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Model-based Dynamic Control and Optimization of Gas Networks

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MODEL-BASED DYNAMIC CONTROL AND OPTIMIZATION OF GAS NETWORKS

Dr. ing. thesis

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Summary

This work contributes to the research on control, optimization and simulation of gas transmission systems to support the dispatch personnel at gas control centres for the decision makings in the daily operation of the natural gas transportation systems. Different control and optimization strategies have been studied. The focus is on the operation of long distance natural gas transportation systems.

Stationary optimization in conjunction with linear model predictive control using state space models is proposed for supply security, the control of quality parameters and minimization of transportation costs for networks offering transportation services. The result from the stationary optimization together with a reformulation of a simplified fluid flow model formulates a linear dynamic optimization model. This model is used in a finite time control and state constrained linear model predictive controller. The deviation from the control and the state reference determined from the stationary optimization is penalized quadratically. Because of the time varying status of infrastructure, the control space is also generally time varying. When the average load is expected to change considerably, a new stationary optimization is performed, giving a new state and control reference together with a new dynamic model that is used for both optimization and state estimation.

Another proposed control strategy is a control and output constrained nonlinear model predictive controller for the operation of gas transmission systems. Here, the objective is also the security of the supply, quality control and minimization of transportation costs. An output vector is defined, which together with a control vector are both penalized quadratically from their respective references in the objective function. The nonlinear model predictive controller can be combined with a stationary optimization. At each sampling instant, a nonconvex nonlinear programming problem is solved giving a local minimum by a structured sequential quadratic programming algorithm of Newton type. Each open loop problem is specified using a nonlinear prediction model. For each iteration of the quadratic programming procedure, a linear time variant prediction model is formulated. The suggested controller also handles time varying source capacity.

Potential problems such as infeasibility and the security of the supply when facing

a change in the status of the infrastructure of the transmission system under a transient customer load are treated. Comments on the infeasibility due to errors such as load forecast error, model error and state estimation error are also discussed.

A simplified nonlinear model called the creep flow model is used to describe the fluid dynamics inside a natural gas transmission line. Different assumptions and reformulations of this model yield the different control, simulation and optimization models used in this thesis.

The control of a single gas transmission line is investigated using linear model predictive control based on instant linearization of the nonlinear model. Model predictive control using a biquadratic optimization model formulated from the creep flow model is also investigated.

A distributed parameter control model of the gas dynamics for a transmission line is formulated. An analytic solution of this model is given with both Neuman boundary conditions and distributed supplies and loads. A transfer function model is developed expressing the dynamics between the defined output and the control and disturbance inputs of the transmission line.

Based on the qualitative behaviour observed from the step responses of the solutions of the distributed parameter model formulated in this thesis, simplified transfer function models were developed. These control models expresses the dynamics of a natural gas transmission line with Neuman boundary control and load. Further, these models were used to design a control law, which is a combination of a Smith predictor and feedforward from a predicted load pattern.

A boundary control model assumed to describe the fluid dynamics of a long distance transmission line was defined. Then, a control model that was an approximation to the defined boundary control model was defined. It was shown that the state solution of the approximated model was in the limit equal to the defined boundary control model. Then, a nominally exponentially stable distributed parameter control system has been designed for a natural gas transmission line control model. The control system is a combination of a feedback controller and a distributed parameter Luenberger state observer combined with a feedforward term from the predicted load forecast. Approximate controllability and approximate observability have been shown. It has also been shown that the defined control model is exponentially stabilizable and exponentially detectable. Further, the distributed parameter linear quadratic optimal regulation problem has been defined using the same control model.

A linear distributed parameter model for gas networks was formulated. This model gives a simplified description of the fluid dynamics close to a defined stationary operation point. The operation point can be determined based on the solution of a defined stationary optimization problem. The formulation gives an expression for the operator that models the dynamics and the couplings of the transmission system. An expression for the semigroup generated by this system operator is also given. This semigroup takes part in the expression for the mild solution of the transmission system state. The state solution is given as the solution of the inhomogenous Cauchy problem. A finite time horizon open loop mathematical programming problem to be used in a model predictive control scheme with quadratic cost function and the derived linear distributed parameter network model was formulated.

A simple time delay quality tracker based on a defined stationary operation point has been proposed. The quality tracker can be used in combination with the non-linear model predictive control scheme suggested in this thesis for the control of quality parameters.

It has been suggested how a simple thermodynamic state equation for a natural gas can be formulated. Identification is used to determine the values of the parameters of this equation.

Preface

This thesis is submitted in partial fulfilment of the requirements for the degree of Dr. Ing. (doktor ingeniør) at the Norwegian University of Science and Technology (NTNU). The research work has been carried out at the Department of Engineering Cybernetics, NTNU during the period January 1996 to December 2000, and was financed by the Research Council of Norway.

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Chapter 1

Introduction

1.1 Introduction and Problem Definition

Natural gas transmission networks such as the off-shore system in the Northern Continental Shelf, are characterised by a few pipelines but with long transportation distances from the sources to the customer terminals. To save high investment costs due to offshore installations, there is hardly any recompression along the transportation route. Compression is only performed at source points. Important quality parameters, such as gross calorific value and concentration of components in the natural gas, can be controlled to some extent at the mixing stations or junctions to reach sales gas quality. Control valves and flow through sources directly connected to the mixing facility are used at the mixing stations, to determine flow ratios and pressure level at input lines and output lines of the mixing station. Different sources have different compositions giving the need for mixing the natural gas before the gas is delivered to the customers. The pressure level of the pipelines is high due the lack of recompression. Cleansing trains are run through the transportation lines at a regular basis for inspection and cleaning, which reduces friction and removes water and other unwanted components. Smaller friction values imply reduced transportation costs. Molecules entering source points can be delayed up to several days before they reach the defined customers terminal points. So, the time delay is extensive. Controlling the pressure at customer terminals by changing the conditions at the source points or at the mixing stations must be done as a predictive action due to the time delay and the need of load forecasts of customer demands. Gas is entered into the transmission system from the processing facilities placed at offshore installations or on-shore or from natural gas storage facilities. A supervisory and control system (SCADA) available to the dispatch personnel provides feedback on network status and interface for gas control. The main task for the operators at a gas dispatch centre is the security of the supply providing the needed customer volumes with customer pressures above contracted limits, the control of quality parameters within the contractual specifications and reducing transportation costs. Handling of contracts is a vital part of the activity for gas dispatchers. The dispatch activity is sometimes divided into two tasks, one for handling the contracts and the other for the physical gas flows. The two tasks are highly interconnected. Operators at the installations of the transportation system gives important status feedback to the personnel at the gas control centre. Future production capacity can be one of these messages to the gas control centre.

Processing facilities with compressor stations producing natural gas to be injected into the transportation system receive hydrocarbons from many different sources. The maximum production capacity is not only limited by the equipment, but also by the status of the sources delivering hydrocarbons to the processing facility. Bad weather at offshore, may lead to the close down or major reduction in production rate yielding a lower supply of hydrocarbons to the gas processing facility. As a result, the supply capacity into the transmission system is time varying. If one supply point faces capacity reduction, another source can back up this lack of supply so that the customer demands can still be met. Contractual agreements specify operational limitations and some directions for this procedure. So, in practice the control space is highly time varying. Gas contracts will usually specify an obliged minimum delivery amount from each source point in compliance with the customer demand.

Transportation costs are mainly due to the use of natural gas for compression in combination with environmental taxes of components that are considered as harmful to the environment. The taxes are defined by the ruling body.

In this thesis, the focus will be on control systems as a support tool for the dispatch personnel for the operation of long distance gas transmission systems with the objective of supply security, quality control and reducing transportation costs while meeting contractual and physical constraints. The transmission system is assumed to provide transportation services for the owners of the gas sources and the gas customers. The control commands determined will mainly be references to low level control systems. Examples of control commands are mass flow references to control valves and mass flows through input-output combinations of a the compressor stations. The dynamics of these control systems will be assumed negligible compared to the pipeline dynamics. It will be assumed that the control commands are piecewise constant. Both system dynamics and the many activities by the dispatch personnel make sampling intervals of fifteen to sixty minutes typical for a gas transmission network.

A gas transmission system consists mainly of source points, customer points, transmission pipelines, junctions, storage facilities which can be treated as source or offtake points, control valves, compressor stations with drivers and mixing facilities. In addition, on/off valves can determine grid configuration of the transmission system. These valves may also be used for the pigging operation and the possibility of closing down parts of the transmission system in case of maintenance or expansion.

In many cases, it is possible to change the compressor station configuration. With this flexibility in place, it can be possible to choose which units that are to be in operation. Different combinations of inputs and outputs of a compressor stations may be possible to configure. Parallel configurations of compressor units can be defined with the objective of increasing flow capacity for some chosen input and output combination of the compressor station. Alternatively, serial configurations of compressor units can be defined with the objective of increasing the compression ratio for some chosen input and output combination of the compressor station. The selection of specific driver units in operation such as gas turbines or electric motors, can have an economic significance depending on the total cost for the energy sources used by the different drivers.

In the work of this thesis, it will be assumed that the network configuration and the configuration of each compressor station are given.

The fluid flow using first principles is complexly described by a coupled set of nonlinear partial differential equations using the conservation laws with an additional thermodynamic state equation. The dispatch personnel uses complex simulators for the supervision of the state of the transmission system. A look ahead model is also available for the prediction of future possible scenarios. Usually, the simulation and prediction model includes the tracking of important quality parameters.

If, for example the customer pressure is below the defined minimum pressure for the optimization algorithm and it is not possible to operate the system in the nearest future so that pressure gets above the lower limit, the system is said to be infeasible. The infeasibility mode must be considered when designing optimization algorithms since one has to expect this situation to occur sometimes. The reasons may be the current status of the system infrastructure such as the lack of source capacity, which makes it difficult to meet customer demands, simplified optimization formulations and errors in load forecasts. A simplified description of fluid behaviour to produce an optimization model which renders an optimization formulation computationally reasonable, can be another cause for infeasibility.

1.2 Previous Work in Dynamic Optimization & Control of Gas Networks

A survey of optimization techniques for large networks of gas and water is given in Osiadacz and Bell (1988c).

In Marques and Morari (1985), an on-line model predictive control scheme in a hierarchical fashion is suggested. First, a model predictive controller with a moving horizon of 24 hours is designed, where the 24 hour predicted gas demand trajectories and source pressures are inserted in the optimizer and control commands for each compressor station with a parametrized control sequence consisting of just one parameter control vector. The second step is a short term dynamic optimization of a moving horizon of four hours, reoptimized every hour with a given control parametrization for the compressor station and pipeline connected to the consumer point. The other stations of the network use the 24 hour optimized control. This reduces computational effort. Since the proposed scheme reoptimizes every hour, the demand predictions are revised every hour. Each open loop problem is non convex. The thesis by Marques (1985), gives further treatment on the above topic.

In Wong and Larsson (1968a), a gas network with a single source and tree structure is stationary optimized for minimizing compression power consumption and meeting the system constraints using dynamic programming. The problem is systematically solved by a sequence of one dimensional dynamic programs (one station is optimized at a time with one variable), taking one transmission line at a time and one section of this line at a time going from the end points to the source.

The paper by Wong and Larsson (1968b) presents a static optimizer using dynamic programming. Dynamic programming is suggested for dynamic short term optimization of the section closest to the offtake customer point, that is, the last compressor station with pipeline connected to offtake point. The combination of the static and dynamic optimizer yields a hierarchical structure.

In Lewandowski *et al.* (1974), a real time optimization algorithm for natural gas transmission systems is suggested. The authors define an objective function consisting of operational costs, failure to meet delivery demands and reliability of the system for each pipeline segment of the transmission system. A pipeline segment consists of a compressor station at the inlet of a pipeline and with a customer point at the outlet together with flow into the next station and segment. The objective parts for all the segments are summed and the complete function is to be minimized. They present two different transmission system structures: radial and ring type respectively. Two simplified linear models of diffusive type to describe the fluid dynamics are used for the optimization with different accuracy. Compressor stations with units of piston type in parallel configuration are used. The authors assume the use of a static optimizer to find the optimal configuration of a compressor station but do not treat this explicitly. All demand points are assumed, for simplicity to be at the end of each pipeline segment. Control commands are mass flow through each compressor station. A customer point is assumed to be interpreted as a large collection of consumers. Local distribution systems are assumed to handle end users. These consumer groups are considered as stochastic processes. The partial differential equations are spatial discretized giving a set of linear ordinary differential equations which are further converted to a discrete time model. The authors consider six to seven differential equations as satisfactory to describe the diffusive behaviour. The pressure distribution for each pipeline is modelled to be within a maximum and minimum limit for each pipeline segment. Flow through a station for a given segment must satisfy a maximum and a minimum flow limit when given discharge and suction pressure over the station. The continuous infinite dimensional control problem is approximated by control parametrization. A linear combination of basis functions of piecewise linear functions for the optimization horizon is used to express a continuous control trajectory. Together with the constraints including the dynamic model for each segment, a resulting quadratic program is formulated. This paper does not generalize to cover all types of networks. For a generalization, a node point model must be included.

In Larsson and Wismer (1971), the concepts of hierarchical system theory is used to develop a control methodology for large complex pipeline systems. The network is divided into subsystems consisting of a single compressor and a pipeline network directly driven by it. A control policy for each individual subsystem is calculated. Then, each optimized subsystem is coordinated using hierarchical system theory to achieve control of the overall pipeline network.

In Osiadacz (1994), a general gas network is dynamically optimized for minimization of operational costs using a simplified linear network model. The cost function, the linear dynamic model and the constraints define a nonlinear program which is reformulated to the Lagrangian Dual Problem solved by the cutting plane method to give the optimal control.

In Sood *et al.* (1971), dynamic optimization of a combined system of a single compressor and a single natural gas pipeline is proposed. The set of partial differential equations for mass balance and a simplified stationary momentum equation have their spatial derivatives numerically approximated, to yield a set of nonlinear differential equations that are linearized and integrated analytically to give a linear discrete pipeline model. A nonlinear programming formulation follows from the discretized energy cost function, the linear pipeline model and the defined constraints. This mathematical program is solved using a gradient search technique giving the control trajectory for the considered optimization horizon. It is suggested that the control scheme can be used on or off-line. Reoptimization as in model predictive control fashion is not suggested.

In Nieplocha (1988), unconstrained discrete linear quadratic optimal control is proposed for optimal control of gas networks. A linear continuous model is developed. This model is integrated using a procedure for taking account of the high stiffness ratio of the system matrix to give a discrete, numerically stable, linear network model. A quadratic objective function is defined to approximate the original operational cost function. The resulting control problem with quadratic objective function and linear model then results in a linear time variant feedback rule from the state vector that follows from the linear quadratic control theory from the solution of the well known Riccati equation plus an additional predicted feedforward term from predicted load. An additional integrator is also suggested to compensate for model errors, etc.

In the paper by Furey (1993), it is developed a sequential quadratic programming algorithm to solve non convex optimization problems for dynamic optimization of gas transmission networks consisting of continuous variables. Emphasis was put on the algorithmic aspects to solve a defined open loop problem.

In Osiadacz (1988a), dynamic optimization using hierarchical methods is considered. Simplified algorithms for the optimization of large scale gas networks are given in Osiadacz and Bell (1981) and Osiadacz and Bell (1986).

The paper by Parkinson and Wynne (1992) looks at modelling and control applied to low pressure gas distribution networks. The paper by Kiuchi *et al.* (1995), suggest an operation support system of large city gas networks. Control of gas transport systems using a multilevel approach is given in Stelter (1988). A computer program for the optimization of natural gas pipeline operation, is suggested in Mantri *et al.* (1996). Gas network operation optimization using singular perturbation methods is suggested in Stelter and Eldin (1987). Modelling and control system design of natural gas supply systems is treated in Fukuda *et al.* (1989). Optimum pressure control in gas networks for the reduction of losses is given in Davenport and Brammelar (1972). The paper by Wynne *et al.* (1988), considers design and implementation of a control system for a gas network with several supplies. See also the paper by Wynne (1991), for modelling and simulation for control system design.

1.3 Contributions

This work contributes to the research on control and optimization of gas transmission systems to support the dispatch personnel at gas control centres for the decision makings in the daily operation of the natural gas transportation systems. Different control and optimization strategies have been suggested. In these strategies, different control, simulation and optimization models have been formulated.

1. Stationary optimization, in conjunction with linear model predictive control using state space models, is proposed for the security of the supply, the control of quality parameters and the minimization of transportation costs for natural gas networks offering transportation services. The stationary optimization problem is solved to a local minimum using an unstructured sequential quadratic programming algorithm. Alternatively, it is mentioned that the problem can be solved to a global minimum if a global optimization algorithm is used to solve the defined optimization problem. It has also been considered how the stationary optimization problem can be formulated for a gas network when a simplified coupled set of the mass, momentum and energy equation together with a state equation are used to describe the stationary fluid flow inside a transmission line. The result from the stationary optimization together with a reformulation of the nonlinear creep flow model define a linear dynamic control model. It is pointed out that if the stationary optimization also considers the energy equation, then stationary complex thermodynamics will be included implicitly in the dynamic control model. The model predictive controller is both control and state constrained. The predicted control space for each open loop problem is assumed generally time varying.

2. Nonlinear model predictive control for the operation of gas transportation systems, is another approach proposed in this thesis. An output vector is defined which is penalized quadratically from an output reference in the objective function. Important quality parameters are included in the output vector and are therefore controlled on a transient basis. A control vector of continuous variables is defined which is penalized quadratically from a defined control reference. The output and control references may be specified based on the result from a stationary optimization problem, just as in Contribution 1. The defined control and output vector are both upper and lower limit constrained. If source compositions are transient, it is pointed out that a quality tracker may be needed in connection with the control scheme to predict the value of important quality parameters entering mixing stations or junction points. The source space can generally be time varying. Each open loop nonlinear programming problem is solved by a structured sequential quadratic programming procedure of Newton type. This procedure uses a linear time variant prediction model formulated from the nonlinear creep flow model.

Simulation examples for both the control structures are presented and the infeasibility for the closed loop controllers are considered. Simulation with a time varying source capacity is presented for the linear model predictive controller. The simulation example shows that the lack of source space for one source for a limited period is taken account of by another source to provide the customer supply security. Simulations also shows that a system in infeasibility mode is brought back to the feasible region. The controller settles the system smoothly to a stationary operation point under a constant load, admissible controls and state initial condition of simulation horizon which is different from the defined current operation point. A Kalman filter that uses the same optimization model as the linear model predictive controller is designed to provide the estimated initial conditions for each open loop problem.

3. The nonlinear creep flow model used as a basis for the control models, the optimization models and the simulation model in Contribution 1 and 2, can also be reformulated to other model representations. The control of a single gas transmission line is investigated using linear model predictive control based on instant linearization of the nonlinear creep flow model. Also, model predictive control using a biquadratic optimization model is investigated. Simulation results for the two model predictive controllers are presented. These control models can be generalized to network representations. A stationary operation point using the stationary equation developed by the American Gas Association is used to provide state and control references.

4. A distributed parameter control model for a transmission line with a linear dynamic inner core is formulated. The control model is nonlinear in the physical variables pressure and mass flow. Given an initial condition, the analytical solution for the state behaviour is given for the case with Neuman boundary conditions. Then, the analytic solution for the case with Neuman boundary conditions and supplies and loads along the transmission line is given. The solution is found after the control model is formulated as an inhomogenous Cauchy problem. To achieve this formulation, the control model is reformulated to an associated form where the boundary conditions are equal to zero.

5. A transfer function model describing the dynamics between the defined control inputs and the defined state outputs and another transfer function model describing the dynamics between the defined disturbance inputs and the defined state outputs, are formulated based on the distributed parameter control model in Contribution 4. The control model increases insight into the dynamics of a long distance natural gas transmission line. Eigenvalues, gains and time constants are explicit functions of fluid properties and pipeline characteristics. The transfer function model can be used to design a control system with the help of multivariable frequency analysis control theory.

6. The dynamics of a natural gas transmission line with the supply of mass at the input boundary and the customer offtake at the output boundary as in point four will be defined. Then, an approximation to the boundary control model is defined. It is shown that the state solution of the approximated boundary control model is equal in the limit to the state solution of the boundary control model. Then, a nominally exponentially stable distributed parameter control system, is designed for a natural gas transmission line. Basis for the controller design is the approximated boundary control model. The control system is a combination of a distributed parameter feedback controller and a distributed parameter Luenberger state observer. It is shown that the defined control model is approximately controllable and approximately observable with the defined available control command and the defined available observation. Then, it is shown that the control model is exponentially stabilizable and exponentially detectable. Finally, it is proved that the defined control system is nominally exponentially stable. The feedback term is combined with a feedforward term from the customer load forecast.

7. A linear distributed parameter network model for long distance natural gas transmission systems has also been derived. The dynamic model gives a local description of the dynamics around a defined stationary operation point. All the boundary conditions for each transmission line are zero. The system operator expressing the dynamics and the couplings of the transmission system is explicitly obtained. Also, an expression for the strongly continuous semigroup generated by this system operator is derived. The state solution is given as the solution to the inhomogenous Cauchy problem.

8. A control system is proposed for a transmission line being a combination of a feedback controller and a feedforward term. The transfer function control model used is developed based on the qualitative behaviour of the step responses of the distributed parameter control model mentioned in Contribution 4. The feedback control part is a Smith predictor that cancels the time delay in the open loop so that the bandwidth of the control system can be increased compared to a traditional series compensator design. An example with simulation result is presented.

9. A simple time delay quality tracker based on a stationary operation point has been proposed. The tracker can be used in combination with the nonlinear model predictive controller, proposed in this thesis, for the control of quality parameters.

10. Identification used to determine the values of the parameters of a proposed simple thermodynamic state equation for a natural gas has also been suggested.

1.4 Outline

Chapter 1 is an introductory chapter. Chapter 2 reviews the fundamental one-dimensional fluid equations and some selected topics with accompanying references of relevance to this research work and in the field of gas transportation. Chapter 3 presents a control scheme which is a combination of stationary optimization and linear model predictive control. In Chapter 4, nonlinear model predictive control of gas transmission systems is considered. Chapter 5 first looks at different control models and control strategies for a single gas transmission line. Chapter 5 then looks at how a gas transmission network can be formulated as a distributed parameter system. The main conclusions and suggestions for future work are given in Chapter 6.

Chapter 2

Natural Gas Transportation

2.1 Introduction

This chapter deals with subjects that are relevant to the work presented in this thesis and in the gas transportation business. The fundamental fluid equations are presented. Assumptions and simplifications of the complex equations to arrive at a simplified description are given. The natural gas supply chain and many of the activities at Gas Dispatching Centres are treated. Many stakeholders are involved in the production, transportation, natural gas market and consumption of natural gas. The optimization objectives may be highly diversified. The terms common carriage, third party access and open access will be explained. As the natural gas market is being less regulated opening up for the possibility of increased free trade, the relevance of these terms is increasing. The European Gas Market is under continuously development. The focus on free trade is increasing. Four different developing stages of a European Gas Market are summarized from Estrada *et al.* (1995). A section on end user load forecasting with some accompanying references is given. References on stationary and transient simulation are presented. Also, references to the problem of optimal pipeline design are given. Previous work in tracking of quality parameters are presented. A simple time delay quality tracker is also proposed.

2.2 Fundamental Fluid Equations

The mass, momentum and energy equations in their complete form for one dimensional fluid flow are given in Eqns. (2.1)-(2.4). See Rist (1996) or Osiadacz (1987) for references. The general form of the mass and momentum equations in three dimensional space for any fluid can be found in White (1994). The vector energy equation is found in Bejan (1993).

Mass balance

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho \upsilon) = 0 \tag{2.1}$$

Momentum balance

$$\rho \cdot \frac{\partial \upsilon}{\partial t} + \rho \upsilon \cdot \frac{\partial \upsilon}{\partial x} + \frac{\partial p}{\partial x} + \frac{\lambda}{D_{\mathrm{h}}} \cdot \frac{\rho}{2} \cdot \upsilon^2 + \rho g \sin\theta - k_{\upsilon} \cdot \frac{\partial}{\partial x}\left(\mu \cdot \frac{\partial \upsilon}{\partial x}\right) \tag{2.2}$$

$$= 0$$

Energy balance

$$\frac{\partial h_{\mathrm{t}}}{\partial t} + \upsilon \cdot \frac{\partial h_{\mathrm{t}}}{\partial x} = \frac{1}{\rho} \cdot \left(\frac{\partial p}{\partial t} + \frac{4k_h}{D_h} \cdot (T_{\mathrm{amb}} - T)\right. \tag{2.3}$$

$$\left. + \frac{\partial}{\partial x}\left(k \cdot \frac{\partial T}{\partial x}\right) + k_{\upsilon}\left[\frac{\partial}{\partial x}\left(\mu \upsilon \cdot \frac{\partial \upsilon}{\partial x}\right) + 4\mu \tan\alpha \cdot \frac{\upsilon}{D_{\mathrm{h}}} \cdot \frac{\partial \upsilon}{\partial x}\right]\right)$$

$$- \upsilon g \sin\theta$$

The total enthalpy is expressed as

$$h_{\mathrm{t}} = h + \frac{\upsilon^2}{2} = u + \frac{p}{\rho} + \frac{\upsilon^2}{2} \tag{2.4}$$

<u>Friction Factor:</u>

Von Karmans equation for fully developed turbulent flow in rough pipes is given in Eqn. (2.5).

$$\frac{1}{\lambda} = 2 \cdot \log\left(\frac{3.7}{\varepsilon / D}\right) \tag{2.5}$$

See Fasold and Wahle (1996a), Rist (1996) or White (1994). The coefficient ε is the absolute wall roughness and D is the diameter of the pipeleg. This equation has its operation range where the inequality $\mathrm{Re} > (\mathrm{Re})_{\mathrm{cr}}$ is satisfied. The Reynolds number is defined as

$$\mathrm{Re} = \frac{\rho \cdot \upsilon \cdot D}{\mu} \tag{2.6}$$

and the critical Reynolds number is calculated as

$$(\mathrm{Re})_{\mathrm{cr}} = \frac{20.9}{\varepsilon/D} \cdot \log\left(\frac{3.7}{\varepsilon/D}\right). \tag{2.7}$$

For natural gas transportation in pipelines on the Norwegian Continental Shelf, the following values are typical (Sjøen,1997)

$$\left.\begin{array}{r} \mathrm{Re} \in [10^7, 10^8] \\ \varepsilon/D \in [10^{-6}, 10^{-4}] \end{array}\right\}.$$

So, there is a fully developed turbulent flow. From the above equations, we see that the friction factor is a constant for fully developed turbulent flow as long as the wall roughness is constant.

Heat transfer coefficient:

For accurate calculations of the heat transfer coefficient k_{h}, the calculations must be based on the materials of the pipe and the composition and the thermal properties of the surrounding medium. Sjøen (1997) gives a formula to calculate the resulting total heat transfer coefficient as in Eqn. (2.8).

$$\frac{1}{k_{\mathrm{h}}} = \frac{1}{h_{\mathrm{i}}} + \frac{D}{2} \cdot \sum_{n=1}^{N} \frac{1}{k_n} \ln\left(\frac{D_{n+1}}{D_n}\right) + \frac{D}{D_{\mathrm{o}} \cdot h_{\mathrm{o}}}. \tag{2.8}$$

The coefficients h_{i}, h_{o} are respectively inner and outer film coefficient inside the pipe in connection to heat transfer by convection. k_n is the heat transfer coefficient in the n - layer. D_n is the inner diameter in the n - layer and there are a total of N layers. The convective heat transfer is given by

$$h = \frac{\mathrm{Nu} \cdot k}{D} \tag{2.9}$$

where Nu is the Nusselt number and k is the thermal conductivity of the fluid. For turbulent flow

$$\mathrm{Nu} = C \cdot (\mathrm{Re})^m \cdot (\mathrm{Pr})^n \tag{2.10}$$

where the constants C, m and n are determined experimentally. The Prandtl number is defined as

$$\mathrm{Pr} = \frac{C_\mathrm{p}(p, T) \cdot \mu}{k} \tag{2.11}$$

where C_p is the heat capacity of the natural gas. Typical range for resulting heat transfer coefficients under different conditions are (Sjøen,1997):

i) For trenched pipes, $k_h \in [5,10]\mathrm{W/m^2/K}$.

ii) For pipes exposed to water, $k_h \in [10,20]\mathrm{W/m^2/K}$.

2.3 Simplification of the Fluid Equations.

In the book by Osiadacz (1987), a study was done for a pipeline at stationary conditions with the objective to compare the relative size of the terms in the momentum equation under isothermal conditions. For the studied example, it was found that the drop in pressure was mainly caused by the friction term and that the acceleration term was negligible. The simulation study was done for a very slow fluid motion with a fluid velocity range of about five to ten m/s for a pipeline with a length of fifty kilometers. It was stated that the acceleration term should be taken account of if the boundary conditions change rapidly, especially for the fluid description close to the boundaries. But for long transmission pipelines, the effect of transients at the boundaries will be effectively dampened by the long pipeline length and friction. Also, usually the boundary flows are piecewise constant indicating that the transient flow change is only for a short period of time going from one constant level to another. A similar study was performed by Guy (1967) and Goldwater and Fincham (1981). Considering fluid flow in slow motion, which is usually the case for natural gas in long distance transmission lines, and neglecting the total acceleration of the fluid particle and the acceleration due to external force and assume isothermal conditions, yields Eqn. (2.12).

$$\begin{aligned} \rho_t + \nabla \rho v &= 0 \\ \nabla p &= \mu \Delta v \\ \rho &= \rho(p) \end{aligned} \tag{2.12}$$

This problem is also called Stokes problem or creep flow. Using the state equation $p = c^2 \rho$, we can write the one dimensional creep flow model given in Eqns. (2.13)-(2.14).

$$\frac{A}{c^2} \cdot \frac{\partial p}{\partial t} = -\frac{\partial W}{\partial x} \tag{2.13}$$

$$\frac{\partial p}{\partial x} = -\frac{2 f \rho v^2}{D} \tag{2.14}$$

The nonlinear model given in Eqn. (2.13) and Eqn. (2.14), will be used as a closed loop simulation model for representing the fluid behaviour in long distance transportation systems which will be the main concern in this thesis. If the gravitational force cannot be neglected, then this additional term will represent a spatially varying constant in correspondence with the spatially varying pipeline angle. As the length of a transmission segment decreases, the contribution from the acceleration terms in the momentum equations will increase (Osiadacz,1987;Goldwater & Fincham,1981;Guy,1967;Stelter,1987).

A simulation model should meet two main requirements. First, the model should represent the effects of interest to us giving correct conclusions. Second, it should be as simple as possible. A compromise between the two contradicting requirements will usually be the case. This conclusion was stated in Osiadacz (1987).

Optimization models for the operation of gas transmission networks will be formulated later in this thesis. The nonlinear creep flow model will be used as a basis. Stationary optimization models will also be formulated.

2.4 Gas Supply Chain

The supply chain for natural gas is an integration and coordination of three different functions: production, transmission and distribution, as illustrated in Figure 2.1. Production, transport and storage of liquified natural gas is omitted in the figure. There are different types of supply sources of natural gas. Purchase gas is gas contracted from producers, operators, brokers and other pipeline companies. The gas is purchased in long and short-term contracts. The natural gas may be well gas, pipeline gas and spot gas. Storage gas is the gas available from underground storages, such as salt caverns and liquified natural gas stored in tanks above ground. This type of gas may, for example, be used for peak shaving and for covering extra demand during the winter. An extremely cold winter day may require the use of a peak shaving facility. Transportation gas is transported from a receipt point on the pipeline to a delivery point on the pipeline. Alternatively, gas is delivered to an end user. If a pipeline has the capacity above the current contracts, there is a possibility for spot sales. Exchange gas between the supplier and the customer is delivered under two contracts. Deferred-exchange gas is physically delivered into a transportation system at one time. Later, the customer receives this amount of gas on request. Simultaneous-exchange gas is received into the pipeline system at one location and delivered to a customer at another location at the same time. For unusual situations that may occur in a transmission system, gas supply may be handled by a category called emergency gas. This type may be necessary, if for example, an important gas source must closed down. Extremely cold weather causing an extra need for gas is another example that may cause the need of emergency gas. Gas stored in underground gas storages might handle the situation.

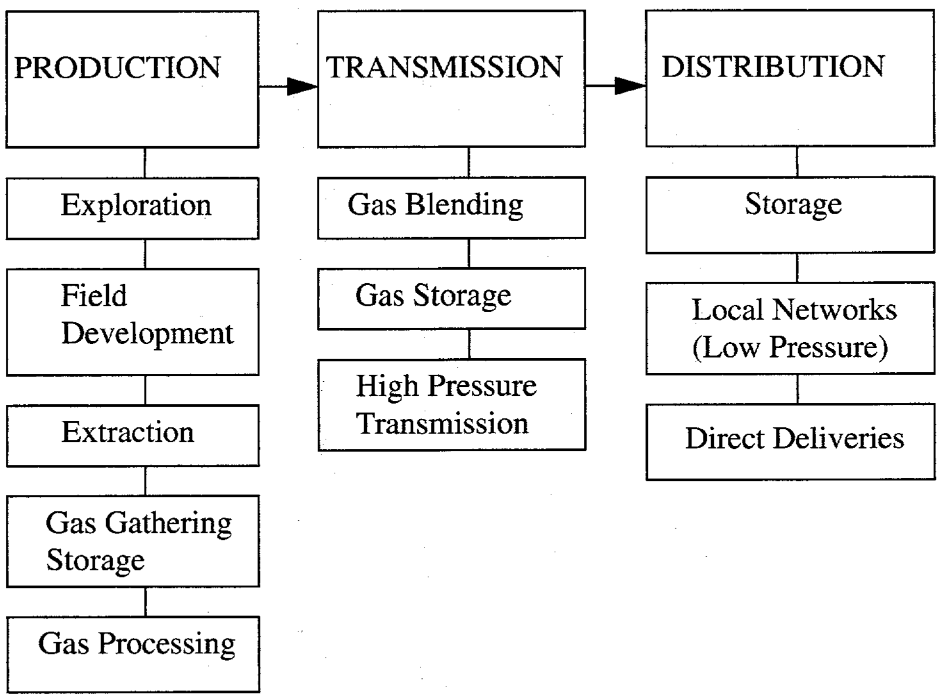


Figure 2.1: Natural gas chain. Source: Estrada *et al.* (1995).

Different types of pipelines or distribution companies may exist within a country. Figure 2.2 from the paper by Ewers and Stackelberg (1993) illustrates the diversity in regional and local gas suppliers in Germany.

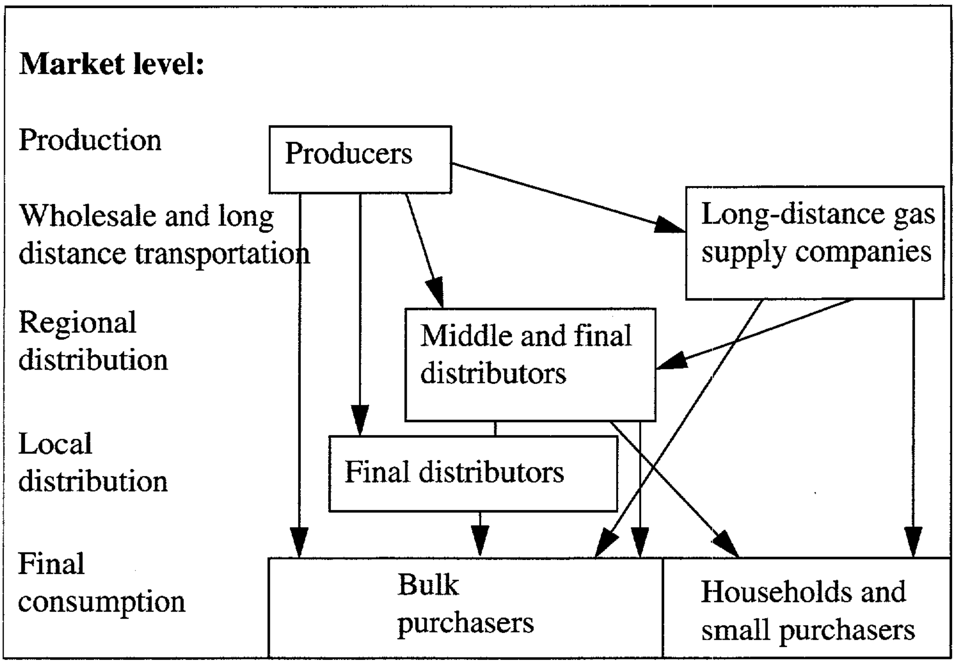


Figure 2.2: Natural gas transportation companies. Source: Ewers and Stackelberg (1993).

2.5 Gas Dispatching Activities

The daily work performed by the personnel at a gas control centre includes many interacting activities. Volume planning, contract handling, grid simulation and control are the main activities. Gas dispatching activities and the available operational tools used to support the dispatchers for the network operation at Ruhrgas AG, are explained in the paper by Holschumacher and Wolf (1991). The paper by Dominican (1990) focuses on the management and the challenges of the English gas transmission system.

The main objective of a transportation system is that it shall receive, transport, process and redeliver the shippers gas delivered under gas sales agreements and according to the conditions defined in the transportation agreement. So, the needs and demands of the shippers specify the duties and tasks for the transport operator.

The transport operator (dispatcher) must have a close relationship with the field operators and the operators of other transportation systems (for example downstream networks). The tasks shall be done in a safe, efficient, prudent and businesslike manner.

The fundamental task is to balance the system's gas requirements with adequate supplies from available sources. If the normal supplies are insufficient to meet the customer demands for natural gas, one may acquire gas from additional supplies or gas storages. If the supply is not adequate to meet the demand, then it is possible to restrict interruptible load. It might be necessary, in some cases, to curtail firm loads so that the transmission system is not depacked to an inventory under accepted operational specifications. The contracts will describe the handling of such a situation including tariff procedures. The gas contracts contain specifications of heating value, gas properties and maximum and minimum limits of certain components in the natural gas. Due to the different properties and compositions of each supply source, gas has to be blended at the mixing stations and junction points to arrive at a quality inside the contractual specifications. Pressures and temperatures at defined customer terminals must also satisfy the contracts. In addition to the handling of physical flows and gas quality, the contractual flow of sales gas between the sellers and the customers must be handled satisfactorily. This non-physical flow is measured in energy units. The physical flow and contractual flow may not be directly correlated. An example is a customer that accepts to receive more gas than needed. The customer pays for the needed amount. This extra amount is gathered into a "paper" storage. The seller may have to pay to the customer for this abstract stored amount. The stored amount can then be transferred as contract flow (and not physical flow) of gas to the customer at a later time. The seller then receives payment for this amount.

In a supply contract between the sellers and the customers, the deliveries are usually defined against specified contract fields. The contract may allow that the deliveries satisfying the customer volumes, can be received from both the contract fields and the other supply fields. Usually, some determined minimum amount must be delivered from the contract fields. This supply diversity increases the degree of freedom for the operators of the transmission system that handles the physical flow and the possibility for the personnel who are responsible for the contract flow to satisfy the contractual supply to the customers. With an increased source space, optimization possibility directed to a specific stakeholder can be increased. As an example, assume the stakeholder is the owner of the pipelines. The contracts and the available production capacity, defines the upper and the lower flow limits of the supply sources. Use of an optimizer finds the optimal supply flows that satisfy the contract with the buyer's natural gas demand and at the same time minimizes the transmission cost.

Different operational and contractual arrangements such as SWAP, operational imbalance, operational flexibility, line pack and operational assistance are carried out at the dispatch centre.

As mentioned above firm customers which are usually non interruptible, have higher priority than interruptible customers. If there is an increased transportation capacity in the system, then the dispatch centre may add spot sales. By overbooking transportation service through a pipeline, the transmission capacity is utilized to a maximum. Service to the interruptible customers may then be reduced so that the transmission capacity is maximized.

Bad weather, status of infrastructure and strikes are examples of causes for "force majoure". The gas dispatchers may in such a case handle shut-downs of the transmission system. This must be done in a safe and prudent manner. When the situation eventually turns to normal, a controlled restart takes place.

The task of gas dispatching for the Norwegian transportation system is divided between the Gas Sales Centre (GSC) and the Transport Control Centre (TCC). GSC handles the contractual flow while the handling of the operation of the physical system is done at TCC. An extensive communication between GSC and TCC is necessary so that both the contracts and physical deliveries are satisfied. Information is updated often and instructions can be revised at any time. The Transport Control Centre sends physical delivery instructions to the production fields. Information about the current and the future production availability is received from operators at production fields. Information out from the Gas Sales Centre are amongst others the instructions on the amounts of natural gas to be delivered at the various delivery points and transport nodes. This instructive information is sent to the Transport Control Centre. GSC receives information from TCC about the field production capacity, the upstream transportation availability line pack, the operational flexibility, etc. GSC has the formal contact with the buyers dispatching representative receiving gas nominations and requests. If the availability does not match the request, after the use of all possible storage and operational arrangements, the GSC splits the "shortfall" on affected contracts according to appliable allocation rules at the relevant delivery points.

Other gas organizations may combine the contractual and physical handling of gas flow in the transportation system into one single gas dispatching centre. But the tasks between different dispatch centres are in many cases quite similar. Differences depend of course on the type of the transportation system and maybe also the size.

Optimization algorithms in use must take into account the constraints set by sales agreements, transportation agreements, storage agreements and operational agreements. Of course, optimization packages finding optimal contracts and optimal buyers can contribute to define many sales agreements.

Gas Nomination Process:

Load forecast is an integrated part of the gas dispatching activities. The load prediction is a sum of sales obligations, transportation and exchange obligations and company uses. The customers come with yearly, monthly weekly, daily and hourly gas requests. A nomination remains in effect until superseded by another nomination or until an end date on the nomination form occurs. The end users starts the nomination process by determining its supply requirements for the particular gas day and initiating the necessary nominations to its suppliers. Subsequent nominations move back towards the wellhead until the owner of the natural gas at the wellhead is reached. Gas nominations through regional and interregional transmission systems complicate the process. Each transporter has his own nomination procedures and nomination deadlines. It is the gas transporter that has the responsibility of ensuring that gas receipts and deliveries are transported to the facilities, satisfying the physical and contractual constraints.

A fiscal measurement measures the calorific energy content of the natural gas denoted $E_{\text{calorific}}$. Both the seller and the buyer have their own measurement. The energy offtake for a gas day is then calculated as in Eqn. (2.15) determining buyers real offtake (BRO).

$$BRO = \int_{6.00}^{6.00+24.00} E_{\text{calorific}}(t)dt \tag{2.15}$$

In a gas day, the buyers real offtake must be larger than a minimum amount $a \cdot DCQ$ and less than a maximum possible offtake (MOP) amount $MOP = b \cdot DCQ$, where DCQ is the abbreviation for Daily Contractual Quantity.

$$a \cdot DCQ \le BRO \le b \cdot DCQ \tag{2.16}$$

The Daily Contractual Quantity is calculated from the annual contracted quantity (ACQ) and the number of days of maintenance as in Eqn. (2.17)

$$DCQ = \frac{ACQ}{365 - N} \tag{2.17}$$

Typical values are $a = 0.4$ and $b = 1.1$. If $BRO \le c \cdot DCQ$, the buyer must pay for $c \cdot DCQ$, which is the agreed take or pay quantity. If $BRO \ge c \cdot DCQ$, the buyer pays for BRO.

The yearly take or pay volume (AMQ) is given by Eqn. (2.18).

$$AMQ = \frac{p}{100}ACQ - D_c - T_c \tag{2.18}$$

where $\{D_c, T_c\}$ are corrections done at the end of the gas year so that the final calorific volume is determined. An example is if the seller has faced reduced regularity so that it was not possible to meet the buyer's request at some delivery day of the year.

Local distribution companies and transmission companies that deliver gas to end users need to develop end user load forecasts. Section 2.6 gives a brief presentation.

Support Tools:

A system for Supervisory Control and Data Acquisition (SCADA) is available for the dispatchers. This system makes it possible to monitor and control the operation of a transportation system from a central control room, namely the dispatch centre. Remote control commands or commands directed to the operators at the installations are possible. On line complex network simulators gives an estimated state of the system. Off-line predictors are used, for example, for look ahead studies to evaluate different control scenarios or responses as a result of some events. Software tools for the contractual handling is also available. Support tools for optimization including software for providing load forecasts from end users may also be available.

Summary:

Control, supervision, communication and coordination are the main tasks at the Dispatch Centre. The dispatchers must be highly qualified personnel to both handle the difficult tasks and to fully utilize the supporting tools that are available to them. The handbook "Gas Control" from 1992 and the book "Natural Gas Transportation - Organization and Regulation" from 1994 are both published by the International Energy Agency, and might be handy for the gas dispatcher.

2.6 End User Load Forecasting

Forecasts of load demands is important in the daily operation of a gas transmission system for supply security and for minimizing transportation costs. With a load forecast with small prediction error, it is possible to operate the transmission system with less line pack. Then, it is possible to reduce the compression costs and still maintain a high level of security of the supply to the customers. With a good load forecast, one may avoid that the delivery terminal pressures gets under the contracted minimum limits. It must be assumed that the source capacities, the transmission system infrastructure and the state of this structure, makes it physically realizable to meet the customer demands.

The amount of gas expected to be required each day is the sum of the sales obligations, transportation and exchange obligations, and company uses. Companies with several large customers, such as other pipeline companies, local distribution companies and end users, estimate a portion of their daily gas requirements by summing the individual requests/estimates from each customer. Fuel used by the transmission company and the storage injection is then added.

Gas companies who provide sales and transportation services to customers whose consumption levels are significantly affected by weather conditions must predict a final increment of their overall gas requirements as a function of the weather conditions. This type of gas requirement typically is generated by a large aggregation of residential and commercial customers and to a lesser extent, by some industrial customers. The smaller portion of the gas requirements of these end users in general, which are the requirements generated by water heating and cooking, which do not fluctuate widely throughout the year is considered base load. The larger portion of the requirements of these end users is considered variable load which is the portion generated by consumption for space heating or cooling, which is a function of the weather conditions, such as temperature, wind speed, cloud cover, precipitation, and the previous day's weather.

The Dispatch Centre evaluates all the information thai is available before finalizing its gas-requirements estimate. The most recent available customer revisions are used. Formal predictive methods as well as the gas dispatcher's experience are used. The final estimate is the volume of gas that must be delivered by the supply sources available. (Source: Handbook for Gas Controllers by the International Energy Agency)

The gas load due to space heating is temperature sensitive and is also time dependent. Space heating is a function of the day of the week. From Monday to Friday, when people are at work or at school, the heating gas consumption is at a lower level. During weekends when people are at home, the space heating increases. The amount of cooking increases in the morning and when people arrives from work. The amount of cooking and drying of clothes remain relatively constant week to week (Flanigan,1995). Small commercial consumers have space heating requirements and also some additional fuel needs. Large commercial customers can have significant process loads and are categorized from the small consumer class by the volume of gas consumed. Small Industrial customers have consumption characteristics that are similar to small commercial consumers. Medium industrial customers have temperature-sensitive loads combined with a significant process load. A very large process load dominates over temperature sensitive load for large industrial customers.

The book by Flanigan (1995) treats underground gas storage facilities. Daily load forecast for each day of the year based on historical weather data is also presented, since this is important for identifying the storage need. The model is based on methods used by distribution companies that handle end consumers. Equation (2.19) gives the gas usage formula.

$$W_{\mathrm{gas}} = N_{\mathrm{g}} \cdot M_{\mathrm{g}} \cdot U \cdot DD \cdot D_{\mathrm{g}} \tag{2.19}$$

N_{g} is the annual average number of customers. The factor M_{g} describes the monthly variation in the number of customers and is obtained by dividing the number of customers for each month by the average annual number of customers. U is the gas usage per customer per degree day per day. DD is the average number of degree days per day for the period. D_{g} is the number of days in the time period. Further details are given in Flanigan (1995).

A typical load pattern on an hourly basis on a cold winter day for a gas company that has a mix of residental, commercial and industrial customers is illustrated in the book by Flanigan. A similar load pattern is given in Figure 2.3.

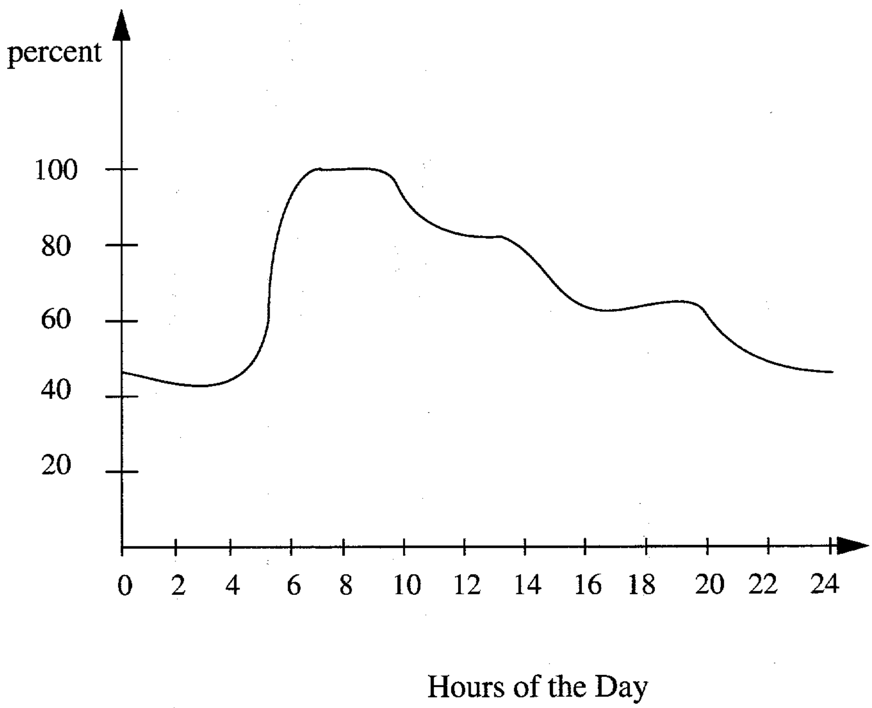


Figure 2.3: Load pattern for a gas company with a mix of residental, commercial and industrial customers.

It is explained how the load changes throughout the day as a function of the residental customer pattern. At night space heating is low. In the morning, thermostats are turned on, commercial establishments open/start up, etc., increasing the load. Combined with the cooking in households, the load peaks at about 8.00 to 9.00. People leaving their houses on their way to work or school and other activities leads to a drop in the load. During the period from 11.30 to 13.30, cooking of lunch increases the load. During the day, the temperature increases so that the gas load drops until 16.30 when the residents return home. Then, the loads increases due to the cooking and this lasts until about 19.30. As the residents go to bed, the load continues to decrease.

