing design 2. Less investigated

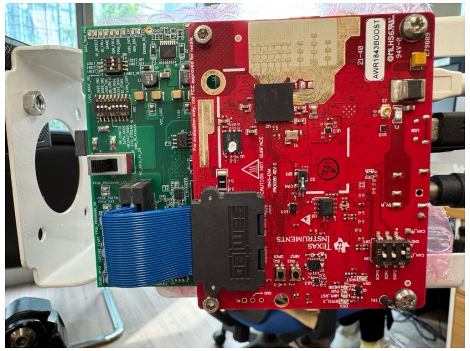
Background | Research Background – General Pipeline

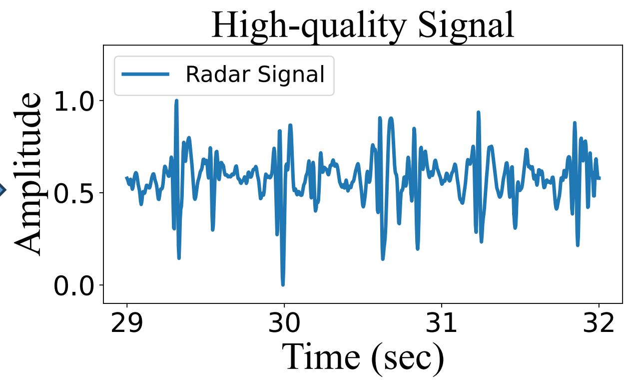


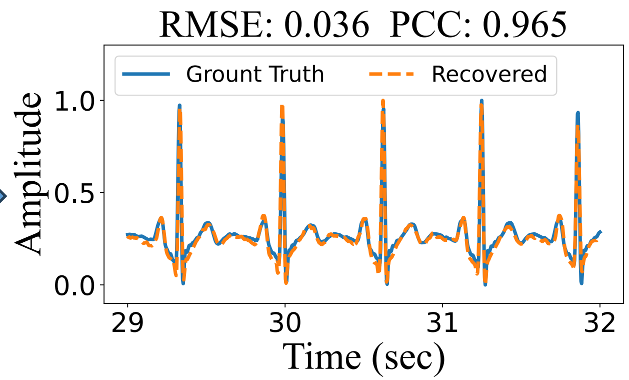
Background | Research Gaps and Contributions – Research Gaps

Illustration for radar-based ECG recovery :









Vital Sign | Radar | Signal Processing | ECG Recovery

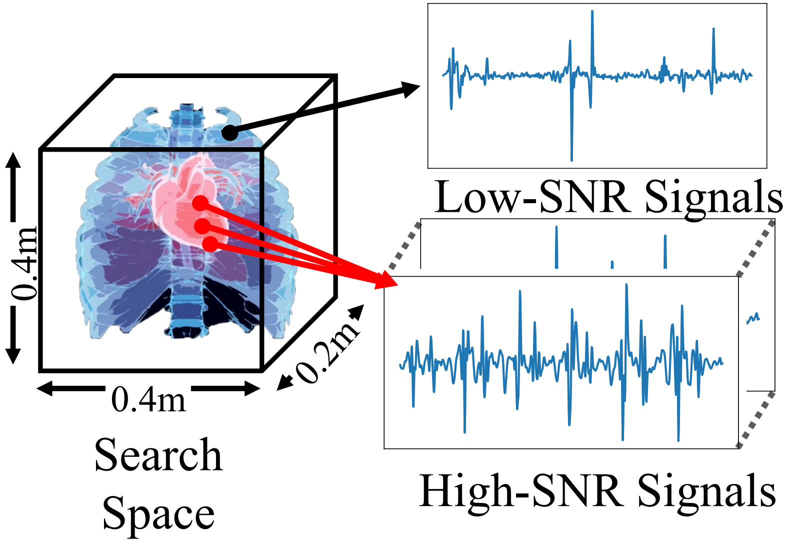
Research gap in radar-based ECG recovery:

1. Design efficient algorithm for high-SNR signal collection.
2. Model and realize the domain transformation from radar to ECG for fine-grained cardiac features.
3. Realize long-term ECG recovery while mitigating the real-world noise (e.g., body movement, constant noise)
4. Alleviate data scarcity for the deployment in new scenarios with limited data.

Research Gaps and Contributions – Contributions

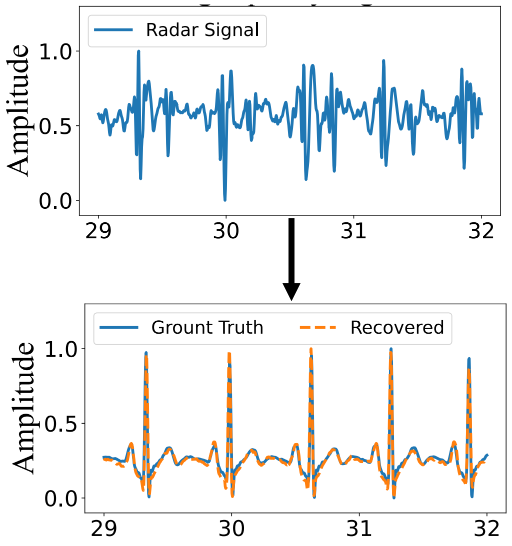
Contributions:

1. Thoroughly review the advanced methods for radar-based cardiac feature extraction (Chapter 2, published on TIM).
2. Propose a derivative-free optimization method to iteratively approach the point with ample cardiac features.
3. Design a signal model to describe the relationship between cardiac activities from different domains (radar to ECG).
4. Design the ODE decoder and multi-task learning framework to realize ECG recovery and resist constant/abrupt noises.
5. Propose an augmentation method and a transfer learning framework to alleviate data scarcity.



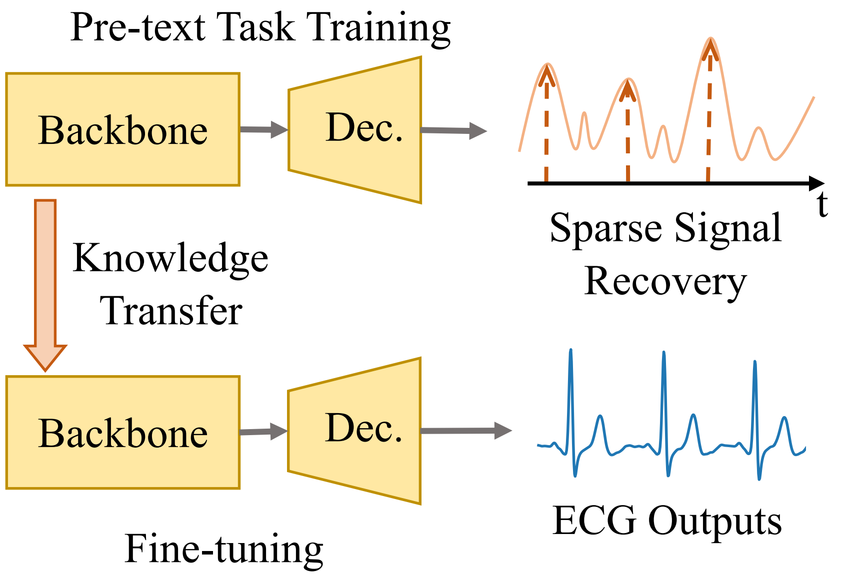
Chapter 3

Submitted to TMC



Chapter 4 and 5

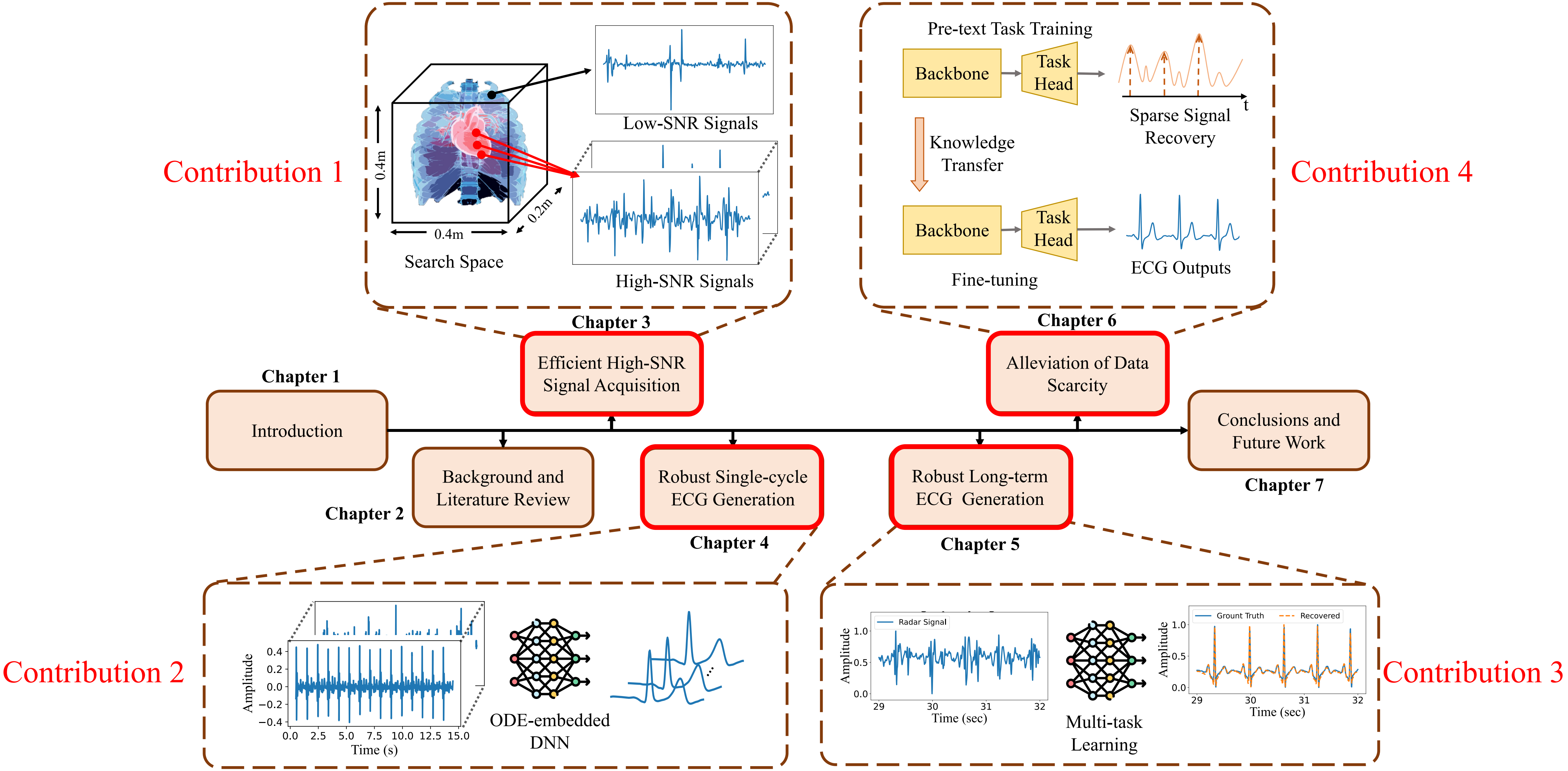
Published in TMC and TIM



Chapter 6

Published in EMBC 2025

Background | Research Gaps and Contributions – Contributions



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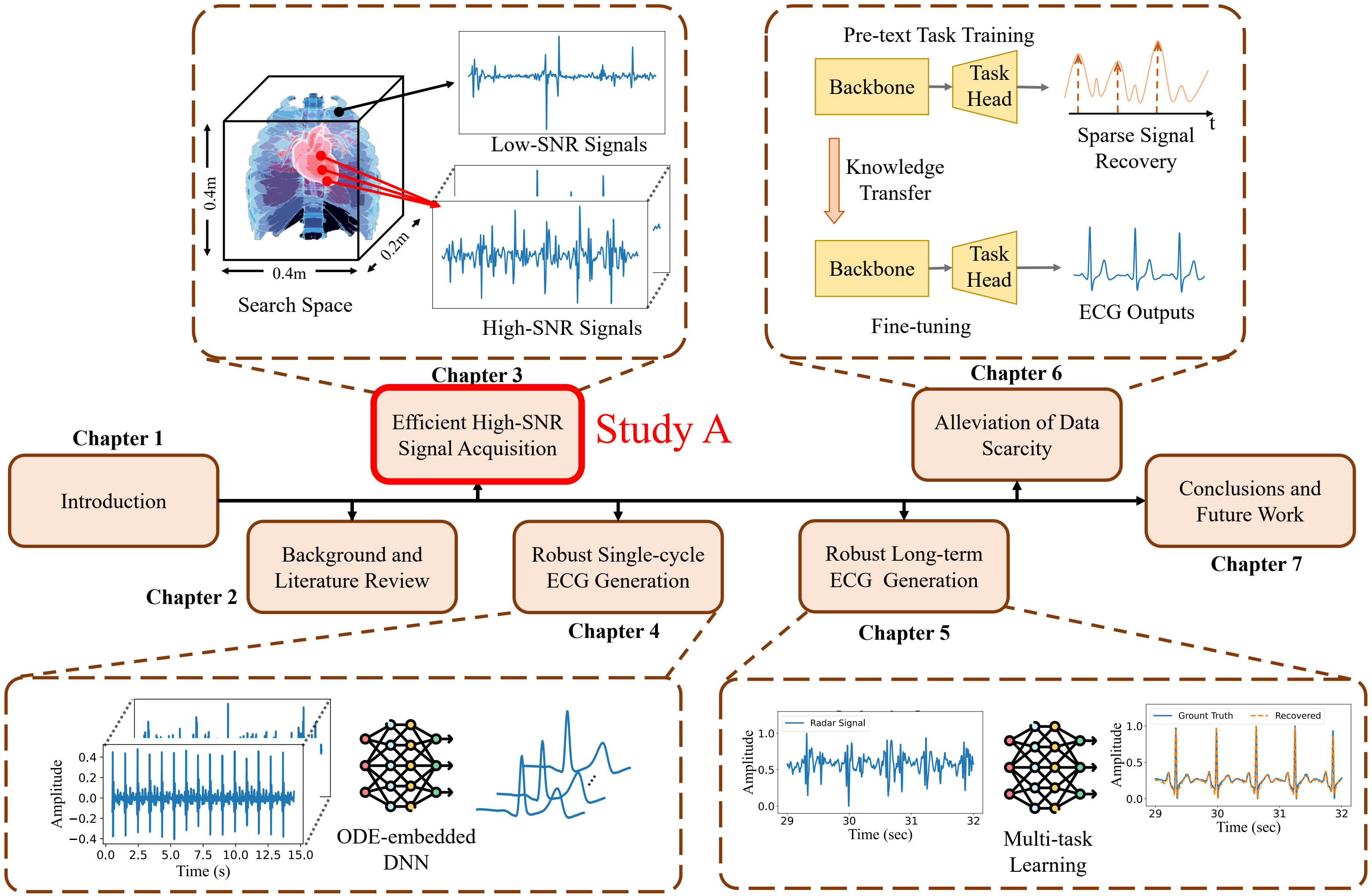
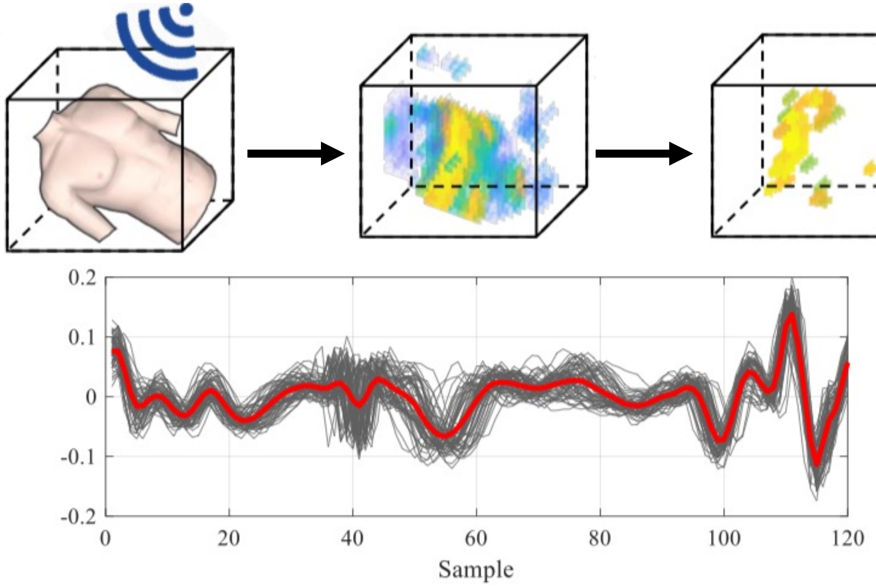
Part 2 – Proposed Methods



Study A ● ○ ○ ○ ○ | Efficient High-SNR Signal Acquisition - Problems

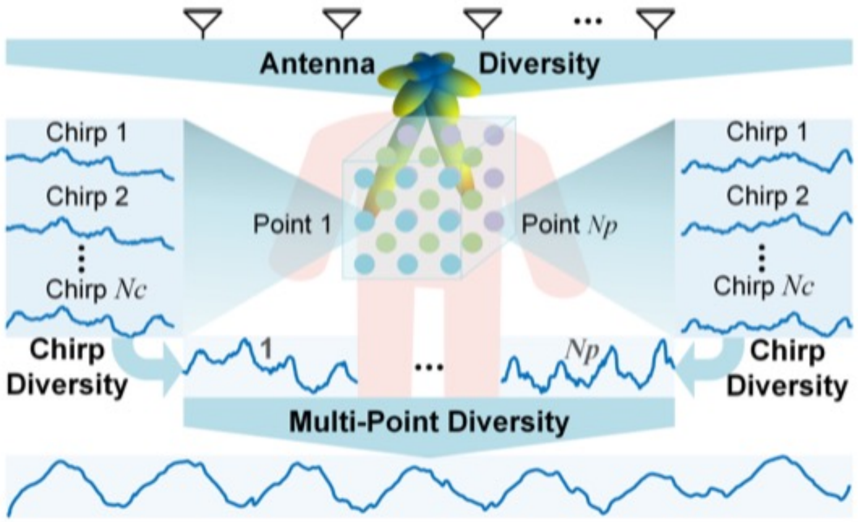
Clustering with exhaustive calculations [1]



Limitations:

1. Time consuming
2. Rely on precise body localization
3. Not suitable for long distance

Signal accumulation for noise suppression [2]



Limitations:

1. Rely on precise body localization
2. Could amplify non-gaussian noise
3. Only used for coarse vital sign, e.g., heartbeat monitoring

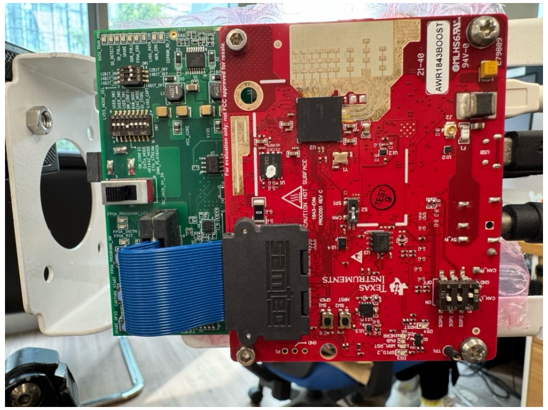
[1] J. Chen, D. Zhang, Z. Wu, F. Zhou, Q. Sun, and Y. Chen, "Contactless electrocardiogram monitoring with millimeter wave radar," IEEE Transactions on Mobile Computing, Dec. 2022.
[2] J. Liu, J. Wang, Q. Gao, X. Li, M. Pan, and Y. Fang, "Diversity-enhanced robust device-free vital signs monitoring using mmWave signals," IEEE Transactions on Mobile Computing, Jun. 2024.

Efficient High-SNR Signal Acquisition - Contributions

Contributions:

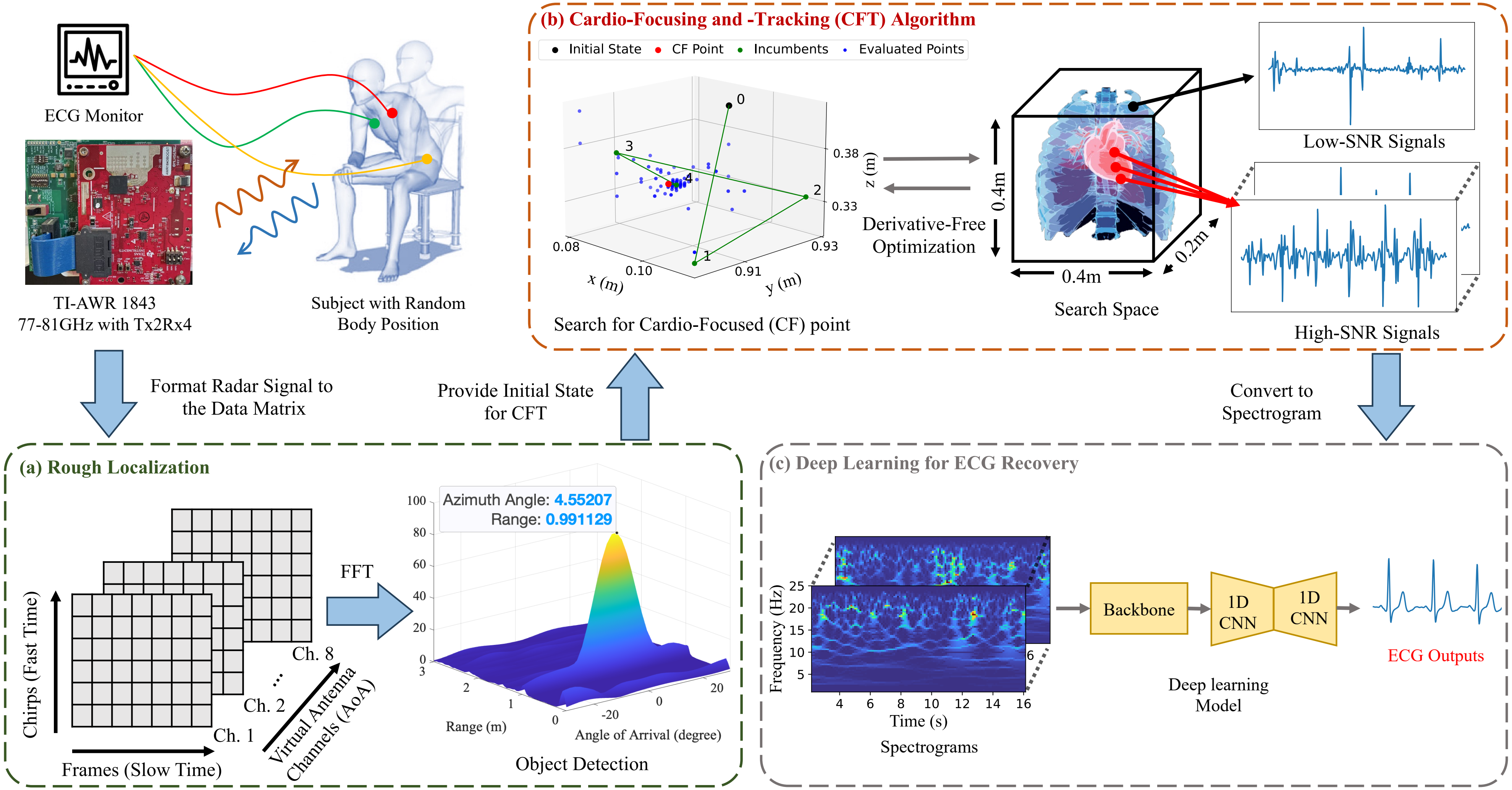
1. Propose cardio-focusing and -tracking (CFT) algorithm to find cardio-focused (CF) points with clear cardiac features from a discontinuous objective space.
2. Provide a faithful approach to assessing SNR without knowing the ground truth, avoiding exhaustive calculations
3. The proposed CFT algorithm has been validated on sitting subjects in various scenarios and could provide radar measurements with better SNR compared with existing methods.

Data Collection scenarios



TI AWR-1843 @77GHz
with 3.8GHz bandwidth

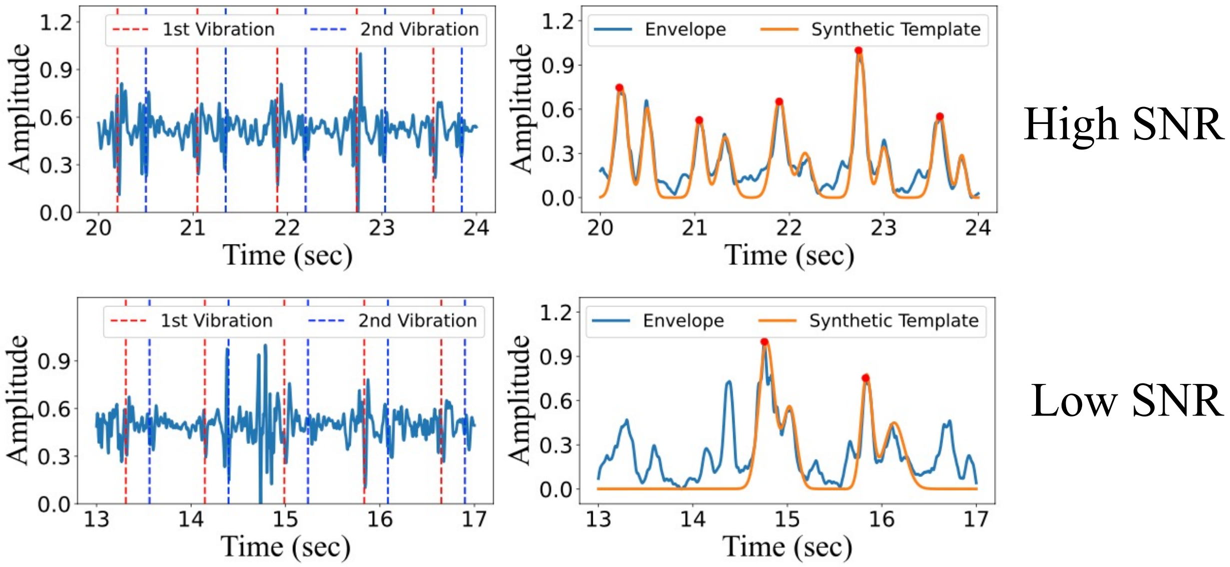
Efficient High-SNR Signal Acquisition – Overview



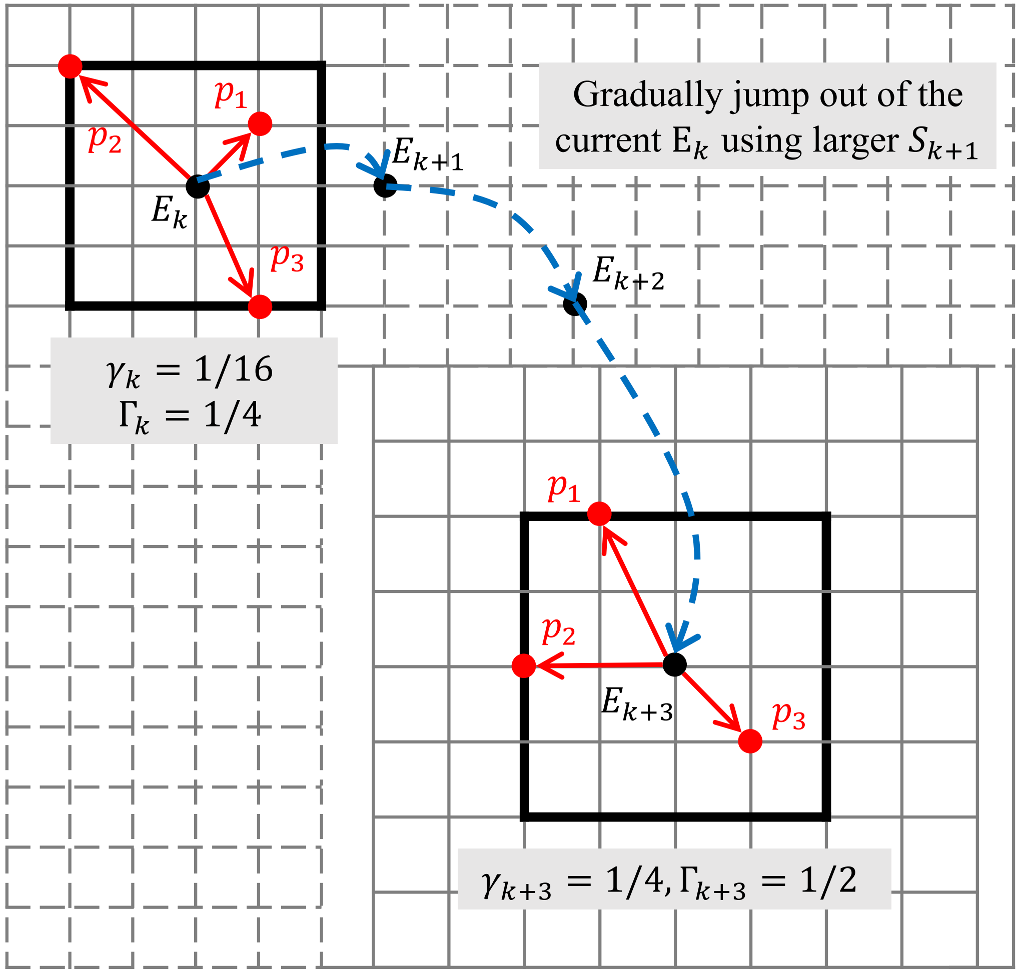
Efficient High-SNR Signal Acquisition – CFT

Advantages of CFT:

1. Provide a faithful approach to assessing SNR without knowing the ground truth, avoiding exhaustive calculations.
2. Suitable for the highly discontinuous objective space and could break local minima by restricting the search space.



Assess signal SNR with universal template

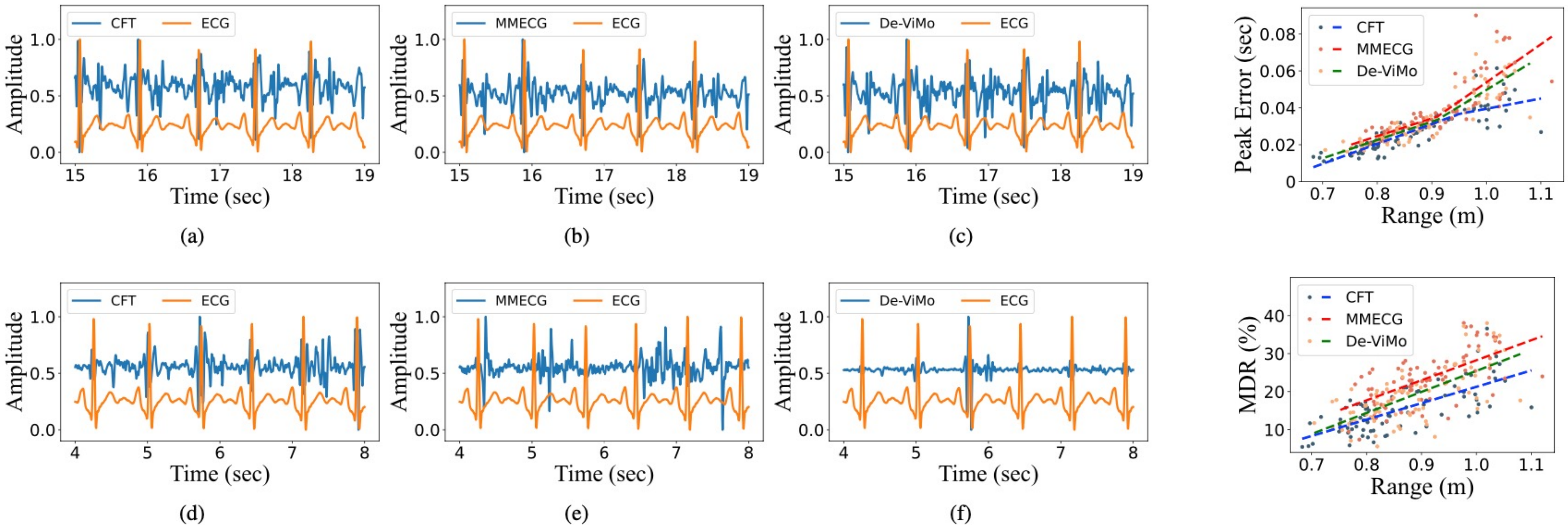


Bold search region based on the spanning grids

Study A | Efficient High-SNR Signal Acquisition – Results

Rough Loc. ≈ CF Point

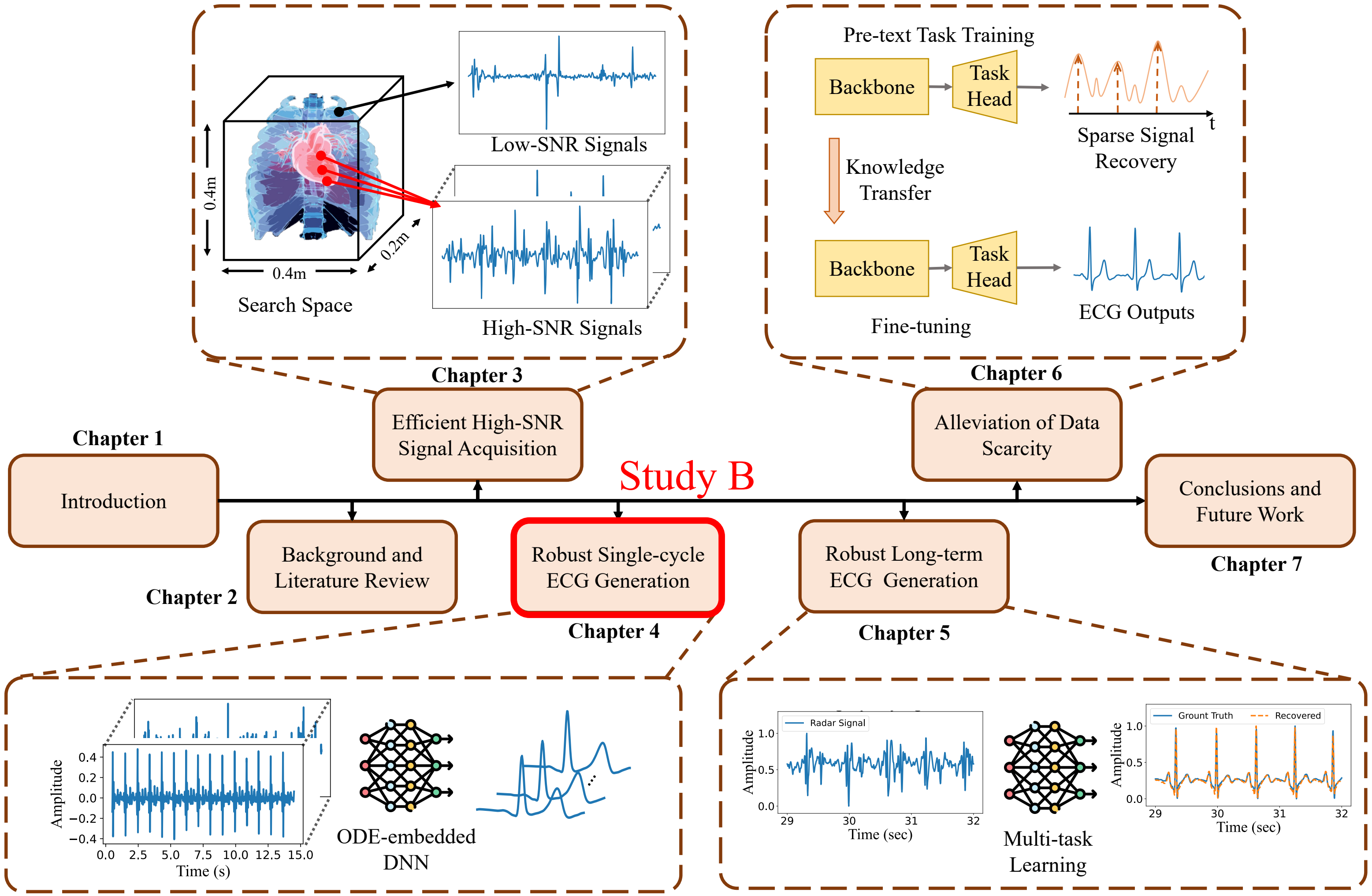
Rough Loc. ≠ CF Point



Remarks:

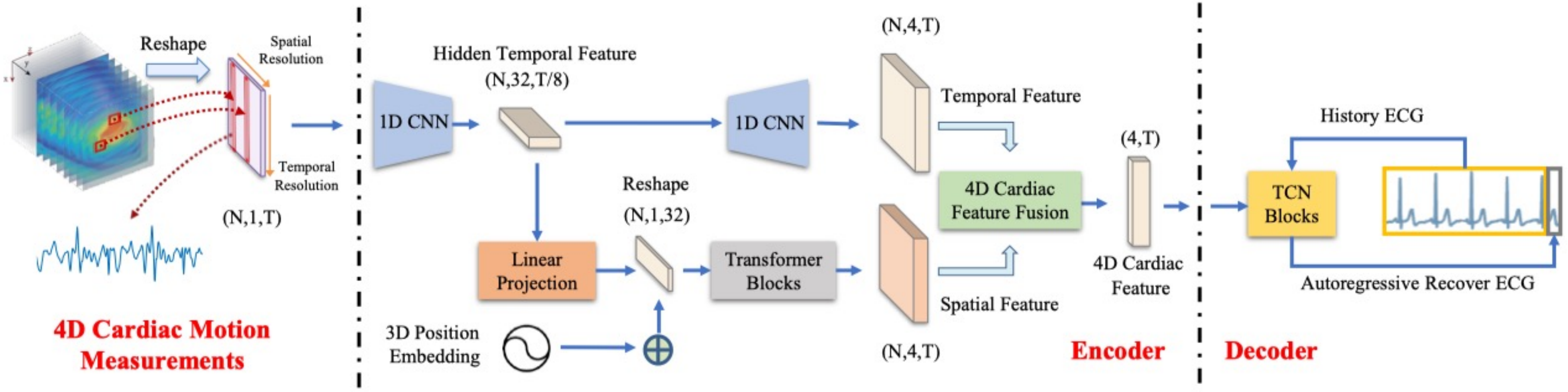
1. Clustering- or accumulation-based method rely on a good initial estimation, but the FMCW signal processing could only provide rough body location.
2. The body posture is subject to change during data collection, causing a deviation of CF point by several decimeters. CFT could track the CF point by reactivating the optimization if SNR is low.