The author mentions that in addition to temperature, other weather factors like percent cloud cover, wind direction and speed and hours of sunshine have been considered. But, these factors have been found to have a small impact on the consumption of gas. The author points out that the day of the week however may be an important correlation factor. The residental load at weekends is usually greater than during the weekdays because more people are at home and therefore use more gas for space heating. During the weekends, some of the commercial customers are closed, consuming less gas. Commercial and industrial customers have an increase in gas consumption on Mondays when the activity is restarted.

Linear regression analysis is used to model the daily gas consumption as a function of temperature and day of the week in Fasold (1994). Statistical uncertainty around the regression curve is also modelled assuming Gaussian normal statistical distribution. Predicted daily load demand for a given day of the year is divided by 24 and then multiplied with an hour factor to give load forecast for the considered hour of the day. Seasonal factors are also included.

An automatic load forecaster is given in Jabbour and Meyer (1989). The paper by Ashouri (1986) describes use of an expert system for predicting load demand.

2.7 Stakeholders and Objectives

The stakeholders specify terms and conditions for the operation of a considered gas utility. The type of stakeholders involved in the business will depend on the type of the system, the organization of the business and the national belonging and interest. Below is example of a list of typical stakeholders for a gas transportation organization consisting of production and transmission to large customer delivery points which are source points of downstream transmission systems. The downstream transmission systems may deliver gas to end users, to local distribution companies or further transport the gas to other transmission systems as part of the route from the producer to the final end consumer.

- National government and parliament, regulators, regional trade alliances
- Owners of a field licence
- Owners of a natural gas transport system licence
- Production field operator
- Transport system operator
- Storage operator
- Gas sellers, gas sellers representative and the representative for implementing gas sale (sellers dispatching representative)
- Pipeline shippers, shippers representative including shippers dispatching representative
- Natural gas buyer

A deregulated organization that opens for third party access, can introduce gas traders as stakeholders buying and selling natural gas and transportation capacity. A special case for the Norwegian gas organization is a committee that negotiates on the existing and new natural gas contracts between the sellers and the buyers. Another committee, consisting of the field owners and operators has a consulting function regarding further expansion, operation and usage of existing production fields and the transport systems on the Norwegian continental shelf.

Optimization objectives for different stakeholders:

In a deregulated market with an increased supply and customer market, the owner constellation of a natural gas source may wish to increase customer gas volume by increasing contract volumes and the number of contracts. A larger number of spot sales may replace or come in addition to long term agreements. Increased volumes imply an increase in the need for transportation services from the source to a defined customer point. For gas contracts where sources and consumers are in different countries, the gas must be transported through national grids. Transportation contracts for each transport system must be signed. The owner of the pipeline system wants to maximize profit from offering transportation services, while the pipeline operator shall meet customer demands and minimize transportation costs and also satisfy the contract handling. Minimizing transportation costs can reduce the burner tip price for a customer. Gas traders want to sign a large number of contracts between other marketers and gas owners and buyers. Governments want to provide conditions for the business increasing economic and environmental efficiency. Reduction of energy costs for the consumer and saving energy at all levels by promoting technology with high level of efficiency is among the targets. Increased economic growth implies increased energy consumption. Therefore, regulations, such as environmental taxes and incentives, with the objective of promoting the development of cleaner technology are used at the production, transportation and consumption levels.

2.8 Common Carriage, Third Party Access and Open Access

Definitions of common carriage, third party access and open access are presented in this section. These are terms relevant to the more liberated natural gas market. This deregulated gas market makes it possible for the natural gas customers to buy natural gas from a large source market and transmission services from possibly many different transportation companies. The owners of gas resources will, with an increased gas grid, be able to sell natural gas to a larger market. The material presented in the rest of this section is taken from Stern (1993).

Common Carriage:

This involves an obligation on the pipeline company to carry gas to the limit of it's capacity. But, if the capacity of the pipeline becomes insufficient to meet demand, the owner must offer capacity pro rata to all parties in proportion to the amounts they tender for shipment, thereby reducing the quantities which parties receive in comparison to what they have requested for. It has been suggested that a common carrier is usually obliged to provide new transmission capacity if required. The distinguishing features of common carriage are: first, that the pipeline company is forced to reduce deliveries to all existing customers to allow new customers access to the system; second, that this is a purely theoretical concept for gas - there are no examples of common carriage in gas anywhere in the world and it has been rejected by the European Commission.

Third Party Access/Liberalisation of Transmission:

Third party access describes a commercial transaction in which owners of transmission pipelines either agree, or are obliged, to carry gas, which they do not own, for a third party. The third party is then required to pay a charge or tariff, for the services provided by the gas (pipeline) company. The role of the gas company is thereby changed from that of a merchant buying from a producer and selling to a consumer to that of a transportation company providing a range of services on a non-discriminatory basis.

Main Reasons for Third Party Access:

Third Party Access introduces competition between gas producers, pressing down the prices and giving the consumers a greater degree of choice. An important element of consumers choice is the possibility of taking a degree of commercial risk in order to gain the potential reward of lower gas prices. An example is to buy natural gas at a low price from a supplier with unreliable supply security.

Difference between Third Party Access and Common Carriage:

The third party access imposes no obligations on the pipeline owners to allow access to additional parties once the capacity of the pipeline has been allocated, that is the different third parties interested in transportation capacity when the pipeline is built. Anyhow, the pipeleg company is not an owner of the gas transported through the network. The allocation of capacity may be based on first come - first served, i.e. customers requesting service after the capacity of the pipeline has been filled cannot be served immediately. Bidding and auction systems provide alternative methods of allocating capacity under conditions of scarcity. The obligation of a transmission company to serve demand over and above the capacity of the pipeline is an additional issue to be decided in any TPA system.

Open Access Market:

All buyers should be able to buy gas from any source if there are transport systems to bring the gas to the gas buyer from the gas source of interest. In a third party access market, certain parties are entitled to participate in direct purchase and sale, while in open access, any party is allowed to participate. Among the major issues which have to be decided before a workable third party access arrangement can be introduced are given in the following list (Stern,1993):

- the parties allowed to participate in a TPA system
- the facilities to which access is to be granted, e.g. pipelines (onshore and offshore), LNG terminals, storage, treatment and blending facilities
- The services which may be involved apart from the transmission, e.g "black-up", "stand-by", like load balancing, pressure balancing, quality management
- the obligation of the gas (pipeline) company to provide these services
- the definition of the capacity that is available and the procedures to be followed when the capacity is not available
- the calculation of charges for services rendered by the gas (pipeline) company
- the relationship (degree of discrimination) between the gas (pipeline) company's own merchant customers and the third parties requesting access
- the regulatory procedure which provides the mechanism by which tariffs are set; a check that monopoly power is not being abused; and a resolution of disputes arising from these transactions

2.9 Driving Forces in European Gas Markets

Estrada *et al.* (1995) separates the driving forces for the change in the organization of the natural gas industry into external and internal forces as illustrated in Figure 2.4.

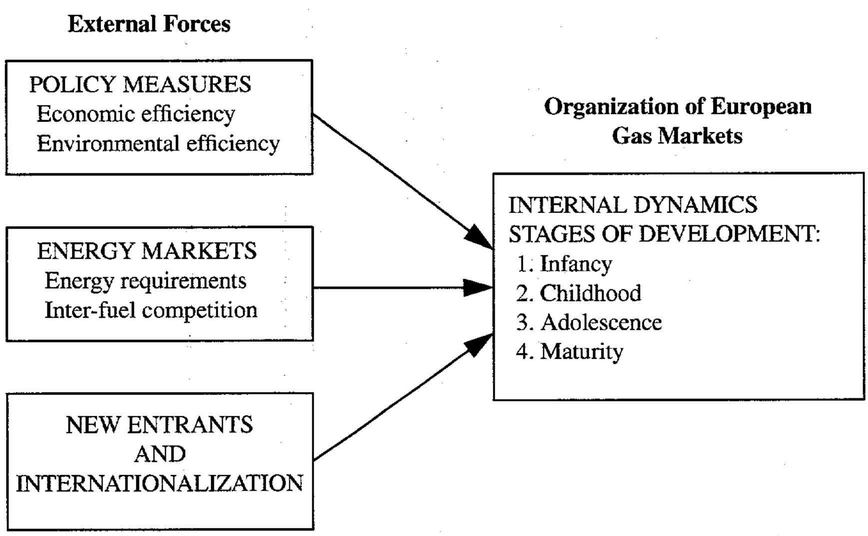


Figure 2.4: Driving forces in European gas markets. Source: Estrada *et al.* (1995).

External forces:

The external forces are policy measures, energy markets and new entrants/ internationalization.

1. Policy measures

Taxes, regulations, subsidies, investments, nationalization/privatization, bilateral agreements, etc. are examples of governmental instruments. They are used to tackle economic, environmental and energy policy concerns. Economic actors must comply with the current rules set by governments and regulators. Instruments to secure a gas industry that have profitable economic benefit not only for the involved actors but also for the interest of the country are formulated by governments. Rules to secure fair competition can be defined. Trade alliances and inter regional conventions aim at removing trade barriers between countries and continents. Governments may decide to disengage the state from those economic activities that can be covered satisfactorily by independent investors. Policy measures in the gas organization are used to improve the environmental side of the business. Taxes on outlets of gases that are expected to have a negative environmental impact is an example of a policy measure. Environmental regulations are placed at production, transformation and consumption stages. Cleaner energy alternatives may gain economic support from the state. Global environmental agreements may force national policy measures that must be satisfied by the actors in the gas industry. The gas industry must be made competitive in regional and international markets. The gas industry must be made attractive resulting in new investments. The energy costs for important industry must be reduced. Removing monopoly and promoting competition can press down the energy prices. International competition can be enhanced by removing subsidies for national energy industries. It is pointed out in Estrada *et al.* (1995) that the regulatory framework usually moves slowly from monopolistic energy companies, in the direction of free markets, through without ever fully reaching this point.

2. Energy markets

The total energy demand and the type of energy will have a large impact on the natural gas requirement. The availability of natural gas and prices compared to other competing fuel alternatives will affect the natural gas share.

3. New entrants and internationalization

Industrial and financial groups in the gas business will have an interest in exploring possibilities of gaining economic profits in the gas markets of other countries. If the protectionism by regulators in the domestic country is gradually removed opening up for free trade, the natural gas can become a commodity where consumers producers, wholesalers and final speculators can trade with gas at their convenience, obviously changing the organization of the gas market.

Internal dynamics of the gas market:

In the book by Estrada *et al.* (1995), the possible evolution of the organization of the European gas market is divided into four main development stages. They are named infancy, childhood, adolescence and maturity. Forces at one stage may give a response resulting in transit to a more complex organizational level. A short description of the different stages is given here. Consult the above reference for the transition forces and a more thorough treatment of the subject.

1. Infancy

At this stage, transmission companies buys gas from the producer, the transport and sell the gas to minor distribution companies, electric utilities or industry as illustrated in Figure 2.5. It is important that the price of the gas at burner tip must be competitive with the other fuels and that the buyers must have a high and stable gas take to avoid retailing and peak load management (gas storages) as is the case for electricity generation sector, large energy intensive industries, district heating and the supply to household sector in the cities that use old networks for other energy carriers. The gas producers will sign contracts with the gas company about future deliveries, minimizing the risk of investments. A transmission company will often be granted monopoly by government for minimizing their risk. The transmission company may be owned by the state. Take or pay agreements are made for guaranteeing income for the gas seller. These volumes may be 80 to 100 percent of expected annual gas deliveries. At the infancy stage, the producer and transporter are highly dependent on each other.

2. Childhood

Customers with varying consumption are included at this stage. Different price policies are apparent for different groups of users for the transmission companies. The price differentiation policy separates the interest of the producer, the transport company, the local gas utilities and the largest consumers. A gas company will buy gas from a number of different consumers and with the increased dynamic load, gas storages are built and the pipeline capacity is increased to meet the increased demand. A larger production base increases competition reducing the source supply prices for the transport company and increasing the source capacity. Therefore, a larger consumer base can be reached. The monopolistic situation for the gas company is strengthened. The producer develops new reserves compensating for the depleted ones. Production costs of new reserves can be reduced if they are close to the old ones or rediscoveries at old reservoirs. In the gas contracts with the gas companies, the producers still want take or pay contracts for security, but the sources that are used may not be specified in the contracts due to the supply diversity.

3. Adolescence

The largest customers are reached at this stage. The transmission company focuses on the part of the customer market giving the highest profit. The transmission company puts efforts in to keeping the obtained customer base but additional expansion may also be of interest. Demand is not driven by prices but by macro-economic activity and technical improvements in the natural gas technology. Negotiation between producers and the transmission company can result in reduced prices and more flexibility in the supply delivery which suits a load varying consumers base. As the supply availability increases, the gas companies can find new customer segments. Large consumers that have a choice between different energy carriers, which makes them less dependent on the firm natural gas deliveries, represent a new consumer group. They are termed interruptible customers. Long term bulk supplies are transformed by the transmission company to smaller volume packages at varying delivery periods and the prices fit the customer's specific need and location. If a company is expected to be competitive in a new geographic region, this expansion of pipelines and market can be financed by the already available core markets. The gas supply network is expanded to reach additionally supply sources or customer groups. A producer that wants to reach a new market but is restricted by a regional transmission company that has a monopoly, can build his own transmission line of a transit market. Several producers can join forces to reach a new customer market. Equivalently, buyers may join forces to build a pipeline to reach new natural gas sources. The number of transmission lines, storage facilities and actors involved in the inter regional gas trade increases. Competition within the gas sector is now apparent and boundaries of exclusive market territories tend to disappear approaching third party access.

4. Maturity

Figure 2.6 illustrates a mature gas market. Gas merchant companies will compete in collecting gas demands from small and large natural gas consumers. These companies are owned by producers, brokers, major consuming companies, private investors, etc. Based on the load demand from the obtained customer base and the amount of that money they can spend, the gas traders have to find suitable supply sources with accompanying supply volumes to cover the demand. Since the consumers are being able to choose between different gas merchants, making load demand uncertain, the traditional long-term contracts are replaced by deliveries of shorter duration. Long term contracts between transmission companies and local distribution companies and large gas users will be removed making the transmission system a public one, which is available for third parties. Consumers and producers are given the possibility to directly trade with each other. Increased competition can reduce the prices but lead to a stop in further investments. The transmission companies, which are paid for providing transmission services, will transform long term bulk supplies into smaller volume packages at varying delivery periods and individual customer prices and varying customer points. Competition between the different suppliers at burner tip will be enhanced. Gas supplies will be focused to market regions with the highest economic value. Consumers will have the possibility of choosing between different energy carriers and different supply sources. Technology will be available to the consumers so that it is possible to choose the cheapest energy carrier from the set of different alternatives available to them. In such a market, the amount of short term deliveries will increase. The short term trade is transformed into a spot market. Competition will increase and consumers are allowed to trade their gas surplus. The source capacity will be larger than the customer need pressing down the prices by increasing the competition. Prices in the lowest value markets will increase as large gas users may be interested in paying extra money for long-term supply contracts. The transportation cost is of course lower for consumers close to the producers. These consumers may be willing to pay a higher price for the natural gas excluding transportation cost. It is valuable for the producers to have a high customer base close to the production point, since less compression is needed. As the reservoirs are depleted, the pressure decreases creating the need for additional compression power. It will be expensive to transport the gas to customers that are placed a long transportation distance from the reservoirs. Customers that have the ability to choose between different energy carriers can sign interruptible contracts.

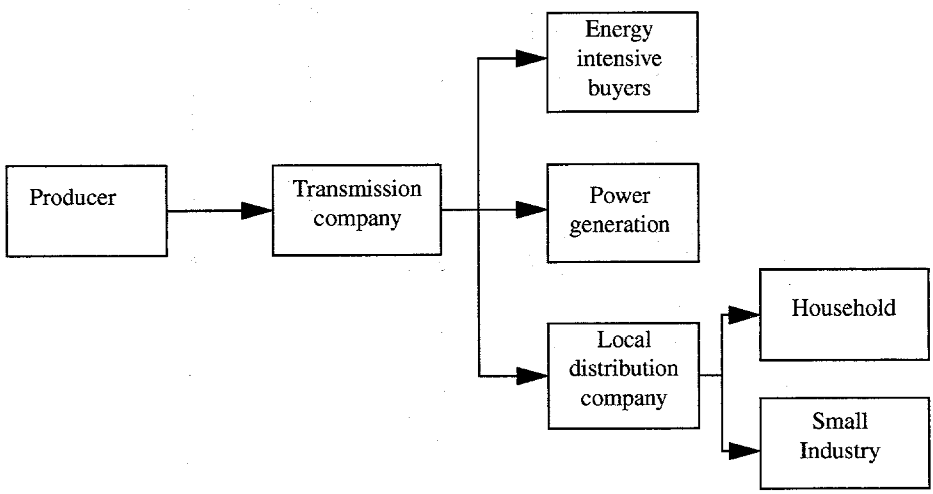


Figure 2.5: Organisation of an infancy gas market. Source: Estrada *et al.* (1995).

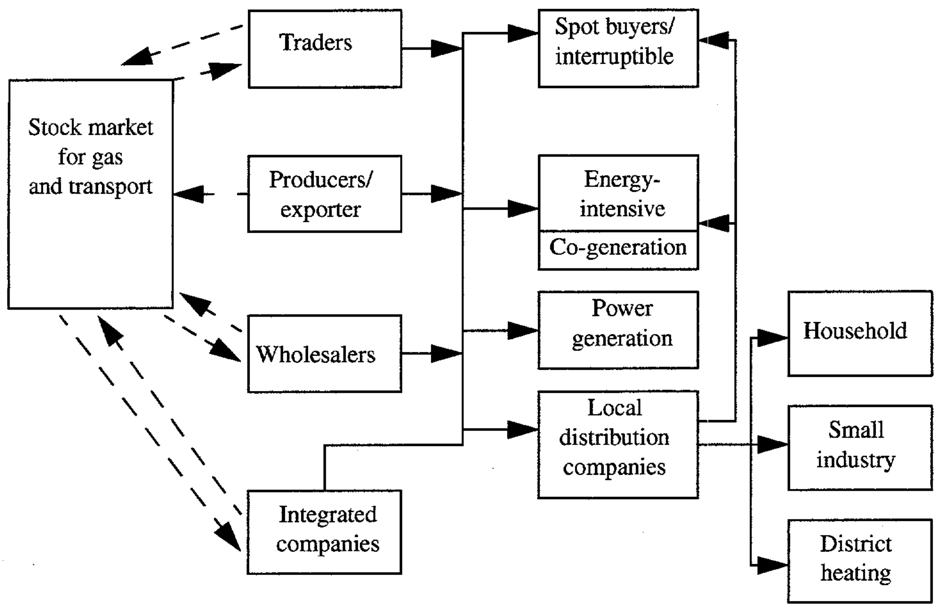


Figure 2.6: Organisation of a mature gas market. Source: Estrada *et al.* (1995).

Discussion on Load Forecasting:

The operation of a gas transmission system will certainly be possible to improve by the increased precision of load forecasting. Each buyer's daily real offtake energy volume must lie within a contractual defined maximum and minimum limit. There are a number of buyers at a delivery point. The total uncertainty of gas needs will then be a result of all the uncertainties for each customer. Therefore, the total uncertainty may be large. This uncertainty will force the transport operator to add extra inventory into the network so that one can be guaranteed that customer pressure is above contracted minimum. This extra line pack represents an increase in transportation costs due to compression and outlet of environmental gases which represent explicit fuel and tax costs.

In a mature gas market, one must expect that the number of short term contracts will dominate over the number of long term contracts. Spot sales will become more common. At each delivery point the number of buyers may be very large. If the transport operator shall provide a service with a high degree of security of supply and at the same time be able to reduce the transportation costs which have both economic and environmental benefits for the gas sellers, transporters, buyers and the State, it is necessary to have a forecast with a low degree of uncertainty. The operator of a specific transportation firm has usually very good knowledge of the delay times in the transportation system. Precise forecasts of time that is necessary before delivery, will certainly be advantageous for all stakeholders involved. This will imply that each gas contract must have a low degree of uncertainty. This can be accomplished by letting the maximum and the minimum possible offtake for each buyer be close to the daily contracted quantity. If this can be made the case, economic and environmental efficiency for a gas transmission utility, can clearly be improved. For example, a reduced transportation cost will reduce burner tip prices. Anyway, good communication between the involved stakeholders with continuously updated natural gas requests will improve operation of the transmission systems.

2.10 Stationary Simulation

Studying different scenarios for planning and operational issues on a stationary basis are some of the activities performed by dispatch personnel at a gas control centre for a transmission system. For that reason, a stationary simulator is needed. A thorough study of stationary simulation is given in the work by Osiadacz (1987). See also Osiadacz and Pienkosz (1988) and Osiadacz and Rudowski (1987). The nonlinear stationary equation developed by the American Gas Association is also extensively used in the gas transportation industry. This equation will also be used in this work.

2.11 Transient Simulation

Some references on dynamic simulation will be mentioned in this section. Different dynamic simulation models for gas transmission systems are described in Osiadacz (1987). Another approach to dynamic simulation is given in the book by Kralic *et al.* (1988), which has led to the commercial gas simulation package SIMONE, which is under continuous development. The researchers that has developed SIMONE are also looking at control and optimization of gas transmission systems. The papers by Osiadacz and Salimi (1988), Osiadacz (1990), Stelter (1987, 1988a), Maier and Schmidt (1986) and Canuto and Quarteroni (1986), give different approaches to transient simulations of gas transmission systems.

2.12 Optimal Pipeline Design

If the topology of a gas transmission network and the expected transportation capacity is defined, a problem is to find the diameter of the pipelines to minimize investment and operating cost. The problem of finding optimal pipeline dimensions is not treated in this thesis. But for completeness, the following references may be consulted: Wolf and Smeers (1996), Bhaduri and Talachi (1988), Manojlovic *et al.* (1994) and Hansen (1988). On optimal configuration, the work by Tayeb *et al.* (1997) may be consulted.

2.13 Quality Tracking

Many transmission systems have the possibility to control values of important quality parameters such as gross calorific value and the concentration of important natural gas components due to junction points and mixing stations. The gas contracts normally include specifications on the accepted ranges for the specified quality parameters. To be able to control the quality parameters, quality tracking is needed.

Quality tracking of important gas components or parameters by an exact solution of partial differential equations describing the processes in a pipeline system is not feasible (Jenicek *et al.*, 1992). The mass balance equation for a component or a quality parameter i is given in Eqn. (2.20).

$$\frac{\partial}{\partial x}W \cdot y_i + A\frac{\partial}{\partial t}\rho y_i = 0 \tag{2.20}$$

where y_i is the weight percent of the component. The mass balance equations for the components or parameters come in addition to the total conservation of mass, momentum, energy and the state equation yielding the mathematical flow and quality dynamics (Jenicek *et al.*, 1992). The complete set of equations is unrealistic to solve as a coupled set of equations due to the need for very small time steps and a large amount of memory. In the same paper, a method is proposed where one model simulates the flow dynamics and the other model tracks the quality parameters in the pipelines from the supply points to the offtake points. The quality tracking is formulated as a task to follow the movement of all "quality flags" entering the network in the supply points and leaving at the offtake points and simulating the mixing at junction points and mixing points. The "quality flag" is characterized by it's position and the values of all the quality parameters that is to be tracked. The flow speed values calculated by the flow dynamics is used to calculate the new positions of all quality flags and the mixing in junctions and mixing stations. The value of the quality parameters are calculated into and out from each node of the network. The quality value for the node is presented as the linear interpolation of the parameter value in to and out from the node. But, this node value is not used for the tracking calculation since it would lead quickly to a simulation error. Simulation examples including the case for quality tracking under an alternating flow is also presented.

An alternative quality tracker is briefly discussed in the paper by Schröder *et al.* (1990). This method is based on the tracking of the movement of separation surfaces. These surfaces can cover a volume with a batch of gas or a slice of gas. See also Weiman *et al.* (1990). A change in the location per unit time is determined from the flow velocity at a considered position using the fluid simulation model.

The simple mixing law is used at junctions and mixing stations. This quality tracker is implemented in the gas network simulation system GANESI.

Design of a simple quality tracker:

Assume that there is a negligible concentration diffusion in the flow direction so that a batch of molecules of a defined control volume is kept together at any position along a pipeline segment when there are no mixing possibilities along the segment. Denote τ to be the time for the batch volume to move from the defined inlet to the defined outlet under generally transient conditions. This time delay will be approximated by the time delay it takes for a batch volume to move from the defined inlet to the defined outlet for a defined stationary operation point of which the system is assumed to operate in the neighbourhood of. This operation point can be determined from a stationary optimization where, for example, the objective is to minimize the transportation cost. The optimization is based on a defined stationary customer load at each terminal of the network. Define the stationary customer load to be the expected average consumption value of the forecasted load pattern for a defined time period in the future. Let t_1 and t_2 be the initial time and the end time respectively of this time period. Calculate the average mass flow load at a customer terminal denoted $\overline{W}$ as in Eqn. (2.21). The variable $\hat{W}$ denotes the forecasted load.

$$\overline{W} = \frac{1}{t_2 - t_1} \cdot \int_{t_1}^{t_2} \hat{W}(t)dt \tag{2.21}$$

In Chapter 3, a detailed description is given of how such a stationary optimization problem for a gas transmission system can be defined. For the current presentation, it holds to assume that a stationary operation point is available and that this is used as a basis for the following calculations. The flow directions are all set for a defined stationary operation point. Consider now a pipeline segment at defined stationary conditions. Let $x = 0$ be the position at inlet and $x = L$ to be the outlet position of the segment. There are no connections between the two boundary positions. Denote W^* to be the stationary mass flow through the pipe segment and $p^*(x)$, $0 \le x \le L$ to be the stationary pressure distribution. Figure 2.7 gives an illustration of such a segment.

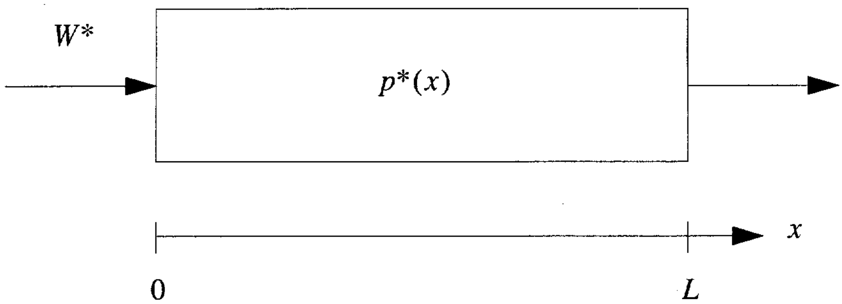


Figure 2.7: Pipeline Segment

Denote $\tilde{\tau}$ to be the approximation of the time delay τ. Neglecting $k_\upsilon \cdot \frac{\partial}{\partial x}\left(\mu \cdot \frac{\partial \upsilon}{\partial x}\right)$ and $\rho\upsilon \cdot \frac{\partial \upsilon}{\partial x}$ in Eqn. (2.2) together with the assumption that $\theta = 0$, we have the simplified the stationary momentum equation in Eqn. (2.22).

$$\frac{d}{dx}p^*(x) + \frac{\lambda}{D} \cdot \frac{\rho^*(x)}{2} \cdot \upsilon^*(x)^2 = 0 \tag{2.22}$$

Under stationary assumptions, the temperature distribution is of course not time varying. Then, we can use the thermodynamic state equation $p = c^2 \cdot \rho$, where c is the speed of sound. From Eqn. (2.1) and using the above state equation, we can express the velocity distribution as in Eqn. (2.23).

$$\upsilon^*(x) = \frac{c^{*2}(x) \cdot W^*}{A} \cdot \frac{1}{p^*(x)} \tag{2.23}$$

The momentum equation can be reformulated as

$$\frac{dp^{*2}}{dx} = -\frac{16 \cdot \lambda(x) \cdot W^{*2} \cdot Z^*(x) \cdot \left(\frac{R}{MW}\right) \cdot T^*(x)}{\pi^2 \cdot D^5}, \tag{2.24}$$

where the relation $c^2 = Z \cdot \left(\frac{R}{MW}\right) \cdot T$ has been used. Assume that λ is constant.

Further, assume that the compressibility factor distribution and temperature distribution are replaced with the average values for the stationary operation point denoted by $Z_{av}{}^*$ and $T_{av}{}^*$ respectively. They can be calculated as suggested in Sjøen (1997). Introduce the convenient definition in Eqn. (2.25).

$$K^* = \frac{16 \cdot \lambda \cdot W^{*2} \cdot Z_{av}{}^* \cdot \left(\frac{R}{MW}\right) \cdot T_{av}{}^*}{\pi^2 \cdot D^5} \tag{2.25}$$

Hence, we have

$$\frac{dp^{*2}}{dx} = -K^* \tag{2.26}$$

This yields the approximate stationary pressure distribution given in Eqn. (2.27)

$$p^{*2}(x) = p^{*2}(0) - K^* x \tag{2.27}$$

Using Eqn. (2.23) and Eqn. (2.27), we obtain the simplified expression in Eqn. (2.28) for the stationary velocity distribution of the pipeline segment.

$$\upsilon^*(x) = \frac{Z_{av}{}^* \cdot \left(\frac{R}{MW}\right) \cdot T_{av}{}^* \cdot W^*}{A \cdot \sqrt{p^{*2}(0) - K^* x}} \tag{2.28}$$

The nonlinear time variant differential equation $\frac{dx}{dt} = \upsilon(x, t)$ is approximated with the nonlinear differential equation in Eqn. (2.29) for the purpose of calculating the approximate time delay.

$$\begin{aligned} \frac{dx}{dt} &= \upsilon^*(x) \\ x(t_0) &= x_0 \end{aligned} \tag{2.29}$$

With the expression in Eqn. (2.28) inserted into Eqn. (2.29), a nonlinear differential equation, which is difficult to solve analytically, is obtained. Instead, we can approximate Eqn. (2.28) as follows:

$$\upsilon^*(x) = \theta_1 + \theta_2 \cdot x, \; 0 \le x \le L. \tag{2.30}$$

Using first order Taylor expansion around the inlet, we get $\theta_1 = \upsilon^*(0)$ from Eqn. (2.28) and $\theta_2 = \left.\frac{d}{dx}\upsilon^*(x)\right|_{x=0}$. Another alternative is to define θ_1 as before and $\theta_2 = \frac{\upsilon^*(L) - \upsilon^*(0)}{L - 0}$. A third alternative is to determine θ_1 and θ_2 from identification using the least squares method and data from $\upsilon^*(x)$ in Eqn. (2.28). Denote $\hat{\theta}_1$ and $\hat{\theta}_2$ to be the determined values of θ_1 and θ_2 by some chosen method. Then, we have the following inhomogenous differential equation

$$\begin{aligned} \frac{dx}{dt} &= \hat{\theta}_2 \cdot x + \hat{\theta}_1 \\ x(t_0) &= x_0 \end{aligned}. \tag{2.31}$$

This system has the well known solution

$$x(t) = e^{\hat{\theta}_2 t} x_0 + \int_0^t e^{\hat{\theta}_2 (t - \eta)} \hat{\theta}_1 d\eta. \tag{2.32}$$

Define $x_0 = 0$, $t = \tilde{\tau}$ and $x(\tilde{\tau}) = L$. Then, we have the approximated time delay between the input and the output of a pipeline segment calculated as in Eqn. (2.33).

$$\tilde{\tau} = \frac{1}{\hat{\theta}_2} \cdot \ln\left(\frac{\hat{\theta}_2}{\hat{\theta}_1} \cdot L + 1\right) \tag{2.33}$$

For each pipeline segment, an approximated time delay is calculated. The time delays are calculated based on the defined stationary operation point of the network. The delay model for quality parameter component i for a pipeline segment with length L is given in Eqn. (2.34).

$$y_i(x = L, t) = y_i(x = 0, t - \tilde{\tau}) \tag{2.34}$$

The value of a quality parameter leaving a junction point or a mixing station is calculated as in Eqn. (2.35)

$$y_{i,\,\mathrm{mix}} = \frac{1}{\displaystyle\sum_{j=1}^{N_{\mathrm{in}}} W_j(x_j = L_j, t)} \cdot \left(\sum_{j=1}^{N_{\mathrm{in}}} W_j(x_j = L_j, t) \cdot y_i(x_j = L_j, t) + \sum_{j=1}^{N_{\mathrm{sources}}} W_{j,\,\mathrm{source}}(t) \cdot y_{ji,\,\mathrm{source}}(t) \right), \tag{2.35}$$

where N_{in} is the number of pipelines into the junction point, N_{sources} is the number of sources supplying the junction point. $W_j(x_j = L_j, t)$ denotes mass flow out from pipeline j into the junction with $y_i(x_j = L_j, t)$ as the value of the quality parameter. $W_{j,\,\mathrm{source}}(t)$ and $y_{ji,\,\mathrm{source}}(t)$ are the source mass flow and parameter value respectively from source j into the junction point.

The simple quality tracker suggested above needs only a minimum of computation which is a great advantage. The value of the quality parameters at the source points may be time varying. These values are assumed known or measured.

2.14 Summary and Discussion

The natural gas industry from hydrocarbon resources to the end users is a combination of a large number of coupled complex systems. Natural gas transportation is one important part of this chain. In a market with an increasing number of short term contracts replacing the traditionally long term contracts, the operational tasks for the gas dispatchers will be more demanding. High quality load forecasts at each customer terminal are very important for supply security and the ability to minimize transportations costs. It is also important to have a clear overview of the available supply sources and their capacity. Reasonable operation of natural gas storage facilities to cover seasonal variations and peak shavings is an important task for the gas dispatchers. Alternative operational strategies must be suggested in the case of a negative change in the status of the infrastructure that may jeopardize the customer supply security. Due to the complexity of the network operation, different computer tools are available to support the gas dispatchers. Optimization formulations and optimization objectives will generally depend on the stakeholder that has the optimization in mind.

Chapter 3

Optimization and Control of Gas Transmission Systems

3.1 Introduction

An on-line optimization and control scheme for the operator support for the gas dispatch personnel will be proposed. The system is a combination of stationary optimization and linear model predictive control. The results from the last performed stationary optimization provide the current operation point for the model predictive controller and state estimator used for on-line dynamic optimization. Each time a new stationary optimization is performed, this provides new state and control references and a new linear optimization model for the model predictive controller and state estimator. The purpose for the on-line controller is security of supply and to bring and keep the state of the system close to the optimal calculated stationary state. The stationary objective function may express, for example, transportation cost or profit. The simulation results and comments for a simple long distance transmission system example is included in this section in order to explain the combined optimization and control scheme in a simple manner. Infeasibility handling and tuning of the controller for the supply security is also considered. Some remarks on the stability issues of the closed loop controller is also treated. All the simulations are performed in closed loop with a simplified nonlinear model to represent the fluid dynamics inside the transmission lines. Some contributions on previous work in stationary natural gas network optimization is also referred to with a short non-exhaustive description of the main features. For details, the referred papers should be consulted.

3.2 Example System

A simple long distance offshore gas transmission system will be used throughout this chapter for illustration purposes. Figure 3.1 illustrates the transmission system. Hydrocarbons supplied to the processing facilities may originate from many different sources. It is assumed that the output pressures from each processing facility are approximately constant. These pressures are input pressures of the two compressor stations that supply the necessary energy to the natural gas for transport to the customer demand point. The compressor stations consist of centrifugal compressors driven by gas turbines. The two source pipelines meet at the junction point where the natural gas then travels along to the customer point. It is assumed that each source produces natural gas of such quality which satisfy or is very close to satisfy the gas quality demands specified in the sales contracts. The transmission system has some degree of mixing ability at the junction point. Important gas quality parameters such as gross calorific value and the concentration of carbon dioxide may then be controlled, to some extent, to reach sales gas quality. The maximum supply capacity into the gas transmission system from the two processing facilities can be time varying. Bad weather off-shore may result in the closure of some off-shore production installations or in a significant decrease in production rate of natural gas. As a result, less gas enters the processing facilities which reduces the maximum source capacity into the transmission network. High costs for building platform installations at sea, is one of the reasons for avoiding intermediate compression. Fewer installations also means less equipment to maintain. As a result, the pipelines are operated at high pressure levels to meet the necessary transportation service capacity. The internal pipeline epoxy coating of the pipelines is used to reduce the pressure drop caused by friction. At regular intervals, the pipelines inner surfaces are cleaned by sending a pigging train through the transmission lines. This reduces pipeline friction and unwanted components including free water. At the same time the pipelines are inspected for corrosion and leakage. Low friction values means reduced transportation costs.

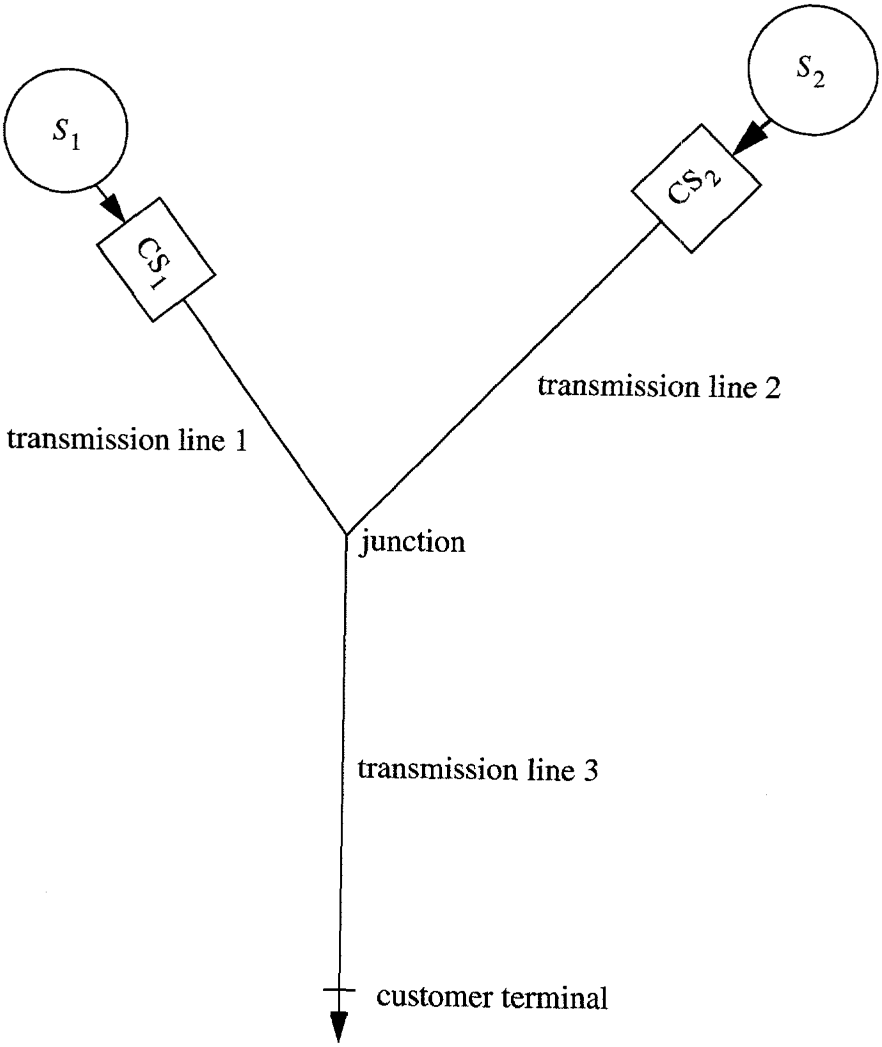


Figure 3.1: Long distance gas transmission system.

3.3 On-Line Optimization and Control of Gas Transmission Systems

Figure 3.2 illustrates the suggested combined optimization and control scheme described in this chapter. The state vector x contains the pressures of the network and x^* is the stationary "optimal" pressure reference. Estimated pressures are denoted by $\tilde{x}$. The control vector of continuous variables is denoted by u_{c} and the control vector of possible discrete variables is denoted by u_{d}. The system example is assumed to have only continuous control variables. But, for generality, discrete variables are also included in the figure. Optimal continuous and discrete control references that follow from a local or global minimum from the stationary optimization, are denoted by u_{c}^* and u_{d}^* respectively. The time of the current sampling instant is denoted by t_k. The vector of current stationary load is denoted by v^*. The predicted open loop load forecast trajectory is the sequence $\{\hat{v}(t_{k+i})\}_{i=0}^{N}$. The component vector $\hat{v}(t_{k+i})$ denotes predicted load demand for a future time point t_{k+i}. The current customer load is expressed as $v(t_k)$. It is assumed that both the control and the load trajectories are piecewise constant with a sampling interval of one hour. The measurement vector is denoted by y. Components of this vector are pressure measurements of the transmission system. These measurements are a subset of the information from the Supervisory Control and Data Acquisition systems available to gas dispatchers.

An average stationary load demand is formulated. This can be the expected average customer gas usage for the next 24 to 48 hours. A transient load forecast is also formulated. The average stationary load demand is used by the stationary optimizer. The transient load pattern is used by the model predictive controller. The load forecast predictions may be based on forecast models, gas contracts, gas request nominations, previous load patterns and most importantly, the dispatch personnel's experience.

A stationary optimization problem is solved providing a solution which is a local or a global minimum of a defined objective function. Also, the solution must satisfy the defined system constraints. In general, this problem may contain both discrete and continuous variables. Discrete variables may originate from the case where one seeks to find optimal compressor station configuration and the optimal number of units in operation. Optimal grid configuration is another example involving discrete control variables. The stationary optimization problems clearly depend on the type of network that is under consideration. Some problems have only continuous control variables while others have a large number control variables of both the continuous and the discrete type.

The solution of the stationary optimization must satisfy the demand that important natural gas quality parameters are kept within the constraint limits. As mentioned, the value of the quality parameters are possible to control at the junction points and the mixing stations of a transmission system. The model predictive controller controls the considered quality parameters implicitly. The state and the control deviation from their respective references are penalized in a quadratic cost function. The controller aims to reach these references to a minimum cost. These references are determined by the stationary optimization where the quality parameters are controlled on a stationary basis. Hence, the quality parameters are implicitly controlled by the model predictive controller.

Optimal pressure distributions and pressures at junction points are obtained from the stationary optimization. Temperature distributions for all the transmission pipelines may also be a result if the stationary energy equation is included in the optimization formulation. In some cases it may be necessary to specify explicitly that the system must operate in a defined region of the pressure-temperature envelope to avoid Hydrate formation or two-phase flow.

If the defined stationary optimization problem has both discrete and continuous optimization variables, then the optimal values of both of these types are obtained. The optimizer determines which on-off valves that are to be opened and which that are to be closed. This specifies the optimal network configuration and possibly the optimal configuration of each compressor station. The optimal number of compressor units that are to be in operation inside a compressor station is also determined. Stationary mass flows for each input-output combination of the compressor stations and mass flows through control valves are determined for the continuous variables. Optimal discharge or suction pressures can be determined instead of mass flows at the stations.

The optimal pressure and temperature distributions, the optimal value of the discrete and continuous control variables and the current stationary average load vector specify the linear dynamic optimization model. This model is used in the model predictive controller and the state estimator. State and control references are also specified from the stationary optimization.

The main goal of the model predictive controller with state estimator is the on-line dynamic operation of the transmission network determining the value of the continuous control variables. State constraints and control constraints must be met. At each sampling instant, an open loop optimization problem is solved and the values of the continuous control variables for the current sampling interval are determined.

The most recent update on the prediction of the future time varying maximum source production capacity and the time varying load forecasts are inserted into the open loop formulation. The minimum obliged source production is also inserted into the problem formulation. The objective is the supply security at the customer points. This means that the customer gas demands must be met and the customer terminal pressures must be kept above the contracted minimum limits. For economic reasons, the model predictive controller aims to bring the system close to the current optimal state and control reference.

Each time a new stationary optimization is being done, the linear dynamic optimization model and state and control references are updated. In this way, the control system has adaptive properties.

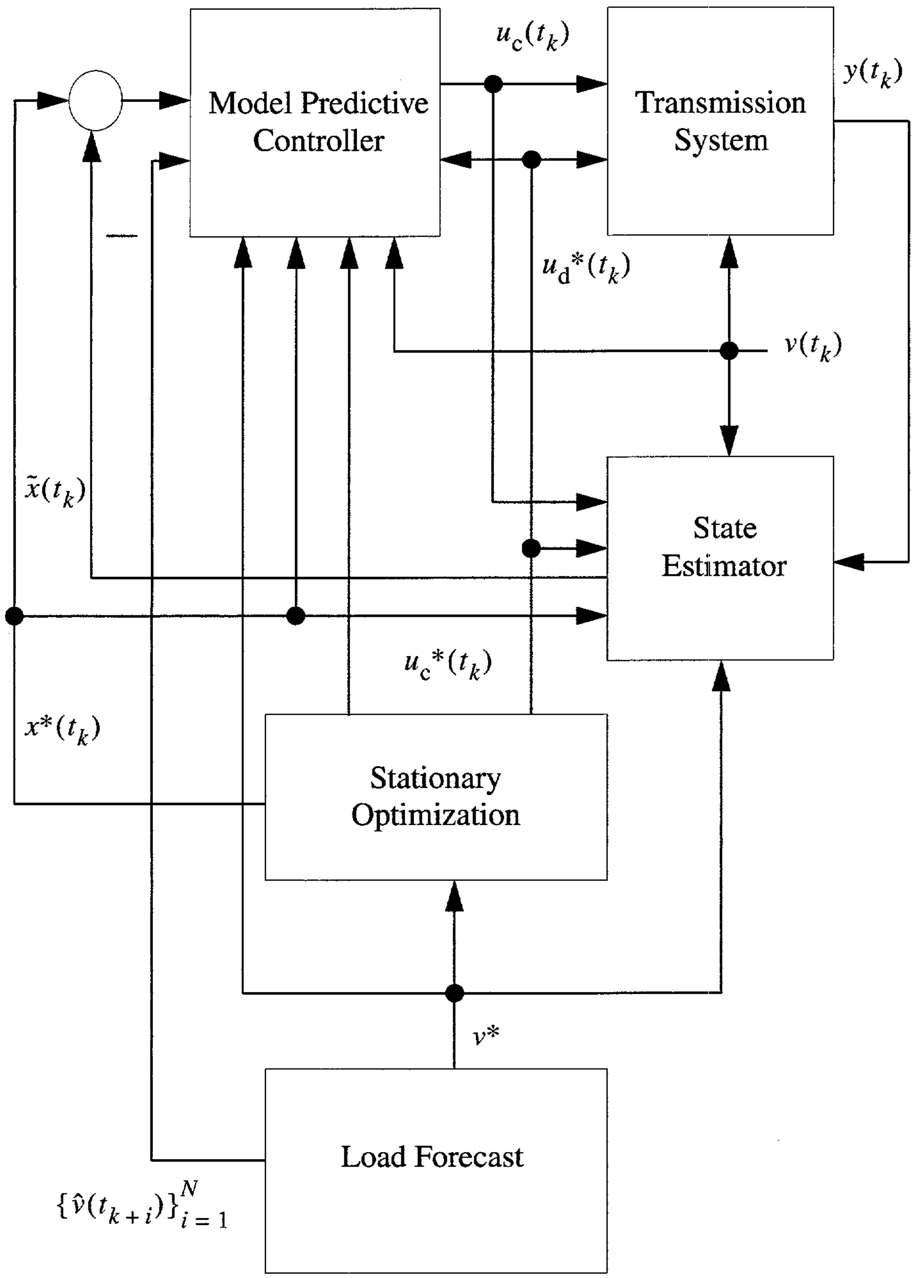
Since a new open loop problem is solved at every sampling instant, taking account of updated information from process measurements, load forecasts and production availability and using results from the latest stationary operation, a major feedback effect from the actual transmission system is obtained.

Important events reported to the gas control centre via the personnel at the different facilities of the transmission system and through the supervisory information handling system are taken into account in the stationary and transient optimization. Event examples may be the reduction in compressor station capacity due to necessary maintenance or a severe reduction in on-shore production capacity due to a lack of incoming natural gas sources. This may be caused by bad weather at sea forcing a reduction or closure of production at one or several off-shore plants.

The dispatch personnel will generally evaluate the suggested continuous control commands calculated by the model predictive controller. Based on predictive simulations and experience of the personnel at a gas control centre, they may choose to modify, fully disregard or implement the proposed continuous control commands to the transmission network. This part is included in the transmission system block.

The transmission system block is also assumed to contain the low level control systems. Examples of such systems may be the flow control systems at compressor stations, control valves and control systems at each natural gas processing facility for changing production rates. Systems that control withdrawal and injection rates of possible storage facilities also belong to this class.

Figure 3.2: Illustration of the combined optimization and control scheme.



Simulation model:

For simulation of the gas transmission system example in closed loop operation, the unsteady gas flow is described by the nonlinear creep flow model. It is also assumed that the pipelines are approximately normal to gravitational force everywhere. The nonlinear creep flow model can be written as in Eqn. (3.1), see Osiadacz (1987).

$$\begin{aligned} \frac{\partial p}{\partial t} &= -\frac{c^2 \rho_n}{A} \cdot \frac{\partial Q_n}{\partial x} \\ \frac{\partial p}{\partial x} &= -\frac{2 f \rho_n^2 c^2 Q_n^2}{D A^2 p} \end{aligned} \tag{3.1}$$

Osiadacz (1987) also shows how Eqn. (3.1) can be transformed to an equation for the rate of change of pressure at a node j, which is given in Eqn. (3.2) in terms of the pressures at the surrounding node points and net mass flow offtake at the node.

$$\frac{V_j}{c^2} \cdot \frac{dp_j}{dt} = \sum_{i=1}^{k} \sqrt{\left(\frac{\left|p_i^2 - p_j^2\right|}{K_i}\right)} \cdot \operatorname{sgn}(p_i - p_j) - W_j \tag{3.2}$$

For the surrounding node i, we have the constant K_i given in Eqn. (3.3).

$$K_i = \frac{64 f_i c^2 \Delta x_i}{\pi^2 D_i^5} \tag{3.3}$$

The geometric volume of node j is given in Eqn. (3.4).

$$V_j = \frac{\pi}{8} \sum_{i=1}^{k} D_i^2 \Delta x_i \tag{3.4}$$

A simplified nonlinear dynamic description for the transmission system example can then be compactly written as in Eqn. (3.5).

$$\dot{x} = f(x, u, v) \tag{3.5}$$

with initial condition of the simulation horizon as $x_0 = x(t_0)$. The components of the state vector x are the pressures at the grid points of the transmission system. The components of the control vector u are the mass flows through the two compressor stations and v is variable for the customer natural gas demand. The ordinary differential equation solver for stiff systems, ode15s in Matlab, was used to solve the simulation model in Eqn. (3.5).

3.3.1 Stationary Optimization

One main advantage of performing a stationary optimization first is the reduction of the number of optimization variables. Also, if one should solve a dynamic optimization problem using the original cost function and nonlinear constraints and nonlinear dynamics, a non convex problem would have to be solved at every sampling instant. The number of optimization variables would be much larger than in the stationary case. This may be too time consuming and not suitable for on-line optimization. Usually, one only has time to use a local optimizer, giving a local minimum. The stationary optimization does not have to be revised at every sampling instant. A reoptimization may only have to take place when considerable changes are occurring or are expected to occur. Until the optimizer has solved the current defined stationary problem to a local or global minimum, the latest available stationary optimization result is used. A calculated stationary operation point may hold for many sampling instants if the transmission system operates transiently in the neighbourhood of this operation point in this period. The results from the stationary optimization defines the optimal stationary operation point. This operation point then specifies the parameters of the linear dynamic control model that is used in the linear model predictive control scheme and for the state estimator. The optimal solution from the stationary optimization problem, is also used to define the reference state vector and reference control vector for the model predictive controller. Each open loop problem for the dynamic optimization will be defined as a strictly convex problem. An effective commercial algorithm will provide fast convergence to a global minimum inside specified tolerances for this problem. This is clearly an advantage in on-line operation which makes the control scheme computationally feasible.

Stationary optimization using AGA equation:

First, we will give a simple stationary optimization formulation for the simple gas transmission system example using the well known equation developed by the American Gas Association for stationary fluid flow (Sjøen,1997). This stationary optimization has been used in the simulations to specify the current "optimal" stationary operation point for the transmission system. Under stationary conditions, we have Eqn. (3.6) and Eqn. (3.7) (Rist,1996;Osiadacz,1987;Sjøen,1997). Note that since we now have equations only as a function of the position, we can replace the partial derivatives operators with ordinary derivatives operators.

$$W = \rho \cdot A \cdot \upsilon = \text{constant}, \forall x \tag{3.6}$$

$$\upsilon \cdot \frac{d\upsilon}{dx} = -\frac{1}{\rho}\frac{dp}{dx} - \frac{\lambda}{D} \cdot \frac{\upsilon^2}{2} - g \cdot \sin\theta \tag{3.7}$$

For fluid flow in pipelines at low velocities (creep flow), by considering the sizes of the terms in Eqn. (3.7), it can be shown that the acceleration term is negligible, see one of the above references. If it is assumed that the pipelines are approximately normal to the gravitational force everywhere, then we get the stationary momentum equation in Eqn. (3.8).

$$\frac{dp}{dx} = -\frac{\lambda}{D} \cdot \frac{\rho\upsilon^2}{2} \tag{3.8}$$

The use of Eqn. (3.6), $A = \pi \cdot \left(\frac{D}{2}\right)^2$ and the state equation $p = \rho Z R' T$ then yields Eqn. (3.9) along a transmission pipeline

$$\frac{dp^2}{dx} = -\frac{16 \cdot \lambda(x) \cdot Z(x) \cdot R' \cdot T(x)}{\pi^2 \cdot D^5} \cdot W^2, \tag{3.9}$$

where $R' = \frac{R}{MW}$. We will assume that $\lambda(x) = \lambda, \forall x$ for a considered pipeline segment. In addition average values for the temperature distribution and compressibility factor distribution will be used between the input and the output of the considered pipeline segment. After integration between the input and the output, the AGA equation expressed in mass flow form is obtained. This equation is given in Eqn. (3.10)

$$p_{\text{in}}^2 - p_{\text{out}}^2 = K \cdot W^2, \tag{3.10}$$

where the constant K is expressed in Eqn. (3.11)

$$K = \frac{16 \cdot \lambda \cdot L \cdot Z_{\text{av}} \cdot \left(\frac{R}{MW}\right) \cdot T_{\text{av}}}{\pi^2 \cdot D^5} \tag{3.11}$$

and where L is the length of the pipeline segment. Note that the average values for the temperature distribution and the compressibility factor distribution used for the AGA equation are also used in the dynamic optimization model used by the model predictive controller. This equation is presented later in this chapter. Assume that we know the inlet temperature T_{in} of the pipeline, the outlet temperature T_{out} and the ambient temperature T_{amb}. Then, we can calculate the average pipeline temperature as in Eqn. (3.12), see Sjøen (1997).

$$T_{av} = \left(\frac{T_{in} - T_{out}}{\ln(T_{in} - T_{amb}) - \ln(T_{out} - T_{amb})}\right) + T_{amb} \tag{3.12}$$

The average compression factor will usually be in the range $[0.8, 1]$ and is a function of the pressure level of the considered pipeline. The input and output pressures are among the optimization variables. So, we do not know the optimal input and output pressure. If we knew these, we could calculate an average pressure for a pipeline as in Eqn. (3.13), (Sjøen,1997).

$$p_{av} = \frac{2}{3} \cdot \left(\frac{p_{in}^3 - p_{out}^3}{p_{in}^2 - p_{out}^2}\right) \tag{3.13}$$

But an approximate guess of the input and the output pressure may be available. We can also solve several stationary optimization problems using different values of the compressibility factor. Then, we may calculate the expected input and output pressures taking the average from these optimizations and then design an average compressibility factor used for the final stationary optimization using Eqn. (3.14), where a state equation is also used.

$$Z_{av} = \frac{p_{av}}{\rho(p_{av}, T_{av}) \cdot \left(\frac{R}{MW}\right) \cdot T_{av}} \tag{3.14}$$

<u>Objective function:</u>

Transportation cost due to compression will be defined to be the objective function for the system example. It will be assumed that each compressor station consists of modules in series where each module consists of equal centrifugal compressors that are in parallel providing the necessary mass flow capacity. The series connection of the parallel modules then provides the necessary pressure increase. The total compression ratio is divided equally among all parallel modules and the total mass flow through the parallel modules is divided equally among the centrifugal units. This is illustrated in Figure 3.3. As the gas volume of the reservoirs decreases, further parallel modules may still be necessary to provide the necessary transportation pressure at a specified production capacity.

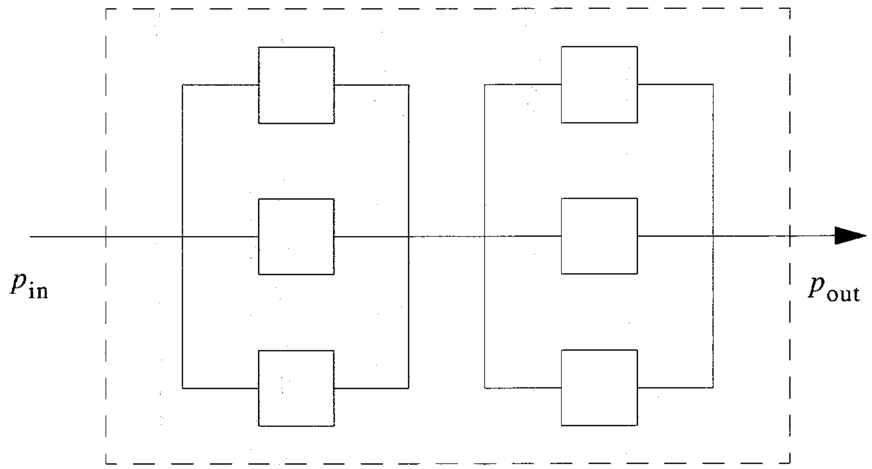


Figure 3.3: Compressor station configuration

The compression power for a turbo compressor is given in Eqn. (3.15). The variable P_{gas}, expresses the increase in enthalpy for the natural gas from the input to the output under isotropic compression. References are Fasold and Wahle (1992b), Osiadacz (1987) and Percell and Van Reet (1986)

$$P_{gas} = \frac{1}{\eta_s} \cdot \left(\frac{\Upsilon}{\Upsilon - 1}\right) \cdot \left(\frac{R}{MW}\right) \cdot Z_{in} \cdot T_{in} \cdot W \cdot \left\{\left(\frac{p_{out}}{p_{in}}\right)^{\frac{\Upsilon - 1}{\Upsilon}} - 1\right\} \quad (3.15)$$

where Υ is the isotropic exponent. The isotropic efficiency factor is denoted by η_s and lies in the region $0.6 \le \eta_s \le 0.87$ for turbo compressors. The mass flow through the compressor unit is denoted by W. The input and output pressures of the compressor unit are denoted by p_{in} and p_{out} respectively. For a real gas, the isotropic coefficient can be written as in Eqn. (3.16).

$$\Upsilon = \frac{v}{p} \cdot \left[\left(\frac{T}{C_v} \cdot \left(\frac{\partial p}{\partial T}\right)_v\right)^2 - \left(\frac{\partial p}{\partial v}\right)_T\right] \quad (3.16)$$

Here v is the molar volume. A typical region for the isotropic exponent is $1.3 \le \Upsilon \le 1.45$. The factors $\left(\frac{\partial p}{\partial T}\right)_v$ and $\left(\frac{\partial p}{\partial v}\right)_T$ are calculated from the state equation that is chosen to represent the thermodynamical properties of the natural gas, for example the BWR equation, see Fasold (1996b). An alternative to use one of the well known historical state equations, is to design a state equation where the parameters of this equation can be determined by practical experiments and identification. This equation is then used to describe the thermodynamics of the natural gas to be used in the transmission system. The practical experiments on the natural gas under consideration are performed in the expected temperature and pressure ranges used in the operation. Simply, we take a known mass of natural gas and inject it inside a container as illustrated in Figure 3.4.

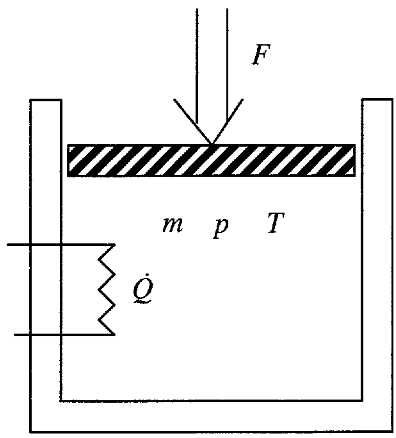


Figure 3.4: Experimental setup to develop a state equation for a natural gas of given composition.

The pressure and temperature are measured inside the natural gas container. We have one control system that controls the temperature and another that controls the pressure. Control variables are heat flow and piston force. At a given pressure and temperature condition inside the container, the volume is measured. Then, enough information is available to calculate the density of the natural gas inside the container. Assume that $p \in [\underline{p}, \bar{p}]$ and $T \in [\underline{T}, \bar{T}]$. Density ρ_i is determined for the pair $\{p_i, T_i\}$ where the index denotes experiment i. Let N_e denote the number of pairs inside the defined operational range. Assume that the state equation is approximated by a second order polynomial model with unknown coefficients as in Eqn. (3.17).

$$\rho = \rho(p_{\mathrm{ref}}, T_{\mathrm{ref}}) + \nabla\rho(p_{\mathrm{ref}}, T_{\mathrm{ref}}) \cdot \begin{bmatrix} p - p_{\mathrm{ref}} \\ T - T_{\mathrm{ref}} \end{bmatrix} \tag{3.17}$$
$$+ \begin{bmatrix} p - p_{\mathrm{ref}} \\ T - T_{\mathrm{ref}} \end{bmatrix}^T \cdot \nabla^2\rho(p_{\mathrm{ref}}, T_{\mathrm{ref}}) \cdot \begin{bmatrix} p - p_{\mathrm{ref}} \\ T - T_{\mathrm{ref}} \end{bmatrix}$$

where the unknown gradient and Hessian are to be determined. Here, $\{p_{\mathrm{ref}}, T_{\mathrm{ref}}\}$ is a reference pair. We can rewrite the above as

$$\Delta\rho_M = \theta_1 \cdot (p - p_{\mathrm{ref}}) + \theta_2 \cdot (T - T_{\mathrm{ref}}) + \theta_3 \cdot (T - T_{\mathrm{ref}})^2 \quad , \tag{3.18}$$
$$+ \theta_4 \cdot (p - p_{\mathrm{ref}})^2 + \theta_5 \cdot (p - p_{\mathrm{ref}}) \cdot (T - T_{\mathrm{ref}})$$

where $\Delta\rho_M = \rho_M - \rho(p_{\mathrm{ref}}, T_{\mathrm{ref}})$ is the modelled density deviation. Denote $y = \Delta\rho$ to be the actual measured density deviation. Define the regression vector for experiment i as in Eqn. (3.19).

$$\varphi(i) \tag{3.19}$$
$$= \left[(p_i - p_{\mathrm{ref}}), (T_i - T_{\mathrm{ref}}), (T_i - T_{\mathrm{ref}})^2, (p_i - p_{\mathrm{ref}})^2, (p_i - p_{\mathrm{ref}}) \cdot (T_i - T_{\mathrm{ref}})\right]^{\mathrm{T}}$$

and the vector of unknown coefficients as in Eqn. (3.20).

$$\theta = \begin{bmatrix}\theta_1 & \theta_2 & \theta_3 & \theta_4 & \theta_5\end{bmatrix}^{\mathrm{T}} \tag{3.20}$$

The model error for a given value of the parameter vector and experiment i is then given as in Eqn. (3.21).

$$\varepsilon_i(\theta) = y_i - \varphi^{\mathrm{T}}(i) \cdot \theta \tag{3.21}$$

Assume that we have N_{e} data giving the augmented error system in Eqn. (3.22).

$$\varepsilon(\theta) = Y - \Phi\theta \tag{3.22}$$

where $\varepsilon = \left[\varepsilon_1\ \varepsilon_2\ \dots\ \varepsilon_{N_e}\right]^T$, $Y = \left[y_1\ y_2\ \dots\ y_{N_e}\right]^T$ and the regression matrix $\Phi = \left[\varphi^T(1)\ \varphi^T(2)\ \dots\ \varphi^T(N_e)\right]^T$. Assume that the parameters are to be determined minimizing the sum of squared errors as in Eqn. (3.23).

$$V(\theta) = \|\varepsilon(\theta)\|^2 = \frac{1}{2}(Y - \Phi\theta)^T(Y - \Phi\theta) \tag{3.23}$$

The experiment is performed so that all the columns of Φ are linearly independent such that $\Phi^T\Phi > 0$, implying a unique minimum of $V(\theta)$ since it is then strictly convex. The parameter estimate is then given in Eqn. (3.24).

$$\hat{\theta} = (\Phi^T\Phi)^{-1}\Phi^T Y \tag{3.24}$$

The approximated state equation is now given as in Eqn. (3.25).

$$\rho = \rho(p, T) = \rho(p_{ref}, T_{ref}) + \varphi^T(p, T; \{p_{ref}, T_{ref}\}) \cdot \hat{\theta} \tag{3.25}$$

A reference for the subject of system identification is Søderstrøm and Stoica (1989).

The driver unit, for example an electric motor or a gas turbine, that drives one or several compressor units, uses more energy than the increased energy amount supplied to the natural gas by the compression process. This is due to mechanical losses. The driver power of the considered compressor unit is given in Eqn. (3.26)

$$P_{driver} = \frac{P_{gas}}{\eta_m}, \tag{3.26}$$

where η_m is the mechanical efficiency factor. The value of the efficiency factor is typically within the region $0.95 \le \eta_m \le 0.99$. The complete power supplied by the driver unit to increase the energy of the natural gas through the compressor unit is then expressed as in Eqn. (3.27).

$$P_{\text{driver}} \tag{3.27}$$

$$= \frac{1}{\eta_m} \cdot \frac{1}{\eta_s} \cdot \left(\frac{\Upsilon}{\Upsilon - 1}\right) \cdot \left(\frac{R}{MW}\right) \cdot Z_{\text{in}} \cdot T_{\text{in}} \cdot W \cdot \left\{ \left(\frac{p_{\text{out}}}{p_{\text{in}}}\right)^{\frac{\Upsilon - 1}{\Upsilon}} - 1 \right\}$$

The outlet temperature can be calculated using Eqn. (3.28), see Fasold (1994) or Percell and Van Reet (1986).

$$T_{\text{out}} = T_{\text{in}} \cdot \frac{Z_{\text{in}}}{Z_{out}} \cdot \left\{ \frac{\left(\frac{p_{\text{out}}}{p_{\text{in}}}\right)^{\frac{\Upsilon - 1}{\Upsilon}} - 1}{\eta_s} + 1 \right\} \tag{3.28}$$

If the outlet gas is temperature controlled, this temperature value is used. The instant energy cost due to compression is given in Eqn. (3.29)

$$K_{\text{cost}} = \xi_{\text{cost}} \cdot P_{\text{driver}}, \tag{3.29}$$

where ξ_{cost} is the money unit cost pr. energy unit. The compression power including mechanical losses for the considered compressor station is given in Eqn. (3.30).

$$P_{\text{station}} = \sum_{j=1}^{N_s} \sum_{i=1}^{M_j} P_{ji} \tag{3.30}$$

Here, M_j is the number of centrifugal compressors in the parallel module j and N_s is the number of parallel modules in series connection. The compression ratio over the compressor station is given in Eqn. (3.31).

$$\varepsilon = \prod_{j=1}^{N_s} \varepsilon_j \tag{3.31}$$

The compression cost is expressed as in Eqn. (3.32)

$$K_{\text{cost}} = \sum_{j=1}^{N_s} \sum_{i=1}^{M_j} \xi_{ji} P_{ji}, \tag{3.32}$$

where ξ_{ji} is the cost in money units per energy unit for driver i of the parallel module j. Here, the cost variable includes the total cost per mechanical energy unit supplied to the driver shaft connected to the compressor unit. Some alternative driver configurations are illustrated in Figure 3.5. P_{mec} is the abbreviation for the mechanical energy supplied to the driver shaft of the compressor unit, P_{cal} is the calorific amount of energy that is bought and is necessary for providing the mechanical energy to the driver including the energy loss. P_{electric} is the electric energy supplied including losses to provide the necessary momentum to the compressor unit.

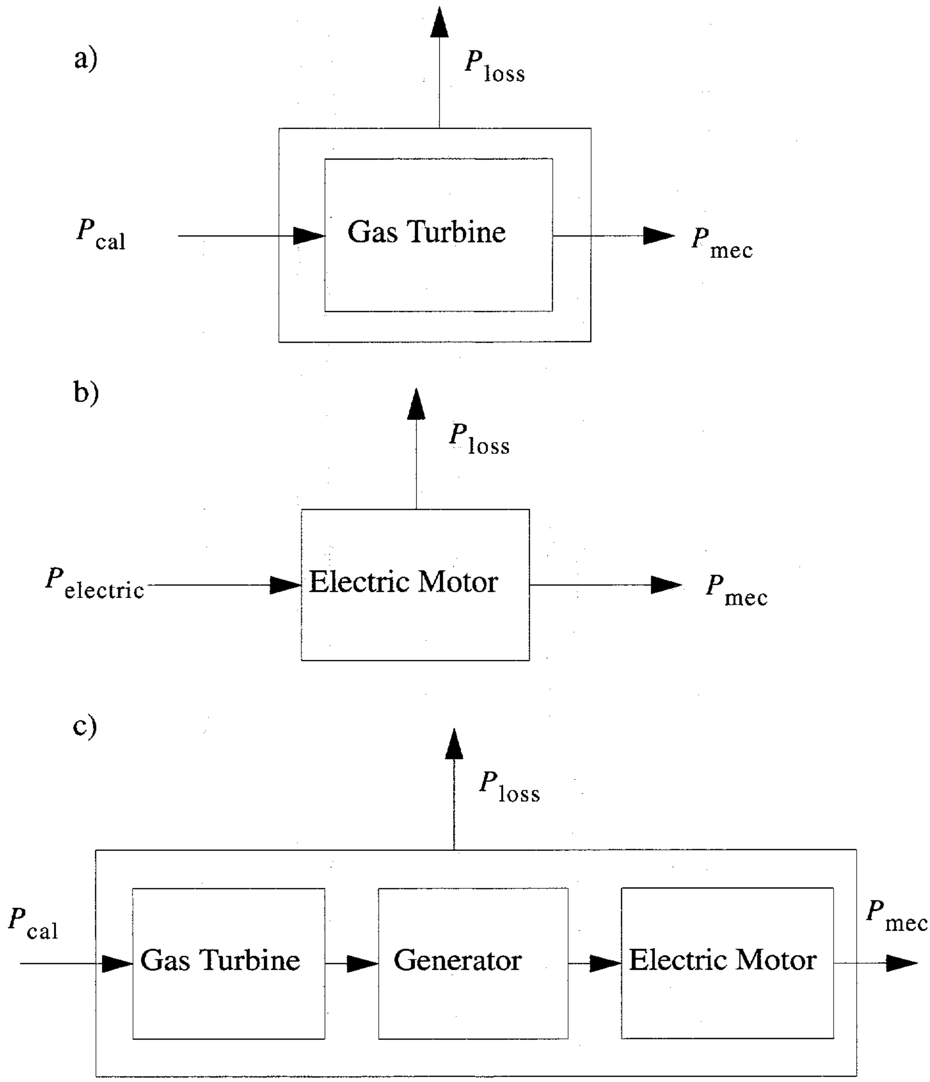


Figure 3.5: Some configurations from the energy source to the energy supplied to the driver shaft of the compressor unit.

In configuration a) of Figure 3.5, a gas turbine is directly connected to the compressor unit. In configuration b), energy is supplied from the electric power line to an electric motor providing the mechanical energy needed for compression. Configuration c) is often used in offshore taking advantage of the natural gas available. Here, the major cost is due to the environmental tax. Both the purchase cost and the environmental taxes may be considerable for a transportation firm that only offers transportation services and thus needs to buy gas from suppliers for the needed compression. Cases where it is possible to switch between different energy sources give the possibility to the choose the cheapest combination. For example, we may have the case that during day time, the cost of natural gas including environmental taxes is cheaper than buying electrical energy for each energy unit of mechanical energy provided to the compressor, while at night, the electric energy from power line from a spot market is the cheapest energy. Choosing the cheapest energy alternative from the spot market should also be encountered. In Eqn. (3.33), we have the possibility to choose between natural gas or electrical energy supplied from power line to provide the mechanical momentum to a compressor unit.

$$\xi_j(t) \in \{\xi_{j,\,\text{electric}}(t), \xi_{j,\,\text{gas}}(t)\} \tag{3.33}$$

<u>Energy cost for transmission example</u>:

Equation (3.34) expresses the compression cost for a transmission system example in Figure 3.1.

$$K_{\text{cost}} = \xi_1 \cdot \sum_{j=1}^{N_{s1}} \frac{1}{\eta_{m,1}} \cdot \frac{1}{\eta_{s,1}} \cdot \left(\frac{\Upsilon_1}{\Upsilon_1 - 1}\right) \cdot \left(\frac{R}{MW_1}\right) \cdot Z_1 \cdot T_1 \cdot u_{1,s} \cdot \left\{\left[\left(\frac{p_1}{p_{\text{suction},1}}\right)^{\frac{1}{N_s}}\right]^{\frac{\Upsilon_1 - 1}{\Upsilon_1}} - 1\right\} + \xi_2 \cdot \sum_{j=1}^{N_{s2}} \frac{1}{\eta_{m,2}} \cdot \frac{1}{\eta_{s,2}} \cdot \left(\frac{\Upsilon_2}{\Upsilon_2 - 1}\right) \cdot \left(\frac{R}{MW_2}\right) \cdot Z_2 \cdot T_2 \cdot u_{2,s} \cdot \left\{\left[\left(\frac{p_2}{p_{\text{suction},2}}\right)^{\frac{1}{N_s}}\right]^{\frac{\Upsilon_2 - 1}{\Upsilon_2}} - 1\right\} \tag{3.34}$$

For simplicity we assume that the output temperature of each parallel module is controlled and equal to the input temperature of the compressor station. It is also assumed that the compression ratio over the compressor station is divided equally among all the parallel modules. All centrifugal compressors inside a parallel module are assumed to be on a common driver shaft and all centrifugal units are of the same type, yielding approximately the same mechanical efficiency for a considered compressor station. Further, the driver cost is assumed to be equal for any of the parallel modules inside a station. Since the output pressure rises between each compression state, the compressibility factor will change. For simplicity it is assumed to be constant for each stage. The isotropic coefficient is also assumed to be constant. Here, $u_{i,\,s}$ is the stationary mass flow through station i. p_1 and p_2 are output pressures of station one and two respectively. $p_{\text{suction},\,i}$ is input pressure of station i. Υ_i is assumed as the isotropic coefficient for all the centrifugal units in station i. The pair $\{\eta_{m,\,i}, \eta_{s,\,i}\}$ are mechanical and isotropic efficiency factors at station i. The triple $\{T_i, Z_i, MW_i\}$ is temperature, compressibility factor and molecular weight at station i.

Transmission system constraints:

Using Eqns. (3.10) and (3.11), stationary fluid flow through the three pipes is given in Eqn. (3.35)

$$\begin{aligned}(p_{\text{junction}}^2 - p_{\text{terminal}}^2) &= K_1 \cdot v^{*2} \\ (p_1^2 - p_{\text{junction}}^2) &= K_2 \cdot u_{1,\,s}^{\;2}, \\ (p_2^2 - p_{\text{junction}}^2) &= K_3 \cdot u_{2,\,s}^{\;2}\end{aligned} \tag{3.35}$$

where the fluid properties are given in Eqn. (3.36).

$$K_i = \frac{16 \cdot \lambda_i \cdot L_i \cdot Z_{i,\,\text{av}} \cdot \left(\frac{R}{MW_i}\right) \cdot T_{i,\,\text{av}}}{\pi^2 D_i^5}, 1 \le i \le 3. \tag{3.36}$$

Here, v^* is the current defined average load. Equation (3.37), expresses the mass balance at junction point.

$$u_{1,\,s} + u_{2,\,s} = v^* \tag{3.37}$$

The terminal pressure, junction pressure and input pressure of the source pipelines must satisfy the constraints in Eqn. (3.38).

$$
\begin{aligned}
\underline{p}_{\text{terminal}} \le p_{\text{terminal}} \le \overline{p}_{\text{terminal}} \\
\underline{p}_{\text{junction}} \le p_{\text{junction}} \le \overline{p}_{\text{junction}} \\
\underline{p}_1 \le p_1 \le \overline{p}_1 \\
\underline{p}_2 \le p_2 \le \overline{p}_2
\end{aligned}
\tag{3.38}
$$

Additionally, we have quality parameter constraints. Assume that the gross calorific value and the concentration of carbon dioxide must be within contractual accepted regions expressed in Equation 3.39 for the junction point.

$$
\begin{aligned}
(\underline{\mathrm{GCV}}) \le \frac{1}{u_{1,\mathrm{s}} + u_{2,\mathrm{s}}} \cdot (\mathrm{GCV}_1 u_{1,\mathrm{s}} + \mathrm{GCV}_2 u_{2,\mathrm{s}}) \le (\overline{\mathrm{GCV}}) \\
(\underline{\mathrm{CO}_2}) \le \frac{1}{u_{1,\mathrm{s}} + u_{2,\mathrm{s}}} \cdot (\mathrm{CO}_{2_1} u_{1,\mathrm{s}} + \mathrm{CO}_{2_2} u_{2,\mathrm{s}}) \le (\overline{\mathrm{CO}_2})
\end{aligned}
\tag{3.39}
$$

Here, $\underline{\mathrm{GCV}}$ and $\overline{\mathrm{GCV}}$ are the lower and upper limits of the accepted region for the gross calorific value while $\underline{\mathrm{CO}_2}$ and $\overline{\mathrm{CO}_2}$ are the limits for carbon dioxide. $\{\mathrm{GCV}_i, \mathrm{CO}_{2_i}\}$ are the gross calorific value and the carbon dioxide value from the processing facility i. The vector of stationary optimization variables is defined in Eqn. (3.40).

$$
x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} p_{\text{terminal}} \\ p_{\text{junction}} \\ p_1 \\ p_2 \\ u_{1,\mathrm{s}} \\ u_{2,\mathrm{s}} \end{bmatrix}
\tag{3.40}
$$

Resulting nonlinear program:

The stationary optimization problem is now compactly written as in Eqn. (3.41).

$$\begin{aligned} &\underset{x}{\text{minimize}} \quad f(x) \\ &\text{subject to} \\ &\qquad h(x) = 0 \\ &\qquad g(x) \le 0 \\ &\qquad \underline{x} \le x \le \bar{x} \end{aligned} \tag{3.41}$$

Stationary optimization formulation also using energy equation:

In some cases, the temperature level must be considered in the optimization formulation. For example, in some cases, we must avoid that the temperature of the fluid inside a pipeline gets under a critical minimum temperature. The temperature along each pipeline will depend on the operation point. At offshore, one may specify some lower accepted temperature limits. These limits must be included in the network optimization so that a stationary feasible operation point is found satisfying the temperature constraints. The Joule Thomson coefficients for natural gas usually have a sign in the expected operation area so that the temperature drops as the pressure drops along the pipeline. Also, the formation of hydrates must be avoided so the fluid state of the transmission system is kept in the allowed region of the temperature pressure envelope for the considered natural gas. Use of a stationary optimization featuring energy equation and a state equation that represents thermodynamical behaviour adequately, then should give a feasible stationary operation point. At stationary conditions, we have the one dimensional mass, momentum and energy equation given as in Eqns. (3.42)-(3.44). References for this material are found in Sjøen (1997), Rist (1996) or Osiadacz (1987).

$$W = A\rho\upsilon = \text{constant} \tag{3.42}$$

$$\upsilon \cdot \frac{dv}{dx} = -\frac{1}{\rho} \cdot \frac{dp}{dx} - \frac{\lambda}{D} \cdot \frac{\upsilon^2}{2} - g\sin\theta \tag{3.43}$$

$$\upsilon \cdot \frac{dh_t}{dx} = \frac{4k_h}{\rho D} \cdot (T_{\text{amb}} - T) - \upsilon g \sin\theta \tag{3.44}$$

W is the stationary mass flow, $\upsilon = \upsilon(x)$ is the fluid velocity along the pipeline, $\theta = \theta(x)$ is the pipeline angle, $\lambda = \lambda(x)$ is Darcys friction factor and the relation between Fannings friction factor f and Darcys friction factor is given by Eqn. (3.45).

$$f = \frac{\lambda}{4} \tag{3.45}$$

The assumed stationary ambient temperature is denoted by $T_{\mathrm{amb}}(x)$, $x \in [0, L]$. The constant k_h is the net heat transfer coefficient from surroundings to the fluid. The variable h_t is the total enthalpy and is expressed in Eqn. (3.46)

$$h_t = h + \frac{\upsilon^2}{2}, \tag{3.46}$$

where h is the internal energy plus the pressure energy given in Eqn. (3.47).

$$h = u + \frac{p}{\rho} \tag{3.47}$$

If we assume that the pipeline is approximately normal to the gravitational force and neglect the acceleration energy $\frac{\upsilon^2}{2}$ in the energy equation, we can assume that $h_t \approx h$. A simplified energy equation is then given in Eqn. (3.48).

$$\frac{dh}{dx} = \frac{4k_h}{\upsilon\rho D} \cdot (T_{\mathrm{amb}} - T), \ x \in [0, L] \tag{3.48}$$

Using the relation in Eqn. (3.49)

$$dh = C_p \cdot (dT - \beta_{\mathrm{JT}} dp), \tag{3.49}$$

where $\beta_{\mathrm{JT}} = \left.\frac{\partial T}{\partial p}\right|_h$ is the Joule-Thomson coefficient, we get Eqn. (3.50).

$$\frac{dT}{dx} = \frac{\pi D k_h}{C_p} \cdot (T_{\mathrm{amb}} - T) + \beta_{\mathrm{JT}} \cdot \frac{dp}{dx}, \ x \in [0, L], \tag{3.50}$$

where the pressure gradient along the pipeline is given by Eqn. (3.51).

$$\frac{dp}{dx} = -\frac{8\lambda W^2}{\pi^2 \rho(T, p) D^5} \tag{3.51}$$

Inserting Eqn. (3.51) into Eqn. (3.50) and using the state equation $\rho = \rho(T, p)$, we have the coupled set of nonlinear ordinary differential equations with initial conditions as in Eqn. (3.52).

$$\begin{aligned}
\frac{dT}{dx} &= \frac{\pi D k_h}{C_p} \cdot (T_{\text{amb}} - T) - \beta_{\text{JT}} \cdot \frac{8\lambda W^2}{\pi^2 \rho(p, T) D^5} \qquad x \in [0, L] \\
\frac{dp}{dx} &= -\frac{8\lambda W^2}{\pi^2 \rho(p, T) D^5} \qquad x \in [0, L] \\
T(x = 0) &= T_{\text{in}},\ p(x = 0) = p_{\text{in, s}}
\end{aligned} \tag{3.52}$$

We see that

$$\begin{aligned}
\frac{dT}{dx} &= f(T, p, T_{\text{amb}}, W) \\
\frac{dp}{dx} &= g(T, p, W)
\end{aligned}$$

The differential equations are solved by a numerical approximation procedure. A treatment of Runge-Kutta and Euler methods are given in Cheney and Kincaid (1999). Use of an explicit discretization method then yields a coupled set of difference equations as in Eqn. (3.53).

$$\begin{aligned}
T_{i+1} &= \varphi_1(T_i, p_i, T_{\text{amb}, i}, W_i) \\
p_{i+1} &= \varphi_2(T_i, p_i, W_i)
\end{aligned} \tag{3.53}$$

Here, we have

$$\begin{aligned}
T_i &= T((i-1) \cdot \Delta x) \\
p_i &= p((i-1) \cdot \Delta x) \\
T_{\text{amb}, i} &= T_{\text{amb}, i}((i-1) \cdot \Delta x)
\end{aligned}$$

where $1 \leq i \leq M$ and M is the number of spatial points along the considered pipeline.

Objective function of the stationary optimization problem:

The objective function for the system example is the same as in Eqn. (3.34) and re- expressed in Eqn. (3.54).

$$K_{\text{cost}} = \xi_1 \cdot \sum_{j=1}^{N_{s1}} \frac{1}{\eta_{m,1}} \cdot \frac{1}{\eta_{s,1}} \cdot \left(\frac{\Upsilon_1}{\Upsilon_1 - 1}\right) \cdot \left(\frac{R}{MW_1}\right) \cdot Z_1 \cdot T_1 \cdot u_{1,s} \tag{3.54}$$

$$\cdot \left\{ \left[\left(\frac{p_{1,1}}{p_{\text{suction},1}} \right)^{\frac{1}{N_s}} \right]^{\frac{\Upsilon_1 - 1}{\Upsilon_1}} - 1 \right\} + \xi_2 \cdot \sum_{j=1}^{N_{s2}} \frac{1}{\eta_{m,2}} \cdot \frac{1}{\eta_{s,2}} \cdot \left(\frac{\Upsilon_2}{\Upsilon_2 - 1}\right)$$

$$\cdot \left(\frac{R}{MW_2}\right) \cdot Z_2 \cdot T_2 \cdot u_{2,s} \cdot \left\{ \left[\left(\frac{p_{2,1}}{p_{\text{suction},2}} \right)^{\frac{1}{N_s}} \right]^{\frac{\Upsilon_2 - 1}{\Upsilon_2}} - 1 \right\}$$

Constraints of the stationary optimization problem:

The ambient temperature is assumed to be uniform along the transmission pipeline. The pressure and temperature distribution along the pipelines one and two are given in Eqns. (3.55) and (3.56).

$$\begin{aligned} T_{1,i+1} &= \varphi_1(T_{1,i}, p_{1,i}, T_{1,\text{amb}}, u_{1,s}) \\ p_{1,i+1} &= \varphi_2(T_{1,i}, p_{1,i}, u_{1,s}) \end{aligned} \quad , 1 \leq i \leq M_1 \tag{3.55}$$

$$\begin{aligned} T_{2,i+1} &= \varphi_1(T_{2,i}, p_{2,i}, T_{2,\text{amb}}, u_{2,s}) \\ p_{2,i+1} &= \varphi_2(T_{2,i}, p_{2,i}, u_{2,s}) \end{aligned} \quad , 1 \leq i \leq M_2 \tag{3.56}$$

The temperature out of the junction point into pipeline three is given in Eqn. (3.57).

$$T_{3,1} = \frac{1}{u_{1,s} + u_{2,s}} \cdot (u_{1,s} \cdot T_{1,M_1} + u_{2,s} \cdot T_{2,M_2}) \tag{3.57}$$

The pressure and temperature distribution along pipeline three is given in Eqn. (3.58).

$$\begin{aligned} T_{3,i+1} &= \varphi_1(T_{3,i}, p_{3,i}, T_{3,\text{amb}}, u_{1,s}, u_{2,s}) \\ p_{3,i+1} &= \varphi_2(T_{3,i}, p_{3,i}, u_{1,s}, u_{2,s}) \end{aligned}, \; 1 \le i \le M_3 \tag{3.58}$$

Mass balance at the junction point is given in Eqn. (3.59).

$$u_{1,s} + u_{2,s} = v^* \tag{3.59}$$

At the junction point, Eqn. (3.60) is satisfied.

$$p_{1,M_1} = p_{2,M_2} = p_{3,1} \tag{3.60}$$

The pressure constraints are defined in Eqn. (3.61)

$$\begin{aligned} \underline{p}_{3,M_3} \le p_{3,M_3} \le \bar{p}_{3,M_3} \\ \underline{p}_{\text{junction}} \le p_{1,M_1} \le \bar{p}_{\text{junction}} \\ \underline{p}_{1,1} \le p_{1,1} \le \bar{p}_{1,1} \\ \underline{p}_{2,1} \le p_{2,1} \le \bar{p}_{2,1} \end{aligned}, \tag{3.61}$$

where we recall Eqn. (3.60). Assume that we have a lower temperature limit of the two source pipelines before entering the junction point and a lower temperature limit constraint at the delivery terminal. These constraints are defined in Eqn. (3.62).

$$\begin{aligned} \underline{T}_{\text{in, junction}} \le T_{1,M_1} \\ \underline{T}_{\text{in, junction}} \le T_{2,M_1} \\ \underline{T}_{3,M_3} \le T_{3,M_3} \end{aligned} \tag{3.62}$$

Additionally, as before, we have quality parameter constraints. Assume that the gross calorific value and the concentration of carbon dioxide must be within contractual accepted regions expressed in Eqn. (3.63) for the junction point.

$$\begin{aligned}(\underline{\mathrm{GCV}}) &\le \frac{1}{u_{1,\,s}+u_{2,\,s}}\cdot(\mathrm{GCV}_1 u_{1,\,s}+\mathrm{GCV}_2 u_{2,\,s}) \le (\overline{\mathrm{GCV}})\\(\underline{\mathrm{CO}_2}) &\le \frac{1}{u_{1,\,s}+u_{2,\,s}}\cdot(\mathrm{CO}_{2_1} u_{1,\,s}+\mathrm{CO}_{2_2} u_{2,\,s}) \le (\overline{\mathrm{CO}_2})\end{aligned} \tag{3.63}$$

Here, $\underline{\mathrm{GCV}}$ and $\overline{\mathrm{GCV}}$ are the lower and upper limits of the accepted region for the gross calorific value while $\underline{\mathrm{CO}_2}$ and $\overline{\mathrm{CO}_2}$ are the limits for carbon dioxide. $\{\mathrm{GCV}_i, \mathrm{CO}_{2_i}\}$ are the gross calorific value and carbon dioxide value from the processing facility i. The input temperatures $T_{1,\,1}$ and $T_{2,\,1}$ are given and are equal to the outlet temperatures of the two compressor stations. The current stationary load v^* is also given.

Optimization vector:

The vector of unknown variables in the optimization problem is defined in Eqn. (3.64).

$$x = \begin{bmatrix} u_{1,\mathrm{s}} \\ u_{2,\mathrm{s}} \\ p_{1,1} \\ p_{1,2} \\ T_{1,2} \\ \cdots \\ p_{1,M_1-1} \\ T_{1,M_1-1} \\ T_{M_1} \\ p_{2,1} \\ p_{2,2} \\ T_{2,2} \\ \cdots \\ p_{2,M_2-1} \\ T_{2,M_2-1} \\ T_{M_2} \\ p_{\mathrm{junction}} \\ T_{3,1} \\ p_{3,2} \\ T_{3,2} \\ \cdots \\ p_{3,M_3} \\ T_{3,M_3} \end{bmatrix} \tag{3.64}$$

Resulting optimization problem:

The optimization problem is written compactly in Eqn. (3.65).

$$\begin{aligned} &\underset{x}{\text{minimize}} \quad f(x) \\ &\text{subject to} \\ &\qquad h(x) = 0 \\ &\qquad g(x) \leq 0 \\ &\qquad \underline{x} \leq x \leq \bar{x} \end{aligned} \tag{3.65}$$

Comments:

From the complex stationary optimization, we get the optimal mass flows, the optimal pressure distributions and the optimal temperature distributions. Hence, by the use of the state equation, the compressibility factor distributions are also obtained. These results can then be used as parameters for the dynamic optimization model of the model predictive control scheme. This will be explained in Section 3.3. In this way, complex stationary thermodynamics is implicitly included in the dynamic control model.

We note that the number of variables have increased considerable for the complex stationary optimization in comparison to the simplified stationary optimization formulation which do not use the energy equation. The increased computational effort is the main drawback for the more complex optimization. The main advantage is that the stationary description is more accurate so that the calculated optimization result may yield lower transportation costs. Another important point already mentioned is that it may be necessary to include the energy equation and extra constraints with the objective to avoid a critical fluid state in the transmission system.

To fully obtain the advantage by using the complex stationary optimization formulation, the global minimum should be obtained. Then, a global optimizer is necessary to use.

Solving the stationary optimization problem:

The optimization problem defined in Eqn. (3.41) was solved using the sequential quadratic programming solver **fmincon** in Matlab, which finds a local minimum. For each iteration in the search for a local minimum, the gradient vector of the objective function and the gradient vectors for each equality and inequality constraints were calculated analytically and supplied to the unstructured optimization algorithm avoiding the need for function evaluations that would otherwise be necessary to perform to calculate the gradients numerically. The local minimum that is found is generally a function of the initial guess for the solution. For the transmission example, we know that the optimal delivery terminal pressure is close to the minimum pressure level. By using the AGA equation and the mass balance at the junction point it is possible to find reasonable initial guesses for the junction pressure, inlet pressures for the source pipelines and source flows. For a description of sequential quadratic programming algorithms, see Bertsekas (1995).

A global optimization algorithm is necessary if we want to be sure that the solution found is a global minimum. The global optimization method alfa branch and bound (αBB) defines convex underestimators for all types of nonconvex function terms yielding a convex program for the subdomain under consideration. A standard convex solver like a sequential quadratic programming solver then finds a global minimum of the convexified problem. This yields a lower bound for the function value for the domain under evaluation. Solving the original optimization problem for the same domain using a local solver provides an upper bound for the function value. By keeping an account of the upper and lower bounds for the different subregions for each optimization iteration, some regions may be fathomed due to the fact that the global solution can not be in this region. Other sub domains must be further partitioned since they may contain the global minimum. The iterative refinement of lower and upper bounds by successively partitioning the initial rectangular region into smaller ones leads eventually to convergence to the global minimum inside the feasibility tolerances and the termination criteria of the objective function. Refinement is only performed in those regions that may contain a global minimum. For a thorough understanding of the alfa branch and bound algorithm, see, Adjiman *et al.* (1998a;1998b) and Androulakis *et al.* (1995).

Some previous work in stationary gas transmission optimization:

In the paper by Mallinson and Fincham (1993), gas transmission systems are solved using the Sequential Augmented Lagrange method for different objective functions. Examples of the objective functions that were used are minimizing line pack in system, minimizing fuel consumption and minimizing supply costs. First, the method was used on the problem where the pipe-flow variables have been eliminated reducing the computational effort. But, it was found that this method was unreliable for large systems. Solving the stationary optimization problem using the full set of variables was found to be reliable, but this was at the expense of a longer calculation time. The thesis by Wong (1988) gives a more thorough description of the optimization method.

The paper by Lo (1984) defines optimization problems in the gas field and attempts to find a solution using successive linear approximation or a Lagrange method.

In Pratt and Wilson (1984), stationary gas transmission optimization problem with both discrete and continuous variables is described. The discrete variables may arise due to the sources and compressors that are to be in use, introducing on-off variables into the problem. The objective function and constraints are linearized around the solution from the previous iteration in the search for a minimum giving a mixed integer linear program. The sequential method is repeated until convergence.

The paper by Luongo *et al.* (1991) uses a nonlinear programming top level optimizer to optimize the physical flows in the transmission network and a linear programming optimizer for the economic optimization handling a large number of contracts when the network flows from the top level optimizer are determined. Fuel minimization and minimization of the cost of fuel are among the considered optimization objectives. The flows in the network are given. Throughput maximization and profit maximization are considered as objectives when a set of supply, delivery and transport contracts are given. The economic optimizer determines the optimum allocation of flows among all of the delivery, supply and transport contracts and uses a modified Simplex method.

In Goslinga *et al.* (1994), a stationary optimization is performed where both the constraints and the objective function are linearized yielding a linear programming problem which was solved using the Simplex method. Different objective functions were treated.

The paper by Ramchandani and Gray (1993) deals with natural gas supply and transmission operation as an economic game with the objective to solve network operation and planning problems.

The main features of a stationary operation package for natural gas transmission networks called ACCORD is presented in the paper by Ostromuhov (1998). This package looks at finding optimal dimensions of transmission networks, minimization of investment costs, selecting facilities, supplies and demands and looks at stationary optimization of a given transmission system. The stationary optimization handles objectives such as profit maximization, network flow maximization and minimization of transport cost. Optimization regarding the injection and withdrawal of gas storage facilities is also treated. The result from the stationary optimization are optimal set-points for supplies, demands and pressures and station configurations to minimize cost due to transportation and for gas purchased so as to maximize profit. Graph theory, integer programming and nonlinear programming and methods to solve a nonlinear system of equations and inequalities are used to solve an optimization problem with continuous and discrete variables.

Linear programming is used in Avery *et al.* (1992) for minimizing cost and satisfying regulatory agencies for pipeline and natural gas distribution companies on both short and long term basis. On the short term basis, the objective was defined to minimize the cost for the available gas supply by choosing the optimal amounts of supply. On the long term basis, the objective was to construct an optimal portfolio of gas sources minimizing purchase cost.

Steady state optimization of large gas transmission systems is treated in Wilson and Furey (1988).

3.3.2 Optimization Model for Model Predictive Controller

A linear control model will be used based on the reformulation of the nonlinear creep flow model and use of the current calculated "optimal" stationary operation point. Eqn. (3.1) can be rewritten to Eqn. (3.66), see Osiadacz (1987)

$$\begin{aligned}\frac{\partial p}{\partial t} &= \frac{c^2}{A\lambda} \cdot \frac{\partial^2 p}{\partial x^2}, \\ W &= -\frac{1}{\lambda} \cdot \frac{\partial p}{\partial x}\end{aligned} \tag{3.66}$$

where λ is given in Eqn. (3.67).

$$\lambda = \frac{2fc^2|W|}{DA^2 p} \tag{3.67}$$

Let W^* be the optimal calculated stationary mass flow through the considered pipeline and $p^* = p^*(x)$ to be the optimal calculated pressure from the stationary optimization at position x.

This gives the following linear model at position x which is given in Eqn. (3.68)

$$\frac{\partial p}{\partial t} = \frac{c^2}{A\lambda^*(x)} \cdot \frac{\partial^2 p}{\partial x^2}, \tag{3.68}$$

where λ^* at the considered pipeline position is given as in Eqn. (3.69).

$$\lambda^*(x) = \frac{2fc^2|W^*|}{DA^2 p^*(x)} \tag{3.69}$$

<u>Comment.</u> For a transmission line with supplies and offtakes along the transmission line, the stationary mass flow will be a function of the position along the transmission line.

For the more complex stationary optimization considering the coupled system of stationary mass, momentum and energy equation along each pipeline of the transmission system, we replace Eqn. (3.69) with Eqn. (3.70)

$$\lambda^*(x) = \frac{2f \cdot \kappa \cdot Z(x) \cdot \left(\frac{R}{MW}\right) \cdot T(x) \cdot |W^*|}{DA^2 p^*(x)}, \tag{3.70}$$

where the relations $c^2 = \kappa Z R' T$ and $R' = \frac{R}{MW}$ have been used. In this way, the simple optimization model includes complex thermodynamical properties implicitly through the stationary optimization.

The numerical method of lines is used to obtain a linear state space model of ordinary differential equations. A fourth order accuracy scheme (Schiesser,1991) was used for the approximation of the second order spatial derivatives, both for the internal points of each pipeline and for the Neuman boundary conditions at the source and load points of the transmission system example. A linear differential equation was used for the description of change in pressure at the junction point of the example.

The spatial differentiation at the inlets of each source pipeline is given in Eqn. (3.71).

$$\frac{\partial^2}{\partial x^2} p(x_1) = \frac{1}{4!\Delta x^2} \cdot \left(-\frac{415}{3} p(x_1) + 192 p(x_2) - 72 p(x_3) + \frac{64}{3} p(x_4) - 3 p(x_5) - 100 \cdot \frac{\partial}{\partial x_1} p(x_1) \cdot \Delta x\right) + O(\Delta x^4) \tag{3.71}$$

The relation $\frac{\partial}{\partial x} p(x_1) = -\lambda^*(x_1) \cdot W(x_1)$ is inserted into Eqn. (3.71)

yielding the Neuman boundary condition. At the load demand point the spatial differentiation in Eqn. (3.72) was used.

$$\frac{\partial^2}{\partial x^2}p(x_M) = \frac{1}{4!\Delta x^2}\cdot\left(-\frac{415}{3}p(x_M) + 192p(x_{M-1})\right. \tag{3.72}$$

$$\left. - 72p(x_{M-2}) + \frac{64}{3}p(x_{M-3}) - 3p(x_{M-4}) + 100\cdot\frac{\partial}{\partial x}p(x_M)\cdot\Delta x\right)$$

$$+ O(\Delta x^4)$$

Similarly, the Neuman boundary condition $\frac{\partial}{\partial x}p(x_M) = -\lambda^*(x_M)\cdot W(x_M)$ is

used at the customer demand point of the transmission system example. At points next to the input grid point, the spatial differentiation given in Eqn. (3.73) is used and the differentiation approximation at the point next to the output grid point of the transmission line is given in Eqn. (3.74).

$$\frac{\partial^2 p}{\partial x^2} = \frac{1}{4!\Delta x^2} \tag{3.73}$$

$$\cdot(20p(x_1) - 30p(x_2) - 8p(x_3) + 28p(x_4) - 12p(x_5) + 2p(x_6))$$

$$+ O(\Delta x^4)$$

$$\frac{\partial^2 p}{\partial x^2} = \frac{1}{4!\Delta x^2}\cdot(20p(x_M) - 30p(x_{M-1}) - 8p(x_{M-2}) \tag{3.74}$$

$$+ 28p(x_{M-3}) - 12p(x_{M-4}) + 2p(x_{M-5})) + O(\Delta x^4)$$

At internal points of each transmission line, the spatial differentiation in Eqn. (3.75) is used

$$\frac{\partial^2}{\partial x^2}p(x_i) = \frac{1}{4!\Delta x^2} \tag{3.75}$$

$$\cdot (-2p(x_{i-2}) + 32p(x_{i-1}) - 60p(x_i) + 32p(x_{i+1}) - 2p(x_{i+2})$$

$$) + O(\Delta x^4)$$

where $3 \le i \le M-2$ for the considered transmission line.