Part 2 – Proposed Methods

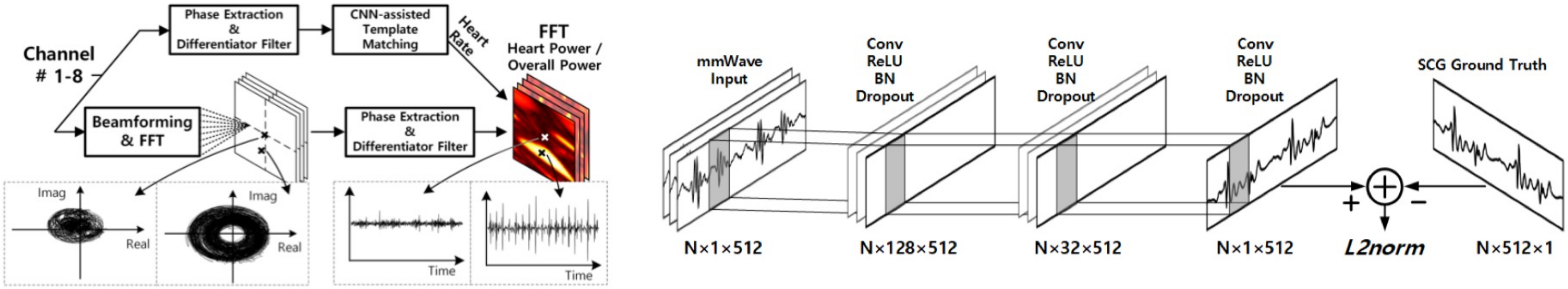


Robust Single-cycle ECG Generation – Existing Methods

[1]



[3]

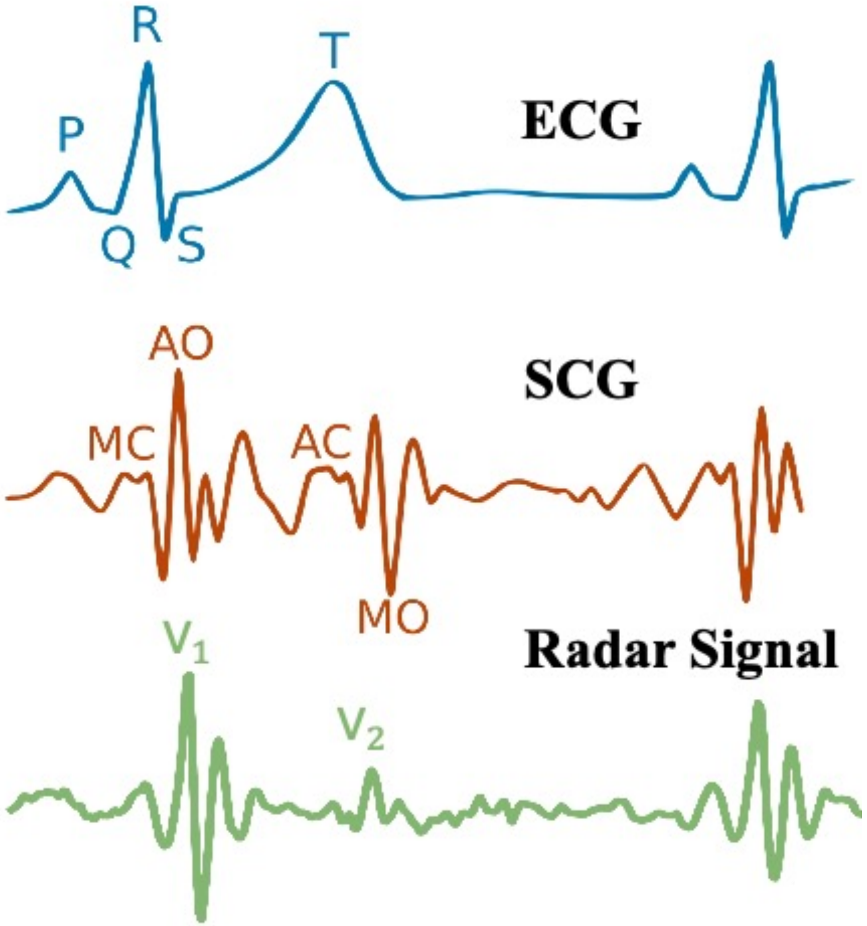


[1] J. Chen, D. Zhang, Z. Wu, F. Zhou, Q. Sun, and Y. Chen, “Contactless electrocardiogram monitoring with millimeter wave radar,” IEEE Transactions on Mobile Computing, pp. 1–17, Oct. 2022.
[3] U. Ha, S. Assana, and F. Adib, “Contactless seismocardiography via deep learning radars,” in Proceedings of the 26th Annual International Conference on Mobile Computing and Networking (MobiCom), pp. 114, Apr. 2020.

Robust Single-cycle ECG Generation – Preliminary

Domain transformation of cardiac features:

1. ECG Feature: P-wave, QRS-complex, T-wave for electrical activities of atrial/ventricular depolarization/repolarization.
2. SCG Features: excitation–contraction coupling caused valve opening/closure (AO/AC) and mitral valve opening/closure (MO/MC)
3. Radar Feature: Similar with SCG but only with two prominent vibrations v1 and v2 induced by AO and AC.
4. Heart muscle contraction has a pulsatile nature, and the bones/tissues in chest area introduce the extra damping into the pulse [4], [5].

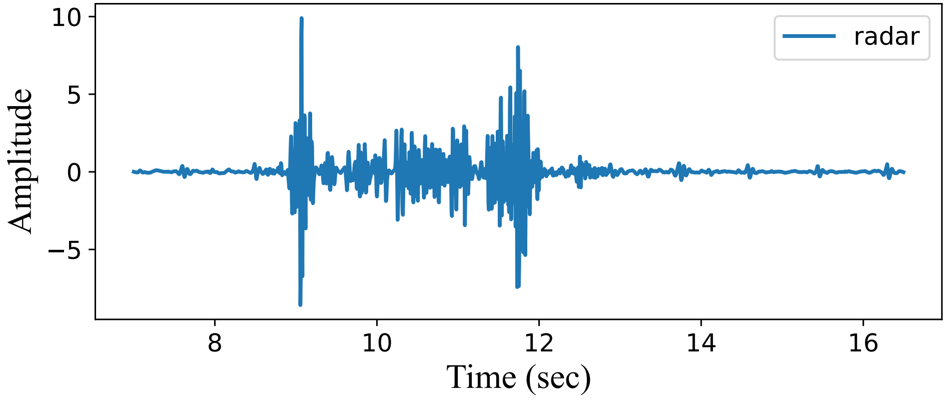


[4] M. Nosrati and N. Tavassolian, “High-accuracy heart rate variability monitoring using Doppler radar based on Gaussian pulse train modeling and FTPR algorithm,” IEEE Transactions on Microwave Theory and Techniques, vol. 66, no. 1, pp. 556–567, Jan. 2017.
[5] D. R. Morgan and M. G. Zierdt, “Novel signal processing techniques for doppler radar cardiopulmonary sensing,” Signal Processing, vol. 89, no. 1, pp. 45–66, Jan. 2009.

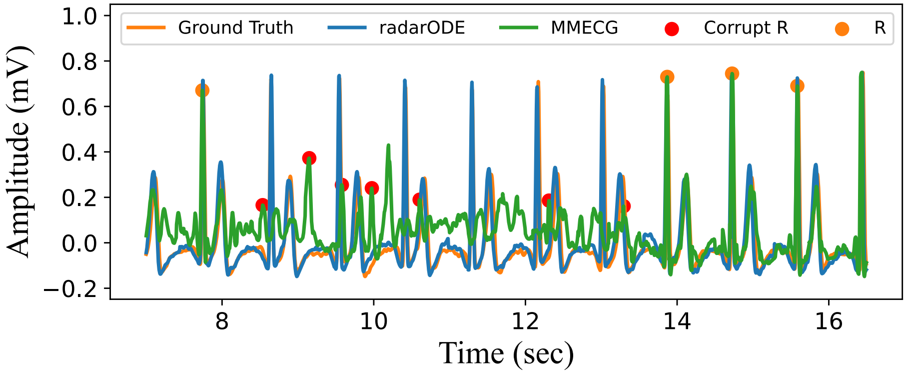
Study B ○ ○ ● ○ ○ ○ Robust Single-cycle ECG Generation – Contributions

Contributions:

1. Propose the signal model in terms of fine-grained cardiac features.
2. Design an ODE-embedded module called single-cycle ECG generator (SCEG) to parameterize the radar signal into sparse representations with morphological meanings.
3. By fusing the extracted morphological and temporal features, the proposed radarODE framework is proven to be robust in the presence of body movement noise and could realize accurate reconstruction of the ECG signal.



Distorted radar signal



Recovered ECG signal

Robust Single-cycle ECG Generation – Signal Model

Proposed signal model:

Based on the previous intuition, we model the cardiac vibration $\tilde{x}(t)$ as two prominent vibrations and other noises as:

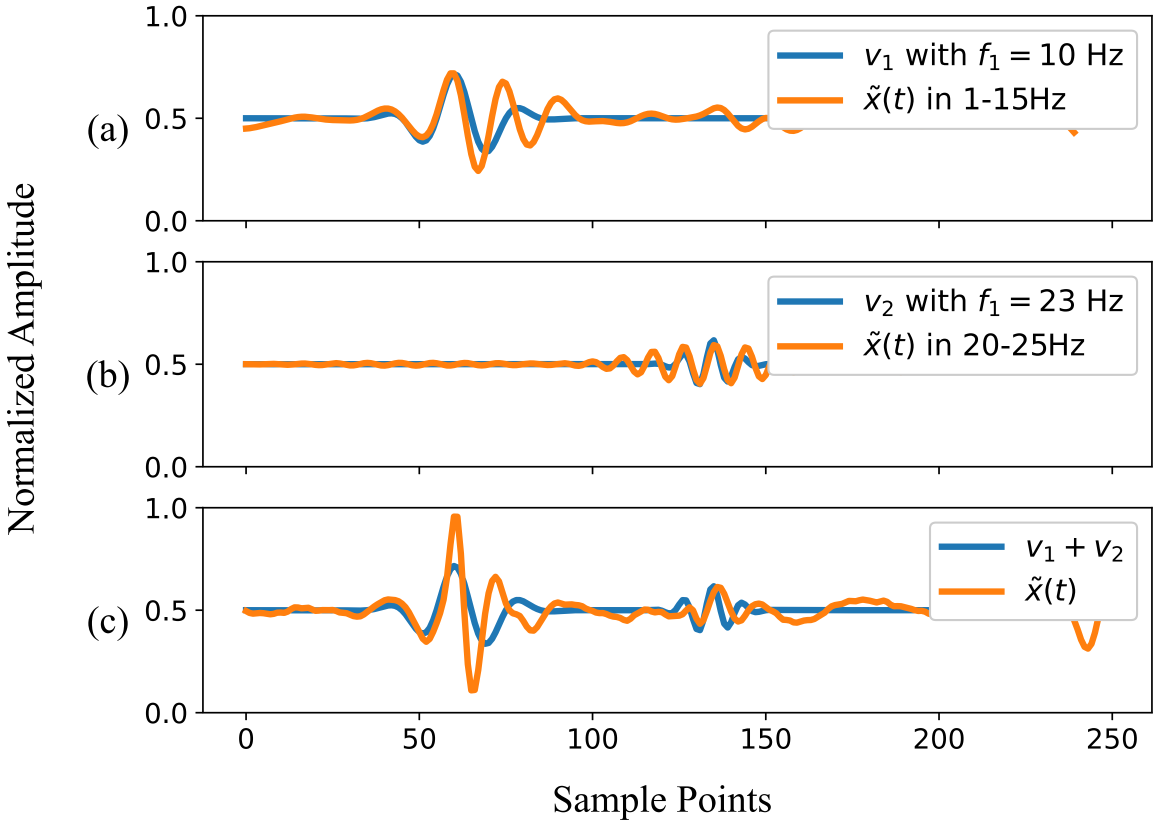
$$\tilde{x}(t) = v_1(t) + v_2(t) + x_n(t)$$

with

$$v_1 = \mathrm{a}_1 \cos(2\pi f_1 t) \exp\left(-\frac{(t-T_1)^2}{{b_1}^2}\right)$$

$$v_2 = \mathrm{a}_2 \cos(2\pi f_2 t) \exp\left(-\frac{(t-T_2)^2}{{b_1}^2}\right)$$

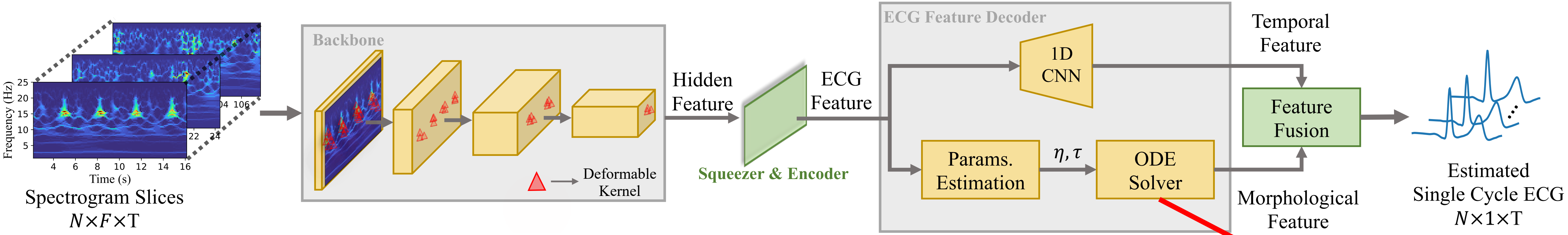
a_1, a_2 → amplitude of the vibrations,
b_1, b_2 → length/width of the vibration,
f_1, f_2 → central frequency of the vibration,
T_1, T_2 → when the vibrations happen.



Synthetic radar signal

Robust Single-cycle ECG Generation – radarODE

Single Cycle ECG Generator (SCEG):



The ODE model describes a $\boxed{\text{3D trajectory}}$ denoted by (x, y, z) [6]

$$\frac{dx}{dt} = \alpha(x,y)x - \omega y$$

$$\frac{dy}{dt} = \alpha(x,y)y + \omega x$$

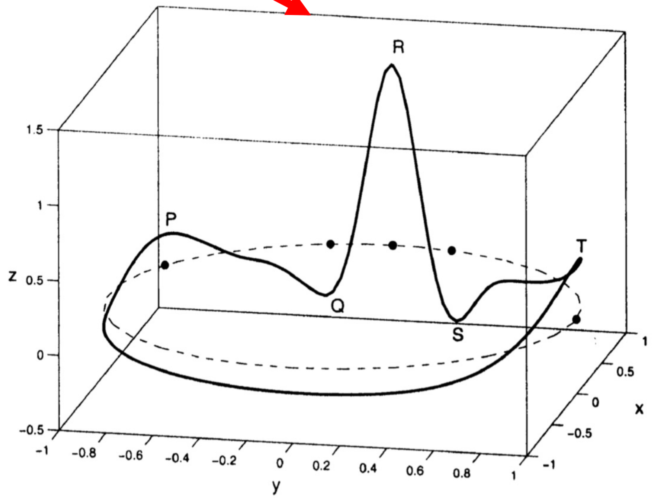
$$\frac{dz}{dt} = -\sum_{e_f \in \mathcal{F}} a_{e_f} \Delta\theta_{e_f}(x,y) e^{-\Delta\theta_{e_f}(x,y)^2/2b_{e_f}^2} - z$$

$$\alpha(x,y) = 1 - \sqrt{x^2 + y^2}$$

$$\Delta\theta_{e_f}(x,y) = \left(\theta(x,y) - \theta_{e_f}\right) \mod 2\pi$$

$$\theta(x,y) = \operatorname{atan2}(y,x)) \in [-\pi, \pi]$$

$$e_f \in \mathcal{F} = \{e_P, e_Q, e_R, e_S, e_T\}$$

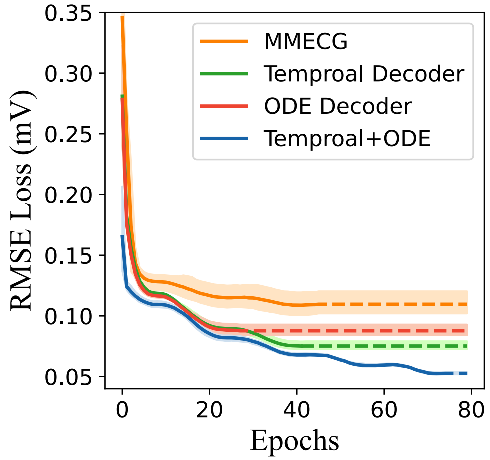


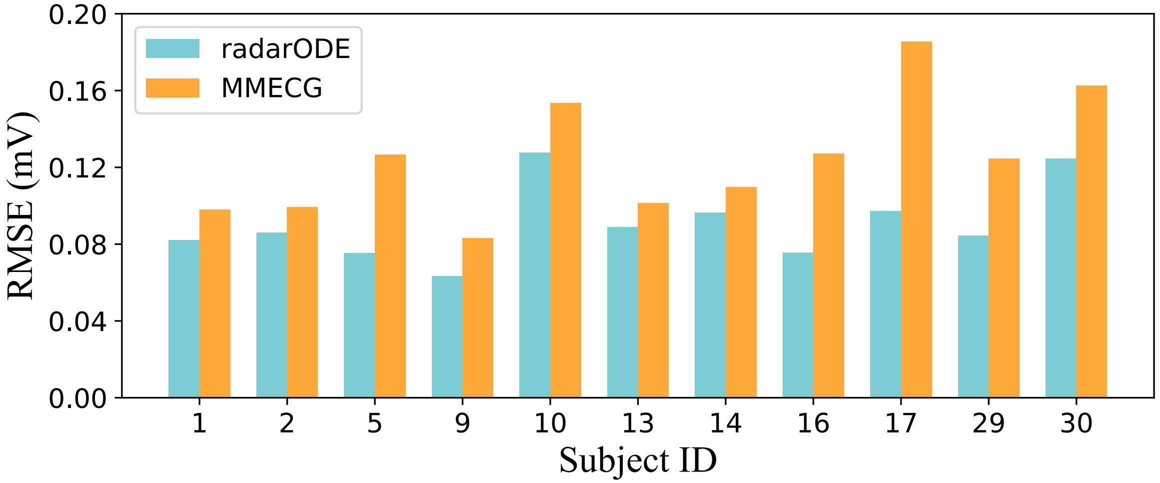
[6] P. E. McSharry, G. D. Clifford, L. Tarassenko, and L. A. Smith, "A dynamical model for generating synthetic electrocardiogram signals," IEEE transactions on biomedical engineering, vol. 50, no. 3, pp. 289–294, Mar. 2003.