The junction model in Eqn. (3.76) from Osiadacz (1987) was used.

$$\frac{V_j}{c^2} \cdot \frac{dp_j}{dt} = \sum_{i=1}^{k} (p_i - p_j) \cdot S_{ij}{*} - W_j \tag{3.76}$$

where $S_{ij}{*} = S_{ji}{*} = \dfrac{1}{\lambda_l{*}\Delta x_l}$ corresponds to pipe l. Here, p_j is the junction pressure and p_i is the pressure at the discretization point incident with section Δx_l. For the transmission system example, W_j is zero.

A more complex way of setting up the junction model for the transmission example which is easily generalized to a junction point including offtakes is described below. The junction point connecting the three pipelines is illustrated in Figure 3.6.

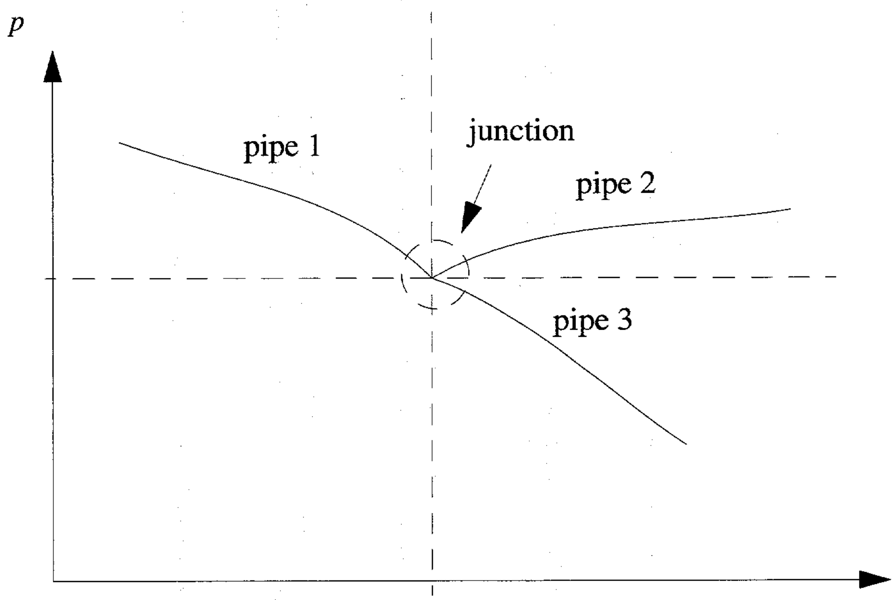


Figure 3.6: Junction point connecting the three pipelines of transmission system example.

The mass balance for the junction point is given in Eqn. (3.77).

$$\frac{dm}{dt} = W(x_1 = L_1, t) + W(x_1 = L_2, t) - W(x_3 = 0, t) \tag{3.77}$$

The mass balance for the junction point needs mass flows out from the two source pipelines into junction point and mass flow out from junction point into the beginning of the offtake pipeline. From Eqn. (3.66) we have the linear relation repeated in Eqn. (3.78) relating mass flow and pressure gradient.

$$W = -\frac{1}{\lambda} \cdot \frac{\partial p}{\partial x} \tag{3.78}$$

Mass flow into the small junction volume from pipe one will be calculated based only on the pressures in pipe 1 and the junction pressure. The same technique applies for pipelines two and three. We use the fourth order accuracy spatial approximation schemes for the spatial derivative $p_x(x)$ from Schiesser (1991).

At the output of pipeline one and two, the approximation in Eqn. (3.79) is used

$$\frac{\partial}{\partial x}p(x_M) = \frac{1}{4!\Delta x}\cdot(6p(x_{M-4}) - 32p(x_{M-3}) + 72p(x_{M-2}) \\ - 96p(x_{M-1}) + 50p(x_M)) + O(\Delta x^4) \tag{3.79}$$

where $p(x_M)$ is the junction pressure. At the inlet of pipe three, we use Eqn. (3.80).

$$\frac{\partial}{\partial x}p(x_1) = \frac{1}{4!\Delta x} \\ \cdot(-50p(x_1) + 96p(x_2) - 72p(x_3) + 32p(x_4) - 6p(x_5)) \\ + O(\Delta x^4) \tag{3.80}$$

Use of the approximations in Eqns. 3.79 and 3.80 with Eqn. 3.78 and the mass balance in Eqn. 3.77 and use of the relation in Equation 3.81,

$$p = c^2 \cdot \rho = \kappa \cdot Z \cdot \frac{R}{MW} \cdot T \cdot \rho \tag{3.81}$$

and $m = \rho \cdot V$, then yields a linear differential equation for the change in junction pressure with accuracy for the spatial derivatives in accordance with the accuracy of the approximations of the second order spatial derivatives.

The sums of the coefficients for the approximations of the second and first order spatial derivatives at the different spatial positions are all zero. This is necessary for obtaining zero when differentiating a constant. Use of the above schemes for the spatial approximation of the second order partial derivatives and use of the junction model and the current calculated "optimal" operation point yields the linear state space model in Eqn. (3.82)

$$\dot{x} = A(x^*(t_k), u^*(t_k), v^*(t_k))x + B(x^*(t_k), u^*(t_k), v^*(t_k))u \\ + C(x^*(t_k), u^*(t_k), v^*(t_k))v \tag{3.82}$$

or in shorter notation omitting the dependence of the "optimal" current stationary operation point as in Eqn. (3.83)

$$\dot{x} = Ax + Bu + Cv \tag{3.83}$$

where we implicitly knows from Eqn. (3.82) that the time invariant prediction model used for the current open loop uses the last available stationary optimization. The state vector x consists of the pressures at the defined spatial points in the network. The control vector u consists of the mass flows from the two sources and v is the customer natural gas offtake. A new stationary optimization yields a new state space model. In this way, the model will adapt to new conditions. For the simulation example, a sampling interval of one hour was used. The load and control command is assumed constant for the entire hour. Integration of Eqn. (3.83) then yields the linear discrete open loop prediction model in Eqn. (3.84)

$$x_{k+i+1|k} = \Phi x_{k+i|k} + \Gamma u_{k+i|k} + \Psi v_{k+i|k} \tag{3.84}$$

where

$$\begin{aligned} \Phi &= e^{A\Delta t} \\ \Gamma &= \int_0^{\Delta t} e^{A\eta} d\eta B \\ \Psi &= \int_0^{\Delta t} e^{A\eta} d\eta C \end{aligned} \quad . \tag{3.85}$$

3.3.3 State Estimator

In the simulations given in this chapter, the model predictive controller, which will described later, reoptimizes every hour. This controller receives estimated states from a discrete Kalman Filter using the same sampling interval as the model predictive controller. The model predictive controller uses the prediction model given in Eqn. (3.84). The same model is also used for the apriori one step prediction in the simulation example system. So, both the model predictive controller and the state estimator will update their models when a new stationary optimization is performed. Based on the prediction model, the following uncertainty system given in Eqn. (3.86) is formulated for designing the time varying filter gain for the state estimator.

$$
\begin{aligned}
x_{k+1} &= \Phi x_k + \Gamma u_k + \Psi v_k + w_k \\
y_k &= H x_k + \eta_k \\
E[w_k] &= 0 \qquad E[w_k w_k^T] = \begin{cases} Q_k, & i = k \\ 0, & i \neq k \end{cases} \\
E[\eta_k] &= 0 \qquad E[\eta_k \eta_k^T] = \begin{cases} R_k, & i = k \\ 0, & i \neq k \end{cases} \\
P_0 &= E[\varepsilon_0 \varepsilon_0^T] = E[(x_k - \tilde{x}_k)(x_k - \tilde{x}_k)^T]
\end{aligned}
\tag{3.86}
$$

The Kalman Filter for the above uncertainty system is given in Eqn. (3.87).

$$
\begin{aligned}
K_k &= \hat{P}_k H^{\mathrm{T}} (H \hat{P}_k H^{\mathrm{T}} + R_k)^{-1} \\
\tilde{x}_k &= \hat{x}_k + K_k (y_k - H \hat{x}_k) \\
\tilde{P}_k &= (I - K_k H) \hat{P}_k \\
\hat{x}_{k+1} &= \Phi \tilde{x}_k + \Gamma u_k + \Psi v_k \\
\hat{P}_{k+1} &= \Phi \tilde{P}_k \Phi^{\mathrm{T}} + Q_k
\end{aligned}
\tag{3.87}
$$

The matrix P_0 and the vector $\hat{x}_0$ are given. The uncertainty matrices R_k and Q_k are given for all k. It was assumed that the process noise and measurement noise uncertainty matrices were constant for the simple transmission example. In a practical application, knowledge about measurement noise and process uncertainty must be used. It may also be necessary to construct filters to model coloured process and measurement noise. With the inclusion of dynamics to model coloured process and measurement noise assuming zero expectation, it is possible to assume that the input and process noise to the uncertainty model of the plant is white with given uncertainty matrices.

The state estimator defined in Eqn. (3.87) is illustrated in Figure 3.7.

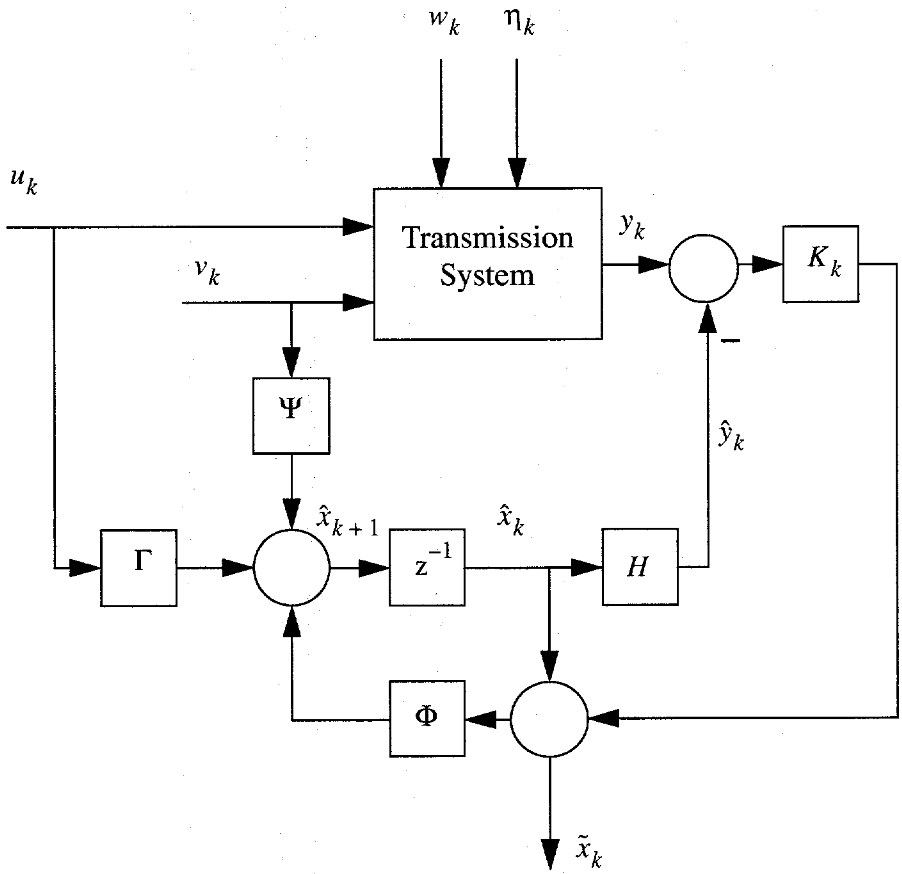


Figure 3.7: Block scheme of the state estimator.

Observability:

Pressures at the inlet of the two source pipelines, the junction pressure and the load pressure were assumed to be measured in the simulations yielding full column rank equal to n of the observability matrix O in Eqn. (3.88), where n is the dimension of the state vector. See Franklin *et al.* (1990) for a description on observability.

$$O = \begin{bmatrix} H \\ H\Phi \\ \dots \\ H^{\mathrm{T}}\Phi^{n-1} \end{bmatrix} \tag{3.88}$$

Flows into the source pipelines were known through the control law from the model predictive controller and the current load demand was assumed to be measured. Both the control command and the load demand were assumed to be piecewise constant. In the simulation, they were constant for one hour before (possibly) changing the value. In practice, the control commands will be references. So, the actual control flows will be transient depending on the closed loop dynamics for the low level control systems.

The discrete Kalman filter was presented for the first time in Kalman (1960). The book by Brown and Hwang (1992) is a modern reference on the material. Other work that looks at state estimation and observers applied especially to gas transmission systems using different approaches are Parkinson (1984), Parkinson and Wynne (1986), Choudary *et al.* (1985), Nichols and Stringert (1994), Carmichael *et al.* (1981) and Chapman *et al.* (1985).

3.3.4 Model Predictive Control Algorithm

In this section, the open loop optimization problem for the model predictive controller will be defined. The open loop problem is defined in Eqn. (3.89). The deviation from the stationary reference pressure distribution and reference stationary control vector is penalized quadratically. We also note that the open loop control vector space is generally time varying. This is necessary because the production availability may be time varying. Also, the minimum value of the control vector may be time varying due to time varying minimum obliged delivery amounts from each source. The linear inequality $Ax_{k+i+1|k} \le a$ is necessary for assuring that compression ratios over intermediate input-output combinations of compressor stations are larger than one, that flows through control valves go in the desired direction by securing that the input pressure is larger than the output pressure in desired flow directions. For the simple transmission system example, this important constraint was unnecessary. The cost function of each open loop problem was defined to be quadratic. The penalty matrices of this cost function were defined to be positive definite. As a result, the cost function is strictly convex. This implies that the defined open loop problem has a unique minimum.

$$\begin{array}{ll}
\underset{\{x_{k+i+1}, u_{k+i|k}\}_{i=0}^{N-1}}{\text{minimize}} & \frac{1}{2}\sum_{i=0}^{N-1} (x_{k+i+1|k} - x^*)^{\mathrm{T}} Q_{k+i+1}(x_{k+i+1|k} - x^*) + (u_{k+i|k} - u^*)^{\mathrm{T}} R_{k+i}(u_{k+i|k} - u^*) \\
\text{subject to} & \\
\end{array}$$

$$\begin{array}{rcll}
x_{k+i+1|k} & = & \Phi x_{k+i|k} + \Gamma u_{k+i|k} + \Psi v_{k+i|k} & 0 \le i \le N-1 \\
\underline{x} \le & x_{k+i+1|k} & \le \bar{x} & 0 \le i \le N-1 \\
\underline{u}_{k+i|k} \le & u_{k+i|k} & \le \bar{u}_{k+i|k} & 0 \le i \le N-1 \\
& Ax_{k+i+1|k} & \le a & 0 \le i \le N-1 \\
x_{k|k} & = & \tilde{x}_{k|k} &
\end{array} \tag{3.89}$$

A term penalizing the change in the control command between neighbouring control vectors in the parametrized control sequence was added to the objective function to obtain a smooth control trajectory. This term has the positive benefit of minimizing equipment wear. For the current open loop, this term is given in Eqn. (3.90).

$$\sum_{i=0}^{M-1} (u_{k+i+1|k} - u_{k+i|k})^{\mathrm{T}} R_{\Delta u}(u_{k+i+1|k} - u_{k+i|k}) \tag{3.90}$$

The augmented form of the open loop control trajectory is given in Eqn. (3.91)

$$u_{\mathrm{ol}} = \left[u_{k|k}^{\mathrm{T}}\ u_{k+1|k}^{\mathrm{T}}\ \dots\ u_{k+N-1|k}^{\mathrm{T}}\right]^{\mathrm{T}} \tag{3.91}$$

and the augmented form of the parametrized control trajectory is given in Eqn. (3.92)

$$u = \left[u_{k|k}^{\mathrm{T}}\ u_{k+1|k}^{\mathrm{T}}\ \dots\ u_{k+M-1|k}^{\mathrm{T}}\right]^{\mathrm{T}} \tag{3.92}$$

The open loop and parametrized control sequence are related as in Eqn. (3.93)

$$\begin{bmatrix} u_{k|k} \\ u_{k+1|k} \\ \dots \\ u_{k+M-1|k} \\ u_{k+M-1|k} \\ \dots \\ u_{k+M-1|k} \end{bmatrix} = \begin{bmatrix} I & & & \\ & I & & \\ & & \dots & \\ & & & I \\ & & & I \\ & & & \dots \\ & & & I \end{bmatrix} \cdot \begin{bmatrix} u_{k|k} \\ u_{k+1|k} \\ \dots \\ u_{k+M-1|k} \end{bmatrix} \tag{3.93}$$

or more compactly written as $u_{\mathrm{ol}} = G_{\mathrm{ol}}u$. The predicted open loop state sequence is written in the augmented form as in Eqn. (3.94)

$$x = \left[x_{k+1|k}^{\mathrm{T}}\ x_{k+2|k}^{\mathrm{T}}\ \dots\ x_{k+N|k}^{\mathrm{T}}\right]^{\mathrm{T}} \tag{3.94}$$

and with the state reference in Eqn. (3.95)

$$x^* = \left[x^{*\mathrm{T}}\ x^{*\mathrm{T}}\ \dots\ x^{*\mathrm{T}}\right]^{\mathrm{T}} \tag{3.95}$$

and the control reference in Eqn. (3.96)

$$u^* = \left[u^{*\mathrm{T}}\ u^{*\mathrm{T}}\ \dots\ u^{*\mathrm{T}}\right]^{\mathrm{T}} \tag{3.96}$$

and the open loop predicted load trajectory is given in Eqn. (3.97).

$$\hat{v} = \begin{bmatrix} v_{k|k}^{\mathrm{T}} & v_{k+1|k}^{\mathrm{T}} & \cdots & v_{k+N-1|k}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \tag{3.97}$$

Using the augmented forms and the control parametrization, the current open loop optimization problem can be written as in Eqn. (3.98).

$$\begin{aligned} & \underset{u}{\text{minimize}} \quad \frac{1}{2}u^{\mathrm{T}}Ku + k^{\mathrm{T}}u \\ & \text{subject to} \\ & \qquad Fu \le f \\ & \qquad \underline{u} \le u \le \bar{u} \end{aligned} \tag{3.98}$$

In the simulations, the open loop problem was scaled using maximum state constraint limit and the predicted time varying maximum open loop control vector constraint limit to obtain a better conditioned open loop problem.

<u>Controllability of transmission example:</u>

The controllability matrix C is given in Eqn. (3.99). With the two source control flows of the transmission system example, the controllability matrix was non singular implying full controllability due to the definition by Kalman *et al.* (1961).

$$C = \begin{bmatrix} \Gamma & \Phi\Gamma & \ldots & \Phi^{n-1}\Gamma \end{bmatrix} \tag{3.99}$$

<u>Supply Security and Infeasibility Handling:</u>

For supply security reasons, the penalty weights for state deviation from the state reference are increased when approaching the delivery points of the transmission system. It is especially important to use large values for state penalization which are close to the customer points when the load demand is time highly transient. The increase of state penalty weights when approaching the customer point for the transmission example in Figure 3.1 is illustrated in Figure 3.8. By this choice, the pressure level at the source points is allowed to change from their references leading to that the pressure level at the junction point is changed so that pressure gradients along the offtake pipeline is changed leading to a change in mass flow distribution for the offtake pipeline also close to the customer point. Then, the transient load pattern at the customer point can be met while still keeping the terminal pressure for most of the time above the minimum terminal pressure value specified as a hard state constraint in the open loop problem. A good load forecast is of course necessary to meet the actual load pattern since predictive action must be performed by the control vector to compensate for the delay effect from source points to the customer point, due to long transmission distances. This is especially the case for a long distance transmission system without recompression, which is the case for many offshore systems. The reason for this is to avoid the expensive installations of recompression at sea. Recompression along the transmission lines will make network operation easier due to the reduced distances between each point where the pressure distribution of the transmission system can be changed. Values for the penalty weights for the control vector deviation from the reference must be low allowing control action for the supply security.

In the simulation section below an example will show how one processing facility (source) increases the production capacity to compensate for the reduction in production availability for the other source so that the customer demand can still be met at the customer point. The open loop controller is, for each sampling instant, updated with a forecast on the maximum production availability for the two sources.

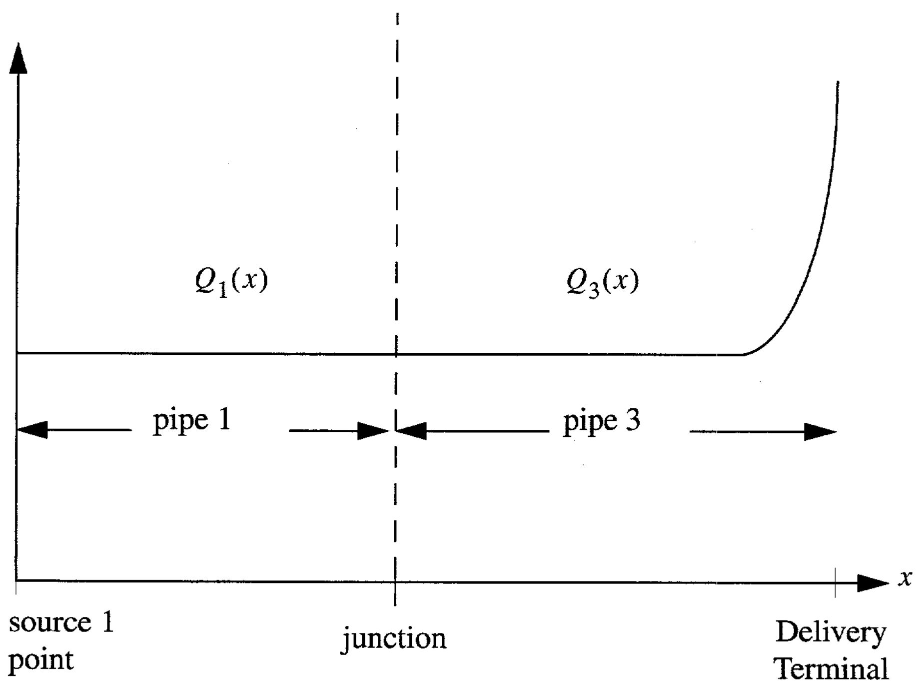


Figure 3.8: State penalty weight illustration for supply security for transmission example along pipeline one and pipeline three of the transmission example in Figure 3.1. The penalty weight function of pipeline two is equal to pipeline 1.

Due to the complexity of transmission systems, infeasibility is a case that must be considered for on-line operation. It is very important also for the supply security and the continuous need for control commands in an on-line application.

An important example of an infeasibility problem in the operation of natural gas transmission systems is when the customer terminal pressure goes under the pressure limit specified in the current open loop problem and there are no admissible controls that can operate the transmission system so that the pressure at the customer terminal goes above the pressure limit for all the future time instants of the open loop horizon. This causes infeasibility for the state constrained model predictive controller.

There may be many reasons for this undesirable case. If the forecasted time varying load demand is underestimating the actual load, then the control vector is not supplying enough gas into the system causing terminal pressure drop under the specified limit in the controller. Incorrect choice of state penalty weights may lead to a lack of enough attention in maintaining terminal pressure at transient load conditions given a transient load forecast to the controller. If the source production fails for some reasons leading to a low supply into transmission system and the line pack is low, infeasibility may be the result. The controller uses estimated states. If the state estimation error is large, this can of course be another factor for the infeasibility. A prediction model that deviates largely from the actual system behaviour may produce closed loop control commands that eventually lead to infeasibility. Reduced flow capacity through a compressor station due to maintenance or malfunction can also cause infeasibility. Also, if source capacity is reduced, this reduces supply into the transmission system. As mentioned earlier, it is very important that the control algorithm calculates a control command also when facing infeasibility.

A simple infeasibility handling algorithm is to drop the upper and lower limit state constraint in the open loop problem in Eqn. (3.89) when the open loop problem is infeasible and use this controller for each open loop under infeasibility to bring the system back to a feasible region. Based on the current network conditions, this or may not may be possible due to available production capacity, status of infrastructure, load forecasts, error etc. At each open loop, the feasibility of the state and control constrained open loop controller is checked. If the open loop problem is feasible, then the state constrained model predictive controller is used to calculate the control command. If not, then the infeasibility open loop controller is used.

A more complex infeasibility handling algorithm is obtained by relaxing the state constraints in the open loop problem of Eqn. (3.89). In the objective function of the infeasibility open loop optimization problem, a term is added to penalize the relaxation vector quadratically. A weight matrix penalizes the relaxation of the constraints differently based on the defined priority. For example, the relaxation of the delivery pressure is penalized high compared to the other pressures of the transmission system. This form of penalization is done because the security of supply at the customer points is very important. Upper and lower limits of the relaxation factors are defined for the open loop problem. Critical physical constraint limits are cathegorized as non relaxable hard constraints. An example, is the maximum allowed fluid pressure limit inside a natural gas transportation line. Zero relaxation is accepted for these types of constraints. The objective function for this open loop infeasibility handler is given in Eqn. (3.100) and constraints with relaxation are given in Eqn. (3.101).

$$\sum_{i=0}^{N-1} (x_{k+i+1|k} - x^*)^T Q_{k+i+1}(x_{k+i+1|k} - x^*) \tag{3.100}$$

$$+ (u_{k+i|k} - u^*)^T R_{k+i}(u_{k+i|k} - u^*)$$

$$+ \begin{bmatrix} \delta_{1,k+i+1|k} \\ \delta_{2,k+i+1|k} \\ \delta_{3,k+i+1|k} \end{bmatrix}^T \cdot \begin{bmatrix} S_{1,k+i+1} & & \\ & S_{2,k+i+1} & \\ & & S_{3,k+i+1} \end{bmatrix} \cdot \begin{bmatrix} \delta_{1,k+i+1|k} \\ \delta_{2,k+i+1|k} \\ \delta_{3,k+i+1|k} \end{bmatrix}$$

$$\begin{aligned}
x_{k+i+1|k} &= \Phi x_{k+i|k} + \Gamma u_{k+i|k} + \Psi v_{k+i|k} && 0 \le i \le N-1 \\
\underline{x} - \delta_{1,k+i+1|k} \le x_{k+i+1|k} &\le \bar{x} + \delta_{2,k+i+1|k} && 0 \le i \le N-1 \\
\underline{u}_{k+i|k} \le u_{k+i|k} &\le \bar{u}_{k+i|k} && 0 \le i \le N-1 \\
Ax_{k+i+1|k} &\le a + \delta_{3,k+i+1|k} && 0 \le i \le N-1 \\
\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \le \begin{bmatrix} \delta_{1,k+i+1|k} \\ \delta_{2,k+i+1|k} \\ \delta_{3,k+i+1|k} \end{bmatrix} &\le \begin{bmatrix} \bar{\delta}_{1,k+i+1|k} \\ \bar{\delta}_{2,k+i+1|k} \\ \bar{\delta}_{3,k+i+1|k} \end{bmatrix} \\
x_{k|k} &= \tilde{x}_{k|k}
\end{aligned} \tag{3.101}$$

The relaxation vector is given in Eqn. (3.102)

$$\delta_{k+i+1|k} = \left[\delta^{\mathrm{T}}_{1,\,k+i+1|k}\ \delta^{\mathrm{T}}_{2,\,k+i+1|k}\ \delta^{\mathrm{T}}_{3,\,k+i+1|k}\right]^{\mathrm{T}} \tag{3.102}$$

where $0 \le i \le N-1$. Note that the penalty matrix may be time varying. As an example we may put priority on the states early in the open loop horizon.

In the thesis by Vada (2000), prioritized infeasibility handling is treated for linear model predictive control. Based on a defined priority list, optimal relaxation for current open loop of infeasibility is calculated by solving a linear programming problem. A relaxation larger than zero is only performed on the hard constraints that are allowed to be relaxed. Critical limits for the plant are not relaxed. For the above problem, this algorithm would then calculate optimal relaxations denoted by $\delta_{k+i+1|k}{}^{*}$, $0 \le i \le N-1$ for the current open loop horizon in infeasibility mode. The objective function of this infeasibility algorithm does not contain a term penalizing the relaxation since it is not necessary as the optimal relaxation has been calculated. Therefore, the infeasibility open loop problem has the same dimension as the original open loop problem for the feasible case, but a linear programming problem must be solved for the current sampling interval. Stability proofs and optimality of the proposed infeasibility algorithm are also treated in the work by Vada (2000). The book by Camacho and Bordons (1999) gives also alternative ways of handling infeasibility for constrained model predictive controllers.

Increased supply security and the resulting reduction in the chance of infeasibility can be obtained by installing a gas storage facility close to a customer point, adding extra line pack into the transmission system or using of extra looping close to the customer point. The disadvantage is increased investment cost or higher operational costs due to the increased pressure level. But infeasibility should be considered anyway. Sooner or later one may have to face the undesired or/and unexpected.

<u>Compressor Station Optimization:</u>

The paper by Percell and Van Reet (1986) discusses the problem of simulating the operation of gas pipeline compressor stations in the environment of a pipeline network simulator. Nonlinear constrained optimization techniques for station optimization are discussed. It was stated that the unit selection can have a large effect on the fuel consumption of the compressor station. One of the conclusions from this paper was that the compressor units operate more efficiently near full load so that one should avoid unnecessary units in operation. No explicit examples of a compressor station and optimization formulation were given in the paper.

Nonlinear programming applied to the optimum control of a compressor station supplied with motor-compressors in parallel driven by combustion gas engines was treated in the paper by Osiadacz (1980). The static and dynamic characteristics of a motor-compressor was formulated. Total costs were defined to be the sum of the number of machine starts and stops and the explicit energy cost due to compression. Suction pressure, necessary discharge pressure and mass flow through the compressor station to meet customer demand was assumed to be given. The number of units in operation and the operation point were determined by the station optimizer.

The papers by Pratt and Wilson (1984) and Ostromuhov (1998) include station optimization in the total stationary optimization formulation of natural gas transmission system yielding mixed integer programs.

Stability and robustness considerations:

The control command at the current sampling instant is a function of the state or the estimated state, the predicted open loop load forecast, the time varying predicted control space, the length of the open loop horizon, the dimension of control parametrization, state and control penalty matrices and the current stationary operation point. The last point in the list is due to control and state references and the current linear dynamic prediction model. Equation (3.103) expresses this functional dependence.

$$u(t_k) = \phi(x(t_k), \{\hat{v}_{k|k+i}\}_{i=0}^{N}, \{\bar{u}_{k|k+i}\}_{i=0}^{N}, \{\underline{u}_{k|k+i}\}_{i=0}^{N} \\ , N, M, Q, R, x^*, u^*, v^*) \tag{3.103}$$

Due to time varying load demands, it is not possible to find closed loop control commands such that the state vector is equal to state reference for all states of the system along all the pipelines and junctions. This is due to the fact that the pressure gradient of the pipelines must vary to meet customer demands and to keep the terminal pressures inside the defined minimum and maximum values and possibly close to their references. Therefore, we may say that the gas transmission system is not controllable. But with source capacity, line pack and status of infrastructure enabling admissible controls to bring the states of the transmission system inside the feasible region, we may say that the transmission system is stabilizable. So, we must generally expect that the sequence of open loop objective functions for the model predictive controller will be larger than zero (zero if reference is reached), but a sequence that is bounded and maybe also non increasing is more important. If the load demand is constant at each customer point, it is possible to reach the stationary state reference pressure distribution everywhere if the status of gas network enables full controllability. A simple example on a controllable system is a single transmission line with a constant load and enough production availability to give a supply trajectory in a closed loop that brings the state to stationary reference pressure distribution under a constant load.

Assume the case for a single transmission line where the production availability has been severely reduced for a longer period of time such that the pipeline is depacked due to a continuously customer demand. Finally, the delivery terminal pressure has gone far beyond the lower contractual accepted limit. The transmission system is now in the infeasible region. If the source production availability is restored to normal capacity it will now be possible to reach the feasible region again by filling the pipeline. It is therefore important that the closed loop controller brings the system finally to the feasible region. For a period of time it will be in the infeasible mode before reaching the feasible. Handling infeasibility is an important property that the controller must posses. It will be shown in the simulation example that this property was satisfied in the closed loop simulations using the proposed linear model predictive controller.

If it is physically impossible to bring the state of the system to the feasible region for a period of time we want, at least, the controller to bring the state of the system as close as possible to the feasible region of the state space.

In some cases, it may be necessary to close some customer terminals to avoid the complete unpacking of system pipelines. These decisions are of course made by the dispatch personnel. The controller may be set into operation again when the network is restored to an acceptable state.

The closed loop controller should also be stable when switching from one stationary operation point to another under transient load.

For finite horizon linear model predictive controllers, one way to achieve stability is to add a terminal constraint such that the state is equal to its reference at the end of the horizon for each open loop problem (Kwon and Pearson,1977). It is assumed that the linear plant is perfectly known which is hardly the case in practice. The method is restricted to controllable systems so that state reference can be met. It is only the open loop predicted state trajectory that converges to the state reference at the end of the open loop horizon, while the closed loop control only converges asymptotically to the state reference. This is discussed in Rawlings and Muske (1993).

The paper by Rawlings and Muske (1993) proves the stability of a state and control constrained infinite horizon linear model predictive controller for both stable and unstable linear plants. It is assumed that the linear plant is perfectly known and that it is time invariant. The objective function is quadratic in state and control vectors. All states are assumed to be available for the controller. For the case where the predictor describes the plant dynamics perfectly, closed-loop stability is proved. It is proved that the sequence of infinite horizon open loop objective functions in closed loop for stable plants is non increasing and bounded below by zero implying convergence also in the presence of constraints and for all initial conditions. This yields for all values of the tuning parameters, namely the control parametrization and the values of the state and control penalty matrices. The proof only assumes that there is enough control space available for stabilizing the system and not the demand that the state must converge to a state reference (controllability) which is more practically relevant. The thesis by Muske (1995) provides a thorough description on model predictive control and stability for linear plants.

The recent paper by Mayne *et al.* (2000) reviews model predictive control of linear and nonlinear systems that are unconstrained, control constrained and both control and state constrained. Four stability axioms are formulated. Nominal stability is guaranteed if these four axioms are all satisfied. It is well known that the use of infinite open loop horizons yields nominal stability but finite horizon controllers are often used for practical implementation. How these finite horizon controllers can be modified so that they have stability properties similar to the infinite time horizon controller is explained in the paper. As mentioned earlier, a model predictive controller of finite horizon with the demand that the state reaches state reference at the end of an open loop for each open loop yields a nominal asymptotically stable closed loop controller for controllable systems. Increasing the length of the open loop horizon will have the same effect as a terminal constraint. Alternatively, the use of a terminal cost is another stabilizing method. Demanding that the model predictive controller brings the open loop terminal state inside a specified space and the use of a local proved stable feedback controller when state of the system reaches this region is a third stabilizing control alternative that satisfies the stability axioms under some necessary conditions. The combination of terminal cost and a constraint set is a fourth alternative for a stabilizing closed loop controller discussed in the paper. The review paper discusses the controller alternatives and demands of each method for satisfying the four stability axioms presented. Two methods using Lyapunov functions are used to prove stability. The first method looks at proving that the sequence of the value of the open loop objective function in a closed loop is non increasing, yielding stabilizing property for a given finite time length of each open loop. The second method looks at the monotonicity property. It is the property that the optimal value of the current open loop horizon, given any initial condition of that open loop, that decreases in value monotonically as the length of the open loop horizon is increased. Robustness issues are also treated in the same paper. Consult also Garsia and Prett (1989) for a survey on model predictive control.

Robustness:

The linear model constructed from the stationary optimization describes the system dynamics well when close to the stationary operation point. It is important that the closed loop controller is also stabilizing when perturbed from this operation point which is the case for most of the time. From the simulations it will be seen that the closed loop control behaves encouragingly also when the state is far away from the state reference.

Conclusion:

As a conclusion, we might state that due to the complexity and non linearity of gas transmission networks and possible events it will be very difficult to prove stability for a closed loop controller covering all aspects.

Closed loop simulations of closed loop control using historical natural gas demands and events and logged predictions of events may give pinpoints on closed loop behaviour. For such complex systems as gas transmission networks, the main decisions must be taken by expert personnel at the gas control centre. So, the control system must rather be seen as a support for operators providing suggestions for operations that need further evaluation. By the use of a detailed complex simulator, the dispatch centre can evaluate the proposed control commands and then accept, modify or reject the calculated proposal.

Control of Centrifugal Compressors:

Gravdahl (1998) considers modelling and nonlinear closed loop control of centrifugal compressors with the objective of avoiding stall and surge when operating close to these critical regions of the operation envelope. A simple anti-surge control system for centrifugal compressors is given in Balchen and Mumme (1988). avoiding that the mass flow into the compressor falls under the surge limit for a given compression ratio by using a bypass around the compressor.

3.4 Simulation Results

The pipeline data are given in Eqn. (3.104) where the average values are used for compressibility factor and temperature along each pipeline.

$$
\begin{aligned}
T_{\mathrm{av}} &= \begin{bmatrix}280 & 280 & 280\end{bmatrix}^{\mathrm{T}}, \text{Kelvin} \\
f &= \begin{bmatrix}0.0025 & 0.0025 & 0.0025\end{bmatrix}^{\mathrm{T}} \\
MW &= 10^{-3} \cdot \begin{bmatrix}80 & 80 & 80\end{bmatrix}^{\mathrm{T}}, \text{(kilograms)/(mole)} \\
L &= 10^{4} \cdot \begin{bmatrix}60 & 30 & 50\end{bmatrix}^{\mathrm{T}}, \text{meters} \\
Z_{\mathrm{av}} &= \begin{bmatrix}0.9 & 0.9 & 0.9\end{bmatrix}^{\mathrm{T}} \\
D &= \begin{bmatrix}0.8 & 0.8 & 1\end{bmatrix}^{\mathrm{T}}, \text{meters} \\
\kappa &= \begin{bmatrix}1 & 1 & 1\end{bmatrix}^{\mathrm{T}} \\
c &= \begin{bmatrix}330 & 330 & 330\end{bmatrix}^{\mathrm{T}}, \text{(meters)/(second)}
\end{aligned}
\tag{3.104}
$$

Twenty spatial grid points were used for each pipeline. The sampling interval was defined to be one hour. The compressor units in the two stations were given the data in Eqn. (3.105).

$$
\begin{aligned}
\Upsilon_1 &= 1.4 & \Upsilon_2 &= 1.44 \\
\eta_{s,1} &= 0.8 & \eta_{s,2} &= 0.85 \\
\eta_{m,1} &= 0.97 & \eta_{m,2} &= 0.99
\end{aligned}
\tag{3.105}
$$

It was assumed that the energy cost per energy unit spent at either of the two compressor stations were identical.

The Gross calorific value and the concentration of carbon dioxide was assumed constant at each source point with data given in Eqn. (3.106).

$$
\begin{aligned}
y_{\mathrm{source,\,GCV}} &= \begin{bmatrix}37 & 45\end{bmatrix}^{\mathrm{T}}, \text{(Megajoule)/(Sm3)} \\
y_{\mathrm{source,\,CO_2}} &= \begin{bmatrix}1 & 3\end{bmatrix}^{\mathrm{T}}, \text{percent}
\end{aligned}
\tag{3.106}
$$

Maximum accepted operational pipeline pressures at the two source pipelines were defined to be 180 bars for both and 140 bars for the offtake pipeline. The stationary optimization used 60 bars as the minimum accepted customer pressure for supply security reasons while the contracted minimum pressure was assumed to be 55 bars.

The quality parameters demand at customer point is defined in Eqn. (3.107).

$$\begin{aligned} 38 \le y_{\mathrm{GCV}} \le 42 \\ 1 \le y_{\mathrm{CO_2}} \le 2.5 \end{aligned} \tag{3.107}$$

with the same units as above. The suction pressure for the two compressor stations was assumed to be 60 bars. Maximum source flows for the stationary optimization was set to be 800 kilograms/second and the average stationary customer load was defined to be 600 kilograms/second.

3.4.1 Case 1

Figure 3.9 shows inlet, junction and customer pressures for a simulation where the transmission system was packed with more gas than the "optimal" pressure distribution from the stationary optimization under a customer load of 600 kg/second. The source flows and constant load flow are illustrated in Figure 3.10. Estimated states were used for each open loop problem. An open loop horizon of time length equal to 12 hours was chosen. The dimension of the parametrized control trajectory was chosen to 12. Since the load was constant, we could reduce the length of the open loop horizon. The closed loop system was stable and the transient response to the stationary operation point was smooth. Figures 3.11-3.13 illustrate the state estimation error for the three transmission pipelines of the system example. The estimates states was initially set equal to the stationary operation point implying an initial state estimation error. The nonlinear creep flow model is used as plant model. The value of the estimated states was initially set to the calculated optimal pressure distribution. It is observed that the state estimator is stable and that the state estimation error decreases fast even with a large set initial estimation error.

3.4.2 Case 2

Now the network started depacked from the "optimal" stationary pressure distribution under a constant load. The system also started with customer pressure under the defined minimum terminal pressure so that the controller works in infeasible mode for the first hours of its operation. The transient response to a stationary operation point was smooth and stable for the complete simulation

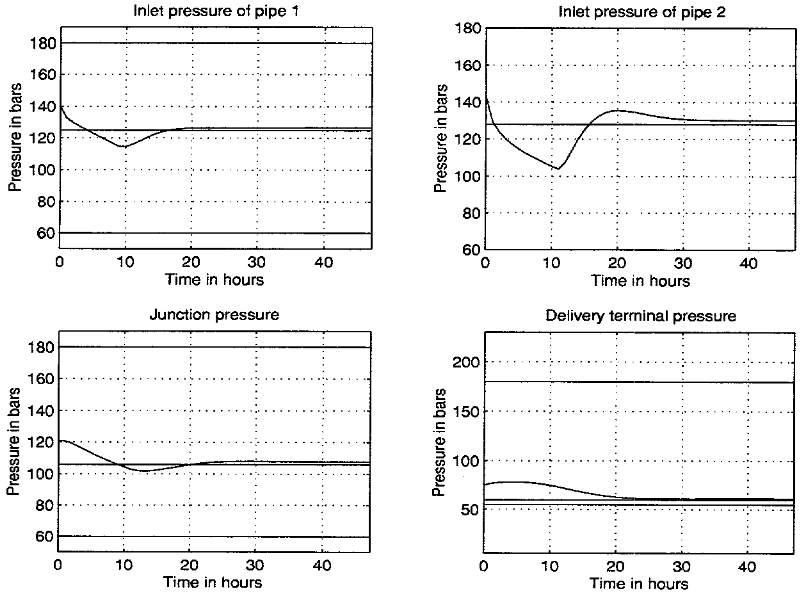


Figure 3.9: Pressures at inlet of the source pipelines, junction point and customer terminal.

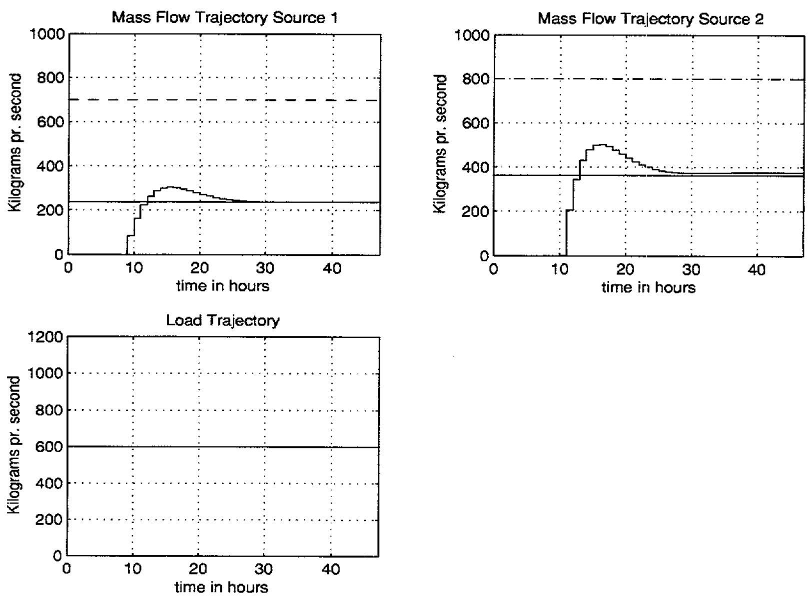


Figure 3.10: Mass flow control commands from source points through compressor stations and customer load.

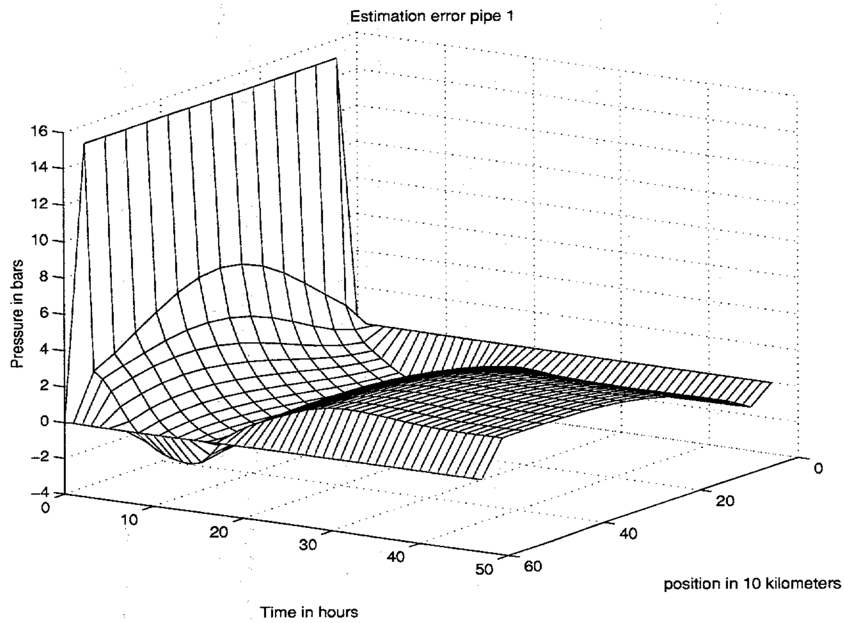


Figure 3.11: Estimation error pipe 1.

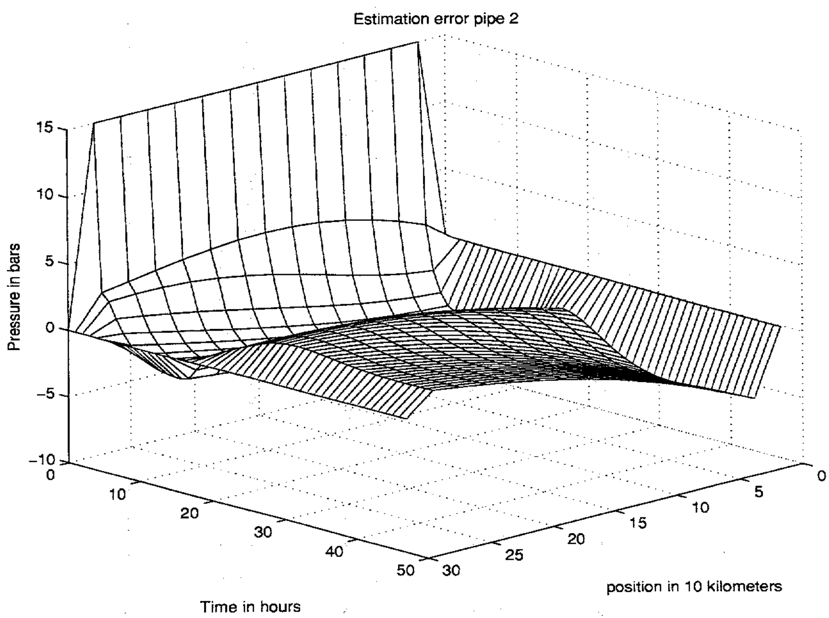


Figure 3.12: Estimation error pipe 2.

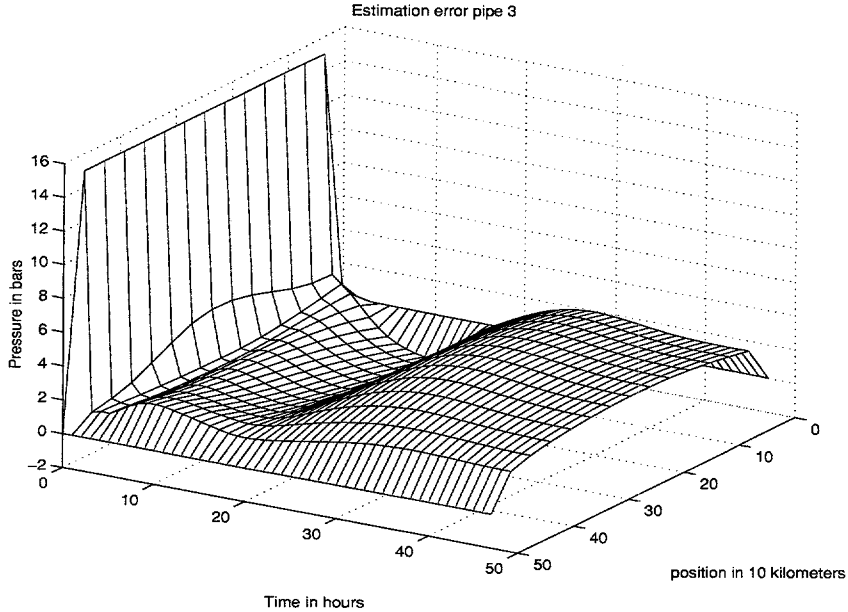


Figure 3.13: Estimation error pipe 3.

horizon. Due to the large transmission system distances, it takes about 20 hours for the customer pressure to reach the feasible region. The length of the open loop horizon and the control parametrization was chosen equal to those stated in Subsection 3.4.1. Figure 3.14 yields selected pressures and Figure 3.15 yields source flows and stationary load. Initial states for each open loop problem was also provided here by the state estimator.

3.4.3 Case 3

In this simulation the load pattern was highly transient. It was assumed that the source capacity was good. State feedback from the states provided by the nonlinear simulation model was used. Figure 3.16 shows the inlet pressures of the two source pipelines, the junction pressure and the customer terminal pressure for the long distance transmission system. Figure 3.17 shows the source flows and transient load pattern. Figure 3.18 yields the value of the two considered quality parameters of the system example at the junction point which then reaches the customer point. Figure 3.19 yields the compression ratios of the compressor stations. An open loop horizon of 48 hours was chosen with a control parametrization of 48 control steps. The rest of the data are given in Eqns. (3.104)-(3.106).

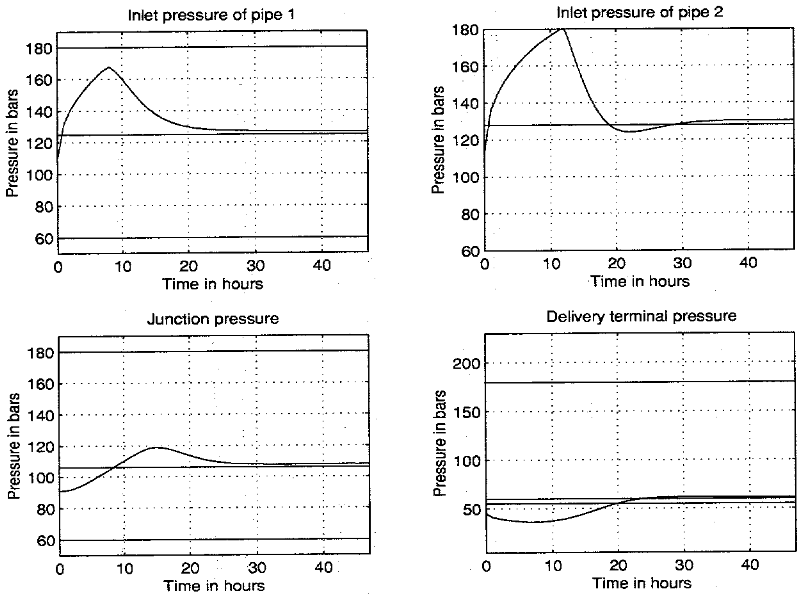


Figure 3.14: Source pressures, junction pressure and customer terminal pressure.

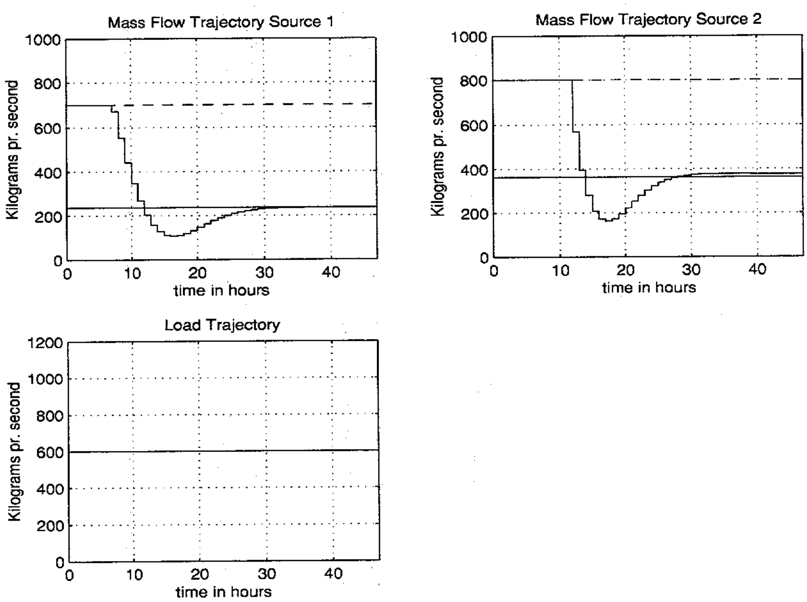


Figure 3.15: Source flows and customer load.

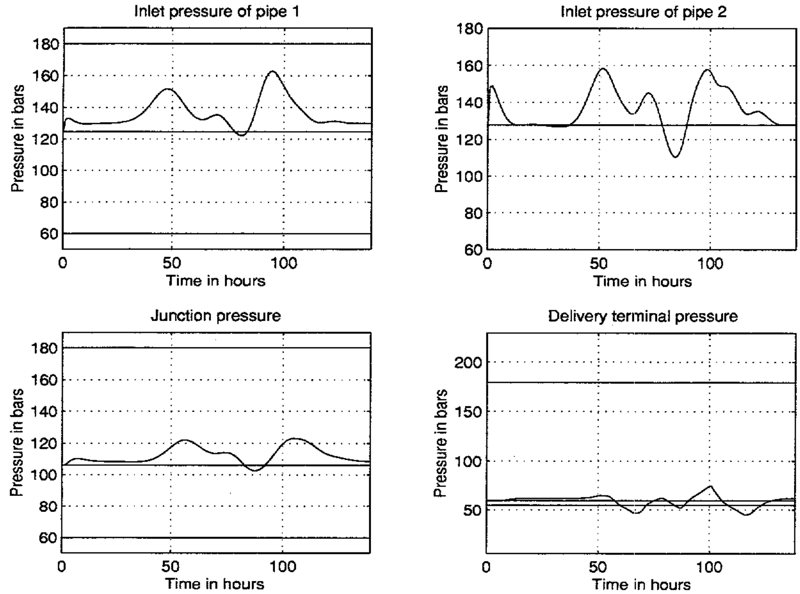


Figure 3.16: Pressures at inlet of the source pipelines, junction point and customer terminal.

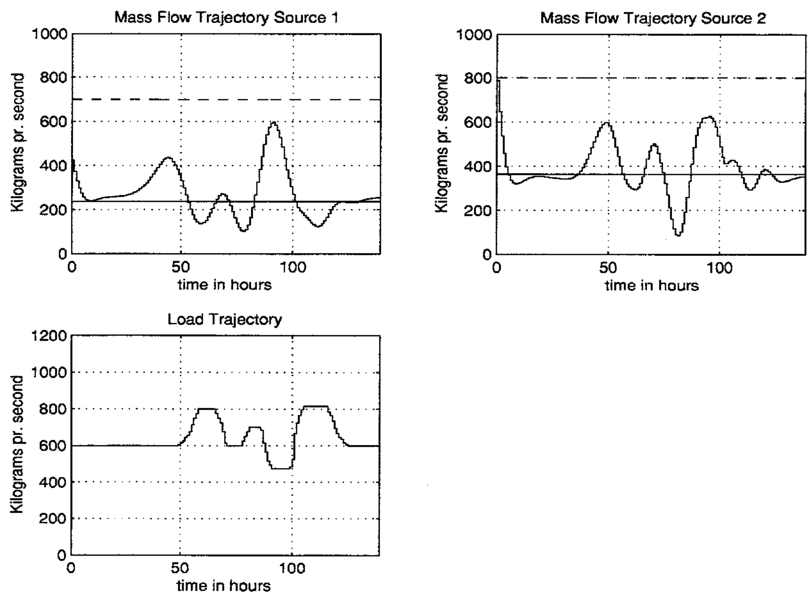


Figure 3.17: Mass flow control commands from source points through compressor stations and transient load pattern.

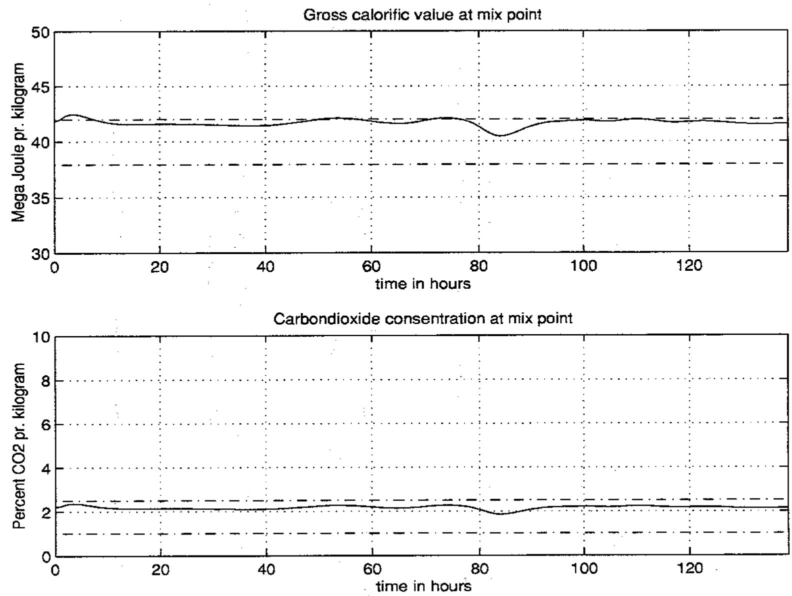


Figure 3.18: The value of quality parameters at junction point.

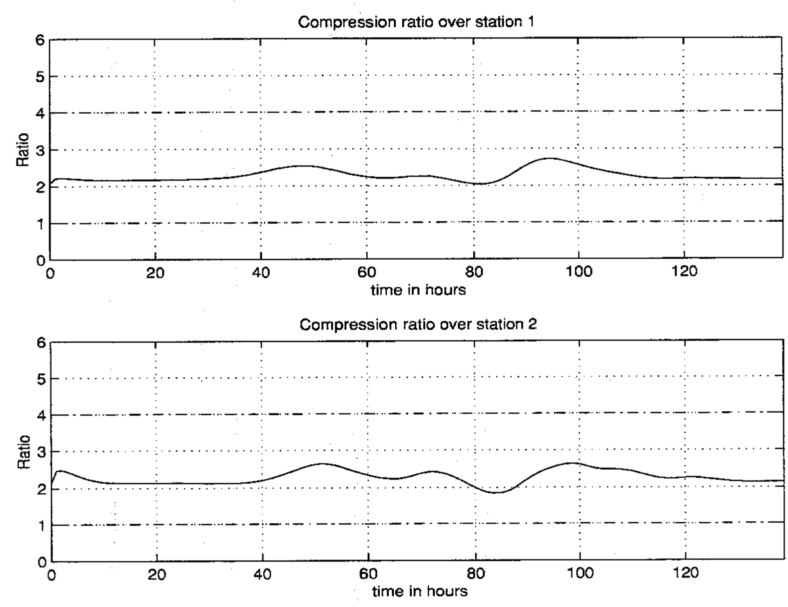


Figure 3.19: Compression ratios over the two compressor stations.

3.4.4 Case 4

In this simulation, we have the case where the maximum production capacity of one of the sources was reduced severely for a period of over 24 hours in the simulation horizon. It was assumed that the maximum source capacity prediction was correct. The reduction in capacity into the pipeline one may be, for example, caused by a reduced number of reservoir sources delivering hydrocarbons to the processing facility supplying pipeline one with natural gas. Additionally, the transmission system started in depacked infeasible mode and with a transient customer load. Figure 3.20 and Figure 3.21 yield the pressures and flows respectively. The value of the carbon dioxide concentration and the gross calorific value at the junction point is seen in Figure 3.22. It is seen that the quality parameters get outside the band in the period of the reduced source capacity since source two must back up source one changing the mixing ratio. Delivering the required volume of natural gas to a customer has higher priority than the value of the quality parameters. The compression ratio for each compressor station is shown in Figure 3.23. The estimated states were used as the initial condition for each open loop optimization problem.

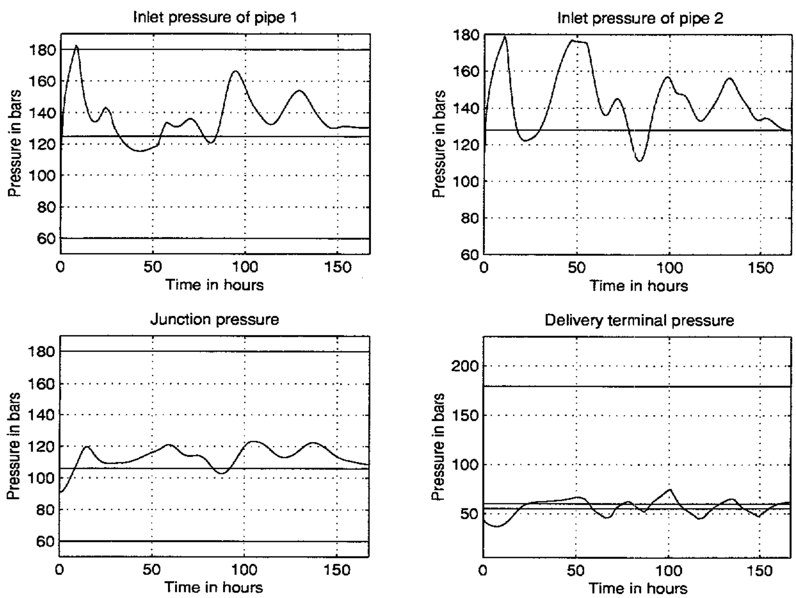


Figure 3.20: Pressures at inlet of source pipelines, junction and customer point.

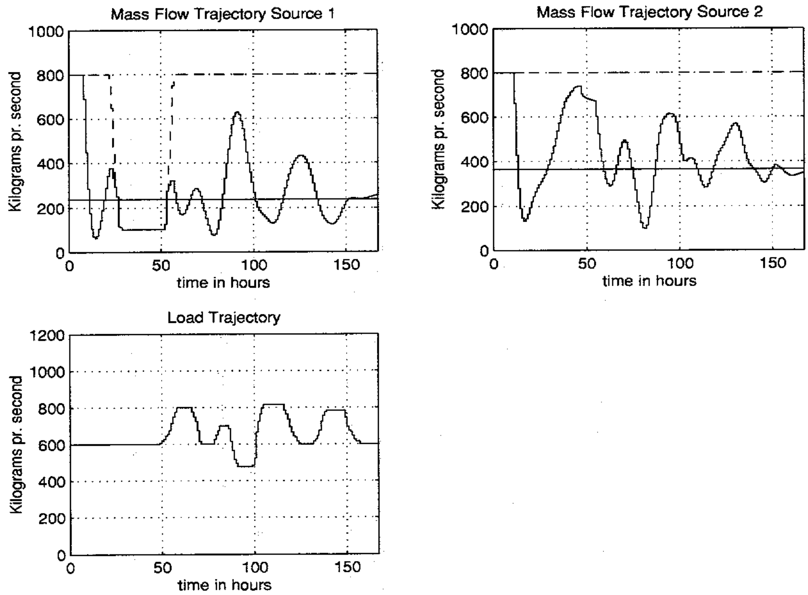


Figure 3.21: Source flows and customer demand.

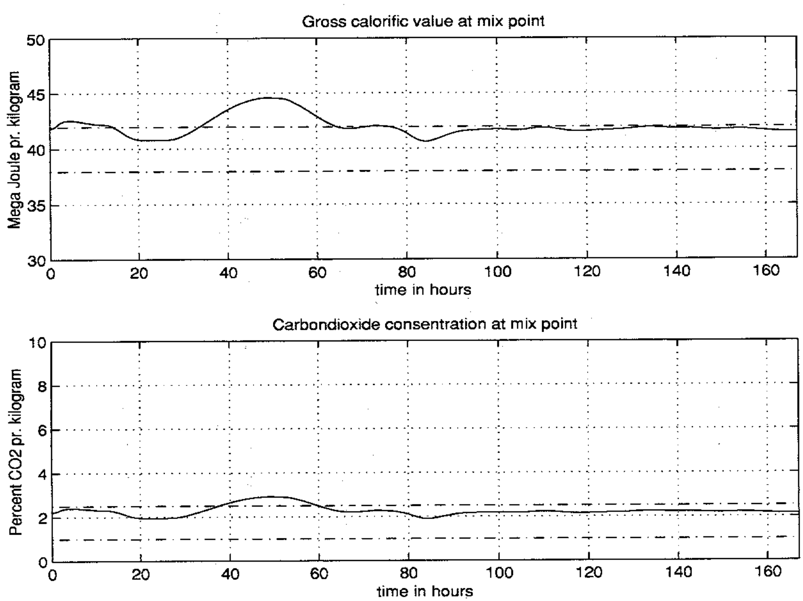


Figure 3.22: Quality parameter values.

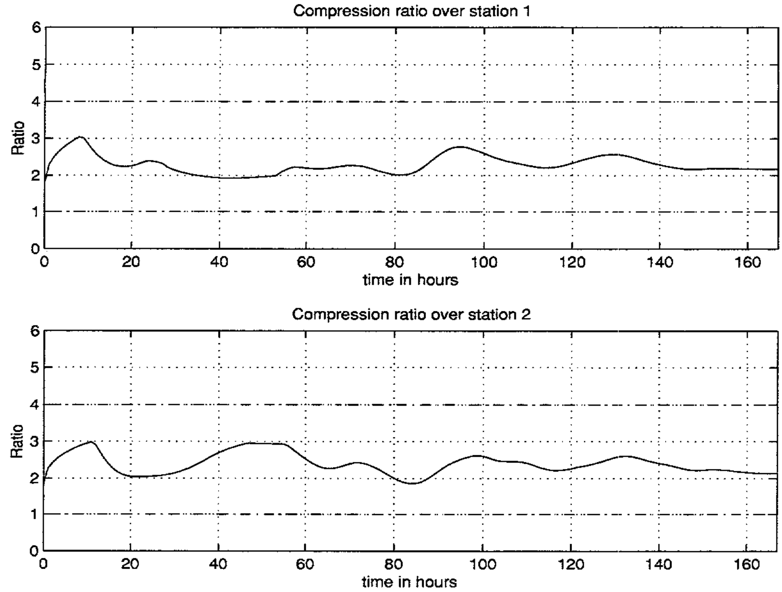


Figure 3.23: Compression ratios over each compressor station.

3.4.5 Case 5

This simulation shows closed loop control with a transient load pattern where the dynamic optimization model for the first 4 days is based on a stationary optimization using an average load of 600 kilograms per second. The dynamic optimization for the next 4 days uses a dynamic model that was constructed from a stationary optimization with an average load of 400 kilograms per second. After this period the closed loop controller switched back to the dynamic model with parameters that were based on a stationary optimization with a load equal of 600 kilograms per second. The minimum accepted customer terminal pressure for the two stationary optimization problems were both set to 60 bars. Figure 3.24 and Figure 3.25 shows pressures and flows respectively. Figure 3.26 and Figure 3.27 show the value of the considered quality parameters out from the junction point and the compression ratios over each station. State feedback from the nonlinear creep flow model was used in the simulations.

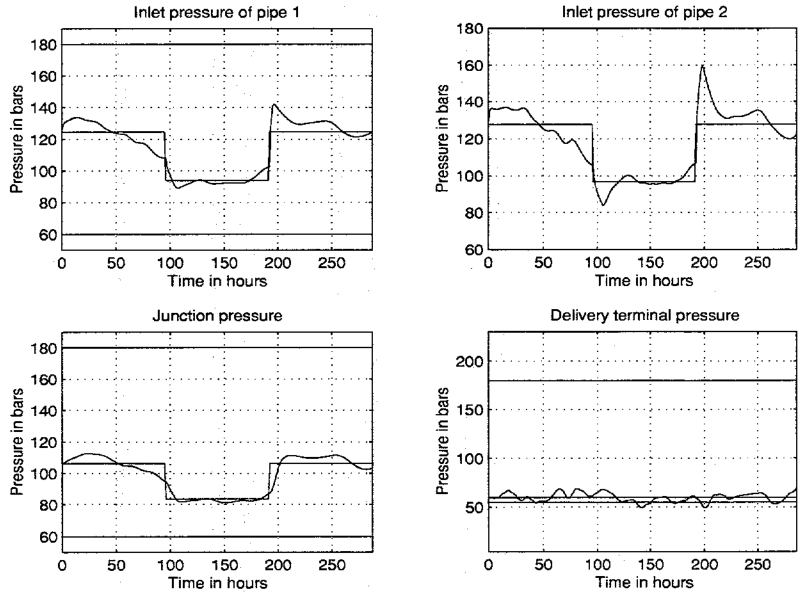


Figure 3.24: Supply, junction and customer terminal pressures.

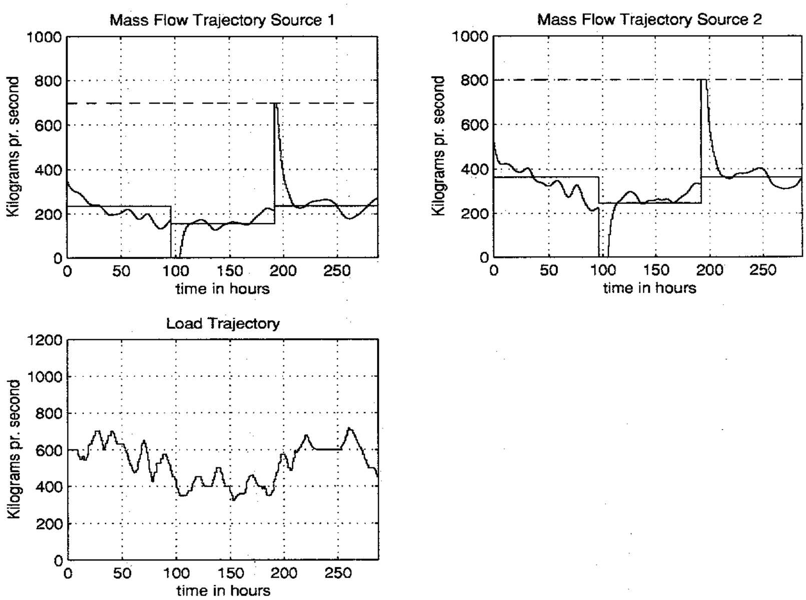


Figure 3.25: Source flows and demand pattern.

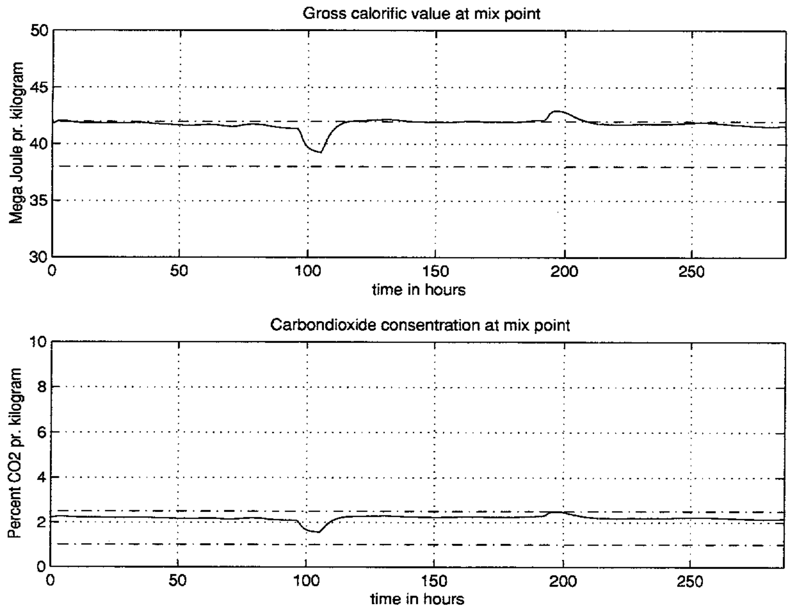


Figure 3.26: Quality parameter values at junction under closed loop control.

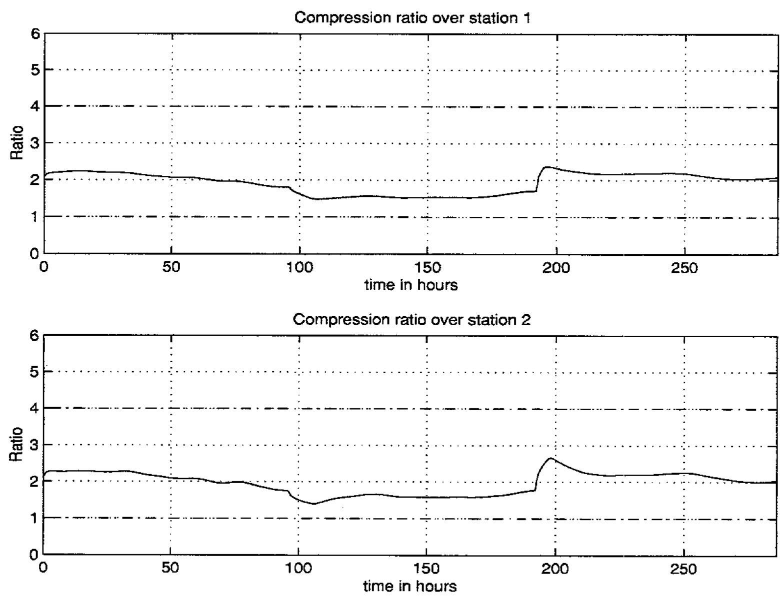


Figure 3.27: Compression ratios.

3.4.6 Simulation Conclusions

It is observed that the control system was stable in all simulations. When the load pattern is expected to change considerably, a new stationary optimization is performed giving new state and control references together with a new optimization and state estimator model. The aim of this strategy is to specify an optimization model that is in close correspondence with the current and expected close future operating conditions. Since the optimization model is a linear model around the current stationary operation point, it will give its best performance when the state of the system is close to this region. This is especially important for meeting the load pattern. The linear optimization model will not be able to perfectly meet the nonlinear behaviour and as a result, it is seen that the customer terminal pressure has some cases of overshoot and undershoot from the reference especially under a transient load. The overshoots imply increased transmission costs but increased supply security while the undershoots means that pressure may be close to or under the contracted minimum. By setting the minimum accepted operational pressure above the contracted minimum, the security of the supply is increased.

The linear model predictive controller presented in this chapter specifies, for each open loop, a strictly convex programming problem where the quadratic programming solver finds a global optimum (using the linear model) inside specified tolerances. Since the problem is convex and therefore not dependent on an initial guess to find a minimum, it makes the closed loop controller computationally feasible. Solving nonlinear non convex open loop problems using the nonlinear dynamics can not still be considered reliable for on-line operations in many cases. Usually there is only enough computation time to use a local solver finding a local minimum. A global optimization method as the alfa branch and bound may be too time consuming for large scale systems. The local optimum found may not be better than the control suggestion by the linear model predictive controller. In some cases the algorithms that are supposed to solve the non convex problems do not succeed in finding a feasible solution even if such a solution does exist. The reason for this could be a bad initial guess for the solution. On a 500 MHz computer, it took about one to fifteen seconds of computation time in Matlab for the system example to solve each of the open loop problems. The computational effort is mainly determined by the choice of the length of the open loop horizon, the dimension of control parametrization, spatial discretization and the size of the transmission system. The computation time can be reduced significantly by a more efficient implementation of computer code.

From the simulations, it can be seen that when one source faced a severe reduction in the production capacity, the other source increased production so that the customer demand was still met. Satisfactory maximum source production capacity forecasts are necessary for successful operation under these conditions. Events such as necessary maintenance of off-shore installations, controlled shut-downs of offshore production platforms, weathers forecasts implying close down, etc. can, in many cases, be highly predictable.

Due to the time delay effect, it takes some sampling instants before the model predictive controller can control the delivery terminal pressure satisfactorily taking into full account the time delay. This is the problem of initialization. This problem may arise during start ups of the operation in a practical application.

It was seen that if the system was initially set in a highly depacked mode, assuming good source production capacity, the closed loop controller brought the system into the feasible region after some hours of operation also under a transient load demand. The infeasibility algorithm determines the closed loop control commands in infeasibility mode. Due to the large volumes and distances of the system example it took some hours of time to reach the feasible region.

From the simulations of the transmission system, it can be seen that the time delay effects are very large. So an open loop horizon up to several days may be necessary. For the transmission system example, a length of 48 hours seemed to be satisfactory for the source points to meet the main effects of the customer pattern taking into account the significant time delay. The dimension of the control parametrization sequence was set high (12-24 vectors in parametrized control sequence) for each open loop to meet the expected future transient load demand.

Setting up an on-line adaptive scheme to improve the linear optimization model by determining some tuning parameters but retaining the model structure may improve the controller performance. At each sampling instant, new values of the model parameters are identified. Tuning parameters may be the fluid temperature and the friction factor. The fluid temperature and friction factors are today used as tuning parameters for complex simulators describing fluid behaviour in gas transmission systems.

Long distance transmission systems without recompression possesses difficulties regarding the maintenance of the customer terminal pressure at the optimal value continuously along the operation time horizon when the gas customer load is highly transient. It is mainly the pressure gradients that determine the mass flow for a long distance transmission systems. The physical properties of the pipeline and the customer demands determine the minimum and maximum pressures. The longer the pipeline gets, the less is the possible pressure gradient along the pipeline. Therefore, it gets harder to meet the transient load pattern and to keep a constant customer pressure since the diffusive mass flow into the pipeline volume close to the customer point can not be instantly fast controlled as for the load pattern. Intermediate compressor stations will clearly improve controllability.

It is difficult to predict the load pattern precisely, let us say for a period of 48 hours. If the actual load is larger than the predicted load, the customer terminal pressure may decrease under a contracted minimum leading to the infeasibility mode. So, in any case, one should have some line pack in the transmission line for the supply security. A peak shaving storage close to the customer point will be very advantageous for the supply security. But this represents high investment and additional operation costs. For long distance transmission networks, the load predictions are very important due to the large time delay effect giving a need for long open loop horizons especially for transient loads. Also, it is more difficult to find controls at the source points so that the customer pressure can be kept close to its reference.

We must expect that prediction errors, error in optimization model, lack of source capacity or minimum gas line pack may lead to the infeasibility mode for some time instants of the system operation. A closed loop controller with an optimization model that underestimates customer pressures will lead to the system being packed with more natural gas than necessary, thus implying security of the supply but increased transportation costs. If the control signals are determined from an open loop optimization model predicting the customer pressure larger than they actually will be in correspondence with the actual system dynamics will lead to a decrease in pressure at customer point. A load forecast that underestimates future actual load have the same effect.

3.5 Summary and Discussion

A combined system of transient optimization and linear model predictive control is suggested for the on-line control of gas transmission systems. The supply security and infeasibility handling have also been considered. Simulations for a simple transmission example shows encouraging closed loop behaviour regarding stability and transient responses in cases of time varying load demands, time varying maximum source space, transient response to stationary operation point and switching between different dynamic controllers determined from correspondingly stationary operation points. The use of a linear optimization model with a quadratic cost function of a time finite horizon length gives a computationally feasible on-line controller scheme. A state estimator based on the same model as the optimization model is also outlined.

Chapter 4

Nonlinear Model Predictive Control of Gas Transmission Systems

4.1 Introduction

Nonlinear model predictive control for the minimization of transmission costs and the control of quality parameters and the maintenance of the security of the supply for a long distance transmission system that provides transportation services will be described in this chapter.

4.2 Nonlinear Model Predictive Control Algorithm

Figure 4.1 illustrates a long distance transmission system. There are three source points, each with a compressor station. The sources are assumed to be processing facilities for natural gas. These facilities may receive hydrocarbons from many different sources. The natural gas is transported along the three source pipelines that are connected to a mixing station. Here, the gas from the three sources is mixed to sales gas quality and further transported along the pipelines connected to the two customer terminals. Control valves are used in the mixing facility. The transmission system has no recompression.

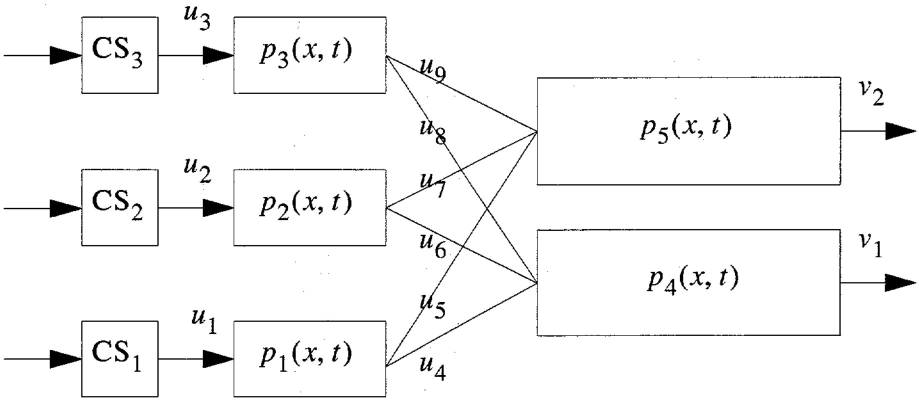


Figure 4.1: Transmission System Example

The control system method will be demonstrated with the objective to minimize transportation costs with emphasis on compression costs. But instead of using the explicit compression cost in the objective function, we will define an output vector. This output vector will be constrained to have values less than a defined upper limit vector and larger than a defined lower limit vector. An output vector reference will be defined. The objective function will also consist of an input vector part with a corresponding input reference. Deviation from input and output references will be penalized quadratically.

For a centrifugal compressor, the instant compression cost is given as in Eqn. (4.1).

$$K_{\text{cost}} = \xi_{\text{cost}} \cdot c_1 \cdot W \cdot \left(\left(\frac{p_\text{d}}{p_\text{s}} \right)^{c_2} - 1 \right) \tag{4.1}$$

From the expression in Eqn. (4.1), it can be seen that when the compression ratio and the flow through the unit is low, the cost is low. The compression ratio over a compressor station will be defined as a component of the output vector. The flow through the station is a part of the input vector. The minimum compression ratio is then defined as the reference for the station compression ratio and the minimum flow through the station is defined as the reference for the mass flow through the station. Different costs at different compressor stations are distinguished by using different values for the penalty weights in the objective function. A low pressure level of the transmission system implies a low compression cost. The customer terminal pressures are components of the output vector. Their corresponding reference values will for the security of the supply be chosen larger than the contractual minimum pressure limits. Each of the control valves are limited by a defined minimum differential pressure limit over the control valve and a defined maximum differential pressure limit over the control valve. The purpose with the minimum limit, is to secure that the flow through the control valve goes in the desired direction and that the control valve has enough driving force to reach a flow capacity above a defined minimum limit. The maximum differential pressure limit is due to physical limitations. The reference value for the differential pressure may be set close to or equal to the defined minimum limit. The minimal pressure drop over the control valves contributes to minimizing the pressure level of the transmission pipelines, thus reducing the compression cost. The differential pressures over the control valves are components of the output vector while the flow through the control valves are components of the control vector. Also, we want to minimize the pressure drop between the input and the output of each transmission pipeline. A maximum and minimum limit is also defined for these output components. The minimum limit ensures the flow direction while the maximum limit is due to the physical limitations. Input and output pressures may also be included in the output vector with defined accepted upper and lower operational pressure limits. At the mixing facility, we can control quality parameters such as gross calorific value and the concentration of gas components within the contract specifications. Considered quality parameters are included in the output vector. The gross calorific value and the concentration of carbon dioxide out from the two mixing points are defined to be the quality parameters. Each of these quality parameters must have values less than a defined upper limit and larger than a defined lower limit.

The resulting input vector for the system example is given in Eqn. (4.2)

$$\begin{aligned} u &= \begin{bmatrix} u_1 & u_2 & u_3 & u_4 & u_5 & u_6 & u_7 & u_8 & u_9 \end{bmatrix}^{\mathrm{T}} \\ &= \begin{bmatrix} W_{\mathrm{CS}_1} & W_{\mathrm{CS}_2} & W_{\mathrm{CS}_3} & W_{14} & W_{15} & W_{24} & W_{25} & W_{34} & W_{35} \end{bmatrix}^{\mathrm{T}} \end{aligned} \tag{4.2}$$

where W_{CS_i} denotes mass flow through compressor station i and W_{ij} denotes mass flow through the control valve bringing gas from pipeline i to pipeline j. The customer load vector is given in Eqn. (4.3).

$$v = \begin{bmatrix} v_1 & v_2 \end{bmatrix}^{\mathrm{T}} \tag{4.3}$$

The components of the customer load vector are the mass flows of natural gas delivered to the customers at the demand points. The output vector is defined in Eqn. (4.4).

$$y = \begin{bmatrix} p_{T_1} \\ p_{T_2} \\ \Delta p_{\text{pipe},4} \\ \Delta p_{\text{pipe},5} \\ \Delta p_{\text{valve},1} \\ \Delta p_{\text{valve},2} \\ \Delta p_{\text{valve},3} \\ \Delta p_{\text{valve},4} \\ \Delta p_{\text{valve},5} \\ \Delta p_{\text{valve},6} \\ \Delta p_{\text{pipe},1} \\ \Delta p_{\text{pipe},2} \\ \Delta p_{\text{pipe},3} \\ \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ y_{GCV_1} \\ y_{CO_{2_1}} \\ y_{GCV_2} \\ y_{CO_{2_2}} \end{bmatrix} \tag{4.4}$$

The customer terminal pressures are denoted by p_{T_1} and p_{T_2}. Input minus output pressure of pipe j is denoted by $\Delta p_{\text{pipe},\,j}$. Differential pressure over the control valve j is denoted by $\Delta p_{\text{valve},\,j}$ and ε_j is the compression ratio between the input and the output of station j. Gross calorific value and the concentration of carbon dioxide out from mixing point j are denoted y_{GCV_j} and $y_{CO_{2_j}}$ respectively.

The nonlinear creep flow model in Eqn. (3.2), given in Chapter 3, is used as simulation and nonlinear prediction model. It can be written more compactly as in Eqn. (4.5).

$$\dot{x} = f(x, u, v) \tag{4.5}$$

The state vector consists of the pressures from the input to the output along the grid points of each pipeline going in an orderly manner from pipeline one to pipeline five. For simplicity, we will assume that the composition from each source is constant but, in general different. A quality tracker must be used to predict the value of the quality parameters at the output of each source pipeline if the source composition varies. The output vector is a function of both the state and the control vectors yielding Eqn. (4.6).

$$y = g(x, u) \tag{4.6}$$

Denote y^* to be the constant output reference and $u^*(t)$ to be the time varying input reference. The control vector reference may be time varying because of the maximum production capacity and also possibly the minimum production capacity and the capacity through the compressor stations may be time varying such that the reference must take into account the changing conditions. A simple rectangular operational envelope is used over each compressor station as illustrated in Figure 4.2

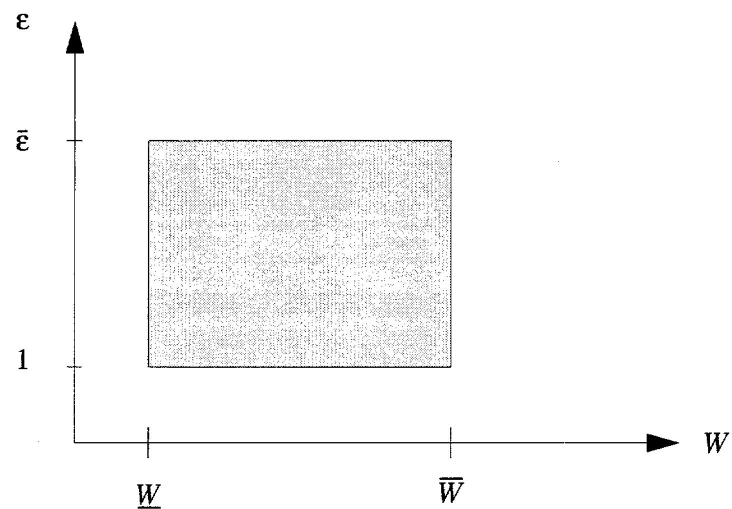


Figure 4.2: Simple rectangular station operation envelope

The model predictive controller solves an open loop optimization problem at each sampling instant. The nonlinear open loop optimization problem in continuous form is defined in Eqn. (4.7).

$$\begin{aligned}
&\underset{u_{[t_k, t_k+\Delta t_{\mathrm{ol}}]},\, y_{[t_k, t_k+\Delta t_{\mathrm{ol}}]}}{\text{minimize}} \quad \frac{1}{2}\int_{t_k}^{t_k+\Delta t_{\mathrm{ol}}} (y-y^*)^{\mathrm{T}} Q (y-y^*) + (u-u^*)^{\mathrm{T}} R (u-u^*) dt \\
&\text{subject to} \\
&\qquad \dot{x} = f(x, u, v) \\
&\qquad y = g(x, u) \\
&\qquad \underline{y} \le y \le \bar{y} \\
&\qquad \underline{u} \le u \le \bar{u} \\
&\qquad x_0 = x(t_k) \\
&\qquad v_{[t_k, t_k+\Delta t_{\mathrm{ol}}]} = \hat{v}_{[t_k, t_k+\Delta t_{\mathrm{ol}}]} \\
&\qquad t_k \le t \le t_k + \Delta t_{\mathrm{ol}}
\end{aligned} \tag{4.7}$$

The time length of open loop horizon is denoted by Δt_{ol}. The predicted load trajectory is denoted by $\hat{v}_{[t_k, t_k+\Delta t_{\mathrm{ol}}]}$ and the control and the output trajectories are denoted by $u_{[t_k, t_k+\Delta t_{\mathrm{ol}}]}$ and $y_{[t_k, t_k+\Delta t_{\mathrm{ol}}]}$ respectively. In the above formulation, the complete state vector is assumed to be available and may be replaced by the current estimated state denoted by $\tilde{x}(t_k)$. The upper and lower limits for the control vector together with the control reference are generally time varying. The control commands and load demand are assumed piecewise constant. Numerical approximation of the system dynamics using an explicit or implicit method and approximating the value of objective function numerically, yields the nonlinear programming problem for the current open loop given in Eqn. (4.8).

$$\underset{\{u_{k+i|k}\}_0^{N-1},\,\{y_{k+i|k}\}_1^N}{\text{minimize}} \frac{1}{2}\sum_{i=0}^{N-1} (y_{k+i+1|k}-y^*)^{\mathrm{T}}Q(y_{k+i+1|k}-y^*) + (u_{k+i|k}-u^*_{k+i|k})^{\mathrm{T}}R(u_{k+i|k}-u^*_{k+i|k})$$

subject to

$$\begin{array}{ll}
h(x_{k+i+1|k}, x_{k+i|k}, u_{k+i|k}, v_{k+i|k}) = 0 & 0 \le i \le N-1 \\
y_{k+i+1|k} = g(x_{k+i+1|k}, u_{k+i|k}) & 0 \le i \le N-1 \\
\underline{u}_{k+i|k} \le u_{k+i|k} \le \bar{u}_{k+i|k} & 0 \le i \le N-1 \\
\underline{y} \le y_{k+i+1|k} \le \bar{y} & 0 \le i \le N-1 \\
v_{k+i|k} = \hat{v}_{k+i|k} & 0 \le i \le N-1 \\
x_{k|k} = x(t_k) &
\end{array} \tag{4.8}$$

The open loop nonlinear programming problem can be written in augmented scaled form as in Eqn. (4.9). The scaling provides better conditioning of the problem. The control trajectory is parametrized to reduce the dimension of the problem.

$$\underset{y_s,\, u_s}{\text{minimize}} \ \frac{1}{2}(y_s - y_s^*)^{\mathrm{T}}\Phi_y(y_s - y_s^*) + \frac{1}{2}(u_s - u_s^*)G_{01}^{\mathrm{T}}\Phi_u G_{01}(u_s - u_s^*)$$

subject to

$$\begin{array}{c}
h_s(y_s, u_s, v, x_0) = 0 \\
\underline{y}_s \le y_s \le \bar{y}_s \\
\underline{u}_s \le u_s \le \bar{u}_s
\end{array} \tag{4.9}$$

The dynamic model is given in more detail in Eqn. (4.10).

$$h_s(y_s, u_s, v, x_0) = \begin{bmatrix} u = M_u^{-1}(\bar{u})u_s \\ h(x, u, v, x_0) = 0 \\ y = g(x, u) \\ y_s = M_y(\bar{y})y \end{bmatrix} = 0. \tag{4.10}$$

The open loop control trajectory is defined as

$$u_{\mathrm{ol}} = \left[u_{k|k}^{\mathrm{T}}\ u_{k+1|k}^{\mathrm{T}}\ u_{k+2|k}^{\mathrm{T}}\ \cdots\ u_{k+N-1|k}^{\mathrm{T}}\right]^{\mathrm{T}}$$

The parametrized control trajectory yields

$$u = \left[u_{k|k}^{\mathrm{T}}\ u_{k+1|k}^{\mathrm{T}}\ \cdots\ u_{k+M-1|k}^{\mathrm{T}}\right]^{\mathrm{T}}$$

The linear transformation between the parametrized control sequence and the open loop control sequence is compactly written as in Eqn. (4.11) and is equal to the transformation in Eqn. (3.93), given in Chapter 3.

$$u_{\mathrm{ol}} = G_{\mathrm{ol}} u \tag{4.11}$$

The scaled parametrized control sequence is given in Eqn. (4.12).

$$u_{\mathrm{s}} = M_u(\bar{u})u = \mathrm{diag}(\bar{u})^{-1} u \tag{4.12}$$

The output trajectory in augmented form is given in Eqn. (4.13).

$$y = \left[y_{k+1|k}^{\mathrm{T}}\ y_{k+2|k}^{\mathrm{T}}\ \cdots\ y_{k+N|k}^{\mathrm{T}}\right]^{\mathrm{T}} \tag{4.13}$$

The scaled output sequence is given in Eqn. (4.14).

$$y_{\mathrm{s}} = M_y(\bar{y})y = \mathrm{diag}(\bar{y})^{-1} y \tag{4.14}$$

The predicted state sequence for current open loop in augmented form yields Eqn. (4.15).

$$x = \left[x_{k+1|k}^{\mathrm{T}}\ x_{k+2|k}^{\mathrm{T}}\ \cdots\ x_{k+N|k}^{\mathrm{T}}\right]^{\mathrm{T}} \tag{4.15}$$

Predicted load trajectory for current open loop in augmented form is given in Eqn. (4.16).

$$v = \left[v_{k+1|k}^{\mathrm{T}}\ v_{k+2|k}^{\mathrm{T}}\ \cdots\ v_{k+N|k}^{\mathrm{T}}\right]^{\mathrm{T}} \tag{4.16}$$

The dynamics in augmented form is given in Eqn. (4.17)

$$h(x, u, v, x_0) \tag{4.17}$$

$$= \begin{bmatrix} h(x_{k+1|k}, x_{k|k}, u_{k|k}, v_{k|k}) \\ h(x_{k+2|k}, x_{k+1|k}, u_{k+1|k}, v_{k+1|k}) \\ \dots \\ h(x_{k+N|k}, x_{k+N-1|k}, u_{k+N-1|k}, v_{k+N-1|k}) \end{bmatrix}$$

where $x_0 = x_{k|k}$. The output sequence is related to the state and the control sequence as in Eqn. (4.18).

$$g(x, u) = \begin{bmatrix} g(x_{k+1|k}, u_{k|k}) \\ g(x_{k+2|k}, u_{k+1|k}) \\ \dots \\ g(x_{k+N|k}, u_{k+N-1|k}) \end{bmatrix} \tag{4.18}$$

The weight matrices for penalizing deviation from the output and the input reference are defined in Eqn. (4.19) and Eqn. (4.20) respectively.

$$\Phi_y = \text{blockdiagonal}(Q, Q, \dots, Q) \tag{4.19}$$

$$\Phi_u = \text{blockdiagonal}(R, R, \dots, R) \tag{4.20}$$

The matrix Φ_y consist of N block matrices and Φ_u consists of M block matrices along the block diagonals.

<u>Solving the open loop nonlinear program to local minimum:</u>

At each sampling instant, a nonlinear program must be solved. A structured sequential quadratic programming algorithm is used to obtain a local minimum. First, a short description follows and then the mathematical details are outlined for the transmission system example.

The nonlinear creep flow model chosen to represent the fluid dynamics is reformulated to a linear time variant prediction model. Parameters for this prediction model at the current iteration of the sequential quadratic programming procedure, are determined after that the mass flow and pressure trajectories for each pipeline are calculated using the parametrized control trajectory calculated at previous iteration and using the nonlinear prediction model in Eqn. (4.5) to predict the state trajectory. The state value or estimated state value at the current sampling instant, is used as initial condition of the current open loop problem. In addition, the output relation given in Eqn. (4.6) is linearized around this same state and control trajectory. The resulting prediction model yields a linear time variant output trajectory predictor for the current iteration of the sequential quadratic programming procedure. This predictor is inserted into the objective function and into the output trajectory constraint expression in the problem defined in Eqn. (4.9). The linear time variant prediction model has replaced the nonlinear prediction model. As a result, a quadratic programming problem in deviation variables from the solution at the previous iteration is obtained. The quadratic programming problem is solved by a standard solver. A line search is then performed in the direction decided by the solution of the quadratic programming problem in order to provide descent properties. The original objective function in Eqn. (4.9) and the nonlinear prediction model defined by Eqns. (4.5) and (4.6) are used in the line search procedure. If the defined termination criteria for a local minimum of the open loop nonlinear programming problem is satisfied, then the first component of the optimal open loop control sequence is applied to the gas transmission system. If the termination criteria is not satisfied, then the parametrized control sequence is used for the next iteration of the sequential quadratic programming procedure. The first iteration of the procedure uses a modification of the optimal parametrized control trajectory of the open loop problem at the previous sampling instant.

<u>Linear time variant output prediction model:</u>

The nonlinear creep flow model yields

$$\frac{A}{c^2} \cdot \frac{\partial p}{\partial t} = -\frac{\partial W}{\partial x} \tag{4.21}$$

$$\frac{\partial p}{\partial x} = -\frac{2 f \rho \upsilon^2}{D} \tag{4.22}$$

The above two equations can be rewritten as a linear time variant partial differential equation as in Eqn. (4.23), see Osiadacz (1987). Equation (4.24) relates the mass flow and the pressure gradient at a point along the pipeline.

$$\frac{\partial}{\partial t}p(x,t) = \alpha(x,t)\cdot\frac{\partial^2}{\partial x^2}p(x,t) \tag{4.23}$$

$$\frac{\partial}{\partial x}p(x,t) = \beta(x,t)\cdot W(x,t) \tag{4.24}$$

Here, $\alpha = \frac{c^2}{A\lambda}$, $\beta = -\lambda$ and $\lambda = \frac{2fc^2|W|}{DA^2p}$.