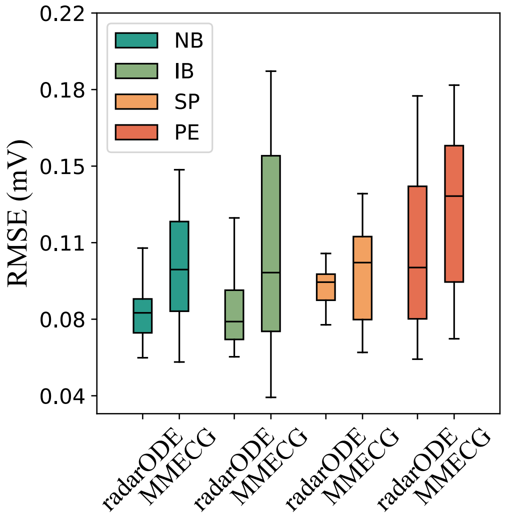
Robust Single-cycle ECG Generation – Results

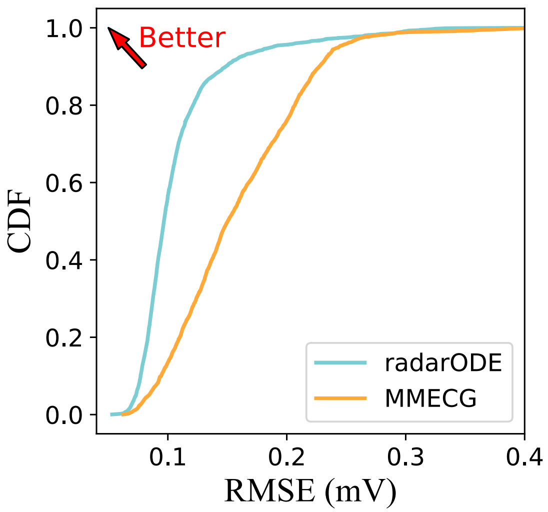
Comparison with benchmark and Ablation study about temporal decoder and ODE decoder (without resampling)

Framework	Backbone	Encoder	Decoder	Fusion Method	RMSE (mV)	Correlation
MMECG [33]	Conv1d + Transformer		Transconv1d + TCN	Multiplication	0.091	87.9%
SCEG	DeformConv2d	Conv1d	Initial + Temporal	-	0.086	89.4%
			Initial + ODE	-	0.092	85.5%
			Initial + Temporal + ODE	Multiplication + Stack + Conv2d	**0.077**	**92.6%**

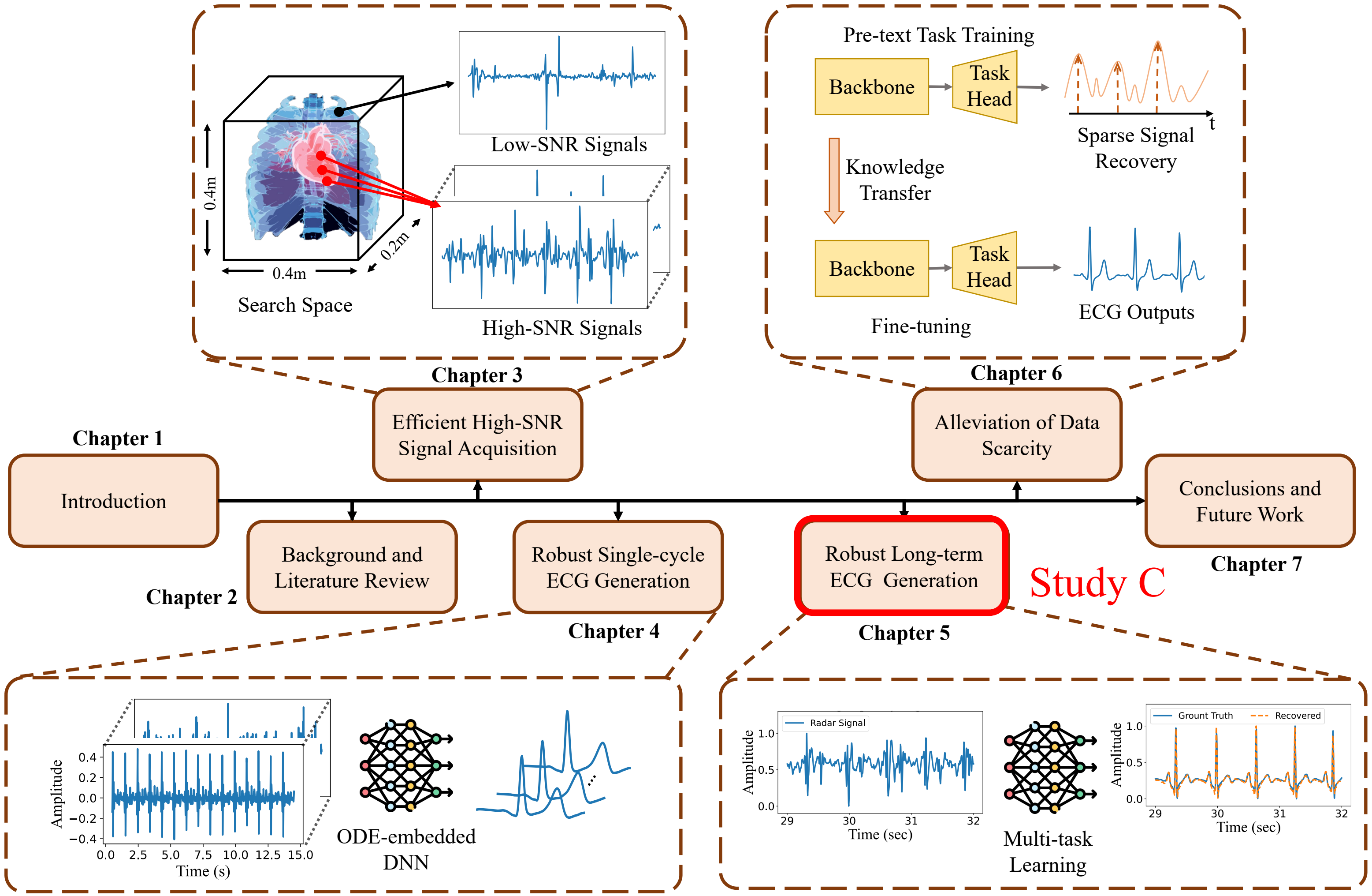








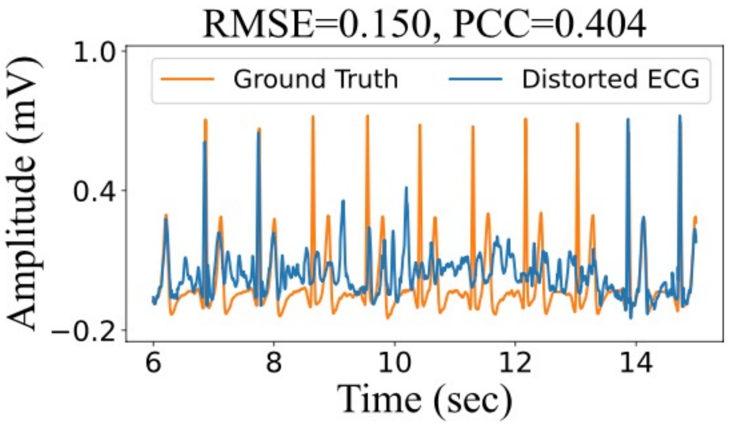
Part 2 – Proposed Methods



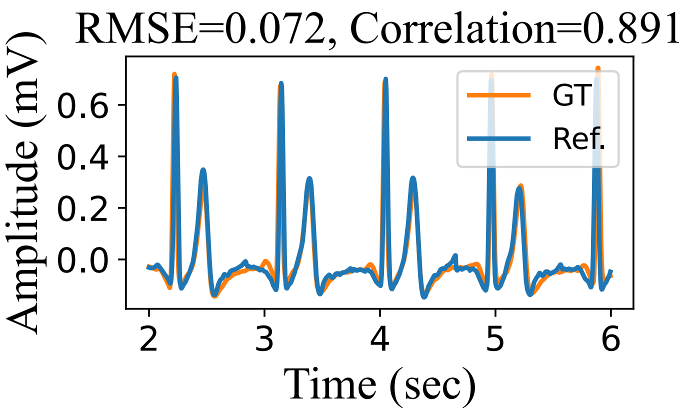
Robust Long-term ECG Generation – Research Gap

Research gap in radar-based ECG recovery:

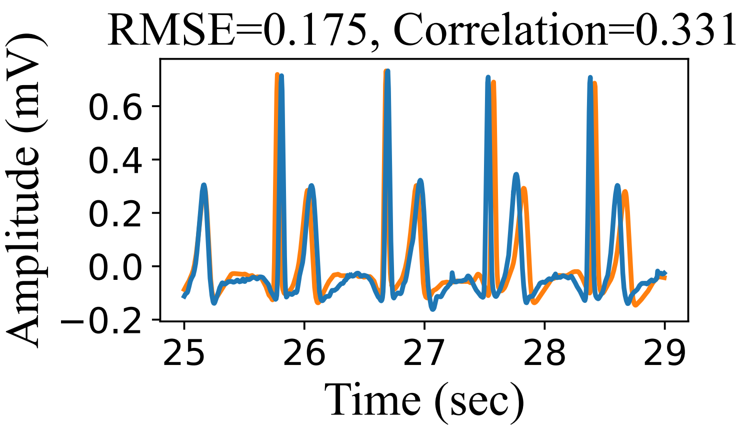
1. The long-term domain transformation (radar to ECG) lacks theoretical explanation.
2. Noise robustness is never considered or validated in ECG recovery, especially for the DL model itself.
3. The misalignment issue in the method based on single cardiac cycle.



Distorted by noise



Well Aligned



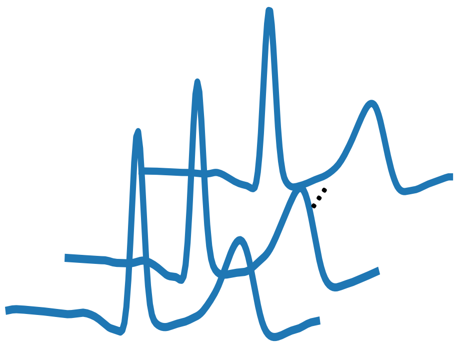
Misaligned

Robust Long-term ECG Generation – Contributions

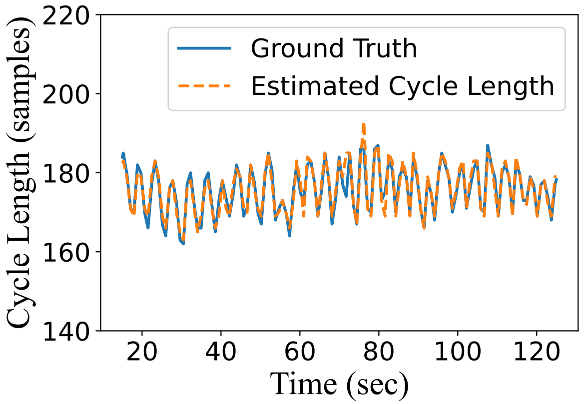
Contributions:

1. Model the radar-based ECG reconstruction into three tasks and design an MTL framework named radarODE-MTL to avoid the reintroduction of the time-domain noises and hence improve the noise robustness.
2. A novel optimization strategy eccentric gradient alignment (EGA) is proposed for MTL learning by eccentrically aligning the task gradients, aiming to adaptively alter the task priority to ensure efficient learning progress for the tasks with disparate complexity while preventing the negative transfer phenomenon at the same time.

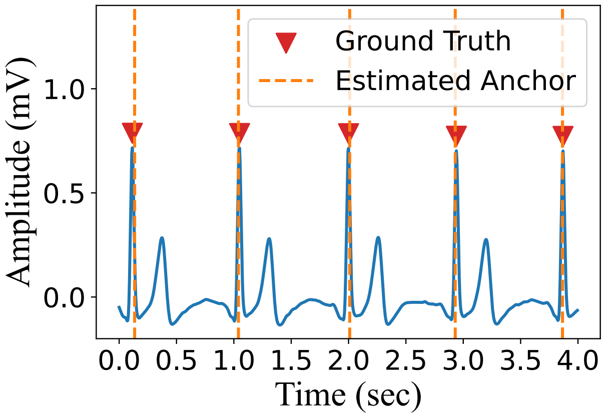
Three Tasks:



Morphological Recovery



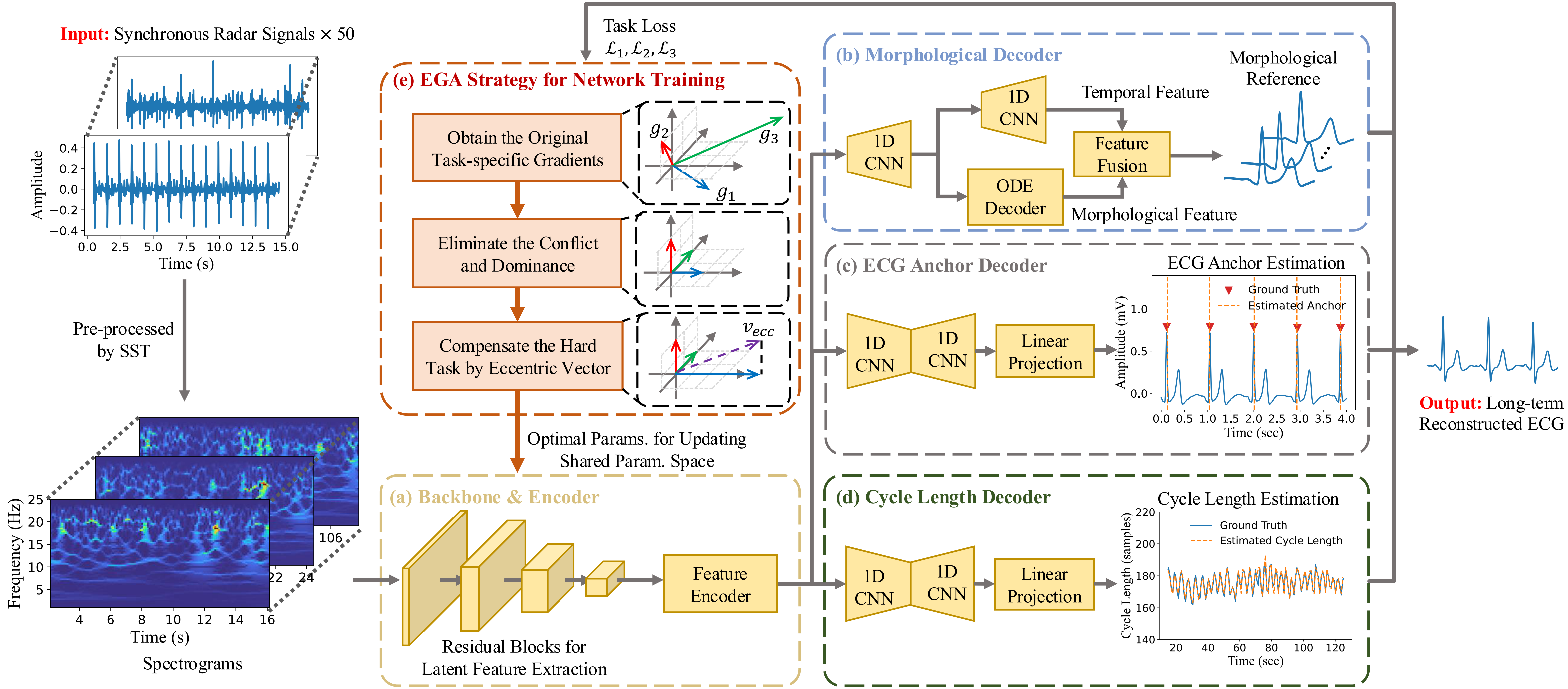
Cycle Length Estimation



ECG Anchor Estimation

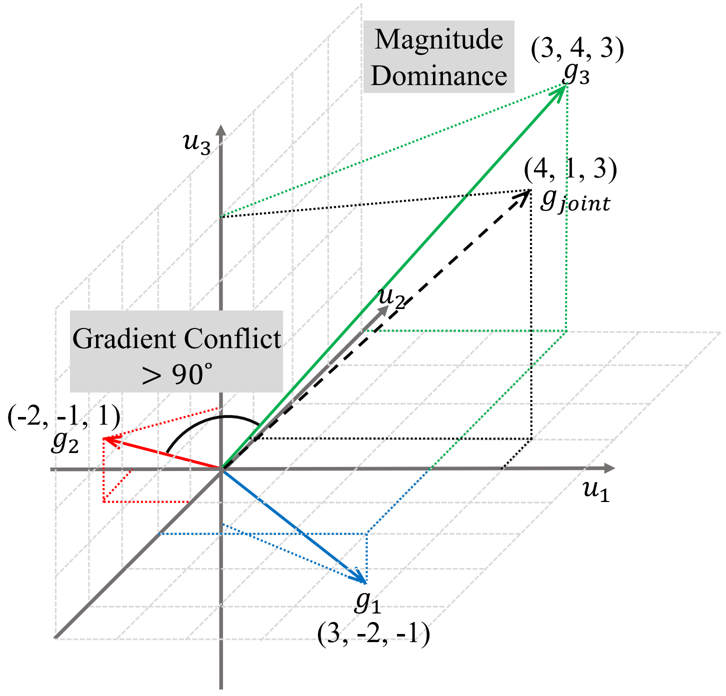
Study C

Robust Long-term ECG Generation – radarODE-MTL

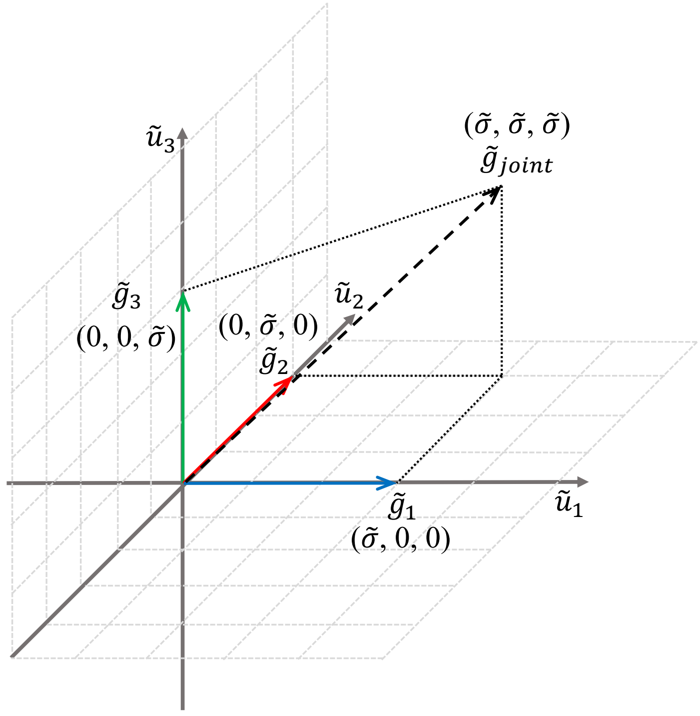


Framework Overview

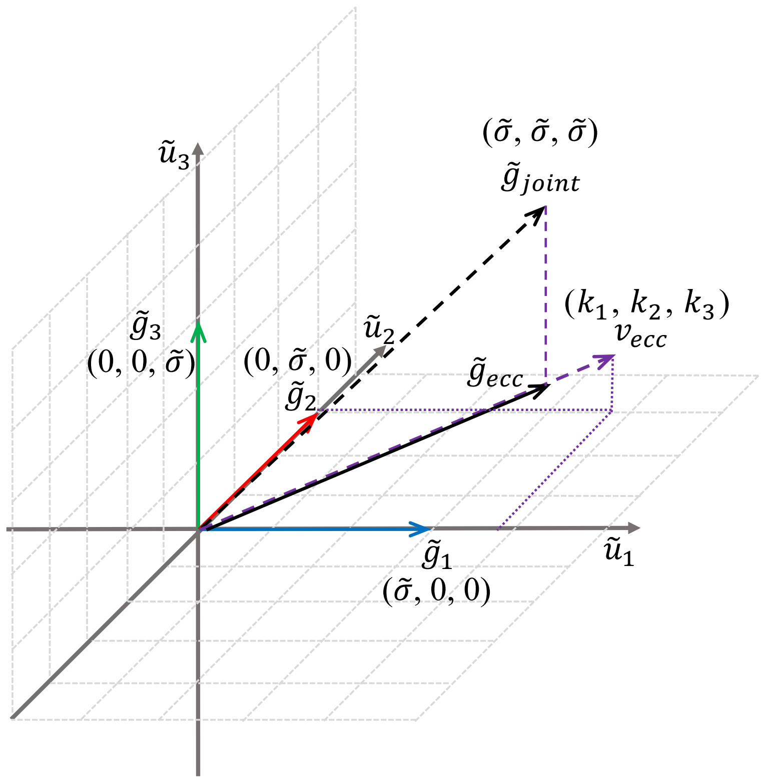
Study C | Robust Long-term ECG Generation – EGA