The time variant coefficients are approximated using the calculated state trajectory from the previous iteration of the sequential quadratic programming procedure using the nonlinear creep flow model in Eqn. (4.25)

$$\begin{aligned}\frac{\partial}{\partial t}p(x,t) &= \tilde{\alpha}(x,t)\cdot\frac{\partial^2}{\partial x^2}p(x,t)\\ \frac{\partial}{\partial x}p(x,t) &= \tilde{\beta}(x,t)\cdot W(x,t)\end{aligned} \tag{4.25}$$

where $\tilde{\alpha}(x,t)$ and $\tilde{\beta}(x,t)$ are given in Eqn. (4.26).

$$\begin{aligned}\tilde{\alpha} &= \frac{AD}{2f}\cdot\frac{\tilde{p}}{|\tilde{W}|}\\ \tilde{\beta} &= -\frac{2fc^2}{DA^2}\cdot\frac{|\tilde{W}|}{\tilde{p}}\end{aligned} \tag{4.26}$$

Here, $\tilde{p}$ and $\tilde{W}$ are the mass flow and the pressure at the given position at the given time instant of the considered open loop horizon. These values are decided by the predicted state trajectory at the current iteration of the sequential quadratic programming procedure. As explained, this state trajectory is calculated using the initial state of the considered open loop, the nonlinear prediction model and the parametrized control trajectory which is the optimization result of the previous iteration after line search of the sequential quadratic programming procedure.

For transmission systems including junction points, we may use the time variant differential equation in Eqn. (3.76), given in Chapter 3, for the pressure at the junction point. The parameters for the junction model is in the same way decided by the nonlinear prediction model and the parametrized control sequence from the previous optimization iteration.

Spatial approximation of the second order spatial derivatives at different spatial positions of fourth order accuracy yields Eqns. (4.27)-(4.31). See Schiesser (1991) for a description of the numerical method of lines.

$$\frac{\partial^2 p_1}{\partial x^2} = \left(\frac{1}{4!\Delta x^2}\right) \tag{4.27}$$

$$\cdot\left(-\frac{415}{3}p_1 + 192p_2 - 72p_3 + \frac{64}{3}p_4 - 3p_5 - 100\frac{\partial p_1}{\partial x}\Delta x\right) + O(\Delta x^4)$$

$$\frac{\partial^2 p_2}{\partial x^2} \tag{4.28}$$

$$= \left(\frac{1}{4!\Delta x^2}\right)\cdot(20p_1 - 30p_2 - 8p_3 + 28p_4 - 12p_5 + 2p_6) + O(\Delta x^4)$$

$$\frac{\partial^2 p_i}{\partial x^2}$$

$$= \left(\frac{1}{4!\Delta x^2}\right)\cdot(-2p_{i-2} + 32p_{i-1} - 60p_i + 32p_{i+1} - 2p_{i+2}) + O(\Delta x^4)$$

for $3 \le i \le M-2$ (4.29)

$$\frac{\partial^2 p_{M-1}}{\partial x^2} = \left(\frac{1}{4!\Delta x^2}\right)$$

$$\cdot(20p_M - 30p_{M-1} - 8p_{M-2} + 28p_{M-3} - 12p_{M-4} + 2p_{M-5})$$

$$+ O(\Delta x^4)$$

and finally (4.30)

$$\frac{\partial^2 p_M}{\partial x^2} = \left(\frac{1}{4!\Delta x^2}\right) \tag{4.31}$$

$$\cdot\left(-\frac{415}{3}p_M + 192p_{M-1} - 72p_{M-2} + \frac{64}{3}p_{M-3} - 3p_{M-4} - 100\frac{\partial p_M}{\partial x}\Delta x\right)$$

$$+ O(\Delta x^4)$$

where, for notational convenience, we have defined

$$p_i = p((x = (i-1)\Delta x), t),\ 1 \le i \le M \tag{4.32}$$

where M is the number of grid points along the considered pipeline. For a transmission pipeline with mass flow at input and output as boundary conditions, we now get the expression for the dynamics for a fluid inside a single gas transmission pipeline as in Eqn. (4.33)

$$\dot{p} = A(t)p + B_{\text{in}}(t)W(0,t) + B_{\text{out}}(t)W(L,t) \tag{4.33}$$

where $p = \begin{bmatrix} p_1\ p_2\ \dots\ p_M \end{bmatrix}^{\mathrm{T}}$. $W(0,t)$ and $W(L,t)$ are mass flows at the defined input boundary and the defined output boundary respectively

The system matrix for the pipeline is given by Eqn. (4.34).

$$A(t) = A_\alpha(t) \cdot A_{\text{diff}},\ A, A_\alpha, A_{\text{diff}} \in R^{M \times M} \tag{4.34}$$

where $A_\alpha(t) = \mathrm{diag}(\tilde{\alpha}_1(t), \tilde{\alpha}_2(t), \dots, \tilde{\alpha}_M(t))$. The constant matrix A_{diff} contains the differentiation coefficients of the second order spatial derivatives at the defined spatial positions along the pipeline. The input and output vectors B_{in} and B_{out} are given by Eqn. (4.35).

$$B_{\text{in}} = \begin{bmatrix} \left(\frac{100c^2}{4!\Delta x A^2}\right) & 0 & \dots & 0 \end{bmatrix}^{\mathrm{T}},\ B_{\text{in}} \in R^M \tag{4.35}$$

and

$$B_{\text{out}} = \left[0 \;\ldots\; 0 \; -\left(\frac{100c^2}{4!\Delta x A^2}\right)\right]^{\mathrm{T}}, \; B_{\text{out}} \in R^M. \tag{4.36}$$

For the network example, using the above method, we now have

$$\begin{aligned}\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} &= \begin{bmatrix} A_1(t) & & & & \\ & A_2(t) & & & \\ & & A_3(t) & & \\ & & & A_4(t) & \\ & & & & A_5(t) \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} \\ &+ \begin{bmatrix} B_{\text{in},1} & & & & \\ & B_{\text{in},2} & & & \\ & & B_{\text{in},3} & & \\ & & & B_{\text{in},4} & \\ & & & & B_{\text{in},5} \end{bmatrix} \cdot \begin{bmatrix} W_{\text{in},1} \\ W_{\text{in},2} \\ W_{\text{in},3} \\ W_{\text{in},4} \\ W_{\text{in},5} \end{bmatrix} \\ &+ \begin{bmatrix} B_{\text{out},1} & & & & \\ & B_{\text{out},2} & & & \\ & & B_{\text{out},3} & & \\ & & & B_{\text{out},4} & \\ & & & & B_{\text{out},5} \end{bmatrix} \cdot \begin{bmatrix} W_{\text{out},1} \\ W_{\text{out},2} \\ W_{\text{out},3} \\ W_{\text{out},4} \\ W_{\text{out},5} \end{bmatrix}\end{aligned} \tag{4.37}$$

where the state vector for pipeline i is

$$x_i = \left[p_{i,1} \; p_{i,2} \; \cdots \; p_{i,M_i}\right]^{\mathrm{T}} \tag{4.38}$$

Mass flows at the input and the output boundary of pipeline i are denoted by $W_{\text{in},i} = W(x_i = 0)$ and $W_{\text{out},i} = W(x_i = L_i)$. The input and output vectors $B_{\text{in},i}$ and $B_{\text{out},i}$ are defined in the same way as in Eqn. (4.35) and Eqn. (4.36).

We write Eqn. (4.37) more compactly as follows:

$$\dot{x} = A(t)x + B_{\text{in}}W_{\text{in}} + B_{\text{out}}W_{\text{out}} \tag{4.39}$$

The input and output mass flow vectors are functions of the control and load vectors through the following linear transformations.

$$\begin{aligned} W_{\text{in}} &= T_1 u \\ W_{\text{out}} &= T_2 u + T_3 v \end{aligned} \tag{4.40}$$

Inserting these transformations yield the state space representation

$$\dot{x} = A(t)x + Bu + Cv \tag{4.41}$$

where $B = B_{\text{in}}T_1 + B_{\text{out}}T_2$ and $C = B_{\text{out}} \cdot T_3$. We assume that the control and load demands are constant for each sampling interval. In addition, we approximate the time variant system matrix to be piecewise constant. Integration for all the time intervals of the open loop horizon yields Eqn. (4.42)

$$x_{k+i+1|k} = \Phi_{k+i|k}x_{k+i|k} + \Gamma_{k+i|k}u_{k+i|k} + \Psi_{k+i|k}v_{k+i|k} \tag{4.42}$$

where

$$\begin{aligned} \Phi_{k+i|k} &= e^{A_{[t_{k+i}, t_{k+i+1}]}T} \\ \Gamma_{k+i|k} &= \int_0^T e^{A_{[t_{k+i}, t_{k+i+1}]}\eta} d\eta B \qquad , 0 \le i \le N-1 . \\ \Psi_{k+i|k} &= \int_0^T e^{A_{[t_{k+i}, t_{k+i+1}]}\eta} d\eta C \end{aligned} \tag{4.43}$$

The matrix $A_{[t_{k+i}, t_{k+i+1}]}$ is the approximated constant value of the time variant system matrix $A(t)$ for the time interval $t \in [t_{k+i}, t_{k+i+1}]$.

We now have

$$x_{k+i+1|k} = \left(\prod_{j=i-1}^{0} \Phi_{k+j|k}\right)\tilde{x}_{k|k} \tag{4.44}$$
$$+ \sum_{j=0}^{i-2} (\Phi_{k+i-1|k} \cdot \ldots \cdot \Phi_{k+j+1|k}\Gamma_{k+j|k})u_{k+j|k}$$
$$+ \Gamma_{k+i-1|k}u_{k+i-1|k}$$
$$+ \sum_{j=0}^{i-2} (\Phi_{k+i-1|k} \cdot \ldots \cdot \Phi_{k+j+1|k}\Psi_{k+j|k})v_{k+j|k}$$
$$+ \Psi_{k+i-1|k}v_{k+i-1|k}$$

In augmented form, the following linear time variant state predictor is then apparent

$$x = Gu_{o1} + Hv + Fx_0 \tag{4.45}$$

where

$$G = \begin{bmatrix} \Gamma_{k|k} & & & & \\ \Phi_{k+1|k}\Gamma_{k|k} & \Gamma_{k+1|k} & & & \\ \Phi_{k+2|k}\Phi_{k+1|k}\Gamma_{k|k} & \Phi_{k+2|k}\Gamma_{k+1|k} & \Gamma_{k+2|k} & & \\ \cdots & \cdots & \cdots & \cdots & \\ \Phi_{k+N-1|k} \cdot \ldots \cdot \Phi_{k+1|k}\Gamma_{k|k} & \cdots & \cdots & \cdots & \Gamma_{k+N-1|k} \end{bmatrix}$$

, (4.46)

$$H = \begin{bmatrix} \Psi_{k|k} & & & & \\ \Phi_{k+1|k}\Gamma_{k|k} & \Psi_{k+1|k} & & & \\ \Phi_{k+2|k}\Phi_{k+1|k}\Gamma_{k|k} & \Phi_{k+2|k}\Gamma_{k+1|k} & \Psi_{k+2|k} & & \\ \cdots & \cdots & \cdots & \cdots & \\ \Phi_{k+N-1|k} \cdot \ldots \cdot \Phi_{k+1|k}\Gamma_{k|k} & \cdots & \cdots & \cdots & \Psi_{k+N-1|k} \end{bmatrix}$$

, (4.47)

$$F = \begin{bmatrix} \Phi_{k|k} \\ \Phi_{k+1|k}\Phi_{k|k} \\ \cdots \\ \Phi_{k+N-1|k}\Phi_{k+N-2|k} \cdot \cdots \cdot \Phi_{k+1|k}\Phi_{k|k} \end{bmatrix} \tag{4.48}$$

and

$$\begin{aligned} u_{\mathrm{ol}} &= \begin{bmatrix} u_{k|k}^{\mathrm{T}} \; u_{k+1|k}^{\mathrm{T}} \; \cdots \; u_{k+N-1|k}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \\ v &= \begin{bmatrix} v_{k|k}^{\mathrm{T}} \; v_{k+1|k}^{\mathrm{T}} \; \cdots \; v_{k+N-1|k}^{\mathrm{T}} \end{bmatrix}^{\mathrm{T}} \\ x &= \begin{bmatrix} x_{k+1|k}^{\mathrm{T}} \; x_{k+2|k}^{\mathrm{T}} \; \cdots \; x_{k+N|k}^{T} \end{bmatrix}^{\mathrm{T}} \end{aligned} \tag{4.49}$$

Using the control parametrization $u_{\mathrm{ol}} = G_{\mathrm{ol}}u$, we get the linear time variant state predictor in Eqn. (4.50)

$$x = Ku_{\mathrm{s}} + k \tag{4.50}$$

where

$$\begin{aligned} K &= GG_{\mathrm{ol}} \\ k &= Hv + Fx_0 \end{aligned} \tag{4.51}$$

We have the nonlinear output trajectory

$$y = g(x, u) \tag{4.52}$$

Define x_0 to be the unscaled optimal state trajectory solution from the previous iteration of the sequential quadratic programming procedure, and u_0 to be the unscaled optimal parametrized control trajectory after line search found from the previous iteration. In addition, we write $u_{\mathrm{s}0}$ to be the scaled optimal parametrized control trajectory from the previous optimization iteration. This control trajectory is calculated as $u_{\mathrm{s}0} = M_u(\bar{u})u_0$. We linearize the output trajectory around the state and control trajectories from the previous iteration of the sequential quadratic programming sequence yielding Eqn. (4.53).

$$y = y_0 + \left.\frac{\partial g}{\partial x}\right|_{x = x_0}(x - x_0) + \left.\frac{\partial g}{\partial u}\right|_{u = u_0}(u - u_0) \tag{4.53}$$

We insert the linear time variant state predictor and use the scaling matrices to obtain the resulting scaled discrete linear time variant output predictor in Eqn. (4.54)

$$y_s = Au_s + a \tag{4.54}$$

where

$$\begin{aligned} A &= M_y(\bar{y})\left(\left.\frac{\partial g}{\partial x}\right|_{x = x_0} K + \left.\frac{\partial g}{\partial u}\right|_{u = u_0}\right)M_u(\bar{u})^{-1} \\ a &= M_y(\bar{y})\left(g(x_0, u_0, \theta_{01}) + \left.\frac{\partial g}{\partial x}\right|_{x = x_0}(k - x_0) - \left.\frac{\partial g}{\partial u}\right|_{u = u_0} u_0\right) \end{aligned} \tag{4.55}$$

Using the parametrized control trajectory from the previous optimization iteration and the nonlinear prediction model provides the pressure distribution trajectory used for the design of the linear time variant prediction model. The mass flow distribution trajectory was calculated using the nonlinear stationary momentum equation of the nonlinear creep flow model given in Eqn. (4.56) together with the pressure distribution trajectory.

$$\frac{\partial p^2}{\partial x} = -\frac{4fc^2}{DA^2} \cdot W^2 \tag{4.56}$$

The Matlab function **ode15s** was used for the open loop prediction of the state and output trajectory using the nonlinear dynamic model defined in Eqn. (4.5) and Eqn. (4.6).

Quadratic programming problem:

With the defined objective function and the defined output constraint and using the output predictor, we get the quadratic program for the current optimization iteration as in Eqn. (4.57)

$$\begin{aligned}
&\underset{u_s}{\text{minimize}} \quad \frac{1}{2}u_s^{\mathrm{T}}\Phi_{\mathrm{QP}}u_s + \varphi_{\mathrm{QP}}^{\mathrm{T}}u_s \\
&\text{subject to} \\
&\qquad A_{\mathrm{QP}}u_s \le a_{\mathrm{QP}} \\
&\qquad \underline{u}_s \le u_s \le \bar{u}_s
\end{aligned} \tag{4.57}$$

where

$$\begin{aligned}
\Phi_{\mathrm{QP}} &= A^{\mathrm{T}}\Phi_y A + G_{\mathrm{ol}}^{\mathrm{T}}\Phi_u G_{\mathrm{ol}} \\
\varphi_{\mathrm{QP}}^{\mathrm{T}} &= -(y_s^* - a)^{\mathrm{T}}\Phi_y(y_s^* - a) + u_s^{*\mathrm{T}}G_{\mathrm{ol}}^{\mathrm{T}}\Phi_u G_{\mathrm{ol}} \\
A_{\mathrm{QP}} &= \begin{bmatrix} A \\ -A \end{bmatrix} \\
b_{\mathrm{QP}} &= \begin{bmatrix} \bar{y}_s - a \\ -(\underline{y}_s - a) \end{bmatrix}
\end{aligned} \tag{4.58}$$

<u>Line Search:</u>

We perform line search on the original objective function using the nonlinear model in the direction $\Delta u_{s,\mathrm{QP}}^* = u_{s,\mathrm{QP}}^* - u_{s0}$ to find a function value such that the descent properties are secured for the sequential quadratic programming sequence.

$$u_s(\alpha) = u_{s0} + \alpha(u_{s,\mathrm{QP}}^* - u_{s0}) \tag{4.59}$$

The local sequential quadratic programming solver **fmincon** in Matlab was used for the line search procedure in the simulations.

We now describe a line search method called the Quadratic Interpolation Method. This method uses polynomial interpolation of three different function points that follows from different values of the α 's. Assume that $f(\alpha)$ is the function to be linearized. Three points $\{\alpha_1, \alpha_2, \alpha_3\}$ are chosen such that $\alpha_1 < \alpha_2 < \alpha_3$ and $f(\alpha_1) > f(\alpha_2)$ and $f(\alpha_2) < f(\alpha_3)$. A local minimum of f must lie between the boundary points α_1 and α_3 of the three point pattern $\{\alpha_1, \alpha_2, \alpha_3\}$. At each iteration, the method fits a quadratic polynomial to the three values $f(\alpha_1)$, $f(\alpha_2)$

and $f(\alpha_3)$, and replaces one of the points $\{\alpha_1, \alpha_2, \alpha_3\}$ by the minimizing point of this quadratic polynomial.

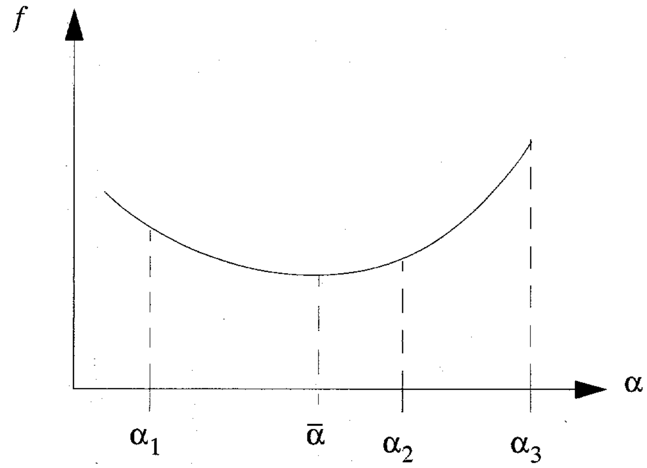


Figure 4.3: Minimum of the quadratic polynomial associated with α_1, α_2 and α_3.

If $\bar{\alpha}$ minimizes the quadratic, a new three point pattern is obtained using $\bar{\alpha}$ and two of the three points α_1, α_2 and α_3. In Figure 4.3 α_1 and α_2 are used. The first step consists of finding three points, α_1, α_2 and α_3, along the search direction such that $\alpha_1 < \alpha_2 < \alpha_3$, $f(\alpha_1) > f(\alpha_2)$ and $f(\alpha_2) < f(\alpha_3)$. Given the current three point pattern α_1, α_2 and α_3 of the line search iteration a quadratic polynomial is fitted to the function values $f(\alpha_1)$, $f(\alpha_2)$ and $f(\alpha_3)$ the unique minimum $\bar{\alpha}$ of the quadratic function inside the interval is given in Eqn. (4.60)

$$\bar{\alpha} = \frac{1}{2} \cdot \frac{f(\alpha_1) \cdot (\alpha_3^2 - \alpha_2^2) + f(\alpha_2) \cdot (\alpha_1^2 - \alpha_3^2) + f(\alpha_3) \cdot (\alpha_2^2 - \alpha_1^2)}{f(\alpha_1) \cdot (\alpha_3 - \alpha_2) + f(\alpha_2) \cdot (\alpha_1 - \alpha_3) + f(\alpha_3) \cdot (\alpha_2 - \alpha_1)} \tag{4.60}$$

Then, a new three-point pattern is generated. If $\bar{\alpha} > \alpha_2$, α_1 or α_3 is replaced by $\bar{\alpha}$ depending on whether $f(\bar{\alpha}) < f(\alpha_2)$ or $f(\bar{\alpha}) > f(\alpha_2)$ respectively. If $\bar{\alpha} < \alpha_2$, α_3 or α_1 are replaced by $\bar{\alpha}$ depending on whether $f(\bar{\alpha}) < f(\alpha_2)$ or $f(\bar{\alpha}) > f(\alpha_2)$ respectively. If $f(\bar{\alpha}) = f(\alpha_2)$ then a special local search near $\bar{\alpha}$ should be conducted to replace $\bar{\alpha}$ by a point $\bar{\alpha}'$ with $f(\bar{\alpha}') \neq f(\alpha_2)$. The computation is terminated when the length of the three-point pattern, is smaller than a specified tolerance expressed in Eqn. (4.61).

$$\alpha_3 - \alpha_1 \le \varepsilon_{\text{line}} \tag{4.61}$$

This method and other line search methods are given in Bertsekas (1995). An alternative reference on line search methods is Luenberger (1984).

There is not a demand that a local or a global minimum is found in the line search procedure. What is important is that the objective function is less than the value at the previous iteration (descent properties). The line search takes place inside a Newton step meaning that the maximum value of the alfa parameter is one and the minimum value is zero. Therefore, we look for a value that ensures descent properties between the solution at the previous iteration and the solution by the quadratic programming problem at the current iteration along the direction determined from the solution of the quadratic programming problem.

Termination criteria:

Denote $J(u_{i-1})$ to be the value of the objective function with the parametrized control trajectory at the optimization iteration $i-1$ and $J(u_i)$ to be the value of the objective function with the parametrized control trajectory at the optimization iteration i for the open loop problem at the current sampling instant denoted by t_k, each after the line search. If the criteria in Eqn. (4.62) is satisfied, then the sequential quadratic programming procedure is terminated and the first component of the sequence u_i is implemented in closed loop operation.

$$J(u_{i-1}) - J(u_i) \le \delta_{\text{Termination}} \tag{4.62}$$

Control command at first iteration:

Denote u_{j-1}^* to be the optimal parametrized control trajectory for the open loop problem at the sampling instant t_{k-1}. The trajectory is given in Eqn. (4.63).

$$u_{j-1}^* = \left[u_{j-1,k-1|k-1}^{\mathrm{T}\,*} \; u_{j-1,k|k}^{\mathrm{T}\,*} \; u_{j-1,k+1|k+1}^{\mathrm{T}\,*} \; \dots \; u_{j-1,k+M-2|k+M-2}^{\mathrm{T}\,*} \right]^{\mathrm{T}} \tag{4.63}$$

The first initial guessed parametrized control trajectory for the current open loop starting at time instant t_k is designed from a modification of Eqn. (4.63) and given in Eqn. (4.64).

$$u_0 = \left[u_{j-1,k|k}^{\mathrm{T}\,*} \; u_{j-1,k+1|k+1}^{\mathrm{T}\,*} \; \dots \; u_{j-1,k+M-2|k+M-2}^{\mathrm{T}\,*} \; u_{j-1,k+M-2|k+M-2}^{\mathrm{T}\,*} \right]^{\mathrm{T}} \tag{4.64}$$

The first control vector from the parametrized control trajectory of the previous optimization horizon has been removed since this has already been implemented as a control command applied to the process. The last component from the optimal parametrized control sequence at the previous open loop horizon has been used twice since we do not have a better suggestion.

Temperature and compressibility factor:

The temperature and the compressibility factor distributions along each pipeline, can be inserted into both the nonlinear and the linear time variant prediction model using the relation $c^2 = \kappa \cdot Z \cdot \frac{R}{MW} \cdot T$ and the temperature and the compressibility factor distributions from the complex simulators that are usually available to the gas dispatchers at a gas control centre. The instant temperature and the compressibility factor distributions at the current sampling instant can be inserted into the nonlinear prediction model and the linear time variant prediction model used in the sequential quadratic programming procedure.

Another approach is to first perform a complex stationary optimization as described in Chapter 3 where the energy equation and a complex thermodynamic equation can be used. The optimal stationary temperature and compressibility factor distributions are then inserted into the speed of sound expression at the correct positions in the optimization model and the nonlinear simulation model. In this way, the nonlinear model predictive controller can also include complex stationary thermodynamics. This may improve the description of the fluid dynamics inside the transmission lines used by the optimization algorithm.

Figure 4.4 illustrates the sequential quadratic programming algorithm for the current sampling instant for a gas transmission system. We note the insertion of the predicted load trajectory and the predicted control space as well as the use of the estimated state at the current sampling instant.

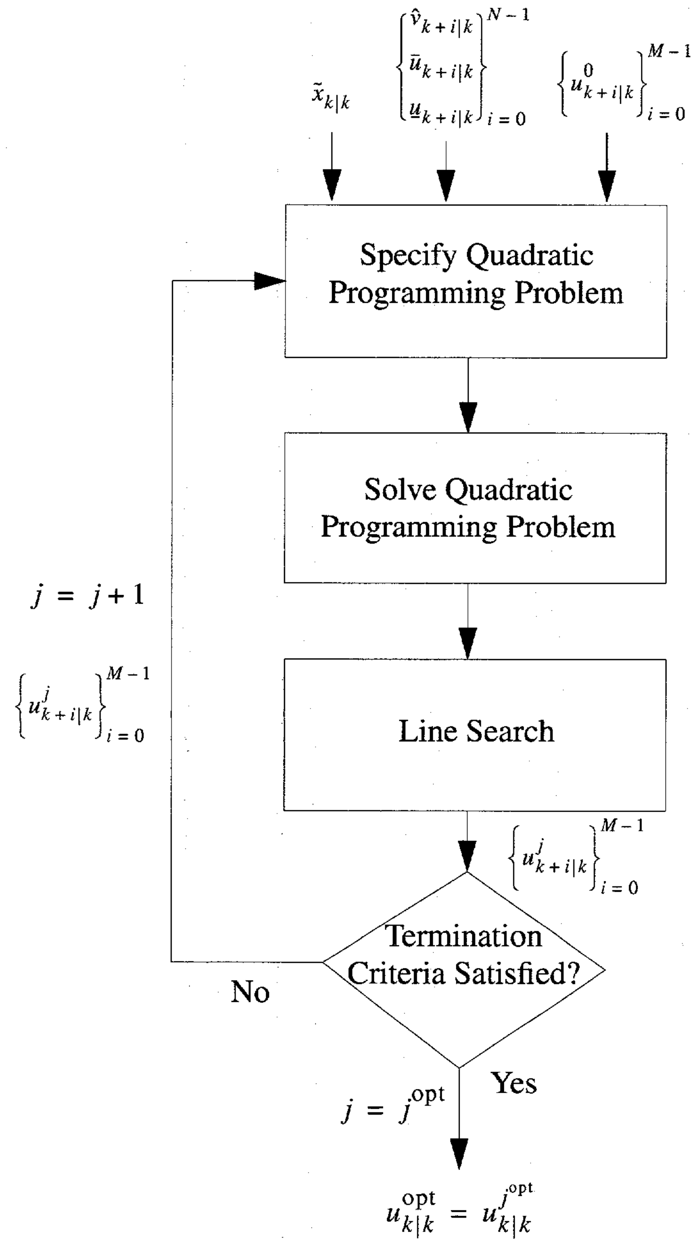


Figure 4.4: Sequential quadratic programming procedure for gas transmission system

The above sequential quadratic programming algorithm is motivated by the ideas presented in the work by Li and Biegler (1989), Oliveira and Biegler (1994,1995), Oliveira (1994), and Oliveira and Morari (1994), called Newton-type algorithms for nonlinear process control. In this thesis, the linear time variant prediction model is developed from a reformulation of the nonlinear creep flow model. In the above papers, the linear time variant prediction model is developed from explicitly linearizing the nonlinear prediction model around a so called "nominal" state trajectory. This trajectory follows from the solution of the nonlinear prediction model given the initial state condition of the open loop horizon and the parametrized control sequence from the previous optimization iteration of the sequential quadratic programming procedure. The linearization for the nonlinear relation between the output vector and the state and the control vector has been done in a similar manner in this thesis as in the above references.

In the paper by Biegler (1998), efficient solutions of the dynamic optimization of constrained nonlinear model predictive control problems are described exploiting the structure of the problem and the use of the sequential quadratic programming procedure. Both sequential and simultaneous nonlinear programming strategies are dealt with. Interior point methods within the framework of sequential quadratic programming for large scale optimization is reviewed and discussed in Gopal and Biegler (1998).

For an introduction to pure and modified Newton methods for both unconstrained and constrained methods, see Luenberger (1984). The book by Bertsekas (1995) gives an introduction to sequential quadratic programming algorithms.

<u>Implicit nonlinear prediction model:</u>

In the simulations, an ordinary differential equation solver in Matlab was used to numerically solve the nonlinear prediction model and for the closed loop simulation. If an implicit discretization method is used for the nonlinear prediction model to ensure a stable solution, it can be represented in augmented form for the entire open loop as in Eqn. (4.65).

$$h(u, x, \hat{v}) = 0 \tag{4.65}$$

Linearization around the state and the parametrized control trajectory from the previous iteration of the sequential quadratic programming procedure yields Eqn. (4.66).

$$\nabla_x h(u, x, \hat{v})\Delta x + \nabla_u h(u, x, \hat{v})\Delta u = 0 \tag{4.66}$$

Rearranging yields the explicit linear time variant state prediction model in Eqn. (4.67)

$$\Delta x = -\nabla_x h(u, x, \hat{v})^{-1} \nabla_u h(u, x, \hat{v}) \Delta u \tag{4.67}$$

and with the output relation $y = g(x, u)$ linearized in Eqn. (4.68)

$$\Delta y = \nabla_x g(x, u) \Delta x + \nabla_u g(x, u) \Delta u \tag{4.68}$$

giving the explicit linear time variant output predictor in Eqn. (4.69)

$$\begin{aligned} &\Delta y \\ &= [-\nabla_x g(x, u) \nabla_x h(u, x, \hat{v})^{-1} \nabla_u h(u, x, \hat{v}) + \nabla_u g(x, u)] \Delta u \end{aligned} \tag{4.69}$$

If the dimension of the transmission network is not too large, it is possible to use the above output predictor in a sequential quadratic programming procedure with the parametrized control trajectory as the only optimization variables. If the dimension is large, inversion of $\nabla_x h(u, x, \hat{v})$ may be too time consuming making the model impractical in an on-line application since this time consuming inversion procedure must be done at every iteration.

One great advantage with the reformulation of the nonlinear creep flow model to a linear time variant prediction model is the avoidance of the above explained inversion making it suitable for large scale transmission systems.

Stationary optimization and nonlinear model predictive control:

A control scheme similar to the method presented in Chapter 3 can be defined. But now, a nonlinear prediction model is used instead of the linear prediction model to represent the dynamics of each open loop problem. Each open loop problem, can then be solved to a local minimum using a structured sequential quadratic programming procedure.

After a stationary optimization has been performed, the result from this optimization defines the state and control references. The objective function of the stationary optimization problem may be chosen to be the minimization of transportation costs. The solution of the stationary optimization problem, if feasible, is found so that all the defined system constraints are satisfied. The constraints that are defined for the defined quality parameters are satisfied on a stationary basis. The objective function of each open loop problem for the nonlinear model predictive controller can be defined to penalize the control and the state deviation from their respective references quadratically. The values of the state vector must be below a defined upper state limit vector and above a lower state limit vector. In addition, a set of linear state inequalities constraints can be defined. The values of the control vector must be below a defined upper control limit vector and above a lower control limit vector. The limit vectors may be time varying due to time varying control space as explained in Chapter 3. Use of the nonlinear creep flow model to represent the fluid dynamics, the quadratic objective function and the defined state and control constraints defines a nonconvex optimization problem for each open loop. The on-line controller handles the security of the supply and the control of the quality parameters by bringing and keeping the state vector close to the reference. The open loop problem for the nonlinear model predictive control scheme in continuous form is defined in Eqn. (4.70).

$$
\begin{aligned}
&\underset{x[t_k,\, t_k + \Delta t_{\mathrm{ol}}],\, u[t_k,\, t_k + \Delta t_{\mathrm{ol}}]}{\text{minimize}} \quad \frac{1}{2} \int_{t_k}^{(t_k + \Delta t_{\mathrm{ol}})} (x - x^*)^{\mathrm{T}} Q (x - x^*) + (u - u^*)^{\mathrm{T}} R (u - u^*) dt \\
&\text{subject to} \\
&\qquad \dot{x} = f(x, u, v) \\
&\qquad \underline{u} \le u \le \bar{u} \\
&\qquad \underline{x} \le x \le \bar{x} \\
&\qquad A_{\mathrm{c}} x \le a_{\mathrm{c}} \\
&\qquad v[t_k, t_k + \Delta t_{\mathrm{ol}}] = \hat{v}[t_k, t_k + \Delta t_{\mathrm{ol}}] \\
&\qquad \underline{u}[t_k, t_k + \Delta t_{\mathrm{ol}}] = \hat{\underline{u}}[t_k, t_k + \Delta t_{\mathrm{ol}}] \\
&\qquad \bar{u}[t_k, t_k + \Delta t_{\mathrm{ol}}] = \hat{\bar{u}}[t_k, t_k + \Delta t_{\mathrm{ol}}] \\
&\qquad x_0 = \tilde{x}(t_k) \\
&\qquad t_k \le t \le t_k + \Delta t_{\mathrm{ol}}
\end{aligned}
\tag{4.70}
$$

We note that the time varying predicted load together with the generally time varying predicted source supply are included in the open loop problem definition. The linear inequality is defined with the objective to secure that flow directions, minimum differential pressures over control valves and compression ratio constraints, etc. are secured. This open loop problem can then be solved to a local minimum by a similarly structured sequential quadratic programming procedure as explained earlier in this chapter. The initial condition for the open loop problem will usually be the estimated states as illustrated in Eqn. (4.70). Since the references are from the current stationary optimization, they are constant throughout the open loop horizon. The formulation in Eqn. (4.70) is in correspondence with the more general formulation in Eqn. (4.7) with $y = g(x, u) = x$.

Security of the supply:

High values for the coefficients in the output penalty matrix Q that correspond with the terminal pressures compared to the value of the coefficients in Q that corresponds with the other components of the output vector contribute to the security of the supply.

High quality load forecasts for the prediction of the customer demands for natural gas are important for the security of the supply, especially when a low pressure level of the transmission system is desired to reduce the transportation costs. An increase in the line pack will increase the need for precise load forecasts.

The length of the open loop horizon must be chosen such that the open loop problem taking into account the delay effects from the points where the state of the system can be manipulated to the customer points. Then, in combination with a precise forecast on the natural gas customer demand, it is possible to meet the customer loads while keeping the customer terminal pressures above the contracted minimum limits for most of the time.

It is also important that the differential pressures (driving force) over the control valves have large enough values to provide satisfactory flow capacities to meet the transient customer demands. The coefficients in the matrix Q penalizing the differential pressures over the control valves are then given high values together with high values for the coefficients in Q penalizing the customer terminal pressures.

The control space is in general time varying. Based on a predicted source capacity forecast, one formulates the time varying control space used by the model predictive controller. If some sources face reduced capacity, then the rest of the sources can increase their delivery into the transmission system so that the security of the supply can still be maintained at the customer terminals while keeping the terminal pressures above the contracted minimum limits.

Infeasibility Algorithm:

Infeasibility in gas transmission networks may occur as discussed in Chapter 3. A simple infeasibility handler is to remove the output constraints when the open loop problem is infeasible. Another approach is to relax the output constraints and penalize the relaxation in the objective function. This is done for the quadratic programmes at each iteration of the sequential quadratic programming procedure for finding a local minimum of the infeasible open loop nonlinear programming problem with the relaxation penalized quadratically in the objective function. Equation (4.71) expresses the relaxed problem of the quadratic programming problem of a given iteration. The matrix S_{relax} expresses the defined priority between the different output constraints. The penalty relaxation has a lower limited which is equal to zero and a defined upper limit. The upper limit vector depends on what is considered to be an acceptable relaxation. Non relaxable hard constraints have an upper limit of zero. The maximum pipeline pressure before the pipeline is destroyed is an example of such a constraint. For the ordinary network operation, the components of the upper relaxation vector should have values below the non relaxable limits. The quality parameters may be fitted into the category of soft constraints or into the class of relaxable hard constraints.

$$
\begin{aligned}
&\underset{u_s,\,\delta_{relax}}{\text{minimize}} \quad \frac{1}{2}u_s^T\Phi_{QP}u_s + \varphi_{QP}^T u_s + \delta_{relax}^T S_{relax}\delta_{relax} \\
&\text{subject to} \\
&\qquad A_{QP}u_s \le b_{QP} + \delta_{relax} \\
&\qquad \underline{u}_s \le u_s \le \overline{u}_s \\
&\qquad 0 \le \delta_{relax} \le \bar{\delta}_{relax}
\end{aligned}
\tag{4.71}
$$

The infeasibility algorithm is stable if the output vector converges to the feasible region. If the source space is too small, we may never reach the feasible region. But, then it is important that the transmission system is operated as close as possible to the feasible region. The matrix S_{relax} and the upper relax limit vector $\bar{\delta}_{relax}$ are chosen so that the important customer points are prioritized. Then, the available resources of natural gas from the available source capacity and the available line pack will be directed against these customer terminals.

At some occasions, when the transmission system is in an infeasible mode, the gas dispatch personnel may have to close down parts of or the entire transmission system. This may be the case if all the extra resources, such as natural gas from the gas storage facilities, the operational agreements and the contractual arrangements all have been stretched to their limits.

We may avoid the introduction of extra optimization variables by specifying the value of the relaxation vector. If we have an idea about the output variables that are the main reason for the infeasibility, we may relax only those variables. If an algorithm can calculate the optimal relaxation factor when the elements of the output vector are put into different priority cathegories, this would be the ideal situation. In the thesis by Vada (2000), optimal relaxation factors are calculated based on a defined priority list for a linear time invariant model predictive control scheme as already discussed in Chapter 3. The theory for nonlinear model predictive control and infeasibility handling was not described. But, assuming that if an algorithm was available and the optimal relaxation δ_{relax}^* for the current sampling instant was available based on a given priority list, the relaxed problem would look like Eqn. (4.72) for the quadratic programming problem of a given iteration of the successive quadratic programming procedure. The number of optimization variables have not increased.

$$\begin{aligned} &\underset{u_s}{\text{minimize}} \quad \frac{1}{2} u_s^T \Phi_{QP} u_s + \varphi_{QP}^T u_s \\ &\text{subject to} \\ &\qquad A_{QP} u_s \le b_{QP} + \delta_{relax}{}^* \\ &\qquad \underline{u}_s \le u_s \le \bar{u}_s \end{aligned} \tag{4.72}$$

Closed loop control system illustration:

Figure 4.5 illustrates the closed loop control system including a state estimator application. The vector $\xi_{k|k}$ is the network observations from the SCADA information handling system supplied to the state estimator. The output value from the state estimator is the pressures along the grid and the junction points of the transmission network at the current sampling instant. The measured load demands are also components of ξ. We also note the load and control space prediction for the open loop optimization.

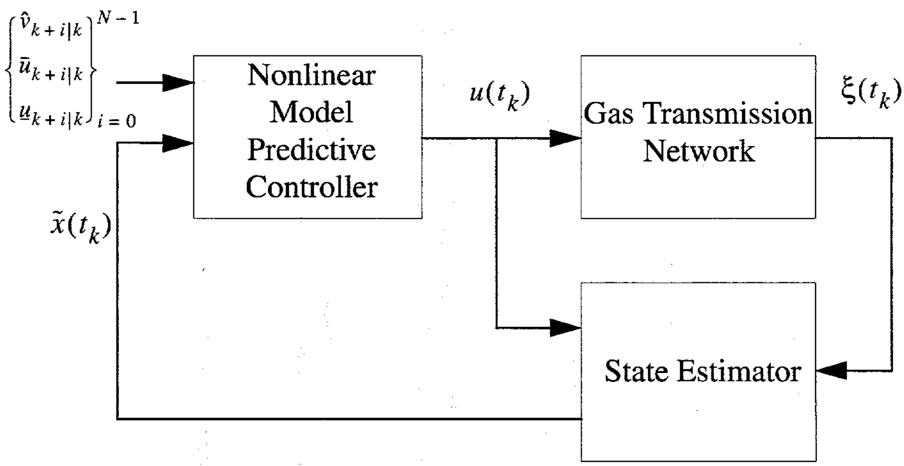


Figure 4.5: Closed loop control system.

Unstructured sequential quadratic programming:

Unstructured sequential quadratic programming algorithms are available for finding a local minimum of a nonlinear programming problem. Such an algorithm finds a local minimum just by using the function values of the objective, equality and inequality constraints in addition to the values of the gradients and the Hessian for the objective function and the constraints for the current optimization iteration without knowing the structure of the problem. The algorithm NPSOL invented by Philip Gill, Walter Murray, Michael Saunders and Maragareth Wright for the minimization of arbitrary smooth functions subject to constraints is an example. Biegler (1998) points out that structured methods provide faster convergence to a local minimum than a generic method. The Newton method converges quadratically while the generic sequential quadratic programming methods converge superlinearly. See Bertsekas (1995) or Luenberger (1984) for definitions of quadratic and superlinearly convergence.

Step response model:

An alternative way of getting a linear time variant prediction model for the sequential quadratic programming procedure is by performing step responses on the nonlinear prediction model or on a complex gas network simulator. The main disadvantage is the time consumption for each nonlinear prediction. A simplification is to assume that the matrix relating the open loop control sequence to the open loop output trajectory is block diagonal. For each sampling interval, one must perform a step response test for each control component observing the output response of each output component around the output trajectory from the previous optimization iteration. For transmission systems with large dimensions of the input and the output vector, this is not a practical procedure.

Stability consideration of nonlinear control algorithm:

Assume that the solution of each open loop problem defined as in Eqn. (4.8) converges to a local minimum. Assume further that each open loop receives states from the nonlinear model. The control command of the nonlinear model predictive controller at the current sampling instant is a function of the state, the predicted open loop load forecast, the time varying predicted control space, the length of the open loop horizon, the dimension of the control parametrization and the defined output and input penalty matrices. If a stationary optimization is performed as in Chapter 3 giving an equilibrium point, the control law is additionally a function of $\{x^*, u^*, v^*\}$. This functional dependence is expressed in Eqn. (4.73).

$$u(t_k) = \phi(x(t_k), \{\hat{v}_{k|k+i}\}_{i=0}^{N}, \{\bar{u}_{k|k+i}\}_{i=0}^{N}, \{\underline{u}_{k|k+i}\}_{i=0}^{N} \\ , N, M, Q, R, x^*, u^*, v^*) \tag{4.73}$$

An equilibrium point satisfies Eqn. (4.74) where it can be seen that the value of the pair $\{u^*, x^*\}$ determines y^*.

$$
\begin{aligned}
f(x^*, u^*, v^*) &= 0 \\
y^* &= g(x^*, u^*)
\end{aligned}
\tag{4.74}
$$

It is important to note that the discrete open loop problem in Eqn. (4.8) is an approximation of the continuous open loop problem in Eqn. (4.7) and this problem is of course further an approximation of the network description with the nonlinear partial differential equations describing the fluid dynamics. The nonlinear creep flow equation is a simplified description of the complex fluid dynamics. The load forecasts will of course not be perfect. The maximum source capacity will be time varying. The prediction of the time varying control space including the maximum source space will of course generally not be perfect. The status of the network infrastructure may be time varying. Obviously, all these factors makes it very difficult to prove robust stability of the closed loop controller.

Assuming that the nonlinear model is perfect, the load is constant and equal to the stationary optimization and with enough control space in relation to the load demand always yielding admissible controls and looking at stability on an infinite time basis, has little practical usefulness for proving closed loop stability. Actual operation conditions are highly time varying and uncertain and with a limited operation time due to the limited natural gas resources. Infeasibility is also an operation mode that one must expect to meet frequently in the practical system operation and the stability must also be considered also in this situation.

Based on the complexity of the gas transmission networks, it seems that the best way of evaluating the controller performance is by simulations evaluating different scenarios. This is also a comment in Chapter 3. Use of the complex gas transmission system simulators that are available commercially and the historical events including load demands simulating closed loop control can give important indications but not proofs of stability and controller performance.

The controller presented in this chapter is motivated by the model predictive controllers presented in Oliveira (1994) and Oliveira and Biegler (1995). The stability proofs, for nominal stability, presented in Oliveira (1994) and Oliveira and Biegler (1995) assumes that the states are available to the controller. It is also assumed that the nonlinear model is perfectly known. Additionally, it is assumed that there is an equilibrium point and that the plant is open loop asymptotically stable in the large and the parameters of the plant are constant. The plant is open loop asymptotically stable in the large if the state of the system converges to the equilibrium point given any initial condition and using the control input in correspondence with the equilibrium point when time goes to infinity. See Kalman and Bertram (1960) and Ioannou and Sun (1996) for stability definitions. It is also assumed that the output vector is only a function of the states. In this chapter the controller is also a function of the input vector due to the quality constraints. The load trajectory is generally transient meaning that the parameters of the plant are not constant. Also, it is assumed in the stability proofs that there is enough control space for the controller. This is not always the case for gas transmission systems. Load forecasts on the demand for natural gas by the customers are usually not perfect.

So, by the above considerations, we see that it is a difficult task to prove stability, both in the feasible mode and in the infeasible mode, and nominally and robustly.

Main Matlab program features:

The specification of a quadratic programming problem together with finding the solution of this problem then followed by a line search are the main operations of the sequential quadratic programming procedure for a considered open loop problem.

In a defined Matlab function, the quadratic program for the current iteration of the sequential quadratic programming procedure was specified. The input value to this Matlab function is the state initial condition (or estimated), the open loop load prediction and the parametrized control sequence after line search from the previous iteration. The nonlinear state prediction model and the output function with the input parameters of the matlab function are used to calculate the open loop state and the output trajectories. The mass flow trajectories along each pipeline are also calculated. The open loop pressure and mass flow trajectories of each pipeleg are used to formulate the continuous linear time variant state space model. The state and the output predictor in augmented form is then formulated. The Hessian matrix and the gradient vector of the quadratic programming problem is formulated and the linear inequality constraints are specified. In addition, the upper and the lower limits of the parametrized control trajectory for the current open loop are specified. This quadratic problem have the parametrized control trajectory as optimization variables. The function **quadprog** in Matlab was used to solve the defined quadratic programming problem of a considered iteration of the current open loop nonlinear programming problem. In a real application, an effective commercial solver should be used. The Matlab function **fmincon** was used for the line search in the direction set by the solution from the quadratic program.

4.3 Simulation Results

Simulation data:

The energy price for the compression process was defined to be 0.1 Dollars per Kilowatt Hour and equal for each of the three compressor stations. Fanning's friction factor was set to 0.0025 for each pipeline assuming that the pipelines have been recently cleaned by a pigging operation. The three source pipelines were each set to be 300 kilometers long and the offtake pipelines were each set to be 400 kilometers long. Suction pressures at the inlet of each compressor station was defined to be 60 bars. An average value for the compressibility factor was used and it was set to the value of 0.9 for all the pipelines. The molecular weight of the natural gas was set to be 90e-3 Kilograms per moles for each of the three sources. An average fluid temperature was defined equal for each pipeline, and set to 285 Kelvin. The diameter of the three source pipelines was set to 0.8 meters and 1 meter for the offtake pipelines. The speed of sound was set to 330 meter per second. The Gross calorific value from the three sources was set to 38, 40 and 42 Megajoule per Kilogram respectively. The concentration of carbon dioxide from source one, two and three was set to one, two and three percent respectively. It was assumed that the source composition was constant. It was specified that the gross calorific value should be within a band with a lower limit of 39 Megajoule per kilograms and an upper limit of 41 Megajoule per kilogram. The reference value for the carbon dioxide concentration out from the mixing facility was set to two percent for both mixing points and with a maximum accepted concentration of carbon dioxide set to four percent. Length of the open loop horizon was chosen to be 24 hours and with dimension of the parametrized control sequence equal to 11. Minimum accepted customer terminal pressures was specified to be 50 bars for each terminal and with reference values set to 55 bars to provide the security of the supply. The output vector penalty weight matrix for the scaled problem penalized the output components differently. Customer pressures was penalized with a factor equal to 1000. Differential pressures over each control valve at the mixing facility was penalized with a factor equal to 100. These two categories were largely penalized due to their importance for the security of the supply. The value of the penalty weight for penalizing the difference in the pressures from the defined difference pressure reference between the input and the output of each transmission line was set to 10 allowing for the variation to meet the customer gas demands and to regulate the differential pressures over each control valve. The compression ratios over each compressor station were penalized using a factor of 10 multiplied with the energy cost ratio which was defined to be equal to one for each station in the presented simulation. For the penalty matrix for use of control power, the penalty weight for the mass flow through each control valve was set to 1 allowing a highly transient customer load demand. The weight for penalizing the mass flow through each compressor station was set to a factor equal to 10 and multiplied with the energy cost ratio. A small compression ratio combined with a small mass flow implies low compression cost. The reference values for the differential pressures over each control valve were set to 10 bars and with a lower limit of 5 bars and an upper limit of 50 bars. The reference value for the difference between the input and the output pressures of each pipeline was set to 10 bars with a minimum limit of 5 bars and a maximum limit of 80 bars. The value of the reference vector for the control vector was set to the minimum reference limit. This lower limit vector was set to be close to zero on the positive side. This small positive limit contributes to securing the computational stability of the algorithm when calculating the gradient of the output trajectory with respect to the parametrized control sequence. Of course, in a real application, supply contracts may specify a minimum supply limit from each source. The maximum production capacity from each of the 3 processing facilities was set to 800 kilograms per hour and with a set maximum control valve capacity larger than 1000 kilograms per second secured by a specified minimum differential pressure over each control valve in the mixing facility. Equations for the compression power where equal to those used in Chapter 3. It was assumed that there were three compressors in series at each station to provide enough compression ratio. Isotropic efficiency factors were given as $\eta_s = \{0.8, 0.85, 0.82\}$ and the mechanical efficiency factors were defined as $\eta_m = \{0.97, 0.99, 0.99\}$ for the three compressor stations. Compression cost was calculated from the closed loop operation.

Simulation results:

Figure 4.6 gives the customer pressures and the difference in pressures between the defined input and the defined output of each of the two "offtake" pipelines. The highly transient customer load demands are given in Figure 4.7. Figure 4.8 gives the differential pressures over each control valve at the mixing station. Figure 4.10 gives the compression ratios over each compressor station at the outlets of the production facilities. Figure 4.11 gives the value of the gross calorific value and the concentration of carbon dioxide out from the mixing station into the two pipelines that transport the natural gas to the customer terminals. Mass flow through the compressor stations are given in Figure 4.12. Mass flows through the control valves at the mixing station are given in Figure 4.13. The input and the output pressures of the transmission lines one, two and three are given in Figure 4.14. The input and output pressures of the transmission lines four and five are given in Figure 4.15. The instant compression cost over each compression station and the total instant compression cost are given in Figure 4.16.

Comments to the simulation results:

Figure 4.8 illustrates that the control system keeps the differential pressures over each of the control valves inside the specified constraints. It is especially the minimum pressure that must be held over a specified minimum value so that one has enough flow capacity through each valve and that the flow goes in the desired direction. It is noted that the customer pressures are kept close to the respective references and over the specified minimum terminal pressure limits for most of the simulation horizon even if the distance to the mixing station is 400 kilometers and the mixing station is the closest point where control is possible and the customer demands for natural gas load were highly transient. The defined quality parameters are controlled at the mixing facility so that they are kept inside the feasible region and close to the references. The pressure level of the network is kept at a low level by keeping the customer pressures close to the reference values and the differential pressures between the defined input and the defined output of each pipeline at a low level close to the reference together with a low level of differential pressures over each control valve at the mixing facility. This operation keeps the compression ratios small. We observe the important fact that the controller in closed loop in the simulation was stable also when the load demand was highly transient. The time length of the open loop horizon was chosen equal to 24 hours which seem to be long enough for the controller to take into account the time delay such that the customer terminal pressures can be kept inside the allowed region and the differential pressures over each control valve at the mixing station are inside the allowed region. It is observed in the simulation that the controller was feasible for most of the time. The infeasibility algorithm took over the control command calculations if the open loop problem was infeasible.

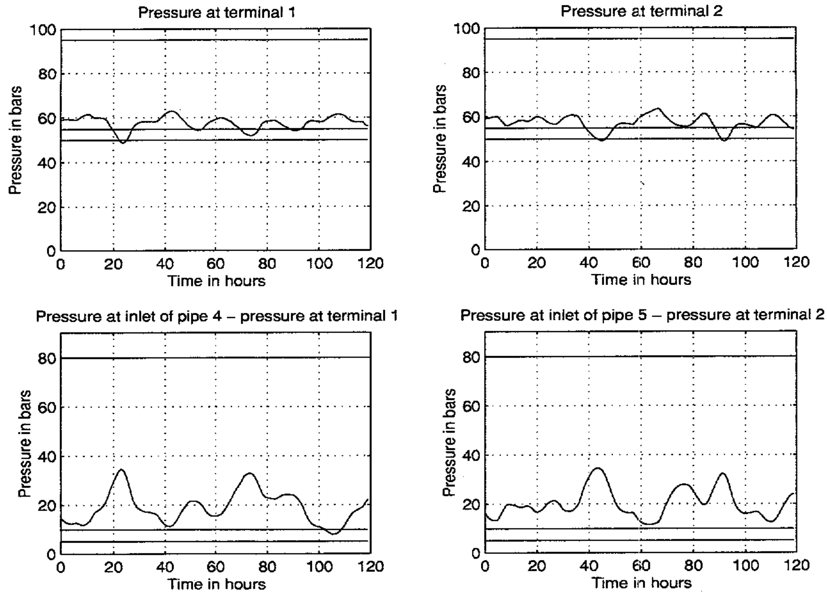


Figure 4.6: Customer terminal pressures and difference between input and output pressures of the offtake pipelines.

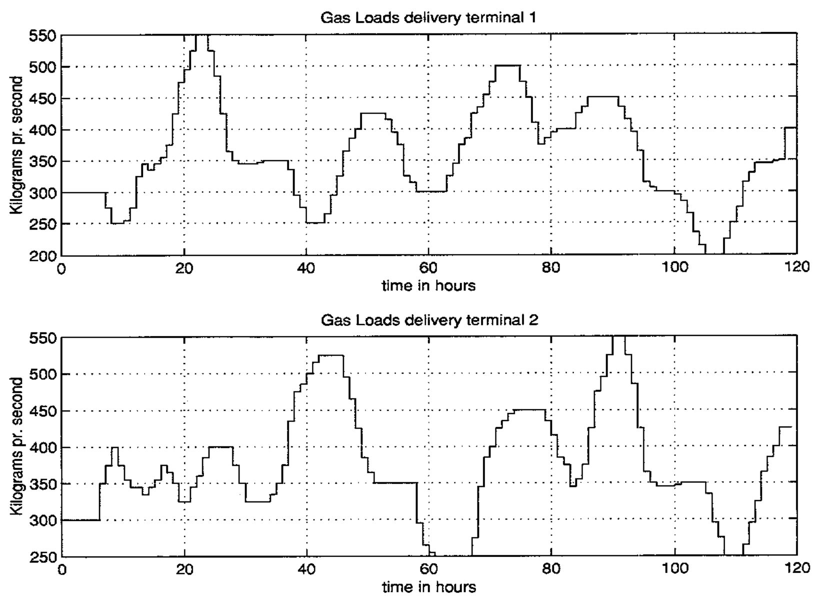


Figure 4.7: Customer load demands.

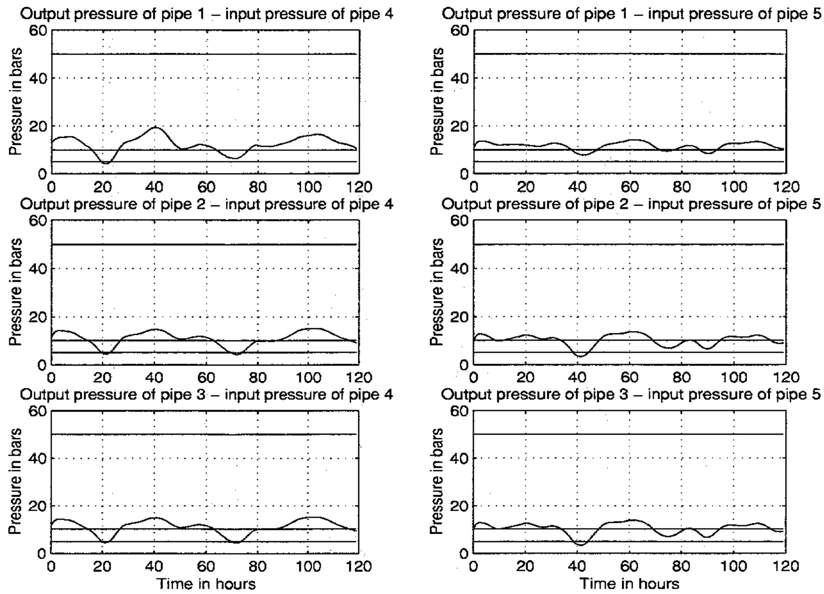


Figure 4.8: Differential pressures over each control valve at mixing station.

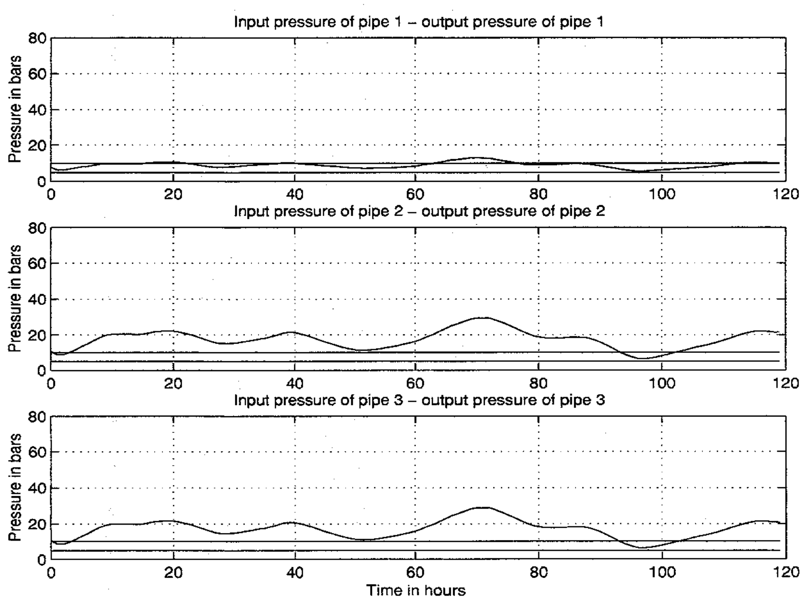


Figure 4.9: Pressure difference between input and output of the three source pipelines.

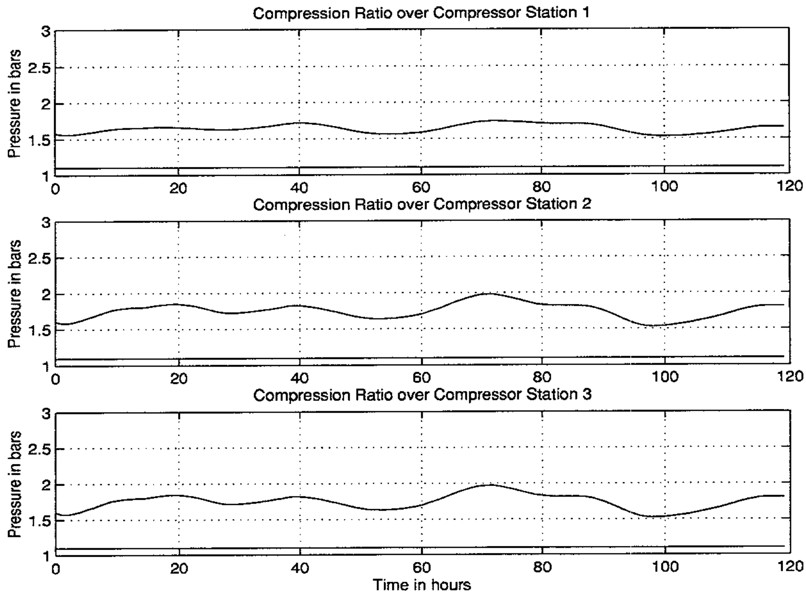


Figure 4.10: Compression ratios.

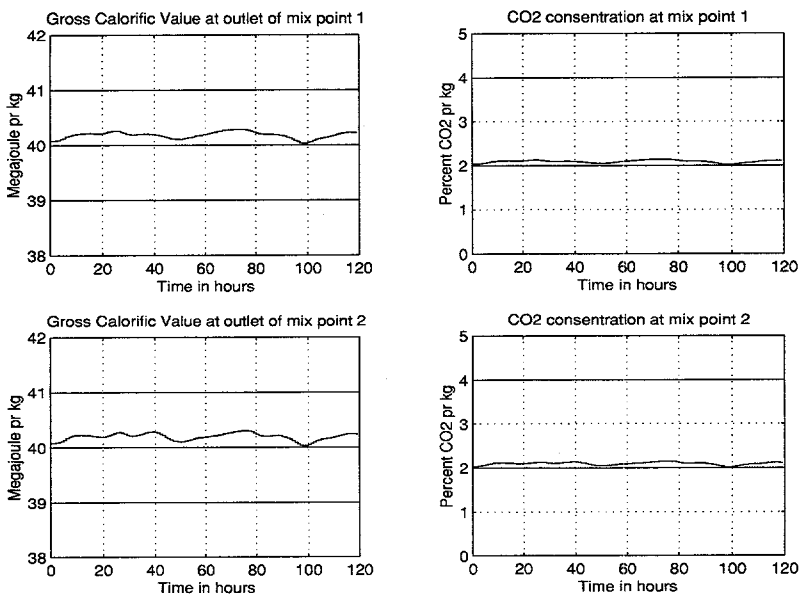


Figure 4.11: Quality parameters out from mixing station.

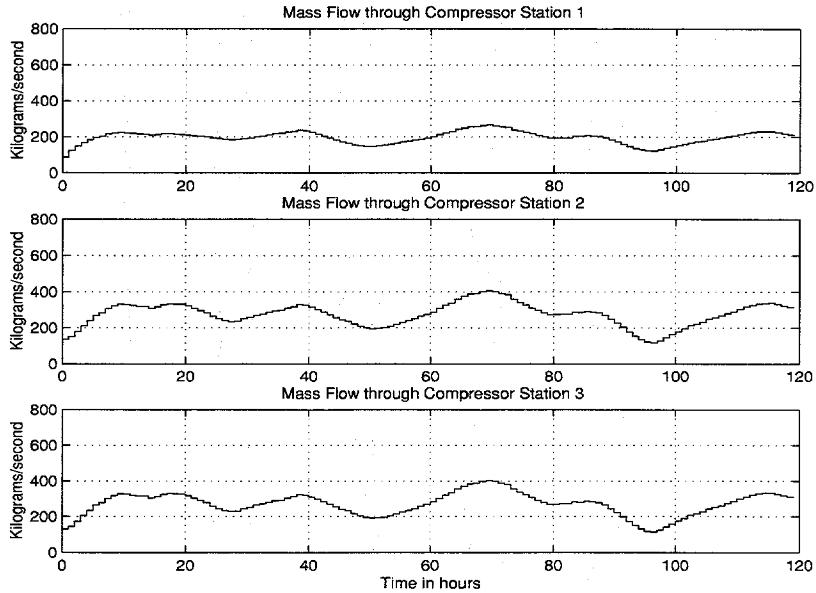


Figure 4.12: Mass flows through compressor stations.

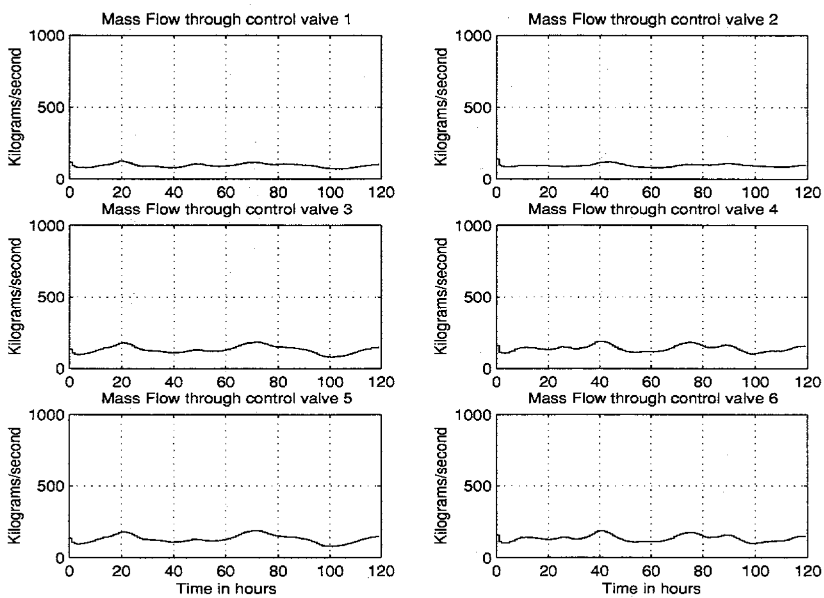


Figure 4.13: Mass flow through control valves at mixing station.

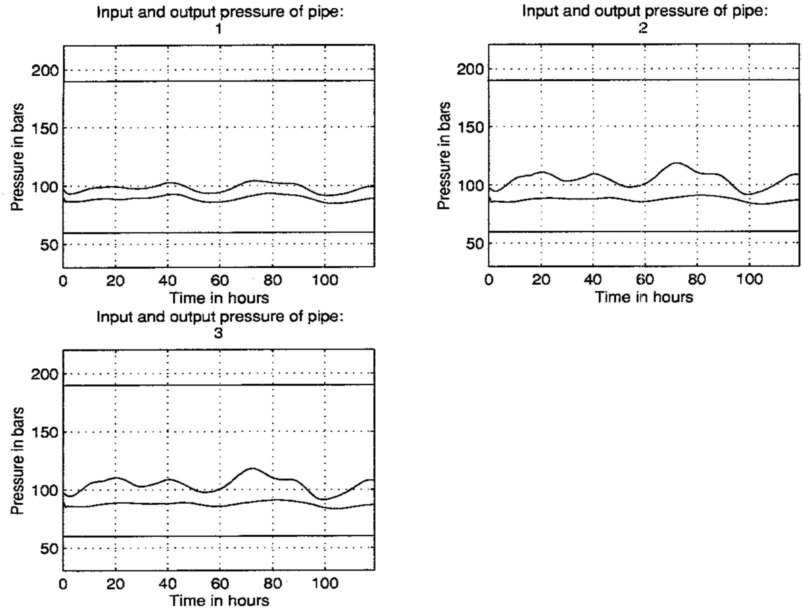


Figure 4.14: Input and output pressures of pipeline one, two and three.

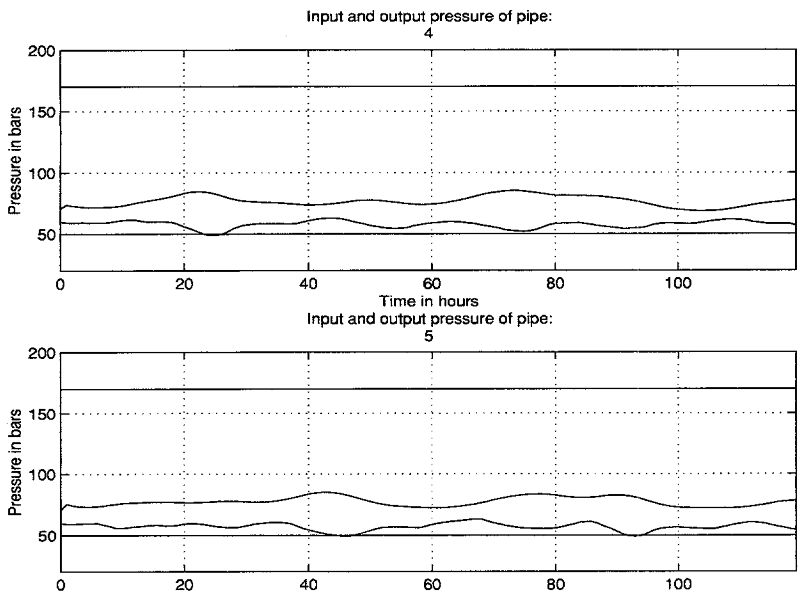


Figure 4.15: Input and output pressure of pipeline 4 and 5.

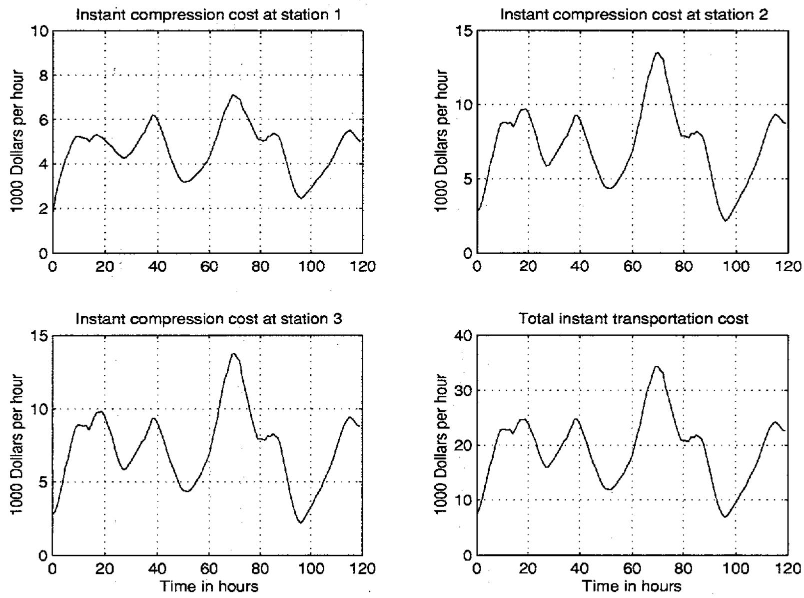


Figure 4.16: Instant compression cost.

4.4 Summary and Discussion

Nonlinear model predictive control is proposed for the security of the supply, the minimization of the transportation costs and the control of quality parameters for long distance natural gas transmission systems providing transportation services.

Each open loop nonlinear programming problem is solved by a structured successive quadratic programming problem of Newton type.

A linear time variant prediction model is developed based on a reformulation of the nonlinear creep flow model and with the parameters in the model given by the use of nominal state and output trajectory for the current optimization iteration. The nominal state and the output trajectory are calculated using the parametrized control trajectory from the previous iteration and the nonlinear open loop prediction model. The linear time variant output predictor is used as a replacement for the nonlinear dynamics in the original nonlinear programming problem yielding a quadratic programming problem. Line search is then performed providing descent properties. The iterative procedure is ended when a defined termination criteria for a local minimum is satisfied or when a maximum specified number of iterations are reached. The first control component of the "optimal" parametrized control sequence is suggested to be implemented to the transmission system. The dispatch personnel takes the final decision on the value of the control command that is to be implemented to the transmission system.

An infeasibility handling algorithm based on soft penalty relaxation of the output constraints is suggested. Another approach discussed is to simply remove the output constraints when the system state is in the infeasibility mode. In this case, the output reference in the objective function will still be a driving force of the closed loop controller to drive the state of the transmission system towards the feasible region.

Supply security at the customer terminals is handled in a suitable manner by putting high penalty weights for the deviations in the customer terminal pressures from their reference values compared to penalty weights for the other components of the output vector. Also, the time length each of open loop horizon must be large enough so that the model predictive controller can compensate for the time delay effects. Precise load forecasts are also necessary for successful operation if one wants security of the supply while minimizing the pressure level in the transmission system. Setting the values of the terminal pressure minimum limits and the reference values for the terminal pressures in the controller slightly higher than the contracted minimum limits provides extra quick access to natural gas resources at the customer points. The transportation costs will therefore increase, but the security of the supply is the main priority. The control algorithm handles time varying natural gas source capacities by the inclusion of capacity predictions defining a time varying control space for the model predictive controller. As explained in Chapter 3, sources with large flow capacity can compensate for the lack of supply for the sources that face reduced capacity so that the customers need for gas is still satisfied.

Simulation results show encouraging results regarding the closed loop stability and the performance of the controller with respect to the security of the supply, the control of quality parameters and the minimization of transportation costs.

On-line computational effort is larger for the nonlinear model predictive controller than for the linear model predictive controller proposed in Chapter 3. The reason for this is the increased computational load due to the sequential quadratic programming procedure. First, a nonlinear prediction and a quadratic programming problem is specified. Then, the specified programming problem is solved followed by a line search. This procedure is repeated for each iteration of the sequential procedure in the search for a local minimum.

The open loop problem was scaled to improve the numerical properties of each quadratic programming problem.

Usually, the gas control centre has access to a complex on-line simulator. The closed loop controller described in this chapter may use pressures received from this simulator as the estimated states defining the initial condition for each open loop problem. Alternatively, one of the state estimators or observers referred to or suggested in Chapter 3 may be used.

The model predictive controller can take account of the different compression costs at each compressor station and the source costs at each source by the choice of the values of the penalty matrices of the control and output vector in the quadratic cost function of the open loop programming problems. Different priorities to maintaining the customer pressures at the customer terminals can also be defined in these matrices. The same goes for the quality parameters where some of the parameters may be considered to have higher priority than others, when it comes to staying close to their defined references. It is also possible to define a priority list for the control valves and the pipelines in the transmission system for the pressure differences over the control valves and the pressure levels and the difference between the input and the output pressure of each transmission line.

A stationary optimization as in Chapter 3 can be used for designing an input and output reference. Then, the nonlinear model predictive controller's main goal is to reach or to keep the value of the output vector and the input vector close to this equilibrium point. The stationary optimization may be based on a simple stationary flow description or it may use a flow description that uses mass, momentum and energy equation together with a state equation. If a more complex flow description is used in the optimization formulation, the solution from this optimization problem, using a local or global solver, can be used to include complex stationary thermodynamics into the open loop nonlinear prediction model and the linear time variant prediction model for a possibly more accurate system description. The stationary optimization may also include discrete variables as commented in chapter 3.

State constraints can be added to the output constraints. A larger number of constraints will then increase the computational effort.

The time length of the open loop horizons must be chosen so that the delay effects due to long distance transmission lines is taken account of just as for the model predictive controller in Chapter 3. Unfortunately, long prediction horizons increase the computational effort.

If the compositions from the production sources are time varying, then a quality tracker must be used to predict the values of the considered quality parameters entering the inlets of the mixing facilities and the junction points of the transmission system. A simple time delay quality tracker as suggested in Chapter 2 may be used to assist the control of the quality parameters.

It is possible to simplify the problem of quality control. We may assume that the values of the quality parameters of the natural gas from the different transmission lines connected to the inlet pipelines of a mixing station are constant. Further, one may assume that these constant values are equal to the defined reference values of the quality parameters at the outlet sides of the control points (junction points and mixing stations) that are connected with pipelines directly to the inlet points of the considered mixing station. The nonlinear model predictive controller is at each mixing station concerned with the objective to control the quality parameters towards the respective reference values.

Chapter 5

Model Based Control & Dynamic Analysis of Transmission Systems

5.1 Introduction

In Chapter 3, a linear control model was used as a basis to develop a controller for a gas transmission network. This control model was derived from the nonlinear creep flow model given in Chapter 2. In Section 5.2, the creep flow model will be used as a basis to formulate a nonlinear control model. It will be shown that this control model can also be used to design a model predictive controller. For simplicity, only a single gas transmission line is considered. The partial differential equation formulated in this section will be further used throughout this chapter as a basis for a distributed parameter control model for both a single gas transmission line and for the formulation of a transmission network. The creep flow model is also a basis for the control model in Section 5.3 which is used to design a model predictive controller based on instant linearization of the creep flow model for a single gas transmission line. The purpose of section 5.2 and Section 5.3 is to show that adequate controllers can be developed based on alternative different reformulations of the nonlinear creep flow model which is used as a basis for the control, optimization and simulation models in this thesis.

The rest of this chapter, will for the most of the time, be concerned with distributed parameter control models or simplified models developed from the behaviour of the distributed parameter control model.

In Section 5.4, a distributed parameter control model is formulated for a gas transmission line. The analytic solution of this model is calculated. Then, a transfer function representation of the dynamics is given. The control model is nonlinear in the physical variables pressures and flows but has a linear inner core.

In Section 5.5, a simple control system for a single gas transmission line with

Neuman boundary conditions is designed. The control system is a combination of a Smith controller and a feedforward from the predicted load demand. Based on the qualitative behaviour of the step responses of the distributed parameter control in Section 5.4, a simple transfer function control model is developed.

In Section 5.6, a distributed parameter model with both boundary control and load together with supplies and offtakes along the transmission line is formulated. Then, the mild analytic solution is given for the formulated control model.

In Section 5.7, it is shown how a nominally exponentially stable linear distributed parameter controller in combination with a distributed parameter state observer can be designed for the operation of a natural gas transmission line. The feedback control command may be combined with a feedforward from the predicted load demand as treated in Section 5.5. The linear quadratic optimal control problem is also defined.

In Section 5.8, a distributed parameter network model formulation is derived. The boundary conditions are all zero in this formulation. Each junction point where pipelines intersect are approximated by a control volume and a differential equation. The transmission lines connected to a junction point are connected through Robin boundary conditions. These boundary conditions are approximated by distributed operators that are part of the partial differential equations describing the pipeline dynamics. An expression for the operator that expresses the dynamics and couplings is obtained. This operator is divided into a "diagonal" operator and a "coupling" operator. The "diagonal" operator generates a strongly continuous semigroup. Use of an operator integral equation gives an expression for the semigroup for the total linear system operator. The state solution is given as the solution to the inhomogenous Cauchy problem. An open loop mathematical programming problem to be used by a linear model predictive control scheme is formulated in Section 5.9. It is assumed that the dynamic model derived in Section 5.8 is used.

5.2 MPC using Model Linear in Squared Pressures and Mass Flows

Figure 5.1 illustrates a transmission system with a compressor station receiving natural gas from a processing facility and a single transmission line delivering gas to a customer point.

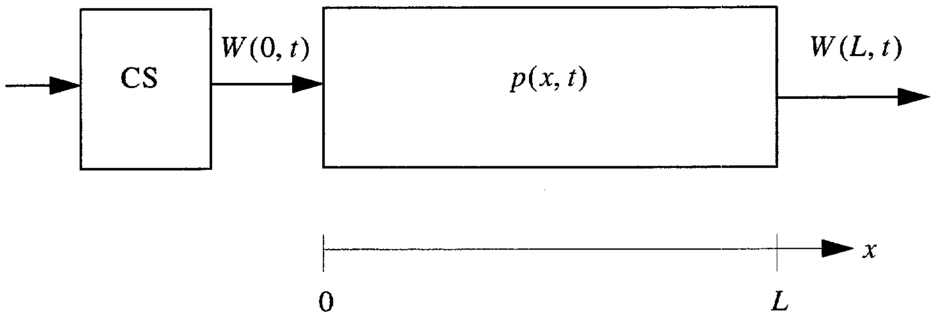


Figure 5.1: Transmission system example.

Recall the nonlinear creep flow model in Chapter 2 repeated in Eqn. (5.1) and Eqn. (5.2).

$$\frac{A}{c^2} \cdot \frac{\partial p}{\partial t} = -\frac{\partial W}{\partial x} \tag{5.1}$$

$$\frac{\partial p}{\partial x} = -\frac{2 f \rho \upsilon^2}{D} \tag{5.2}$$

Given the mass flow which is denoted by W and the normal conditions

$$p_{\mathrm{n}} = 1 \cdot 10^5 \mathrm{N/m^2}$$

and

$$T_{\mathrm{n}} = 288\mathrm{K},$$

the volumetric flow under these conditions is given in Eqn. (5.3).

$$Q_{\mathrm{n}} = \frac{Z \cdot \dfrac{R}{MW} \cdot T_{\mathrm{n}}}{p_{\mathrm{n}}} \cdot W \tag{5.3}$$

Then, the mass flow is expressed as in Eqn. (5.4)

$$W = \frac{p_{\mathrm{n}}}{c_{\mathrm{n}}^2} \cdot Q_{\mathrm{n}} = \rho_{\mathrm{n}} \cdot Q_{\mathrm{n}} = \rho \cdot Q \tag{5.4}$$

where the normal density ρ_{n} is constant. Since we have the relation in Eqn. (5.5)

$$\frac{\partial}{\partial x} \rho_{\mathrm{n}} Q_{\mathrm{n}} = \rho_{\mathrm{n}} \cdot \frac{\partial Q_{\mathrm{n}}}{\partial x}, \tag{5.5}$$

the result in Eqn. (5.6) follows

$$\frac{\partial p}{\partial t} = -\frac{c^2 \rho_{\mathrm{n}}}{A} \cdot \frac{\partial Q_{\mathrm{n}}}{\partial x}. \tag{5.6}$$

Inserting the relation

$$\upsilon^2 = \frac{W^2}{\rho^2 \cdot A^2} = \frac{\rho_{\mathrm{n}}^2 \cdot Q_{\mathrm{n}}^2}{\rho^2 \cdot A^2}$$

into Eqn. (5.2) yields

$$\frac{\partial p}{\partial x} = -\frac{2 f \rho_{\mathrm{n}}^2 c^2 Q_{\mathrm{n}}^2}{D A^2 p}. \tag{5.7}$$

Rearranging, we get Eqn. (5.8).

$$\frac{\partial p^2}{\partial x} = -\frac{4 f \rho_{\mathrm{n}}^2 c^2 Q_{\mathrm{n}}^2}{D A^2} \tag{5.8}$$

Differentiating Eqn. (5.8) with regards to x yields Eqn. (5.9).

$$\frac{\partial^2 p^2}{\partial x^2} = -\frac{4 f \rho_n^2 c^2 Q_n}{DA^2} \cdot \frac{\partial Q_n}{\partial x} \tag{5.9}$$

Use of the relation in Eqn. (5.10)

$$-\frac{A}{c^2 \rho_n} \cdot \frac{\partial p}{\partial t} = \frac{\partial Q_n}{\partial x} \tag{5.10}$$

into Eqn. (5.9) yields Eqn. (5.11).

$$\frac{\partial^2 p^2}{\partial x^2} = \frac{4 f \rho_n Q_n}{DA} \cdot \frac{\partial p}{\partial t} \tag{5.11}$$

Then, use of Eqn. (5.12)

$$\rho_n Q_n = \rho Q = \frac{p}{c^2} \cdot Q, \tag{5.12}$$

gives the biquadratic model in Eqn. (5.13).

$$\frac{\partial p^2}{\partial t} = \frac{DAc^2}{4fQ} \cdot \frac{\partial^2 p^2}{\partial x^2} \tag{5.13}$$

Use of $W = \rho_n \cdot Q_n$ into Eqn. (5.8) yields Eqn. (5.14).

$$\frac{\partial p^2}{\partial x} = -\frac{4fc^2}{DA^2} \cdot W^2 \tag{5.14}$$

The above procedure was similarly done in Osciadacz (1987). Eqn. (5.14) defines the Neuman boundary conditions at the defined input and the defined output of the considered transmission line. Denote W^* to be the calculated optimal stationary mass flow and p^* to be the "optimal" stationary pressure at a given position along the pipeline. Use of $W^* = \rho^* Q^*$ and $p^* = c^2 \rho^*$ into Eqn. (5.13) yields Eqn. (5.15).

$$\frac{\partial p^2}{\partial t} = \frac{DAp^*(x)}{4fW^*} \cdot \frac{\partial^2 p^2}{\partial x^2} \tag{5.15}$$

The stationary pressure distribution function is calculated as in Chapter 3 where also the energy equation and the use of a complex state equation may be included in the stationary optimization.

The state vector is defined to be the deviation in squared pressures from a defined stationary operation point in squared pressures along the transmission pipeline. The control vector u is defined to be the deviation in squared mass flow from the squared mass flow reference resulting from the stationary operation. The load vector v is the deviation in squared mass flow from the current defined average load squared.

Use of the numerical method of lines, as in Chapter 3, yields a state space model linear in squared deviation pressures and squared deviation mass flows at input and output boundary of the transmission line. The state space model is given in Eqn. (5.16)

$$\dot{\tilde{x}} = A\tilde{x} + B\tilde{u} + C\tilde{v} \tag{5.16}$$

where the tilde indicates deviation variables. Integrating Eqn. (5.16) when assuming that the load demand and the control command are constant for an entire defined sampling interval yields Eqn. (5.17).

$$\tilde{x}_{k+i+1|k} = \Phi\tilde{x}_{k+i|k} + \Gamma\tilde{u}_{k+i|k} + \Psi\tilde{v}_{k+i|k} \tag{5.17}$$

The open loop control problem is defined in Eqn. (5.18). The defined state and control vector are penalized quadratically for the deviation from a defined equilibrium point.

$$\begin{aligned}
&\underset{\{\tilde{x}_{k+i+1},\, \tilde{u}_{k+i|k}\}_{i=0}^{N-1}}{\text{minimize}} && \frac{1}{2}\sum_{i=0}^{N-1} \tilde{x}_{k+i+1|k}^{\mathrm{T}} Q \tilde{x}_{k+i+1|k} + \tilde{u}_{k+i|k}^{\mathrm{T}} R \tilde{u}_{k+i|k} \\
&\text{subject to} && \\
&\tilde{x}_{k+i+1|k} = \Phi\tilde{x}_{k+i|k} + \Gamma\tilde{u}_{k+i|k} + \Psi\tilde{v}_{k+i|k} && 0 \le i \le N-1 \\
&\underline{\tilde{x}} \le \tilde{x}_{k+i+1|k} \le \bar{\tilde{x}} && 0 \le i \le N-1 \\
&\underline{\tilde{u}}_{k+i|k} \le \tilde{u}_{k+i|k} \le \bar{\tilde{u}}_{k+i|k} && 0 \le i \le N-1
\end{aligned} \tag{5.18}$$

We note that the constraint limits are also in deviation variables squared using the operation point in squared pressures and squared mass flows and the use of the upper and lower pressure and flow constraint limits. The source space may also be time varying due to the possible time varying production availability as discussed in Chapter 3 and Chapter 4. As usual, for model predictive controllers, only the first control step from the optimal determined control sequence calculated for the current open loop problem is applied to control the plant for the next sampling interval.

The optimal control command applied to control the fluid state of the transmission line denoted by $u_{k|k}^{\mathrm{opt}}$ is then given in Eqn. (5.19).

$$u_{k|k}^{\mathrm{opt}} = \sqrt{\tilde{u}_{k|k}^{\mathrm{opt}} + {u^*}^2} \tag{5.19}$$

The control law for state and control constrained linear model predictive controllers is nonlinear (Muske & Rawlings, 1993;Muske, 1995). Using the linear model in squared pressures and squared mass flows makes the control law even more nonlinear. A simple infeasibility handler was designed by removing the state constraints if the open loop control problem is in the infeasible mode resulting in a control constrained model predictive controller. The change in the control command between each discrete time instant of the open loop is penalized quadratically as in Eqn. (5.20) for the parametrized open loop control sequence.

$$\sum_{i=0}^{M-1} (u_{k+i+1|k} - u_{k+i|k})^{\mathrm{T}} R_{\Delta \mathrm{u}} (u_{k+i+1|k} - u_{k+i|k}) \tag{5.20}$$

Simulations will be performed with two pipelines of lengths 40 kilometers and 100 kilometers. The rest of the data is given in Eqn. (5.21).

$$\begin{aligned} f &= 0.0025 & & \\ MW &= 80\times10^{-3} & & \text{(kilograms)/(mole)} \\ D &= 0.8 & & \text{meters} \\ Z &= 1 & & \\ T &= 285 & & \text{Kelvin} \end{aligned} \tag{5.21}$$

Figure 5.2 and Figure 5.3 yield the transient response for the supply and the customer pressure and the supply flow and the demand flow in closed loop simulation for a pipeline of length 40 kilometers.

Figure 5.4 and Figure 5.5 yields the transient response for the supply and the customer pressure and the supply and the demand flow in a closed loop simulation

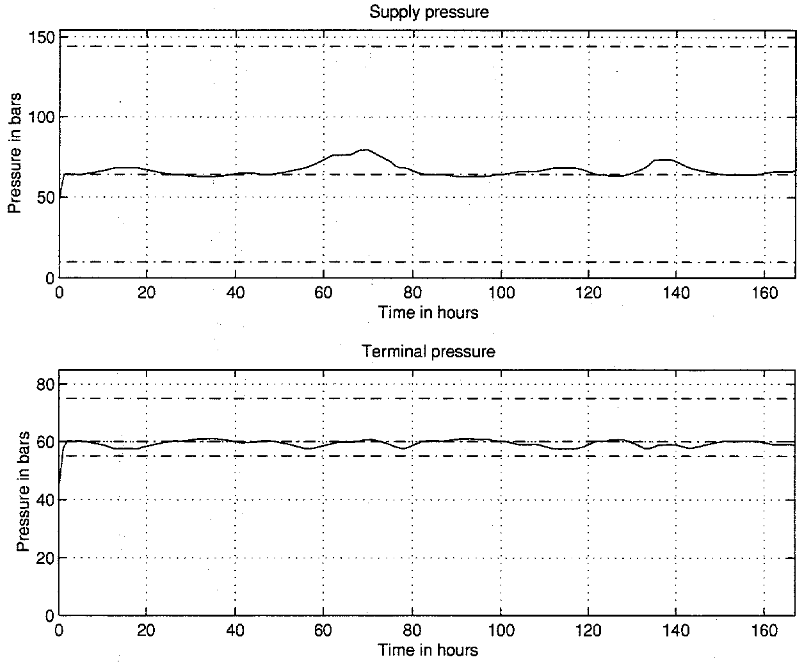


Figure 5.2: Supply and customer pressure.

for a pipeline of length 100 kilometers. It is seen that the time delay effects are becoming more visible for the 100 kilometers pipeline than for the 40 kilometers case. It is observed that the model predictive controller compensates for the time delay.

In order to meet a highly transient load pattern and to keep the customer pressure close to its reference it is necessary to use a high dimension of the parametrized open loop control sequence. By increasing the state penalty when approaching the customer point is also very important. The model predictive controller was clearly stable in the above simulations with good performance with regards to keeping the terminal pressure close to its reference.

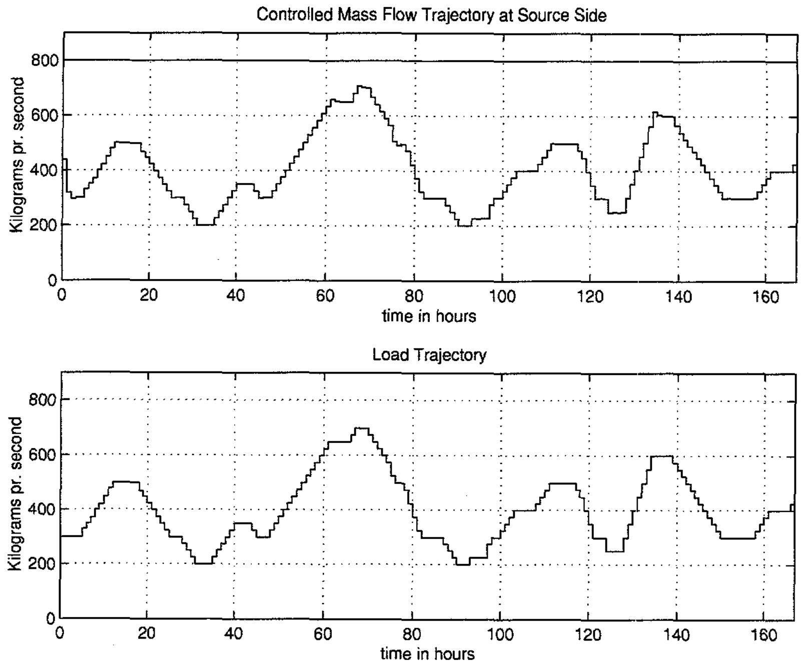


Figure 5.3: Supply and demand flow.

5.3 MPC using Instant Linear Prediction Model

Recall the reformulated creep flow model in Eqn. (3.66) of Chapter 3 repeated for convenience in Eqn. (5.22).

$$\begin{aligned} \frac{\partial p}{\partial t} &= \frac{c^2}{A\lambda} \cdot \frac{\partial^2 p}{\partial x^2} \\ W &= -\frac{1}{\lambda} \cdot \frac{\partial p}{\partial x} \end{aligned} \tag{5.22}$$

where λ is given in Eqn. (5.23).

$$\lambda = \frac{2fc^2|W|}{DA^2 p} \tag{5.23}$$

At the current sampling instant, we have, the instant state of the transmission line

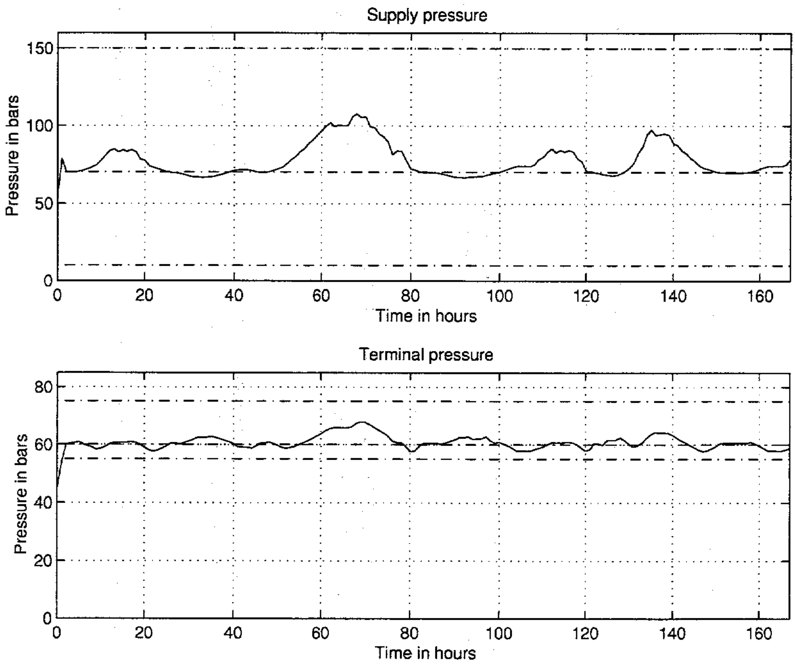


Figure 5.4: Supply and customer pressure.

from the complex simulator available to the dispatch personnel. The instant mass flow and the pressure distribution along the pipeline is inserted into Eqn. (5.22) and Eqn. (5.23). Numerical method of lines as in Chapter 3 is used to give a linear continuous state space model for the current open loop horizon. By assuming that the load and the control trajectory are piecewise constant and then integrating for a sampling instant yields the discrete prediction model in Eqn. (5.24).

$$x_{k+i+1|k} = \Phi x_{k+i|k} + \Gamma u_{k+i|k} + \Psi v_{k+i|k} \tag{5.24}$$

The open loop problem is defined in Eqn. (5.25). N is the number of discrete time steps in the open loop horizon. As noted, the source space may be time varying as discussed in Chapter 3 and Chapter 4.

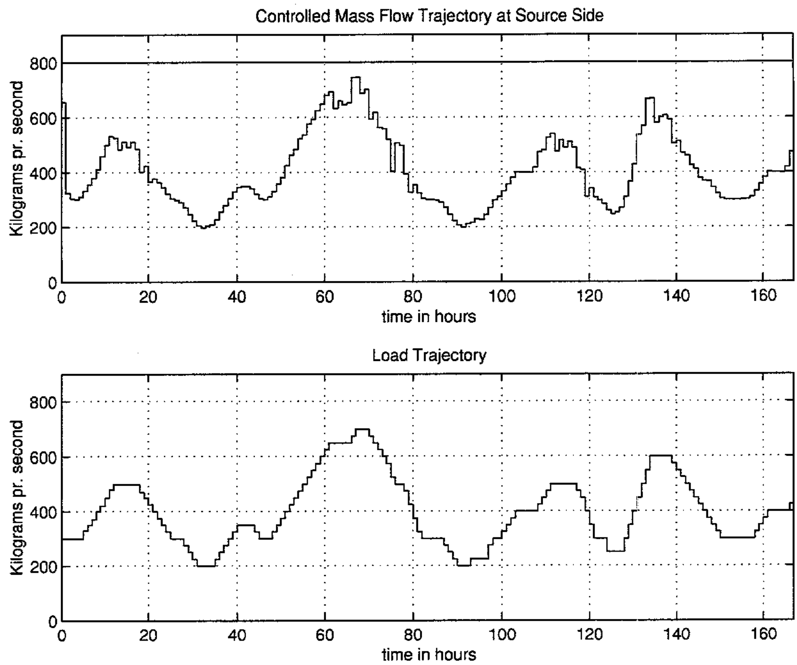


Figure 5.5: Supply and demand flow.

$$\begin{aligned}
&\underset{\{x_{k+i+1},\, u_{k+i|k}\}_{i=0}^{N-1}}{\text{minimize}} \quad \frac{1}{2}\sum_{i=0}^{N-1}\left((x_{k+i+1|k}-x^*)^{\mathrm{T}}Q(x_{k+i+1|k}-x^*)+(u_{k+i|k}-u^*)^{\mathrm{T}}R(u_{k+i|k}-u^*)\right)\\
&\text{subject to}\\
&\qquad x_{k+i+1|k} = \Phi x_{k+i|k}+\Gamma u_{k+i|k}+\Psi v_{k+i|k} \qquad 0\le i\le N-1\\
&\qquad \underline{x}\le x_{k+i+1|k}\le \bar{x} \qquad 0\le i\le N-1\\
&\qquad \underline{u}_{k+i|k}\le u_{k+i|k}\le \bar{u}_{k+i|k} \qquad 0\le i\le N-1
\end{aligned} \tag{5.25}$$

The weight matrices are chosen so that they satisfy $Q > 0$ and $R > 0$. The control trajectory is parametrized just as in Chapter 3 and Chapter 4. The quadratic penalty for the change in the control command was also added to the objective function just as for the model predictive controller in Section 5.2 to provide a smooth control trajectory.

Figure 5.6 and Figure 5.7 illustrates the closed loop performance for the model predictive controller using the instant linearized model for a pipeline of length 40 kilometers. The pipeline data are given in Eqn. (5.21). The same load pattern as in Section 5.2 was used in the simulations. We see that the terminal pressure was kept close to the reference and the pressures were above the minimum pressures for the complete simulation horizon. The controller was clearly stable in the simulations. The nonlinear creep flow model was used as the simulation model. Figure 5.8 and 5.9 yields the transient response and the controller performance for a transmission line of length 100 kilometers. In addition, the system starts initially from a highly depacked infeasibility situation. It is observed that the terminal pressure converges quickly to the feasible region and its reference and the customer pressure is kept close to the defined reference during the complete simulation period under the highly transient load pattern. It is seen that the closed loop controller is stable in the simulation.

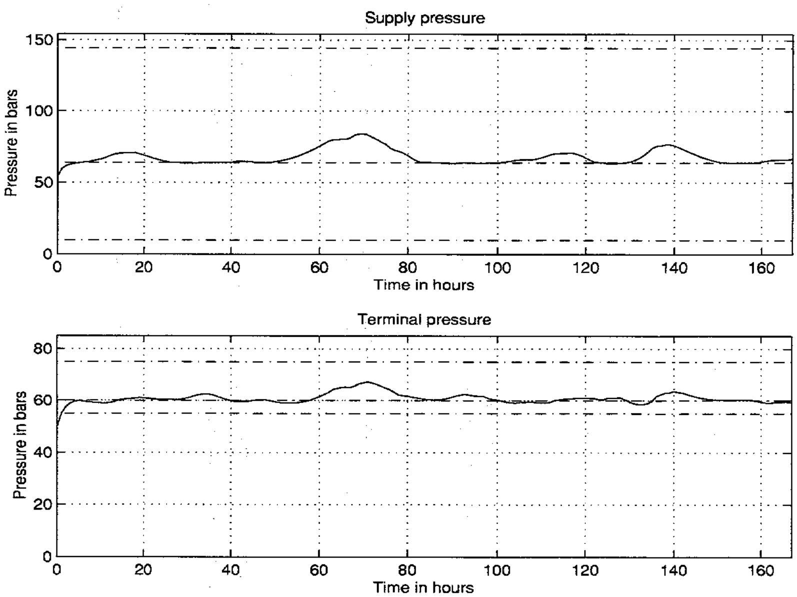


Figure 5.6: Supply and customer terminal pressure.