(a) Original gradient space with gradient conflict and magnitude dominance

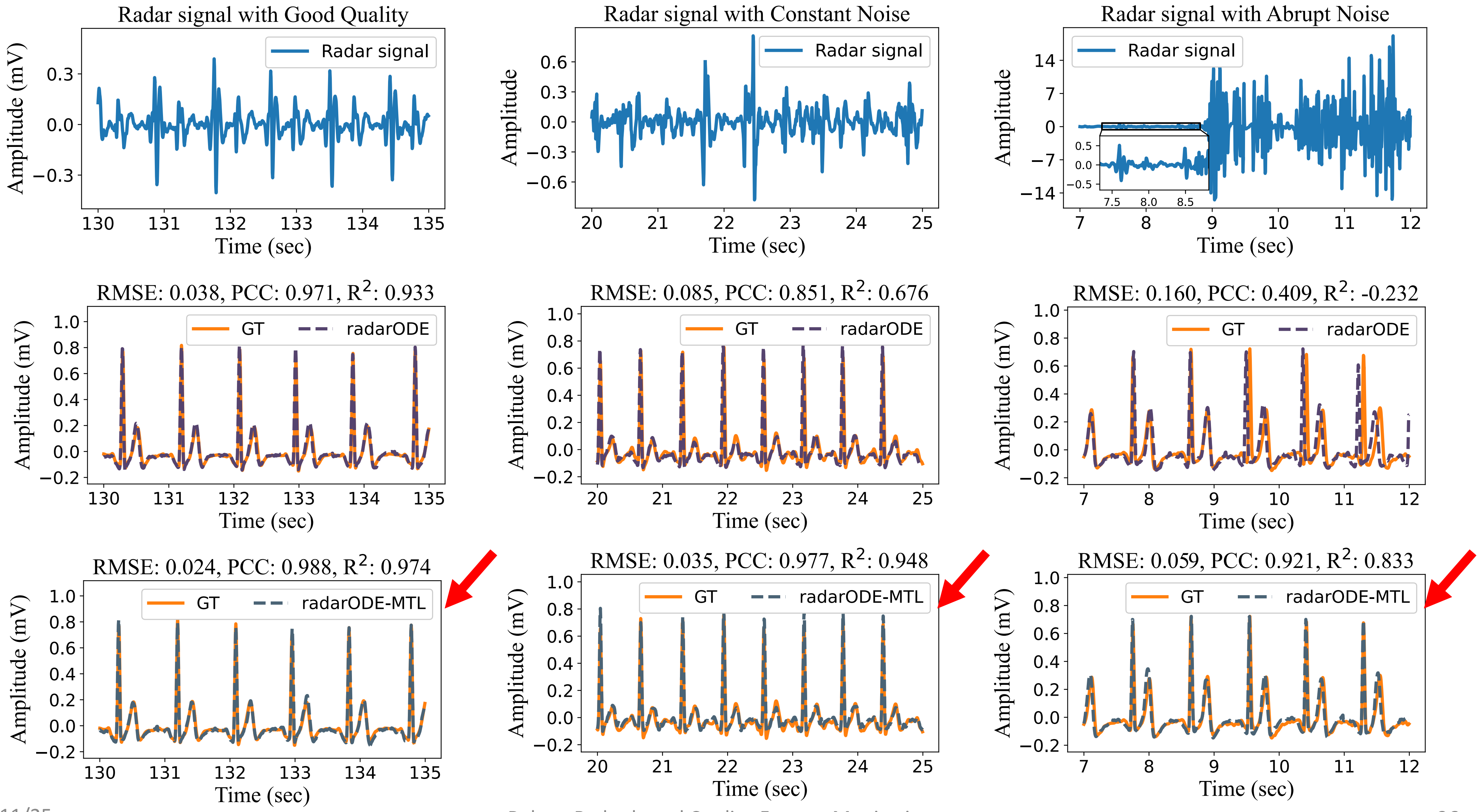


(b) The projection of the original gradient space into the orthogonal space with equal "learning rate"

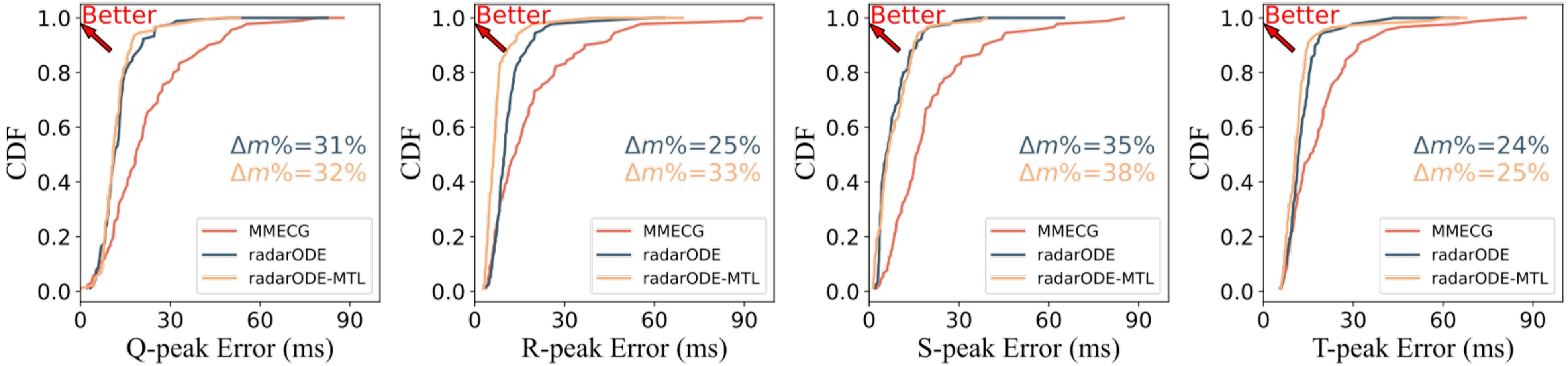


(c) The implementation of eccentric gradient alignment to slant the joint gradient to the hard task by introducing the eccentric vector

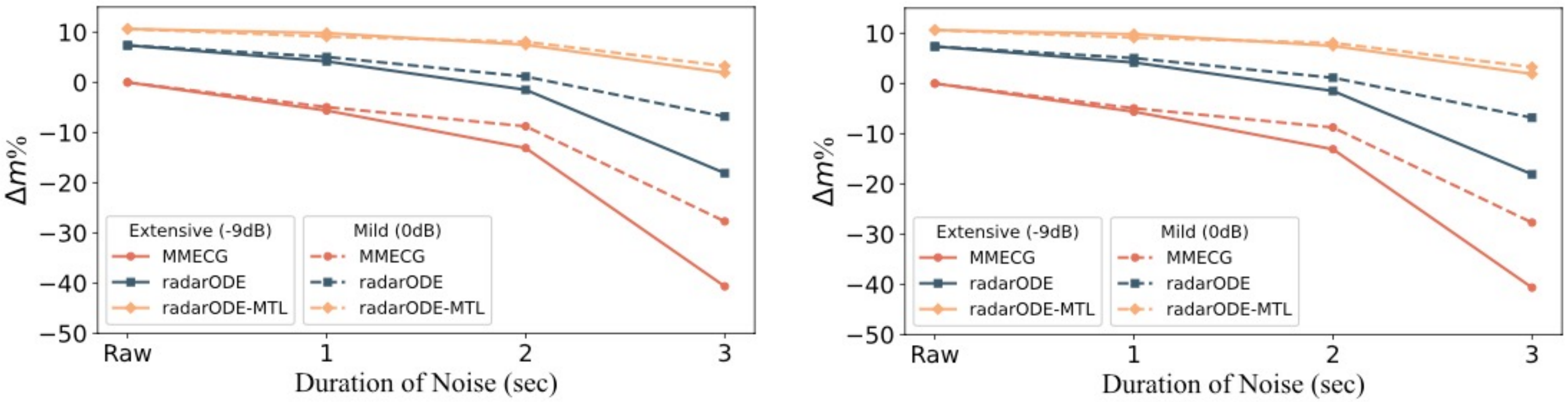
Study C | Robust Long-term ECG Generation – Results



Study C | Robust Long-term ECG Generation – Results

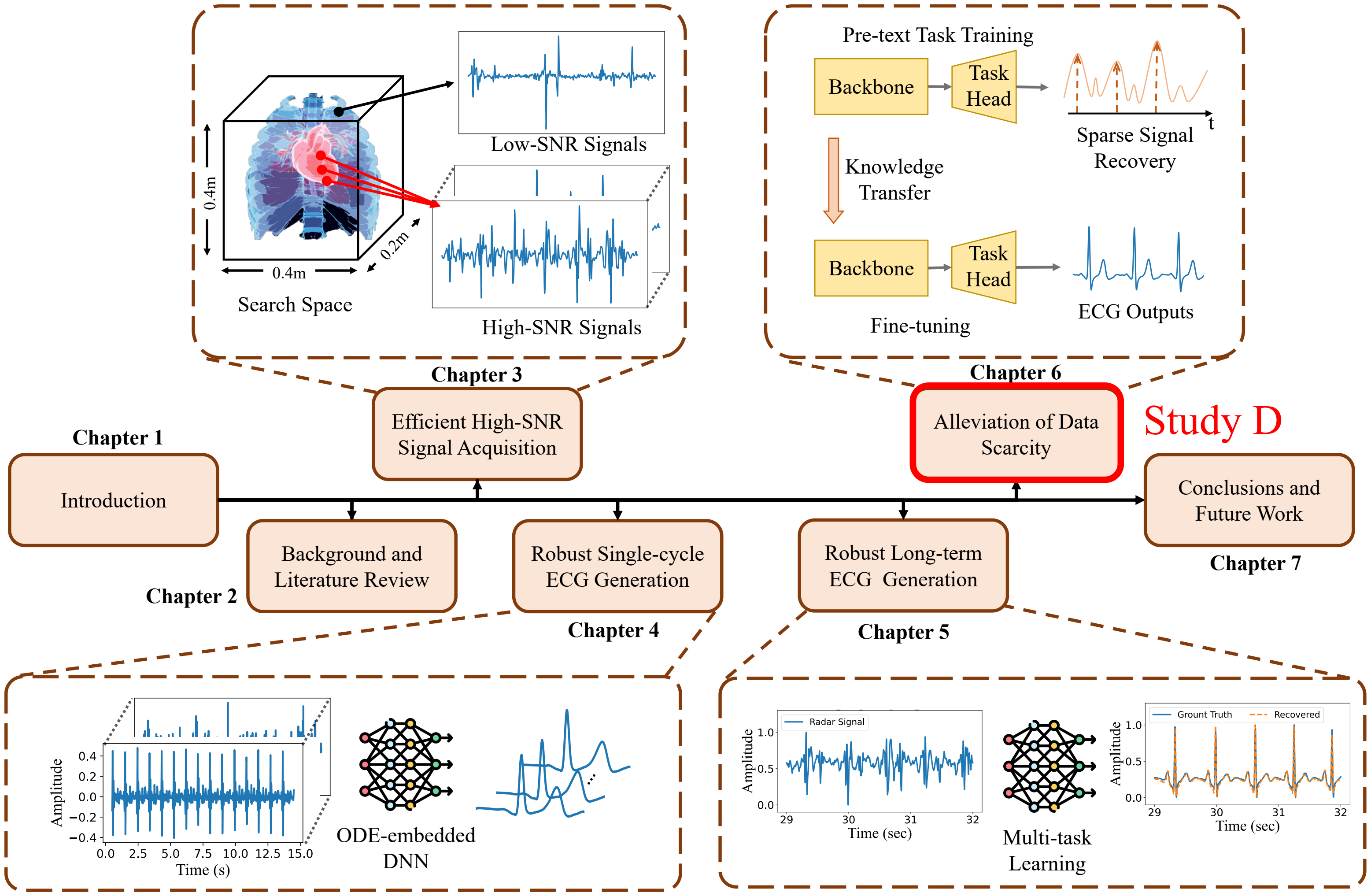


Peak accuracy



Noise robustness test

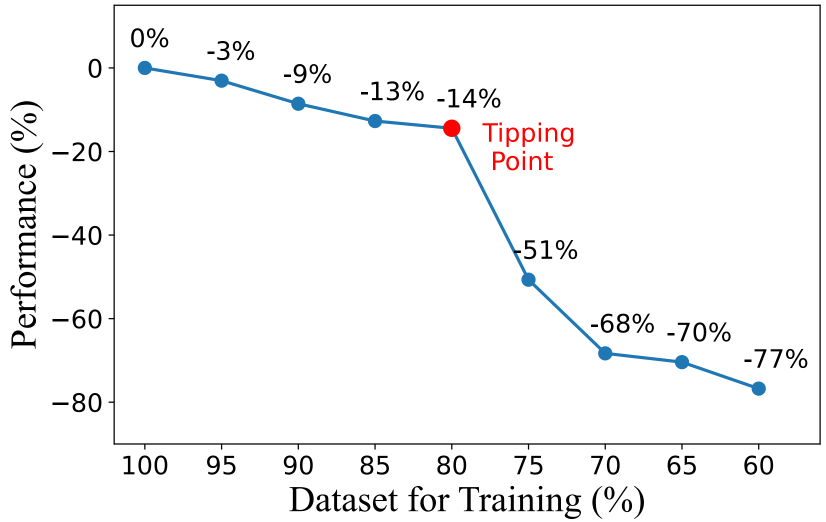
Part 2 – Proposed Methods



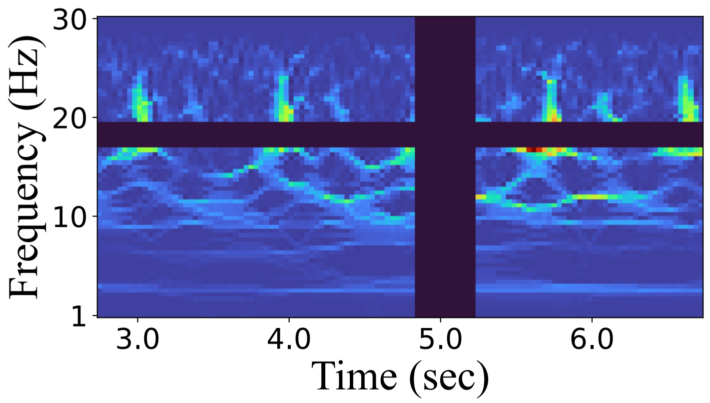
Alleviate Data Scarcity – Research Gap

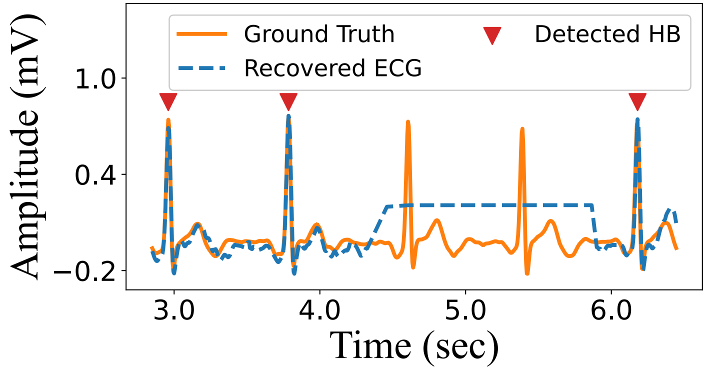
Research gap in radar-based ECG recovery:

1. Limited data for deep learning model training, especially for new scenarios.
2. Unfaithful data augmentation for regression task. (e.g., mask, recombination, shift, flip, etc.)
3. Do not make full use of unlabeled radar signal for self-supervised learning (SSL).



Limited data may cause significant degradation



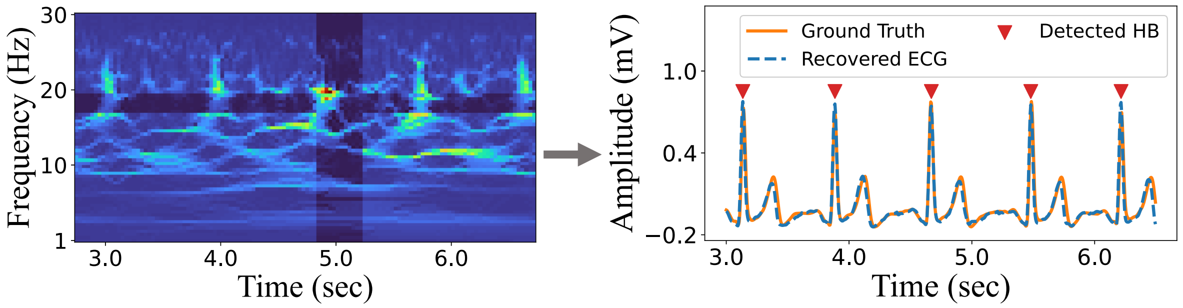


Traditional zero mask may cause missed detection

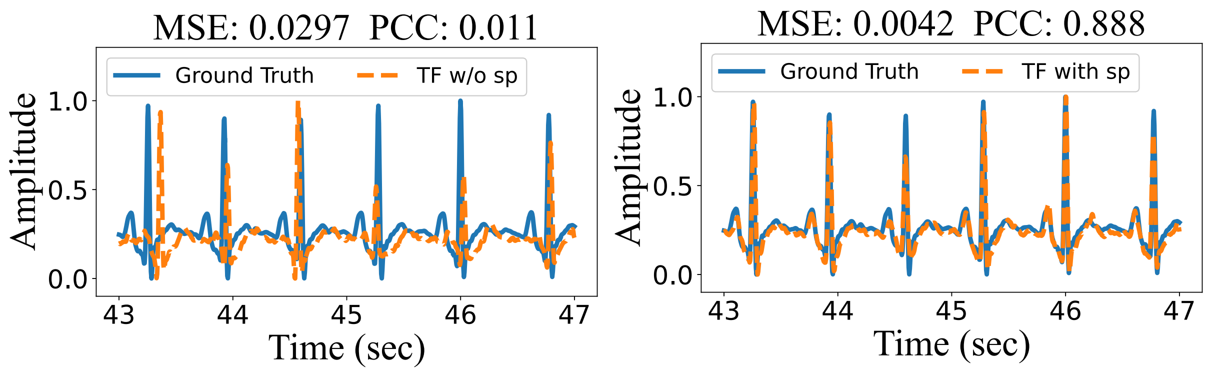
Alleviate Data Scarcity – Contributions

Contributions:

1. Propose Horcrux enhances the diversity of the limited dataset while preserving the intrinsic time consistency in the original signal and restricting the potential distribution shifts introduced into the augmented dataset.
2. Propose a transfer learning framework RFcardi following an SSL paradigm to effectively learn the latent representations from radar signals by leveraging an appropriate pre-text task.

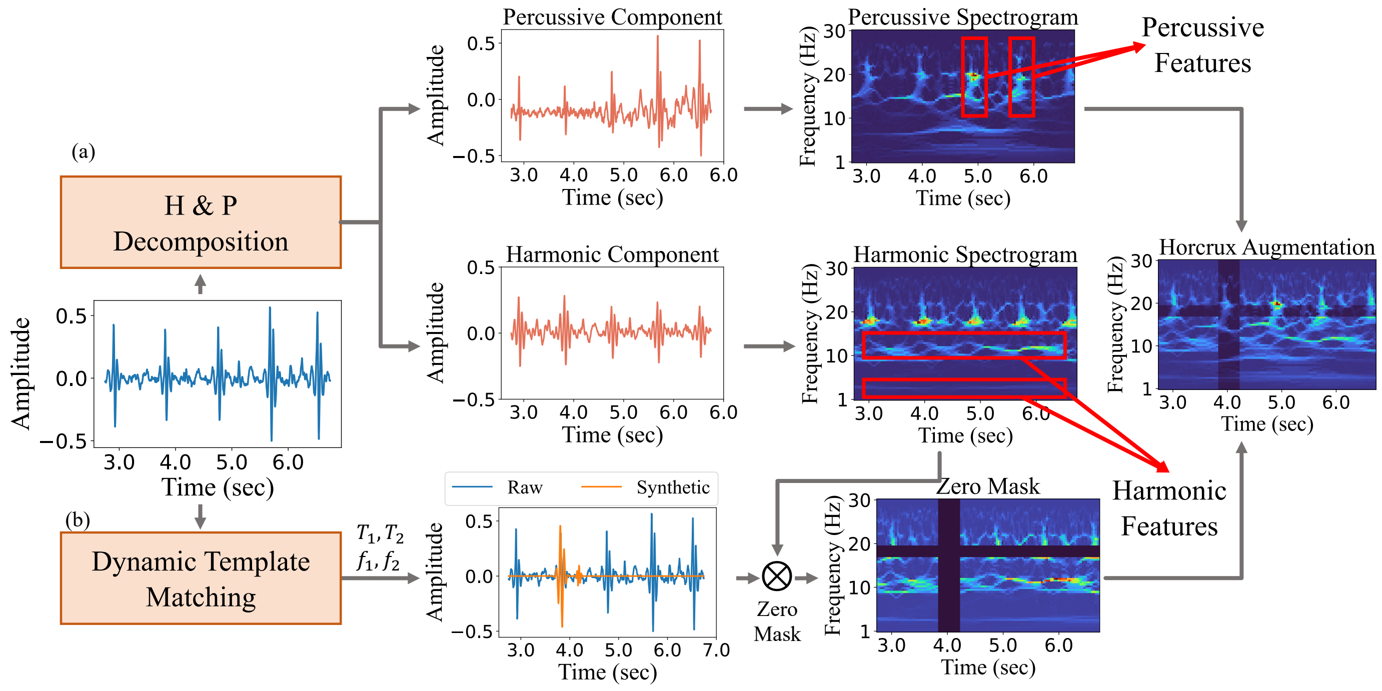


Time-domain consistency preserved using proposed Horcrux

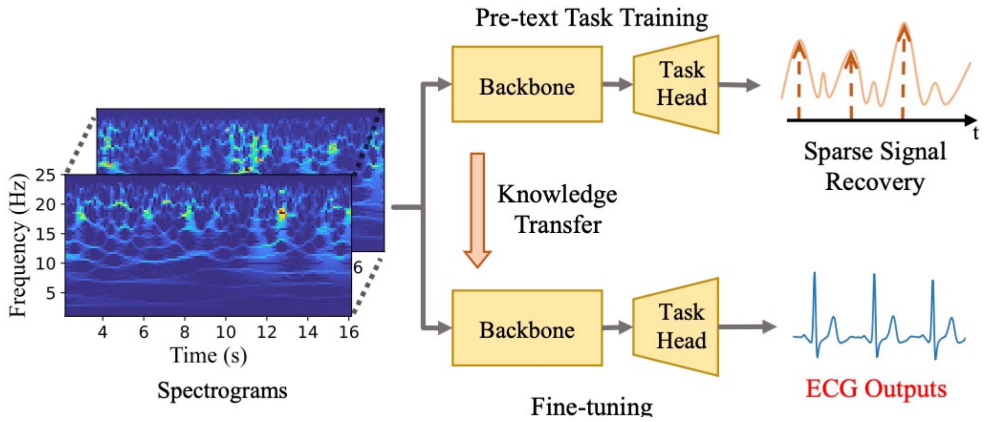


ECG Recovery w/o or with pre-training

Alleviate Data Scarcity – Horcrux and RFcardi



Data augmentation pipeline (Horcrux)



Transfer learning framework (RFcardi)

SSL with pre-text Task

1. Sparse Signal Recovery

$$h = \Phi x + n$$

2. Loss function with sparse penalty from $[0,\ \lambda_s)$

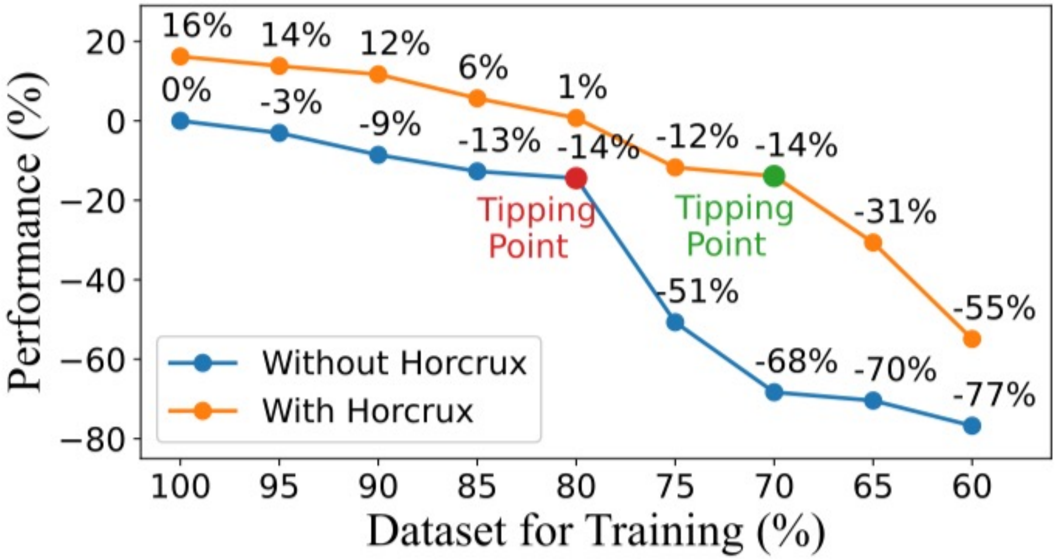
$$\mathcal{L} = \|x - x'\|_2 + \underbrace{\lambda_s \frac{\|x\|_1/\|x\|_2 - 1}{\sqrt{m} - 1}}_{\text{sparse penalty}}$$

Alleviate Data Scarcity – Results of Horcrux

TABLE I
COMPARISON OF DIFFERENT AUGMENTATION METHODS

Methods	RMSE (mV) ↓	PCC ↑	H. E. (ms) ↓	MDR ↓	$\Delta m\%$
Baseline	0.096	82.65%	8.82	6.73%	0.0%
C-Mixup [13]	<u>0.088</u>	82.25%	7.55	6.14%	7.80%
ADA [8]	0.097	81.28%	7.20	6.69%	4.12%
RC-Mixup [14]	0.089	83.36%	7.72	5.79%	8.69%
Horcrux (10%)	0.096	82.24%	6.89	6.67%	5.63%
Horcrux (15%)	0.087	<u>84.75%</u>	6.96	**5.19%**	<u>14.02%</u>
Horcrux (20%)	**0.086**	**85.41%**	<u>6.21</u>	<u>5.30%</u>	**16.20%**
Horcrux (25%)	0.092	81.99%	**6.16**	6.36%	9.81%
Horcrux (30%)	0.094	80.18%	6.55	6.59%	6.78%

Bold and <u>underline</u> represent the best and the second best results.



Alleviate Data Scarcity – Results of RFcardi

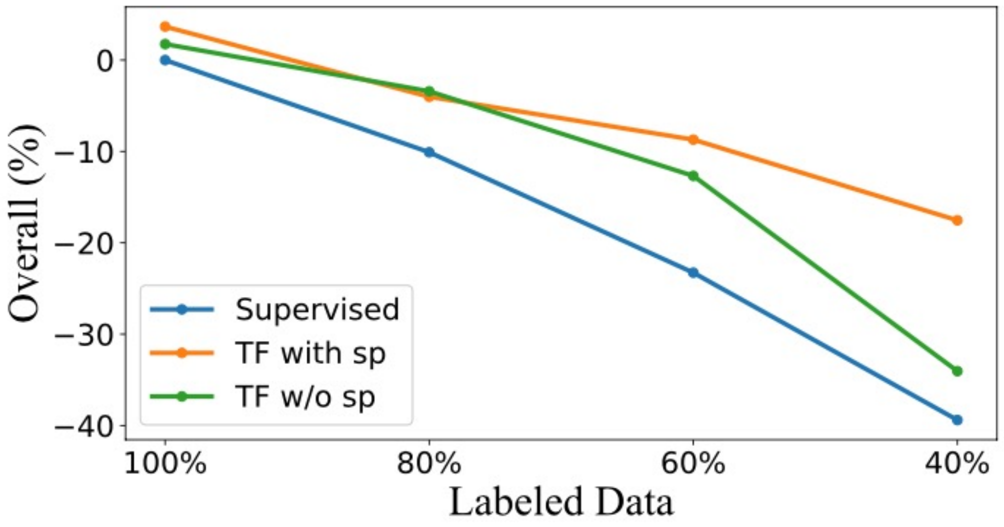


TABLE IV
PERFORMANCE OF ECG RECOVERY USING DIFFERENT PERCENTAGES OF LABELED DATA

Methods	MSE ($\times 10^{-2}$) ↓	PCC ↑	Peak Error (ms) ↓	MDR ↓	Overall ↑	MSE ($\times 10^{-2}$) ↓	PCC ↑	Peak Error (ms) ↓	MDR ↓	Overall ↑
	100% Labeled					**80% Labeled**				
Supervised	0.80	85.47%	7.61	6.85%	0.00%	0.84	84.60%	8.90	8.04%	−10.09%
TF w/o sp*	0.81	85.35%	8.46	5.51%	1.75%	0.82	86.36%	8.35	7.02%	−3.42%
TF with sp	0.80	85.51%	8.40	5.14%	3.66%	0.81	84.29%	8.31	7.14%	−4.02%
	60% Labeled					**40% Labeled**				
Supervised	0.93	79.91%	10.65	8.93%	−23.27%	0.98	75.89%	11.15	12.15%	−39.4%
TF w/o sp	0.85	83.74%	8.84	8.65%	−12.68%	0.97	76.56%	10.87	10.99%	−34.05%
TF with sp	0.86	84.92%	8.58	7.72%	−8.71%	0.93	78.72%	8.70	9.02%	−17.54%

*TF for transfer learning and sp for sparse penalty

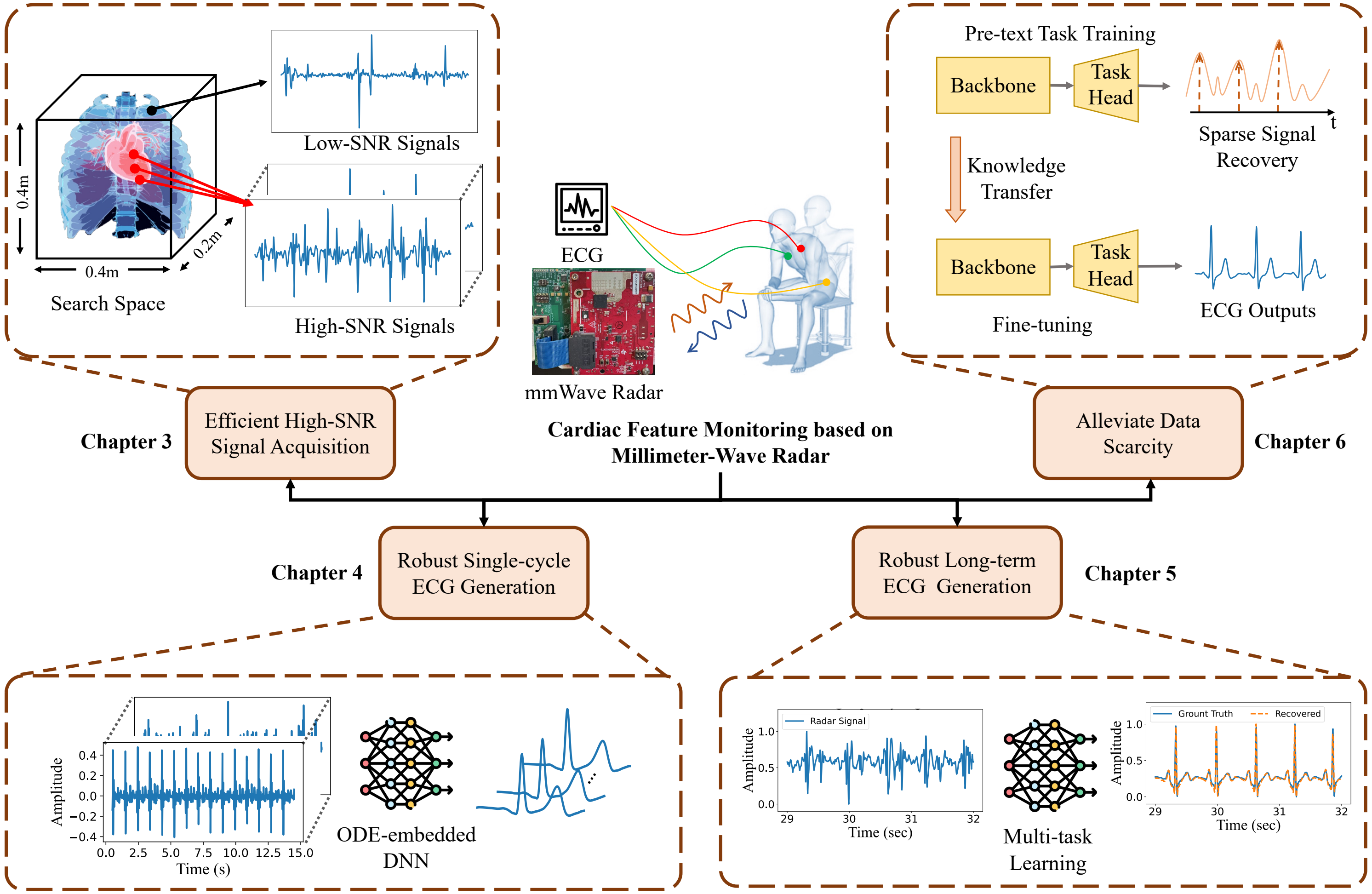
Part 3 – Conclusions and Future Work

- Part 1 – Summary of contributions
- Part 3 – Publications
- Part 2 – Future work

Conclusions | Summary of contributions

Contributions:

1. Efficient data collection with ample cardiac features
2. Model the transformation between fine-grained cardiac features (mechanical domain to electrical domain).
3. Realize the long-term radar-based ECG recovery even under the impact of different noises.
4. Alleviate the data scarcity for the deployment in new scenarios with limited data.



Publications

Publications as first author:

1. **Yuanyuan Zhang**, Rui Yang, Yutao Yue, Eng Gee Lim, Zidong Wang, "An Overview of Algorithms for Contactless Cardiac Feature Extraction From Radar Signals: Advances and Challenges", ***IEEE Transactions on Instrumentation and Measurement***, Jul. 2023. (Chapter 2)
2. **Yuanyuan Zhang**, Haocheng Zhao, Sijie Xiong, Rui Yang, Eng Gee Lim, Yutao Yue, "From High-SNR Radar Signal to ECG: A Transfer Learning Model with Cardio-Focusing Algorithm for Scenarios with Limited Data", ***IEEE Transactions on Mobile Computing***. (**Under Major Revision**) (Chapter 3)
3. **Yuanyuan Zhang**, Runwei Guan, Lingxiao Li, Rui Yang, Yutao Yue, Eng Gee Lim, "radarODE: An ODE-Embedded Deep Learning Model for Contactless ECG Reconstruction from Millimeter-Wave Radar", ***IEEE Transactions on Mobile Computing***, Apr. 2025. (Chapter 4)
4. **Yuanyuan Zhang**, Rui Yang, Yutao Yue, Eng Gee Lim, "radarODE-MTL: A Multi-Task Learning Framework with Eccentric Gradient Alignment for Robust Radar-Based ECG Reconstruction", ***IEEE Transactions on Instrumentation and Measurement***, Apr. 2025. (Chapter 5)
5. **Yuanyuan Zhang**, Sijie Xiong, Rui Yang, Eng Gee Lim, Yutao Yue, "Recover from Horcrux: A Spectrogram Augmentation Method for Cardiac Feature Monitoring from Radar Signal Components", in *Proceedings of the 47th Annual International Conference of the IEEE Engineering in Medicine & Biology Society (**EMBC2025**)*, Jul. 2025. (Chapter 6)

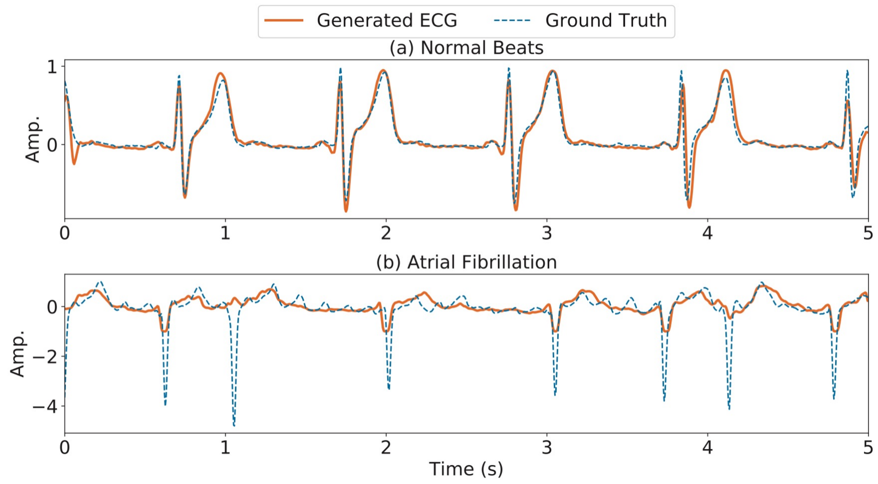
Publications as co-author:

6. Sijie Xiong, **Yuanyuan Zhang**, Cheng Tang, Haoling Xiong, Yiding Li, Atsushi Shimada, "U-MA: A Unified Framework with Differential Mamba under Parallel U-Net Scheme for Time Series Forecasting", ***Engineering Applications of Artificial Intelligence***. (Under Review)
7. Sijie Xiong, Cheng Tang, **Yuanyuan Zhang**, Haoling Xiong, Youhao Xu, Atsushi Shimada, "CME-Mamba with Enhancing Nonlinear Dependencies for Time Series Forecasting", ***Applied Soft Computing***. (Under Review)
8. Sijie Xiong, **Yuanyuan Zhang**, Cheng Tang, Fumiya Okubo, Yinlong Hu, Atsushi Shimada, "PIDTracker: An Efficient 2D Controllable Holt-Winters Model for Glucose Monitoring", in ***Proceedings of the AAAI Conference on Artificial Intelligence***. (Under Review)
9. Sijie Xiong, Jianing Wang, **Yuanyuan Zhang**, Cheng Tang, Fumiya Okubo, Atsushi Shimada, "GlucoMixer: An Efficient Continuous Glucose Monitoring Model with Mixers", in ***Proceedings of the AAAI Conference on Artificial Intelligence***. (Under Review)

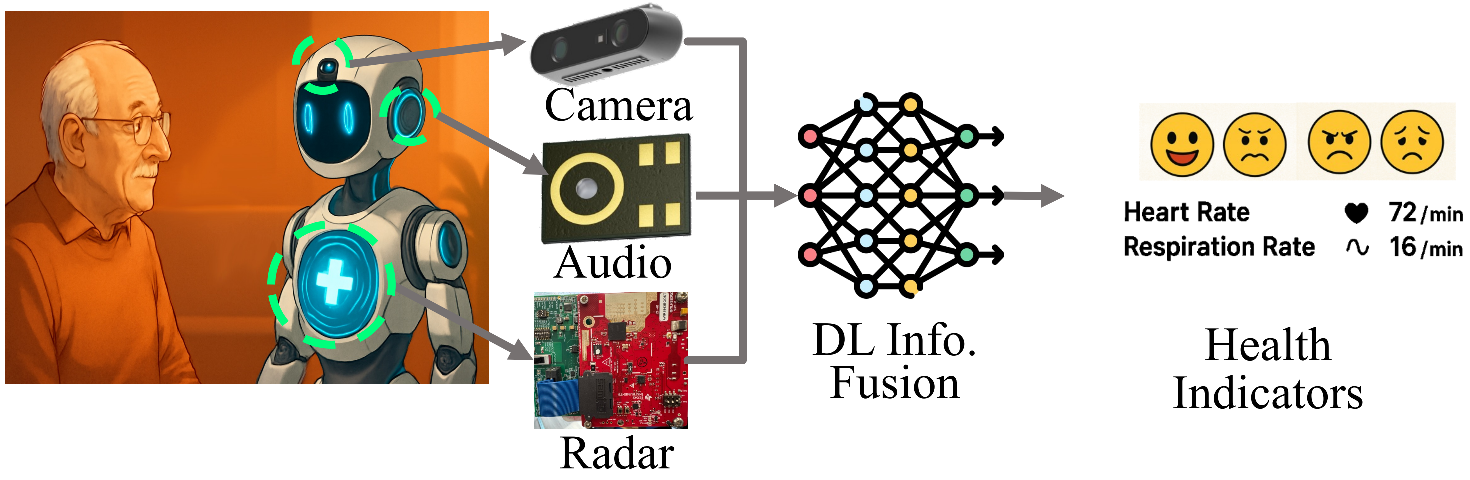
Conclusions | Conclusions – Future work

Future work:

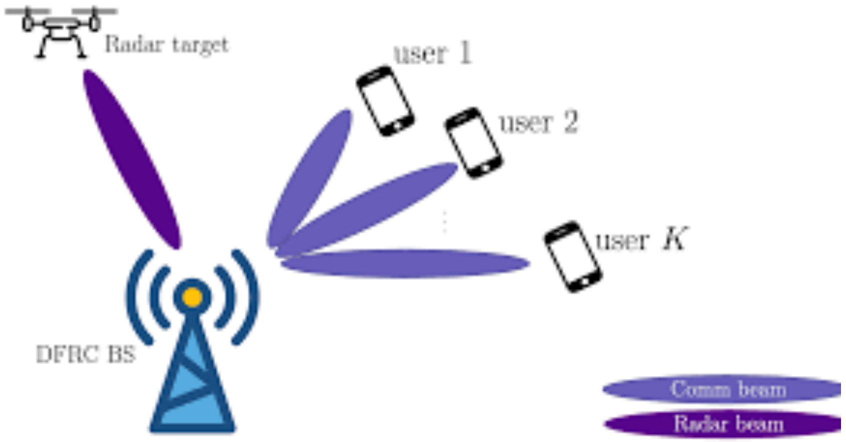
1. Multi-sensor fusion for advanced healthcare monitoring.
2. Evaluation on the dataset for patients.
3. Integrated sensing and communication (for 6G).



ECG signal for patients



Multi-senor system for healthcare monitoring



ISCA

Q&A

Thanks for your time!

Assess signal SNR with universal template

1. Propose the dynamic template

$$h_m(t) = a_1 \exp(-\frac{(t-b_1)^2}{2c_1^2}) + a_2 \exp(-\frac{(t-b_2)^2}{2c_2^2})$$

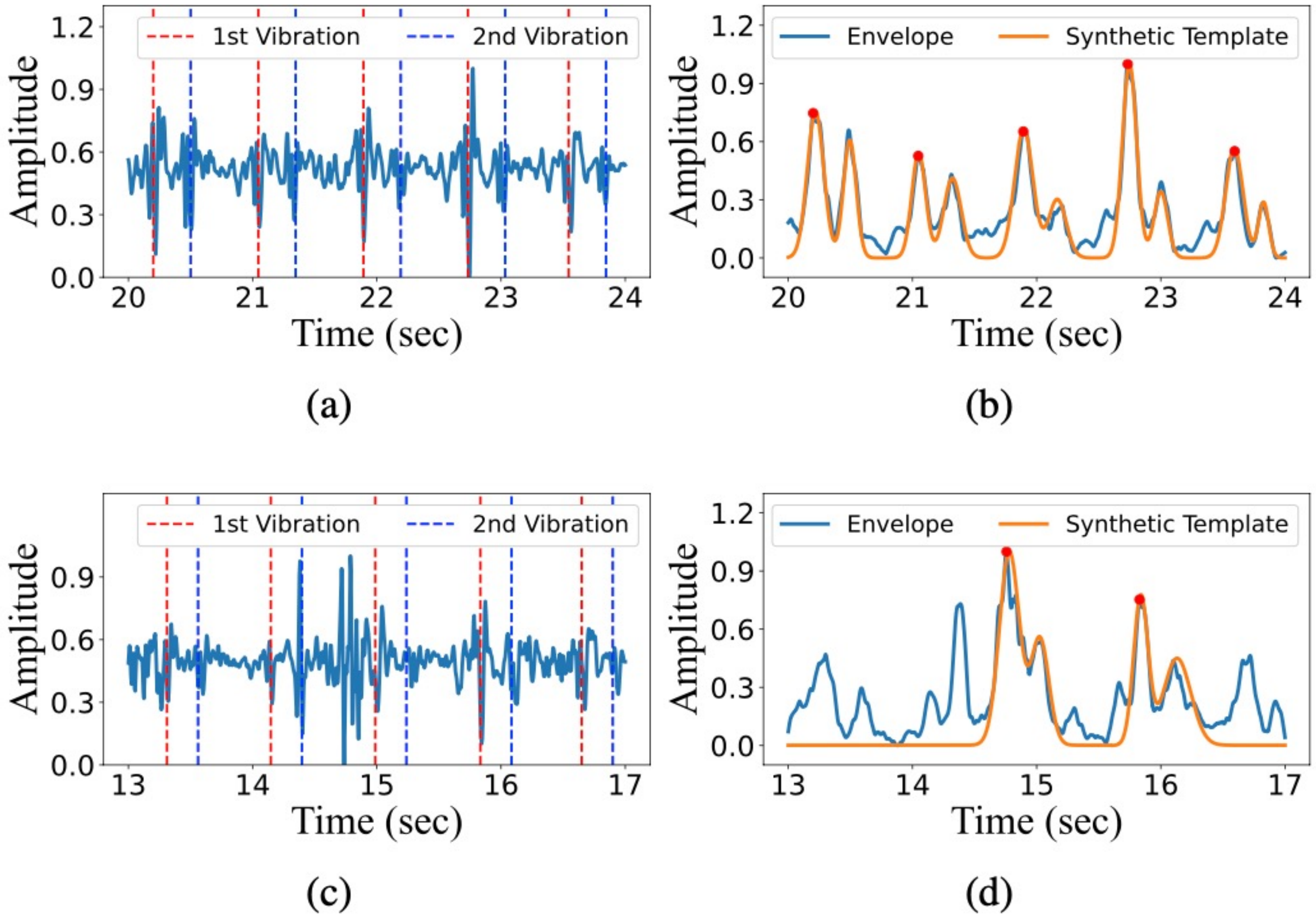
a_1, a_2 → amplitude of the vibrations
b_1, b_2 → when the vibrations happen
c_1, c_2 → length/width of the peak

2. Identify incumbent peaks (red dots)

Peak position and amplitude → a_1, b_1

3. Run template matching

$$\theta^* = \underset{\theta=\{a_2,b_2,c_1,c_2\}}{\arg\min} \|h(E,t) - h_m(t,\theta)\|_2$$



Supplementary Materials – Chapter 3

Range and Angle FFT

1. Received FMCW signal

Phase

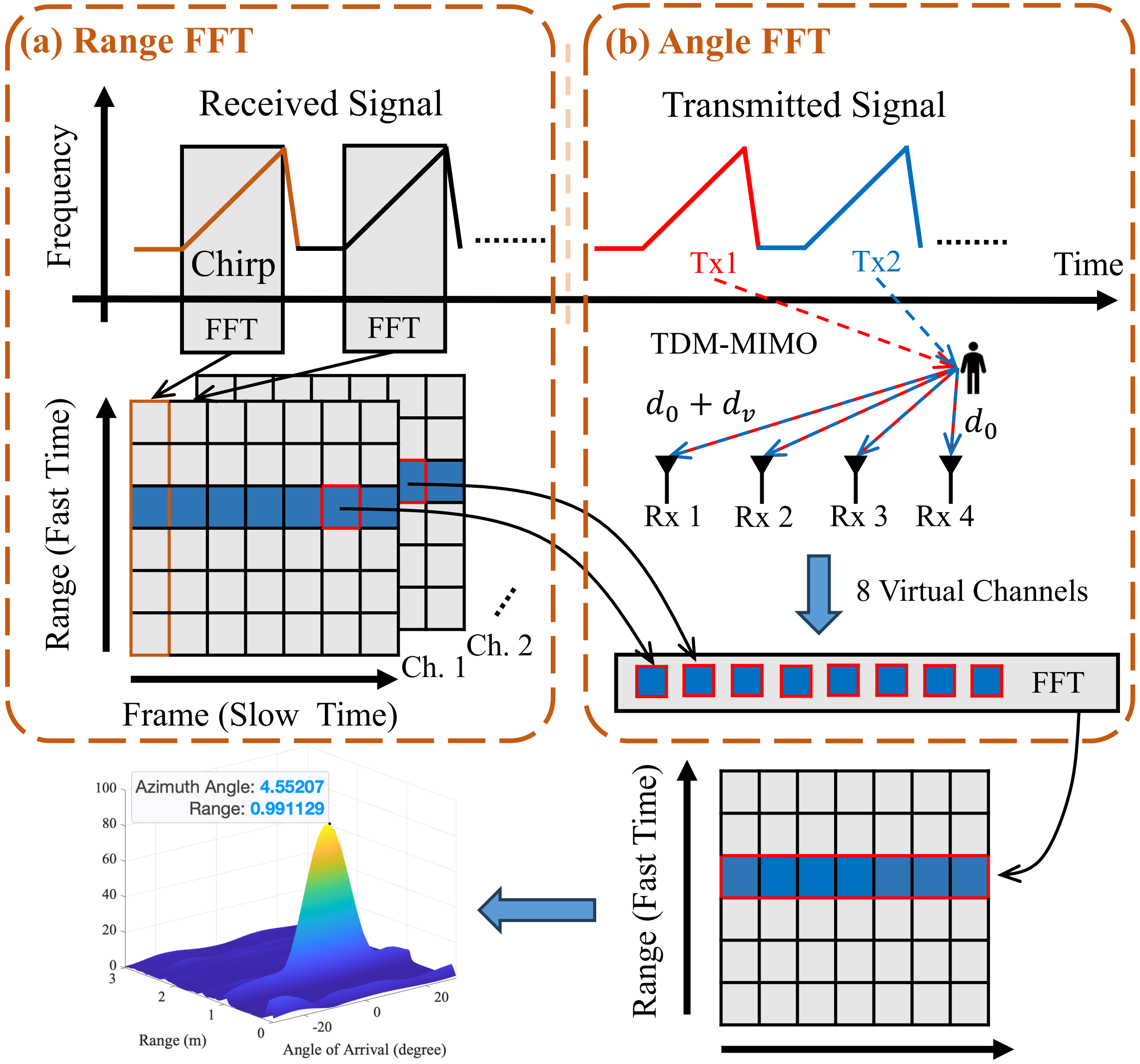
$$x_R(t) = A_R \exp\left\{-j\left[2\pi f_c(t-t_d) + \boxed{\pi \frac{B}{T_c}(t-t_d)^2}\right]\right\}$$

2. Radar-human distance introduce phase shift

$$\phi_s = \frac{4\pi d_0}{\lambda}$$

3. Distance between antennas introduce extra phase shift w.r.t. angle of arrival θ (AoA)

$$\Delta\phi_v = \frac{4\pi d_v}{\lambda}$$
$$d_v = l\ sin(\theta)$$



RA map for single frame

Supplementary Materials – Chapter 3

Objective

$$E_b = \underset{E\in\mathbb{R}^n}{\arg\min}\,\{\mathcal{F}(E) : E \in \Omega\} \quad \text{for each iteration } k$$

Initialization

1. Initialization of grid G_k

$$G_k := \{E_k + \gamma_k D\} \subset \mathbb{R}^n$$

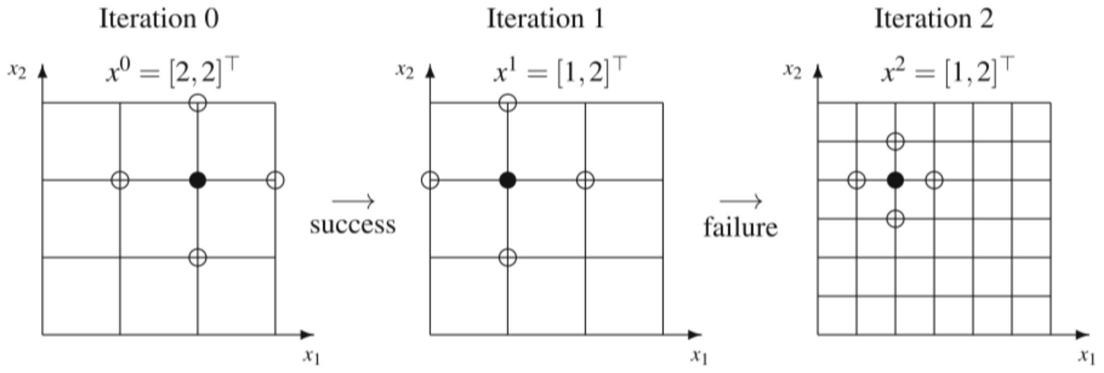
with grid size parameter γ_k and search direction set D

2. Introduce search region S_k

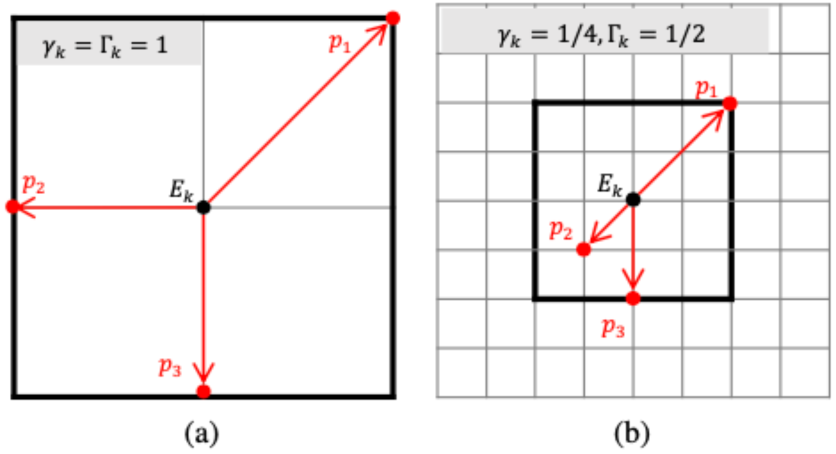
$$S_k := \{E \in G_k : \|E - E_k\|_\infty \le \Gamma_k a\}$$

$$a = \max\{\|a'\|_\infty : a' \in D\}$$

with search size parameter Γ_k



Common coordinate search [3]



Proposed CFT Algorithm

[3] J. Larson, M. Menickelly, and S. M. Wild, "Derivative-free optimization methods," Acta Numerica, vol. 28, pp. 287–404, Jun. 2019.

Supplementary Materials – Chapter 3

Main Iteration

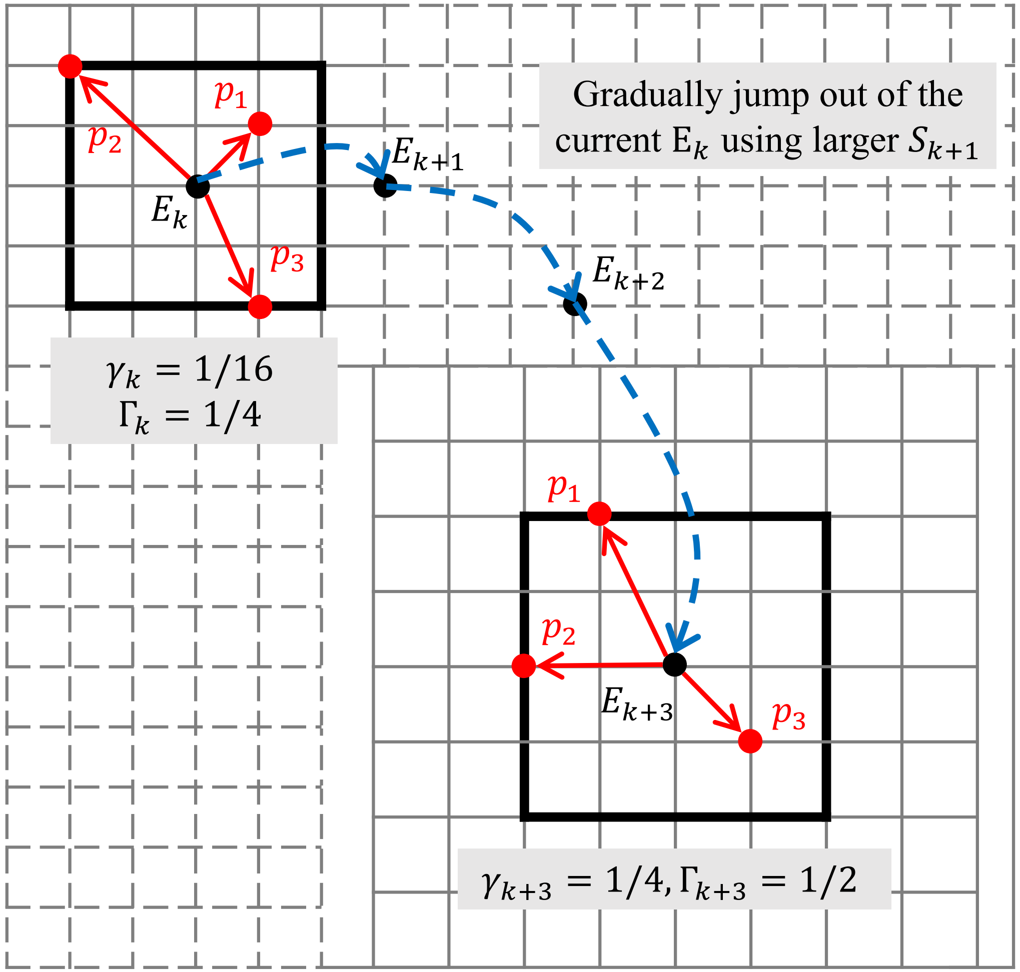
1. Initial Search Stage

   Evaluate $\mathcal{F}(E)$ on a subset of grids G_k based on any sampling algorithm (Latin hypercube sampling)

2. Resizing Stage

- If success, move to new point and double Γ_k
- If failed, run the same search inside search region S_k
  - If success, move to new point and double Γ_k
  - If failed, half Γ_k

Set $\gamma_{k+1} \leftarrow \min(\Gamma_k, {\Gamma_k}^2)$



Supplementary Materials – Chapter 5

Define a MTL problem for n tasks:

$$\theta^* = \arg\min_{\theta \in \mathbb{R}^m} \left\{ \mathcal{F}(\theta) \triangleq \frac{1}{n} \sum_{i=1}^{n} \mathcal{L}_i(\theta) \right\} \quad \theta \in \mathbb{R}^m$$ for shared parameter space

The original task-specific gradient and the gradient matrix :

$$\boldsymbol{g}_i = \nabla_\theta \mathcal{L}_i(\theta), i \in [n] \qquad \boldsymbol{G} = \{\boldsymbol{g}_1, \cdots, \boldsymbol{g}_i\} \in \mathbb{R}^{n \times m}$$

Project to orth. space

1. Formulate the projection problem:

$$\min \|\boldsymbol{g}_{joint} - \tilde{\boldsymbol{g}}_{joint}\|_2^2 \ \text{s.t.} \ \tilde{\boldsymbol{G}}\tilde{\boldsymbol{G}}^\intercal = \boldsymbol{I}$$ with $$\boldsymbol{g}_{joint} = \boldsymbol{G}^\intercal \boldsymbol{w} \qquad \tilde{\boldsymbol{g}}_{joint} = \tilde{\boldsymbol{G}}^\intercal \boldsymbol{w} \qquad \boldsymbol{w} = [1, \cdots, 1]^\intercal$$

Then we have:

$$\left\|\boldsymbol{g}_{joint} - \tilde{\boldsymbol{g}}_{joint}\right\|_2^2 = \left\|\boldsymbol{G}^\intercal \boldsymbol{w} - \tilde{\boldsymbol{G}}^\intercal \boldsymbol{w}\right\|_2^2 \leq \|\boldsymbol{G}^\intercal - \tilde{\boldsymbol{G}}^\intercal\|_F^2 \|\boldsymbol{w}\|_2^2$$

2. The final projection problem:

$$\min_{\tilde{\boldsymbol{G}}} \|\boldsymbol{G} - \tilde{\boldsymbol{G}}\|_F^2 \quad \text{s.t.} \quad \tilde{\boldsymbol{G}}\tilde{\boldsymbol{G}}^\intercal = \boldsymbol{I}$$

Algorithm 1 EGA Optimization Strategy for MTL

Input: Loss values for n tasks $[\mathcal{L}_1, \cdots, \mathcal{L}_i], i \in [n]$,
T for softmax and t_{warm} for warmup epoch
INITIALIZATION:
- Unit eccentric vector $\boldsymbol{v}_{ecc} = [1, \cdots, 1]^\intercal \in \mathbb{R}^n$
FOR EACH ITERATION:
- Get the current epoch as t
- Get task-specific gradient $\boldsymbol{g}_i = \nabla_\theta \mathcal{L}_i(\theta), i \in [n]$
- Form gradient matrix $\boldsymbol{G} = \{\boldsymbol{g}_1, \cdots, \boldsymbol{g}_i\} \in \mathbb{R}^{n \times m}$
- Eigen decomposition of Gram matrix as in (13):
$\boldsymbol{G}\boldsymbol{G}^\intercal = \boldsymbol{U}(\boldsymbol{\Sigma}\boldsymbol{\Sigma}^\intercal)\boldsymbol{U}^\intercal$ with eigenvalues $\boldsymbol{\lambda}$
- Get scaling factor for gradient normalization:
$\tilde{\sigma} = \min(\sqrt{\boldsymbol{\lambda}})$
- Calculated the orthogonal and normalized gradient matrix as in (15): $\tilde{\boldsymbol{G}} = \tilde{\sigma}\boldsymbol{U}\boldsymbol{\Sigma}^{-1}\boldsymbol{U}^\intercal\boldsymbol{G}$

if $t = t_{warm}$ **then**
- Record the loss values for all the tasks $\mathcal{L}_i(t_{warm})$

else if $t > t_{warm}$ **then**
- Calculate the task difficulty weights as in (17):
$k_i(t) = \text{softmax}(lr_i(t-1))$
- Form eccentric vector $\boldsymbol{v}_{ecc} = [k_1, \cdots, k_i]^\intercal$

end if

Output: $\tilde{\boldsymbol{g}}_{ecc} = \tilde{\boldsymbol{G}}^\intercal \boldsymbol{v}_{ecc}$ for optimization

Supplementary Materials – Chapter 5

3. Find the solution for:

$$\min_{\tilde{\boldsymbol{G}}} \|\boldsymbol{G} - \tilde{\boldsymbol{G}}\|_F^2 \quad \text{s.t.} \quad \tilde{\boldsymbol{G}}\tilde{\boldsymbol{G}}^\intercal = \boldsymbol{I}$$

4. Borrow the solution of orthogonal Procrustes problem using SVD:

$$\begin{array}{lll} \min & \|\mathrm{A}\Omega - \mathrm{B}\|_\mathrm{F} & \mathrm{A}, \mathrm{B} \in \mathrm{R}^{\mathrm{m}\times\mathrm{n}} \\ \text{s.t.} & \Omega^\top\Omega = \mathrm{I} & \mathrm{C} = \mathrm{B}^\top \mathrm{A} \end{array} \quad \text{with solution} \quad \begin{array}{c} \mathrm{U}\Sigma\mathrm{V}^\top = \mathrm{C} \\ \Omega = \mathrm{V}\,\mathrm{U}^\top \end{array}$$

5. The new gradient matrix (using eigen decomposition) is:

$$\tilde{\boldsymbol{G}} = \boldsymbol{U}\boldsymbol{V}^\intercal \quad \text{with} \quad \boldsymbol{G}\boldsymbol{G}^\intercal = \boldsymbol{U}\left(\boldsymbol{\Sigma}\boldsymbol{\Sigma}^\intercal\right)\boldsymbol{U}^\intercal$$

6. The final solution is:

$$\tilde{\boldsymbol{G}} = \boldsymbol{U}\boldsymbol{\Sigma}^{-1}\boldsymbol{U}^\intercal\boldsymbol{G} \quad \text{with unit singular values}$$

The gradient conflict resolved □

Algorithm 1 EGA Optimization Strategy for MTL

1: **Input:** Loss values for n tasks $[\mathcal{L}_1, \cdots, \mathcal{L}_i], i \in [n]$,
T for softmax and t_{warm} for warmup epoch
INITIALIZATION:
2: - Unit eccentric vector $\boldsymbol{v}_{ecc} = [1, \cdots, 1]^\intercal \in \mathbb{R}^n$
FOR EACH ITERATION:
3: - Get the current epoch as t
4: - Get task-specific gradient $\boldsymbol{g}_i = \nabla_\theta \mathcal{L}_i(\theta), i \in [n]$
5: - Form gradient matrix $\boldsymbol{G} = \{\boldsymbol{g}_1, \cdots, \boldsymbol{g}_i\} \in \mathbb{R}^{n\times m}$
6: - Eigen decomposition of Gram matrix as in (13):
$\boldsymbol{G}\boldsymbol{G}^\intercal = \boldsymbol{U}\left(\boldsymbol{\Sigma}\boldsymbol{\Sigma}^\intercal\right)\boldsymbol{U}^\intercal$ with eigenvalues $\boldsymbol{\lambda}$
7: - Get scaling factor for gradient normalization:
$\tilde{\sigma} = \min(\sqrt{\boldsymbol{\lambda}})$
8: - Calculated the orthogonal and normalized gradient matrix as in (15): $\tilde{\boldsymbol{G}} = \tilde{\sigma}\boldsymbol{U}\boldsymbol{\Sigma}^{-1}\boldsymbol{U}^\intercal\boldsymbol{G}$
9: **if** $t = t_{warm}$ **then**
10: - Record the loss values for all the tasks $\mathcal{L}_i(t_{warm})$
11: **else if** $t > t_{warm}$ **then**
12: - Calculate the task difficulty weights as in (17):
$k_i(t) = \text{softmax}(lr_i(t-1))$
13: - Form eccentric vector $\boldsymbol{v}_{ecc} = [k_1, \cdots, k_i]^\intercal$
14: **end if**
15: **Output:** $\tilde{\boldsymbol{g}}_{ecc} = \tilde{\boldsymbol{G}}^\intercal \boldsymbol{v}_{ecc}$ for optimization

Supplementary Materials – Chapter 5

Gradient Normalization

1. The magnitude of each gradient ∝ eigenvalues $\boldsymbol{\lambda}$ [4]

$$\tilde{\boldsymbol{G}} = \boldsymbol{U}\boldsymbol{V}^\intercal \quad \text{with} \quad \boldsymbol{G}\boldsymbol{G}^\intercal = \boldsymbol{U}\left(\boldsymbol{\Sigma}\boldsymbol{\Sigma}^\intercal\right)\boldsymbol{U}^\intercal$$

2. Get scaling factor for gradient normalization:

$$\tilde{\sigma} = \min(\sqrt{\boldsymbol{\lambda}})$$

3. The new gradient matrix is:

$$\tilde{\boldsymbol{G}} = \tilde{\sigma}\boldsymbol{U}\boldsymbol{\Sigma}^{-1}\boldsymbol{U}^\intercal\boldsymbol{G}$$

The magnitude dominance resolved □

Algorithm 1 EGA Optimization Strategy for MTL

Input: Loss values for n tasks $[\mathcal{L}_1, \cdots, \mathcal{L}_i], i \in [n]$,
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- Unit eccentric vector $\boldsymbol{v}_{ecc} = [1, \cdots, 1]^\intercal \in \mathbb{R}^n$
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- Calculated the orthogonal and normalized gradient matrix as in (15): $\tilde{\boldsymbol{G}} = \tilde{\sigma}\boldsymbol{U}\boldsymbol{\Sigma}^{-1}\boldsymbol{U}^\intercal\boldsymbol{G}$

if $t = t_{warm}$ **then**
  - Record the loss values for all the tasks $\mathcal{L}_i(t_{warm})$
else if $t > t_{warm}$ **then**
  - Calculate the task difficulty weights as in (17): $k_i(t) = \text{softmax}(lr_i(t-1))$
  - Form eccentric vector $\boldsymbol{v}_{ecc} = [k_1, \cdots, k_i]^\intercal$
end if
Output: $\tilde{\boldsymbol{g}}_{ecc} = \tilde{\boldsymbol{G}}^\intercal \boldsymbol{v}_{ecc}$ for optimization

[4] D.Senushkin,N.Patakin,A.Kuznetsov,andA.Konushin,"Independent component alignment for multi-task learning," in Proceedir
Recognition, Sep. 2023

Supplementary Materials – Chapter 5

Introduce eccentric vector

1. Run several warmup epoch until the epoch t_{warm}

2. Record the loss values for all the tasks $\mathcal{L}_i(t_{warm})$

3. Calculate learning rate:

$$lr_i(t-1) = \frac{\mathcal{L}_i(t-1)}{\mathcal{L}_i(t_{warm})}$$ (small lr_i for fast learning rate)

4. Calculate task difficulty weight and $\boldsymbol{v}_{ecc}$:

$$k_i(t) = \mathrm{softmax}(lr_i(t-1)) = \frac{n\exp\left(lr_i(t-1)/T\right)}{\sum_{j=1}^{n}\exp\left(lr_j(t-1)/T\right)}$$

$$\boldsymbol{v}_{ecc} = [k_1, \cdots, k_i]^\intercal$$ (small T enlarge discrepancy）

5. Get the final joint gradient:

$$\tilde{\boldsymbol{g}}_{ecc} = \tilde{\boldsymbol{G}}^\intercal \boldsymbol{v}_{ecc}$$

The difficulty imbalance resolved ☐

Algorithm 1 EGA Optimization Strategy for MTL

Input: Loss values for n tasks $[\mathcal{L}_1, \cdots, \mathcal{L}_i], i \in [n]$,
T for softmax and t_{warm} for warmup epoch
INITIALIZATION:
- Unit eccentric vector $\boldsymbol{v}_{ecc} = [1, \cdots, 1]^\intercal \in \mathbb{R}^n$
FOR EACH ITERATION:
- Get the current epoch as t
- Get task-specific gradient $\boldsymbol{g}_i = \nabla_\theta \mathcal{L}_i(\theta), i \in [n]$
- Form gradient matrix $\boldsymbol{G} = \{\boldsymbol{g}_1, \cdots, \boldsymbol{g}_i\} \in \mathbb{R}^{n\times m}$
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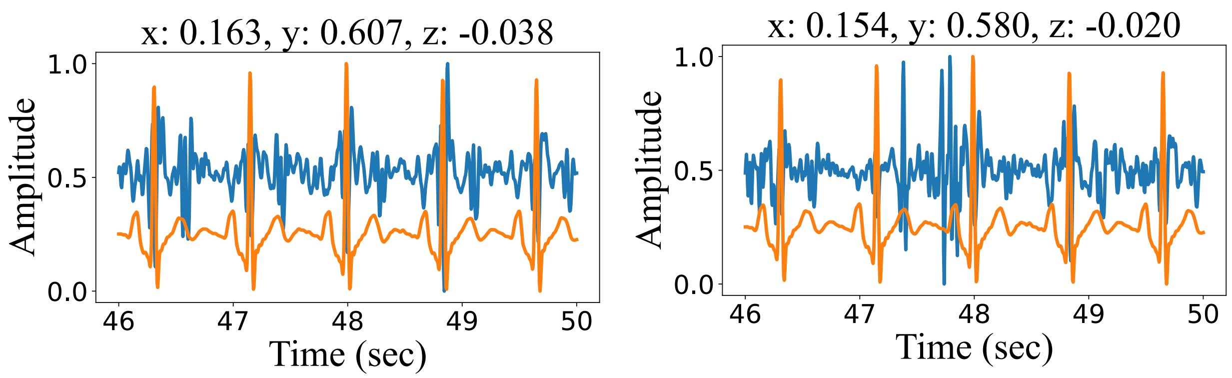
- Calculated the orthogonal and normalized gradient matrix as in (15): $\tilde{\boldsymbol{G}} = \tilde{\sigma}\boldsymbol{U}\boldsymbol{\Sigma}^{-1}\boldsymbol{U}^\intercal\boldsymbol{G}$
if $t = t_{warm}$ **then**
    - Record the loss values for all the tasks $\mathcal{L}_i(t_{warm})$
else if $t > t_{warm}$ **then**
    - Calculate the task difficulty weights as in (17): $k_i(t) = \mathrm{softmax}(lr_i(t-1))$
    - Form eccentric vector $\boldsymbol{v}_{ecc} = [k_1, \cdots, k_i]^\intercal$
end if
Output: $\tilde{\boldsymbol{g}}_{ecc} = \tilde{\boldsymbol{G}}^\intercal \boldsymbol{v}_{ecc}$ for optimization

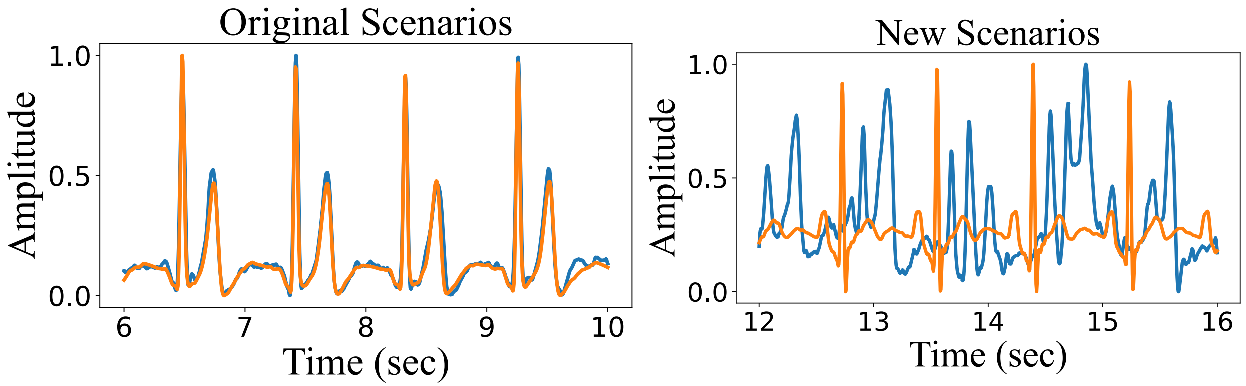
Supplementary Materials – Chapter 6

Challenges:

1. Efficiently extract high-SNR radar signals with ample cardiac features:
   - Assess signal SNR without ground truth.
   - Search for the high-SNR signal within a highly discontinuous space.
2. Learning common representations from unlabeled radar signals to assist supervised training with limited data.



Signal extracted form adjacent points (distance = 3cm)



Inference for the radar signal collected from new scenario