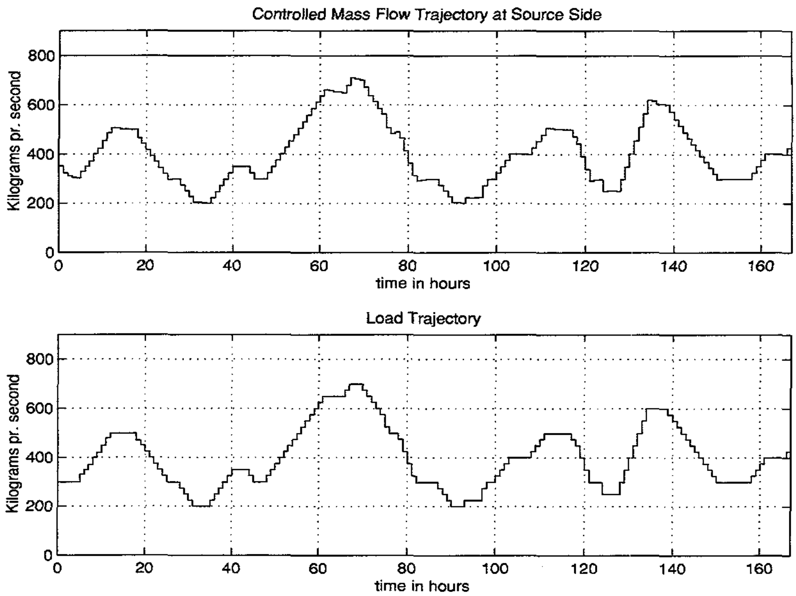


Figure 5.7: Supply and demand flow trajectories.

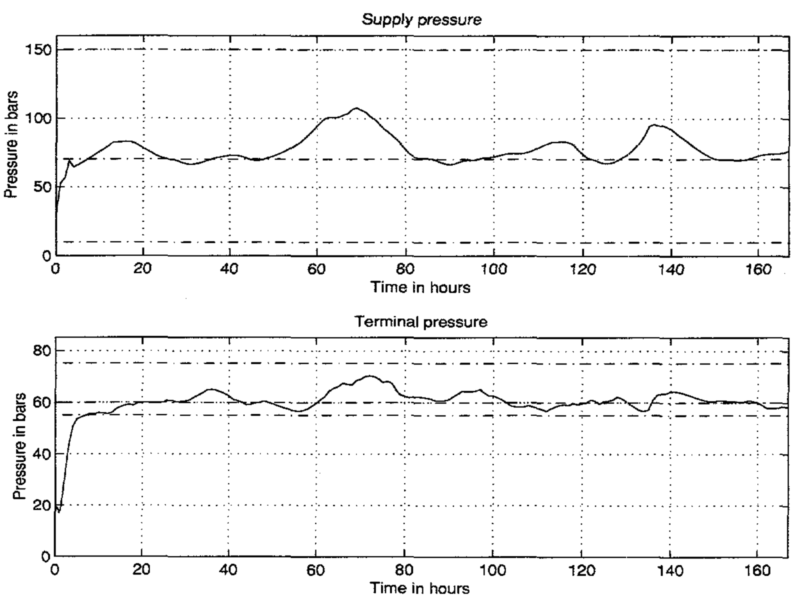


Figure 5.8: Supply and customer terminal pressure.

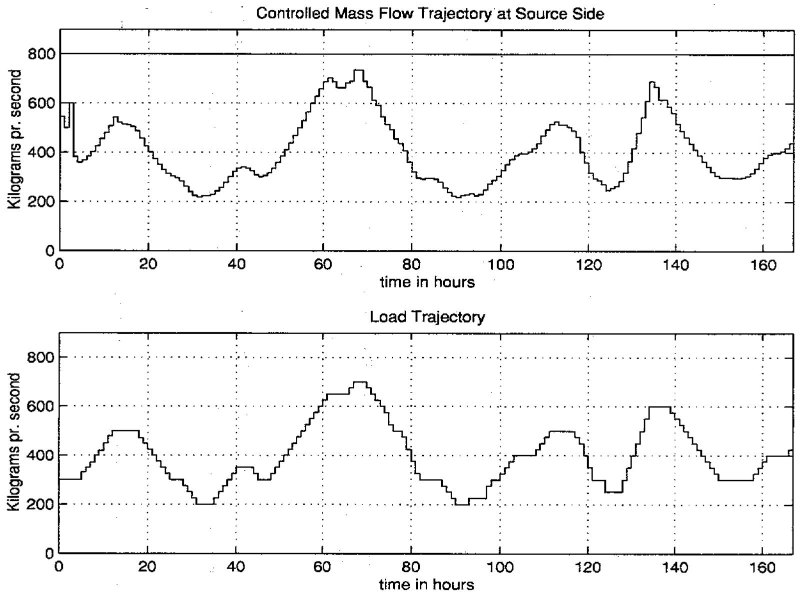


Figure 5.9: Supply and load trajectories.

5.4 Control Model with Neuman Boundary Conditions

In this section, an analytic solution to a distributed parameter control model of a transmission line will be calculated. This model is then Laplace transformed to provide a transfer function model description of the pipeline dynamics.

Using Eqns. (5.13) and (5.14) and changing from volumetric flow in Eqn. (5.13) to mass flow in the same equation yields the model defined in Eqn. (5.26)

$$\begin{aligned}
\frac{\partial}{\partial t}p^2(x,t) &= \frac{DAp(x,t)}{4fW(x,t)} \cdot \frac{\partial^2}{\partial x^2}p^2(x,t) \\
\frac{\partial}{\partial x}p^2(0,t) &= -\frac{4fc^2}{DA^2} \cdot W^2(0,t) \\
\frac{\partial}{\partial x}p^2(L,t) &= -\frac{4fc^2}{DA^2} \cdot W^2(L,t) \\
p^2(x,t) &= p_0^2(x)
\end{aligned} \tag{5.26}$$

where $0 \le x \le L$. Also, we assume that the friction factor, the speed of sound and the cross section area are constant along the transmission line. Mass flow supplied at the defined inlet side is the physical control variable and mass flow at the defined outlet of the transmission line is the customer natural gas offtake. Assume that a stationary optimization has been performed. Then, a stationary pressure distribution function $p^*(x)$, $0 \le x \le L$ is defined together with a stationary mass flow W^*. The control model will be developed around a defined stationary operation point. Define the average model coefficient α^2 in Eqn. (5.27)

$$\alpha^2 = \frac{1}{L} \cdot \int_0^L \frac{DAp^*(x)}{4fW^*}dx \tag{5.27}$$

and β in Eqn. (5.28)

$$\beta = \frac{4fc^2}{DA^2}. \tag{5.28}$$

Then, we have the model in Eqn. (5.29).

$$
\begin{aligned}
\frac{\partial}{\partial t}p^2(x,t) &= \alpha^2 \cdot \frac{\partial^2}{\partial x^2}p^2(x,t) \\
\frac{\partial}{\partial x}p^2(0,t) &= -\beta \cdot W^2(0,t) \\
\frac{\partial}{\partial x}p^2(L,t) &= -\beta \cdot W^2(L,t) \\
p^2(x,t) &= p_0^2(x)
\end{aligned}
\tag{5.29}
$$

Now, subtract the defined stationary operation point from the model defined in Eqn. (5.29) so that Eqn. (5.30) is obtained.

$$
\begin{aligned}
\frac{\partial}{\partial t}p^2(x,t) - 0 &= \alpha^2 \cdot \frac{\partial^2}{\partial x^2}p^2(x,t) - \alpha^2 \cdot \frac{\partial^2}{\partial x^2}p{*}^2(x) \\
\frac{\partial}{\partial x}p^2(0,t) - \frac{\partial}{\partial x}p{*}^2(0) &= -\beta \cdot W^2(0,t) - \beta \cdot W{*}^2 \\
\frac{\partial}{\partial x}p^2(L,t) - \frac{\partial}{\partial x}p{*}^2(L) &= -\beta \cdot W^2(L,t) - \beta \cdot W{*}^2 \\
p^2(x,0) - p{*}^2(x) &= p_0^2(x) - p{*}^2(x)
\end{aligned}
\tag{5.30}
$$

Define the state variable $y(x,t)$ in Eqn. (5.31)

$$
y(x,t) = p(x,t)^2 - p{*}(x)^2, \tag{5.31}
$$

the control variable $u(t)$ in Eqn. (5.32)

$$
u(t) = -\beta \cdot (W^2(0,t) - W{*}^2) \tag{5.32}
$$

and the disturbance variable $v(t)$ in Eqn. (5.33).

$$
v(t) = -\beta \cdot (W^2(L,t) - W{*}^2) \tag{5.33}
$$

Using the definitions in Eqns. (5.31)-(5.33) into Eqn. (5.30) yields the boundary control model in Eqn. (5.34).

$$
\begin{aligned}
\frac{\partial}{\partial t}y(x,t) &= \alpha^2 \cdot \frac{\partial^2}{\partial x^2}y(x,t) \qquad 0 \le x \le L \\
\frac{\partial}{\partial x}y(0,t) &= u(t) \\
\frac{\partial}{\partial x}y(L,t) &= v(t) \\
y(x,0) &= y_0(x)
\end{aligned}
\tag{5.34}
$$

The control model is revised if a new stationary operation point is defined. The supply mass flow is calculated as in Eqn. (5.35).

$$
W(0,t) = \sqrt{-\frac{DA^2}{4fc^2} \cdot u(t) + W*^2} \tag{5.35}
$$

Given a customer mass flow, the disturbance variable v is calculated as in Eqn. (5.36).

$$
v(t) = -\frac{4fc^2}{DA^2} \cdot (W(L,t)^2 - W*^2) \tag{5.36}
$$

Given an initial condition function $y(x,0) = y_0(x)$ and assuming that the control command satisfy $u = 0$ and the disturbance variable satisfy $v = 0$, the autonomous solution is found using the method of separation of variables and is given in Eqn. (5.37)

$$
\begin{aligned}
y(x,t) &= \frac{1}{L} \cdot \int_0^L y_0(x)dx \\
&+ \sum_{n=1}^{\infty} e^{-\left(\frac{n\pi\alpha}{L}\right)^2 \cdot t} \cdot \frac{2}{L} \cdot \int_0^L y_0(z) \cdot \cos\left(n\pi \cdot \frac{z}{L}\right)dz \cdot \cos\left(n\pi \cdot \frac{x}{L}\right)
\end{aligned}
\tag{5.37}
$$

where $0 \le x \le L$. The eigenvalues are $\lambda_n = -\left(\frac{n\pi\alpha}{L}\right)^2, n \ge 0$. For long distance transmission lines the eigenvalues get small since we have $1/L^2$. This implies that the time for the state to settle to an equilibrium point is large. So, if the friction is large and the transmission line is long, the eigenvalues get small implying slow dynamics. An orthonormal basis is given by $\phi_n(x) = \sqrt{\frac{2}{L}}\cos\left(n\pi \cdot \frac{x}{L}\right), n \geq 1$ and $\phi_0(x) = \frac{1}{\sqrt{L}}$ since $\langle \phi_n, \phi_m \rangle = 1$ when $n = m, \forall n, n \geq 0$ and $\langle \phi_n, \phi_m \rangle = 0$ when $n \neq m, \forall n, m \geq 0$. The functions $\phi_n(x)$ can be interpreted as eigenfunctions in a similar way as for the nontrivial solutions of the Sturm-Liouville problem. The pressure distribution along the pipeline is calculated as in Eqn. (5.38).

$$p(x, t) = \sqrt{y(x, t) + p^*(x)^2}, 0 \leq x \leq L. \tag{5.38}$$

The autonomous system can be written in the abstract form as a differential equation on a separable complex Hilbert space $Y = L_2(0, L)$ being the state space as in Eqn. (5.39)

$$\begin{aligned} \frac{d}{dt}y(t) &= \mathcal{A}y(t) \\ y(0) &= y_0 \end{aligned} \tag{5.39}$$

where $y(t) = y(\cdot, t) = \{y(x, t) | 0 \leq x \leq L\}$ is the state trajectory segment. So, $\frac{dy}{dt}$ means the time derivative of the state segment function.

The linear operator $\mathcal{A}$ is defined in Eqn. (5.40)

$$\mathcal{A}h = \alpha^2 \cdot \frac{d^2 h}{dx^2} \tag{5.40}$$

and can be written as the Riesz-spectral operator, see Curtain and Zwart (1995).

$$\mathcal{A}y = \sum_{n=0}^{\infty} \lambda_n \cdot \langle y, \phi_n \rangle \cdot \phi_n = \sum_{n=1}^{\infty} -\left(\frac{n\pi\alpha}{L}\right)^2 \cdot \int_0^L y(z,t) \cdot \sqrt{\frac{2}{L}} \cos\left(n\pi \cdot \frac{z}{L}\right) dz \cdot \sqrt{\frac{2}{L}} \cos\left(n\pi \cdot \frac{x}{L}\right) + 0 \cdot \int_0^L y(z,t) \cdot \frac{1}{\sqrt{L}} \cdot dz \cdot \frac{1}{\sqrt{L}} \qquad y \in D(\mathcal{A}) \tag{5.41}$$

The domain of the operator $\mathcal{A}$ denoted by $D(\mathcal{A})$ is given in Eqn. (5.42).

$$D(\mathcal{A}) = \left\{ h \in L_2(0, L) \middle| h \in C, \frac{dh}{dx} \in C, \frac{d^2 h}{dx^2} \in L_2(0, L), \frac{d}{dx}h(0) = \frac{d}{dx}h(L) = 0 \right\} \tag{5.42}$$

The space $L_2(0, L)$ consists of functions that are all measurable. A function is said to be measurable in the domain Ω if the Lebesgue measure given in Eqn. (5.43) is satisfied. C is the space of continuous functions.

$$\int_{\Omega} |y(x)|^2 dx < \infty \tag{5.43}$$

The statement $\frac{d^2 h}{dx^2} \in L_2(0, L)$ in Eqn. (5.43) is equivalent to

$$\sum_{n=1}^{\infty} \left(\frac{n\pi\alpha}{L}\right)^2 \cdot |\langle y, \phi_n \rangle|^2 < \infty. \tag{5.44}$$

The solution $y(x, t)$ on Y can be written abstractly as

$$y(t) = T(t)y_0 \tag{5.45}$$

where $T(t)y_0$ is given as

$$T(t)y_0 = \sum_{n=0}^{\infty} e^{\lambda_n t} \cdot \langle y_0, \phi_n \rangle \cdot \phi_n = \frac{1}{L} \cdot \langle y_0, \phi_0 \rangle \tag{5.46}$$

$$+ \sum_{n=1}^{\infty} e^{-\left(\frac{n\pi\alpha}{L}\right)^2 \cdot t} \cdot \int_0^L y(z, 0) \cdot \sqrt{\frac{2}{L}} \cos\left(n\pi \cdot \frac{z}{L}\right) dz \cdot \sqrt{\frac{2}{L}} \cos\left(n\pi \cdot \frac{x}{L}\right)$$

with Riesz basis $\phi_n = \sqrt{\frac{2}{L}} \cos\left(n\pi \cdot \frac{x}{L}\right)$, $n \geq 1$ and $\phi_0 = \frac{1}{\sqrt{L}}$.

The semigroup $T(t)$ satisfies

$$T(t) : Y \to Y \,,\ T(0) = I \tag{5.47}$$

$$y(t) = T(t)y_0. \tag{5.48}$$

The above conditions expresses a transformation from $y_0 \in Y$ to $y(t) \in Y$. The condition $T(0) = I$ implies that $y(0) = y_0$.

The semigroup $T(t)$ in Eqn. (5.46) satisfies the following conditions

$$T(t+s) = T(t) \cdot T(s) \text{ for } t, s \geq 0 \tag{5.49}$$

$$T(0) = I \tag{5.50}$$

$$\|T(t)y_0 - y_0\| \to 0 \text{ as } t \to 0^+ \,,\ \forall y_0 \in Y \,, \tag{5.51}$$

and is therefore called a strongly continuous semigroup denoted by C_0. For proof of that a strongly continuous semigroup satisfy the conditions in Eqns. (5.49)-(5.51), see Curtain and Zwart (1995) or Barbu (1994). A C_0 - semigroup, $T(t)$ on a Hilbert space Y is exponentially stable if there exist positive constants M and β such that Eqn. (5.52) is satisfied.

$$\|T(t)\| \leq Me^{-\beta t} \text{ for } t \geq 0 \tag{5.52}$$

As mentioned for long distance transmission systems, the eigenvalues are small implying a small decay rate. This means that β is small if L is large. The control model in Eqn. (5.34) is a parabolic differential equation with Neuman boundary conditions. It is formulated as an abstract boundary control model in Eqn. (5.53)

$$\begin{aligned} \frac{dy}{dt} &= \mathcal{A}y \\ \mathcal{B}y &= u \\ \mathcal{C}y &= v \\ y(0) &= y_0 \end{aligned} \tag{5.53}$$

where $\mathcal{A} = \alpha^2 \cdot \frac{d^2}{dx^2}(\cdot)$, $\mathcal{B} = \frac{d}{dx}(\cdot)\Big|_{x=0}$, $\mathcal{C} = \frac{d}{dx}(\cdot)\Big|_{x=L}$ and $\mathcal{A}$: $D(\mathcal{A}) \subset Y \to Y$, $u(t) \in U$, $v(t) \in V$. The spaces U and V are separable Hilbert spaces. The boundary operator $\mathcal{B}$: $D(\mathcal{B}) \subset Y \to U$ satisfies $D(\mathcal{A}) \subset D(\mathcal{B})$ and the disturbance input boundary operator $\mathcal{C}$: $D(\mathcal{C}) \subset Y \to U$ satisfies $D(\mathcal{A}) \subset D(\mathcal{C})$. We want to reformulate the control system to another representation with the same solution where the associated system has boundary conditions with values equal to zero.

Define the variable z in Eqn. (5.54)

$$z = y - Bu - Cv \tag{5.54}$$

with the control input map defined in Eqn. (5.55)

$$\begin{aligned} Bu &= b(x) \cdot u = -\frac{1}{2} \cdot \frac{1}{L} \cdot (x-L)^2 \cdot u = \sum_{n=0}^{\infty} \langle b, \phi_n \rangle \phi_n \\ &= -\frac{L}{6} \cdot u - \sum_{n=1}^{\infty} \frac{2}{L} \cdot \frac{1}{\left(\frac{n\pi}{L}\right)^2} \cdot \cos\left(n\pi \cdot \frac{x}{L}\right) \cdot u \end{aligned} \tag{5.55}$$

and the disturbance input map defined in Eqn. (5.56).

$$Cv = c(x) \cdot v = \frac{1}{2} \cdot \frac{1}{L} \cdot x^2 \cdot v = \sum_{n=0}^{\infty} \langle c, \phi_n \rangle \phi_n \quad . \tag{5.56}$$

$$= \frac{L}{6} \cdot v + \sum_{n=1}^{\infty} \frac{2}{L} \cdot \frac{(-1)^n}{\left(\frac{n\pi}{L}\right)^2} \cdot \cos\left(n\pi \cdot \frac{x}{L}\right) \cdot v$$

Inserting Eqns. (5.54)-(5.56) into Eqn. (5.53) yields

$$\begin{aligned} \frac{dz}{dt} &= \mathcal{A}z - B\dot{u} + \mathcal{A}Bu - C\dot{v} + \mathcal{A}Cv \\ \mathcal{B}z + \mathcal{B}Bu + \mathcal{B}Cv &= u \\ \mathcal{C}z + \mathcal{C}Bu + \mathcal{C}Cv &= v \\ z_0 = y_0 &= Bu(0) - Cv(0) \end{aligned} \quad . \tag{5.57}$$

With the defined B and C we have $\mathcal{B}B = -\frac{1}{L} \cdot (0 - L) = 1$, $\mathcal{B}C = \frac{1}{L} \cdot 0 = 0$, $\mathcal{C}B = -\frac{1}{L}(L - L) = 0$ and $\mathcal{C}C = \frac{L}{L} = 1$. Inserting these results yields the associated abstract Cauchy formulation in Eqn. (5.58)

$$\begin{aligned} \frac{dz}{dt} &= \mathcal{A}z - B\dot{u} + \mathcal{A}Bu - C\dot{v} + \mathcal{A}Cv \\ z(0) &= z_0 \end{aligned} \tag{5.58}$$

where

$$D(\mathcal{A}) = \left\{ h \in L_2(0, L) \middle| \right. \quad . \tag{5.59}$$

$$\left. h \in C, \frac{dh}{dx} \in C, \frac{d^2h}{dx^2} \in L_2(0, L), \frac{d}{dx}h(0) = \frac{d}{dx}h(L) = 0 \right\}$$

We can give the above system the following representation

$$\frac{dy_e}{dt} = \mathcal{A}_e y_e + \mathcal{B}_e u_e + \mathcal{C}_e v_e$$
$$y_e(0) = y_{e0} \tag{5.60}$$

where $y_e = \begin{bmatrix} u & v & z \end{bmatrix}^T$, $y_e(0) = \begin{bmatrix} u(0) & v(0) & z_0 \end{bmatrix}^T$, $u_e = \dot{u}$ and $v_e = \dot{v}$. The matrices $\mathcal{A}_e$, $\mathcal{B}_e$ and $\mathcal{C}_e$ are given as

$$\mathcal{A}_e = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \mathcal{A}B & \mathcal{A}C & \mathcal{A} \end{bmatrix}$$
$$\mathcal{B}_e = \begin{bmatrix} I \\ 0 \\ -B \end{bmatrix} \tag{5.61}$$
$$\mathcal{C}_e = \begin{bmatrix} 0 \\ I \\ -C \end{bmatrix}$$

The inhomogenous Cauchy problem is defined as

$$\frac{dz}{dt} = \mathcal{A}z + f$$
$$z(0) = z_0 \tag{5.62}$$

where $z_0 \in D(A)$ is a Hilbert space and the operator A generates the strongly continuous semigroup $T(t)$. The solution is given in Eqn. (5.63). For proofs of the solution in Eqn. (5.63) for the system in Eqn. (5.62) see, for example, the references Curtain and Zwart (1995), Barbu (1994) or Li and Yong (1995).

$$z(t) = T(t)z_0 + \int_0^t T(t-\eta)f(\eta)d\eta \tag{5.63}$$

Using $f = -B\dot{u} + \mathcal{A}Bu - C\dot{v} + \mathcal{A}Cv$ from Eqn. (5.57), $z = y - Bu - Cv$ and Eqn. (5.63) then gives the mild solution in Eqn. (5.64).

$$y(t) = T(t)y_0 + Bu(t) - T(t)Bu(0) - \int_0^t T(t-\eta)B\dot{u}(\eta)d\eta \tag{5.64}$$
$$+ \int_0^t T(t-\eta)\mathcal{A}Bu(\eta)d\eta + Cv(t) - T(t)Cv(0)$$
$$- \int_0^t T(t-\eta)C\dot{v}(\eta)d\eta + \int_0^t T(t-\eta)\mathcal{A}Cv(\eta)d\eta$$

The initial condition is, as before, given in Eqn. (5.65).

$$T(t)y_0 = \frac{1}{L} \cdot \int_0^L y_0(x)dx \tag{5.65}$$
$$+ \sum_{n=1}^{\infty} e^{-\left(\frac{n\pi\alpha}{L}\right)^2 t} \cdot \frac{2}{L} \cdot \int_0^L y_0(z) \cdot \cos\left(n\pi \cdot \frac{z}{L}\right)dz \cdot \cos\left(n\pi \cdot \frac{x}{L}\right)$$

$Bu(t)$ and $Cv(t)$ are given in Eqns. (5.66) and (5.67) respectively.

$$Bu(t) = -\frac{L}{6} \cdot u(t) - \sum_{n=1}^{\infty} \frac{2}{L} \cdot \frac{1}{\left(\frac{n\pi}{L}\right)^2} \cdot \cos\left(n\pi \cdot \frac{x}{L}\right) \cdot u(t) \tag{5.66}$$

$$Cv(t) = \frac{L}{6} \cdot v(t) + \sum_{n=1}^{\infty} \frac{2}{L} \cdot \frac{(-1)^n}{\left(\frac{n\pi}{L}\right)^2} \cdot \cos\left(n\pi \cdot \frac{x}{L}\right) \cdot v(t) \tag{5.67}$$

From the calculations in Eqns. (B.1) and (B.2) in Appendix B, $T(t)Bu(0)$ and $T(t)Cv(0)$ are given as in Eqns. (5.68) and (5.69) respectively.

$$T(t)Bu(0) \tag{5.68}$$

$$= -\frac{L}{6} \cdot u(0) - \sum_{n=1}^{\infty} e^{-\left(\frac{n\pi\alpha}{L}\right)^2 \cdot t} \cdot \frac{2}{L} \cdot \frac{1}{\left(\frac{n\pi}{L}\right)^2} \cdot \cos\left(n\pi \cdot \frac{x}{L}\right) \cdot u(0)$$

$$T(t)Cv(0) \tag{5.69}$$

$$= \frac{L}{6} \cdot v(0) + \sum_{n=1}^{\infty} e^{-\left(\frac{n\pi\alpha}{L}\right)^2 \cdot t} \cdot \frac{2}{L} \cdot \frac{(-1)^n}{\left(\frac{n\pi}{L}\right)^2} \cdot \cos\left(n\pi \cdot \frac{x}{L}\right) \cdot v(0)$$

From Eqn. (B.3) we have

$$\int_0^t T(t-\eta)\mathcal{A}Bu(\eta)d\eta = -\frac{\alpha^2}{L} \cdot \int_0^t u(\eta)d\eta. \tag{5.70}$$

and from the calculations in Eqn. (B.4) the result yields

$$\int_0^t T(t-\eta)\mathcal{A}Cv(\eta)d\eta = \frac{\alpha^2}{L} \cdot \int_0^t v(\eta)d\eta. \tag{5.71}$$

Furthermore, we have from Eqn. (B.5) the relation given in Eqn. (5.72).

$$\int_0^t T(t-\eta)B\dot{u}(\eta)d\eta = Bu(t) - T(t)Bu(0) \qquad . \tag{5.72}$$

$$+ \sum_{n=1}^{\infty} 2 \cdot \frac{\alpha^2}{L} \cdot \cos\left(n\pi \cdot \frac{x}{L}\right) \cdot \int_0^t e^{-\left(\frac{n\pi\alpha}{L}\right)^2 \cdot (t-\eta)} u(\eta)d\eta$$

Finally, from the calculations in Eqn. (B.6) we have the following result

$$\int_0^t T(t-\eta)C\dot{v}(\eta)d\eta = Cv(t) - T(t)Cv(0) \tag{5.73}$$
$$-\sum_{n=1}^{\infty} 2\cdot\frac{\alpha^2}{L}\cdot(-1)^n\cdot\cos\left(n\pi\cdot\frac{x}{L}\right)\cdot\int_0^t e^{-\left(\frac{n\pi\alpha}{L}\right)^2\cdot(t-\eta)}v(\eta)d\eta$$

Inserting Eqns. (5.65)-(5.73) into Eqn. (5.64) yields the analytic solution in Eqn. (5.74).

$$y(x,t) = \frac{1}{L}\cdot\int_0^L y_0(x)dx \tag{5.74}$$
$$+\sum_{n=1}^{\infty} e^{-\left(\frac{n\pi\alpha}{L}\right)^2\cdot t}\cdot\frac{2}{L}\cdot\int_0^L y_0(z)\cdot\cos\left(n\pi\cdot\frac{z}{L}\right)dz\cdot\cos\left(n\pi\cdot\frac{x}{L}\right)$$
$$-\frac{\alpha^2}{L}\cdot\int_0^t u(\eta)d\eta - \sum_{n=1}^{\infty} 2\cdot\frac{\alpha^2}{L}\cdot\cos\left(n\pi\cdot\frac{x}{L}\right)\cdot\int_0^t e^{-\left(\frac{n\pi\alpha}{L}\right)^2\cdot(t-\eta)}$$
$$\cdot u(\eta)d\eta + \frac{\alpha^2}{L}\cdot\int_0^t v(\eta)d\eta$$
$$+\sum_{n=1}^{\infty} 2\cdot\frac{\alpha^2}{L}\cdot(-1)^n\cdot\cos\left(n\pi\cdot\frac{x}{L}\right)\cdot\int_0^t e^{-\left(\frac{n\pi\alpha}{L}\right)^2\cdot(t-\eta)}\cdot v(\eta)d\eta$$

where $0 \le x \le L$ and $t \ge 0$.

The step response $y(x, t)$, when given the initial condition $y_0(x) = 0$, $0 \le x \le L$ and a step in the control and the disturbance input yields Eqn. (5.75)

$$y(x, t) = -\frac{\alpha^2}{L} \cdot t \cdot U - \sum_{n=1}^{\infty} \frac{2\alpha^2}{L} \cdot \frac{\cos\left(n\pi \cdot \frac{x}{L}\right)}{-\left(\frac{n\pi\alpha}{L}\right)^2} \tag{5.75}$$

$$\cdot \left(e^{-\left(\frac{n\pi\alpha}{L}\right)^2 \cdot t} - 1\right) \cdot U + \frac{\alpha^2}{L} \cdot t \cdot V$$

$$+ \sum_{n=1}^{\infty} \frac{2\alpha^2}{L} \cdot \frac{(-1)^n \cdot \cos\left(n\pi \cdot \frac{x}{L}\right)}{-\left(\frac{n\pi\alpha}{L}\right)^2} \cdot \left(e^{-\left(\frac{n\pi\alpha}{L}\right)^2 \cdot t} - 1\right) \cdot V$$

where U and V are the step sizes of u and v respectively.

The Laplace transform of the step response yields

$$y(x, s) = h_1(x, s) \cdot \frac{U}{s} + h_2(x, s) \cdot \frac{V}{s} \tag{5.76}$$

with the transfer function between the control variable and the state variable given as

$$h_1(x, s) = -\frac{\alpha^2}{L} \cdot \frac{1}{s} - \sum_{n=1}^{\infty} \frac{K_1(n, x)}{T(n) \cdot s + 1} \tag{5.77}$$

and with the transfer function between the disturbance variable and the state variable

$$h_2(x, s) = \frac{\alpha^2}{L} \cdot \frac{1}{s} + \sum_{n=1}^{\infty} \frac{K_2(n, x)}{T(n) \cdot s + 1} \tag{5.78}$$

where the gains and the time constants are given as

$$K_1(n, x) = \frac{2 \cdot \frac{\alpha^2}{L} \cdot \cos\left(n\pi \cdot \frac{x}{L}\right)}{\left(\frac{n\pi\alpha}{L}\right)^2}, \tag{5.79}$$

$$K_2(n, x) = \frac{2 \cdot \frac{\alpha^2}{L} \cdot (-1)^n \cdot \cos\left(n\pi \cdot \frac{x}{L}\right)}{\left(\frac{n\pi\alpha}{L}\right)^2}, \tag{5.80}$$

and

$$T(n) = \left(\frac{L}{n\pi\alpha}\right)^2. \tag{5.81}$$

The gain factors, the time constants of the infinite sum together with the integrator terms characterises the system dynamics. We see that $K_1(n, x)$ and $K_2(n, x)$ decrease with the increased time constants. The time constant $T(n)$ increases quadratically with an increase in the transportation length and linearly with an increase in the pipeline friction. The dimension of the nominator is one dimension lower than the dimension of the denominator implying that the system is proper. For long distance transportation pipelines, we see from the summing of the infinite number of first order systems that the frequency response of $h_1(x, jw)$ will have a large negative phase angle which implies that when designing a feedback controller using frequency response loop shaping, the bandwidth of the closed loop control system will be unsatisfactorily low. That is why some sort of predictive control action taking account of the time delay effect, like in Chapter 3 and Chapter 4, may be necessary. Due to the time delay effect, a feedforward from the predicted load pattern is necessary.

Some simulations of the state solution given in Eqn. (5.74) will now be shown.

<u>Pipeline data for Simulation Case 1 and 2:</u>

The length of the transmission line is 750 kilometers. The diameter is 0.9 meters, the friction is 0.0025, the speed of sound is 330 meters per second, the average compressibility factor is 0.9, the defined stationary customer pressure is 60 bar, the defined stationary mass flow is 500 kilograms per second, the average temperature is 285 Kelvin and the molecular weight is 90×10^{-3} kilograms per mole. The average value α used in the simulations was equal to 1031 $(\text{meters}^2/\text{second})^{1/2}$. The average pressure for the transmission line was calculated using the average value of the stationary input and output pressure. Another alternative formula to calculate an average pressure value is to use Eqn. (3.13). The stationary pressure distribution was calculated using Eqns. (3.9) and (3.11). With the available stationary pressure distribution function, the value of α

can be calculated by solving the integral in Eqn. (5.27).

Simulation case 1:

Figure 5.10 gives the state response $y(x, t)$, $0 \le x \le L$ for a step change in the control input u from the stationary value equal to zero down to the value equal to -3.29×10^8 $(\text{Newton/meters}^2)^2/(\text{second})$. Initial condition is $y_0(x) = 0$, $0 \le x \le L$. Disturbance input v is equal to zero. Figure 5.11 gives the pressure response for the corresponding step change in the supply mass flow from the stationary value of 500 kilograms per second to the mass flow value equal to 600 kilograms per second. The initial condition is equal to the defined stationary pressure distribution.

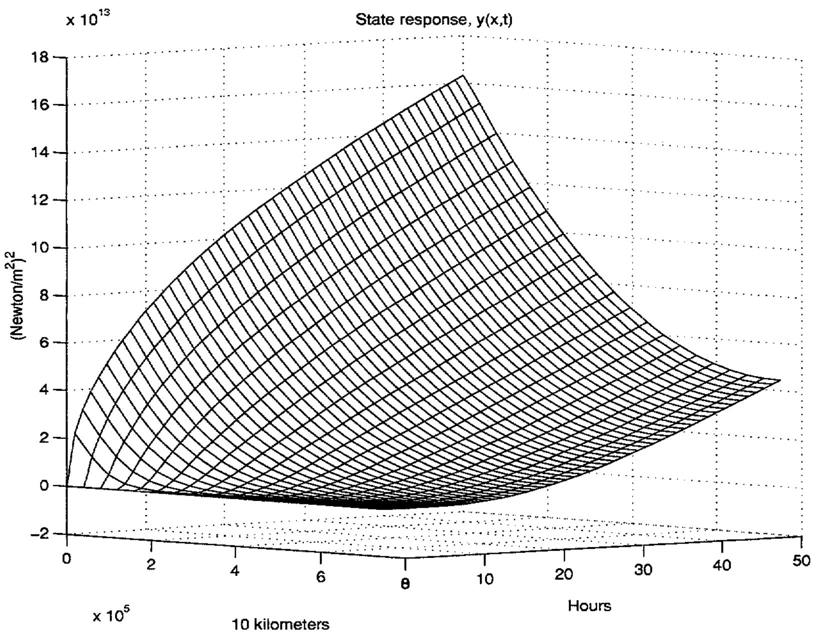


Figure 5.10: State response $y(x, t)$, $0 \le x \le L$ - Case 1.

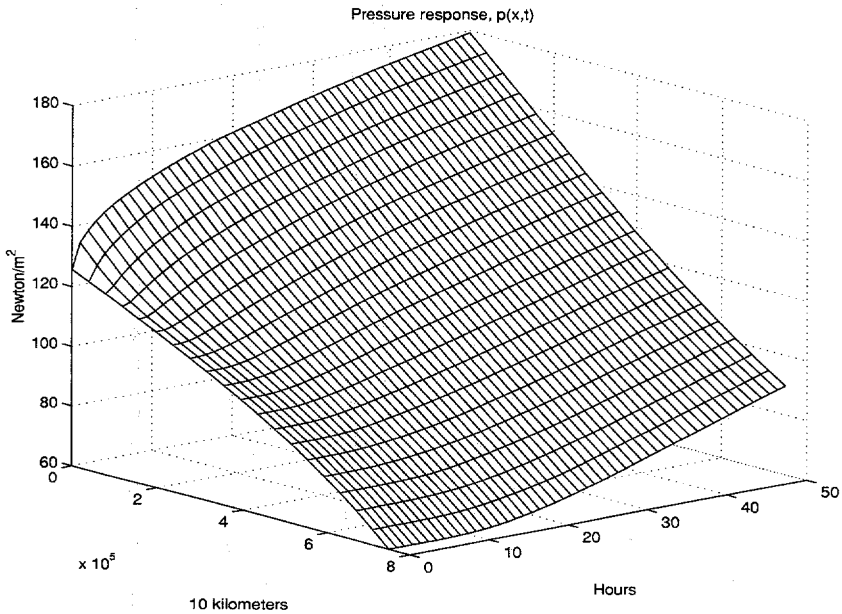


Figure 5.11: Pressure response $p(x, t)$, $0 \le x \le L$ - Case 1.

Simulation case 2:

Figure 5.12 gives the state response $y(x, t)$, $0 \le x \le L$ for a positive step change in the disturbance input v from the stationary value equal to zero up to the value equal to 2.69×10^8 $(\text{Newton/meters}^2)^2/(\text{second})$. Initial condition is $y_0(x) = 0$, $0 \le x \le L$. Control input u is equal to zero. Figure 5.11 gives the pressure response for the corresponding step change in the offtake mass flow from the stationary value of 500 kilograms per second down to the mass flow value equal to 400 kilograms per second. The initial condition is equal to the defined stationary pressure distribution.

Simulation case 3:

To show the significance of the parameter α on the dynamic properties, we show in Figure 5.14 and Figure 5.15 the state and pressure responses, respectively with a value of α defined to be equal to 750 and with rest of the data as in case one.

Remarks:

The extensive time delay between the supply side and the offtake side is clearly seen in the simulation plots. This tells us that a predictive action will be necessary in a control application for long distance transmission lines. The time delay effect between the input and the output is a function of the transmission line length and, of course, the value of the α parameter. A small value of α gives slow dynamics and increased time delay while a large value of the parameter gives fast dynamics as seen when comparing case one and three. From this comparison we also observe that the pressure level in the transmission line is much smaller in case three. The impact on the state solution by the value of α is seen from the analytic state solution given in Eqn. (5.74). From Eqn. (5.27), we see that the α parameter is a function of the pipeline friction, the pipeline diameter, the defined average pressure level and the defined stationary mass flow. Identification can be used to determine the value of α for an expected operation region. Data from a nonlinear transmission simulator with few simplifications of the conservation equations together with available measurement data from a transmission line, for the expected operation region, can be used in the identification process. Then, the value of the parameter α can be determined, for example, with the objective to minimize the sum of the squared errors.

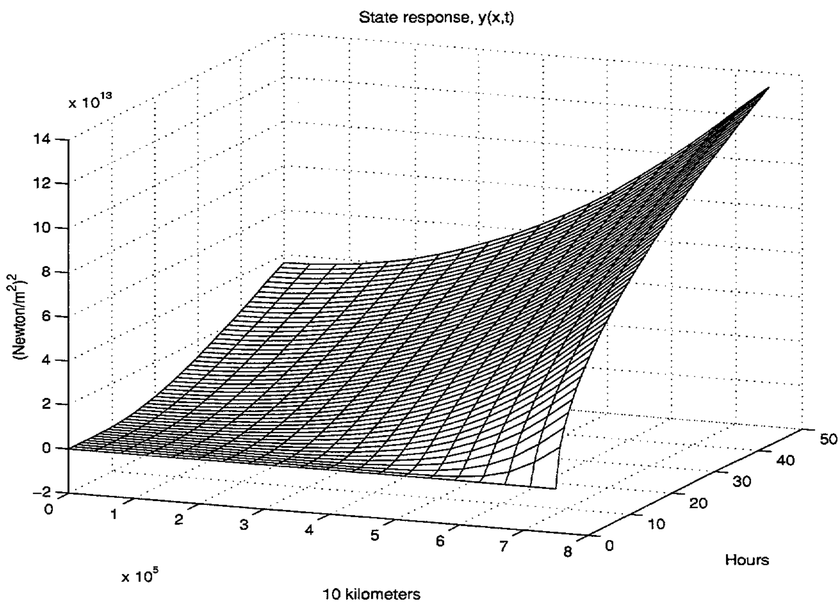


Figure 5.12: State response $y(x, t)$, $0 \leq x \leq L$ - Case 2.

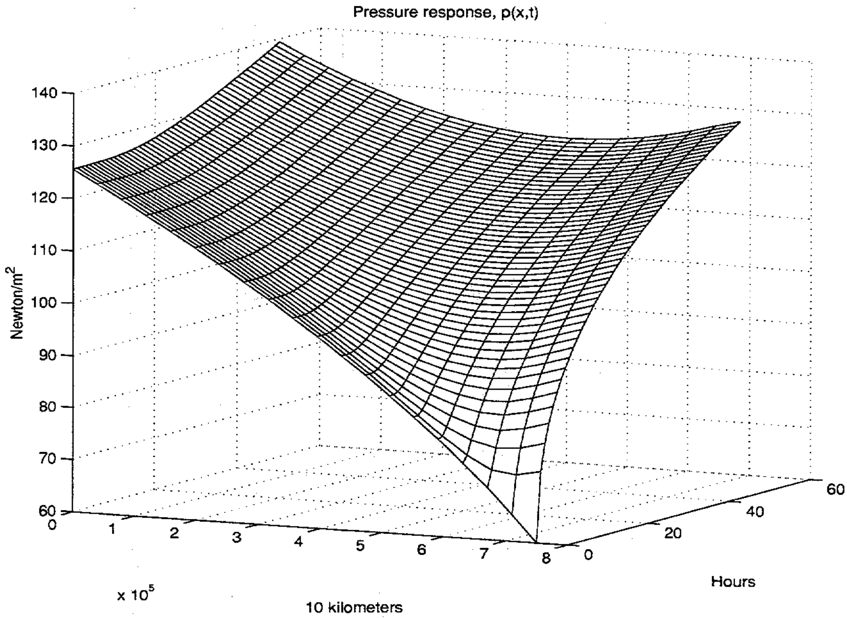


Figure 5.13: Pressure response $p(x, t)$, $0 \leq x \leq L$ - Case 2.

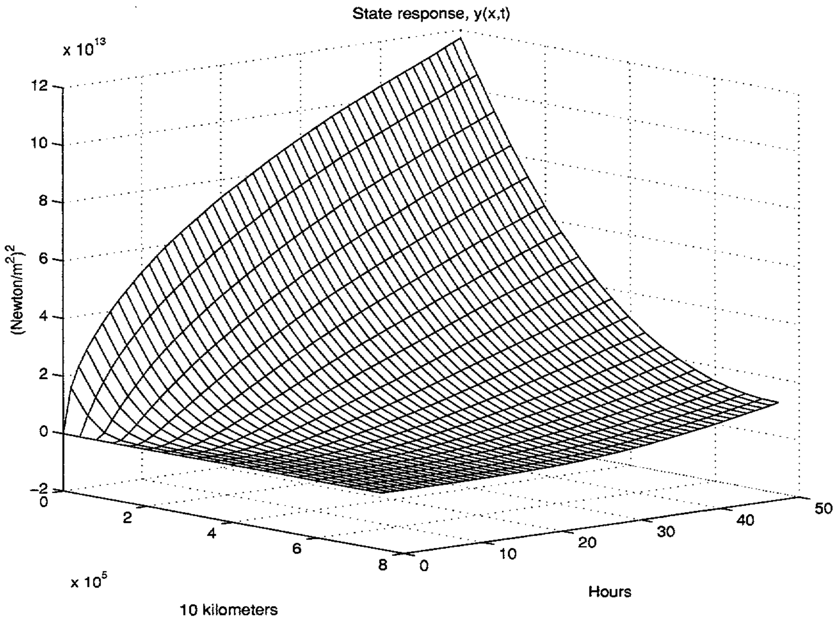


Figure 5.14: State response $y(x, t)$, $0 \leq x \leq L$ - Case 3.

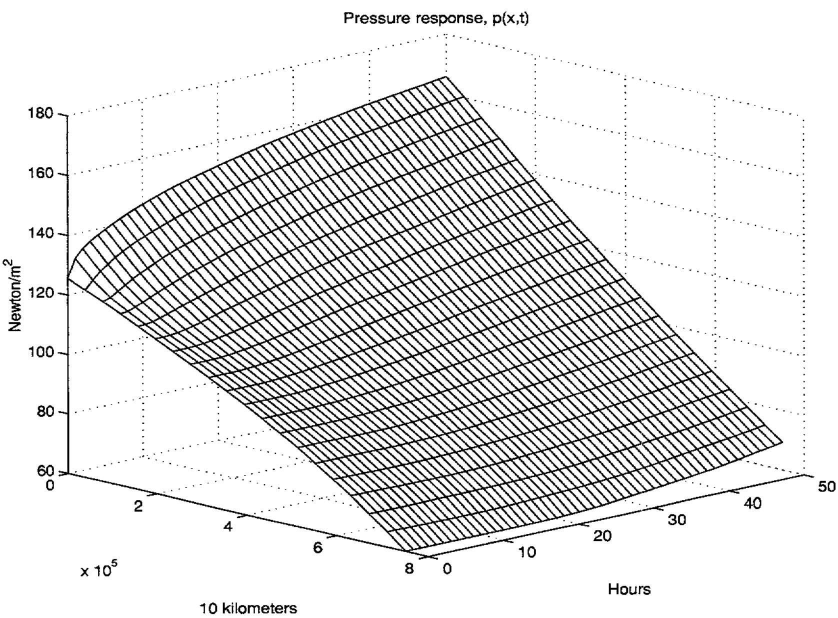


Figure 5.15: Pressure response $p(x, t)$, $0 \leq x \leq L$ - Case 3.

5.5 Smith Predictor and Feedforward Controller

5.5.1 Introduction

In this section we want to develop a control system for a gas transmission line that is a combination of a feedback and a feedforward term. If the dynamics at the source point, the customer point and the measurement dynamics cannot be neglected which is the case for short distance transportation lines, then the pipeline transfer function model can be combined with additional transfer function models of these facilities. A model for the dynamics at a gas dispatching centre can also be included. Personnel at the gas dispatch centre uses time to evaluate the proposed control commands. It also takes time to formulate a future load trajectory. This dynamic part can then be considered to be a part of the control model. The supply and the demand references are assumed to be held constant for an entire sampling interval before changing the value. Figure 5.16 illustrates the transmission system.

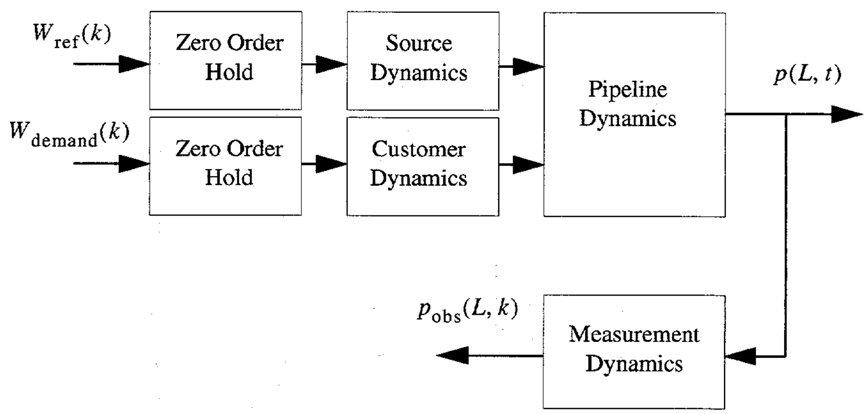


Figure 5.16: Transmission system.

The variable p_{obs} in Figure 5.16 denotes the observed customer pressure received from the SCADA supervision system. The variables W_{ref} and W_{demand} denotes the production mass flow reference and the customer demand reference respectively.

Control Task:

The main objective for the controller is to satisfy the customer demand while keeping the customer pressure close to a defined "optimal" reference value which is above or equal to a contracted minimum. The controller that is developed will be a combination of a feedback and a feedforward term.

5.5.2 Control Model

Source Dynamics:

A simplified model of the closed loop dynamics of the natural gas processing facility with a compressor station is suggested in Eqn. (5.82). A time constant is used to describe the closed loop dynamics of the processing facility and another time constant is for the closed loop dynamics of the compressor station. It may take some time from when the operators receives the mass flow reference from the dispatch centre until they actually change the references on the control systems.

$$\frac{W_{\text{ref}}(s)}{W_{\text{supply}}(s)} = M_{\text{s}}(s) = \frac{e^{-\tau_s s}}{(T_{\text{p}}s + 1) \cdot (T_{\text{cs}}s + 1)} \tag{5.82}$$

The mass flow entering the pipeline is denoted by W_{supply} and the supply reference received from the dispatch centre is denoted by W_{ref}.

Customer dynamics:

The closed loop customer dynamics at the customer terminal for changing a mass flow demand set point at their control valve or compressor station is modelled as a first order system with a time delay.

$$\frac{W_{\text{offtake}}(s)}{W_{\text{demand}}(s)} = M_{\text{c}}(s) = \frac{e^{-\tau_c s}}{(T_{\text{c}}s + 1)} \tag{5.83}$$

The mass flow leaving the transmission line is denoted by W_{offtake} and the customer demand reference is denoted by W_{demand}.

Comment. The control command reference and the load demand reference is assumed to be held constant through the entire sampling interval before possibly changing value implying a zero order hold element.

Measurement dynamics:

Filtering and error compensation of the measured output pressure may be performed by the Scada system. This may take some time in addition to the delay time for transferring of the data. Some dynamics of the measurement devise may also be apparent. Define the simple model in Eqn. (5.84).

$$\frac{p_{\text{obs}}(L, s)}{p(L, s)} = M_{\text{obs}}(s) = \frac{e^{-\tau_{\text{obs}} s}}{T_{\text{obs}}s + 1} \tag{5.84}$$

where $p_{\text{obs}}(L, s)$ is the observed pressure.

Pipeline dynamics:

The control model in Section 5.4 is used as a basis. From the observations of the analytical solutions of the step responses shown in Section 5.4 we will formulate simplified transfer function models for the dynamics and using the variables $\{u, v, y\}$ as defined in Section 5.4. The transfer function approximating the dynamics between the control input $u(s)$ and the output $y(L, s)$ is given in Eqn. (5.85).

$$\frac{y(L,s)}{u(s)} = h_{\text{p}}(s) \approx -\frac{1}{s(T_1 s+1)} \cdot e^{-\tau s} \tag{5.85}$$

The time constant is for the dynamic transition between the time delay and the pure integrator. As noted from the Figures 5.10-5.15, the time delay between the input and the output for long distance transmission lines is of considerable size. The low dimension transfer function in Eqn. 5.86 is used to approximate the dynamics between the disturbance $v(s)$ and the output $y(L, s)$. The time constants in the nominator approximates the derivative effect observed in Figure 5.12.

$$\frac{y(L,s)}{v(s)} = h_2(L,s) \approx \frac{(T_2 s+1)(T_3 s+1)}{s(T_4 s+1)} \tag{5.86}$$

The transfer function model including supply, customer and measurement dynamics considering $y(L, t)$ is given in Eqn. (5.87)

$$y(L,s) = h_{\text{p}}(s) \cdot u_{\text{ref}}(s) + h_{\text{v}}(s) \cdot v_{\text{ref}}(s) \tag{5.87}$$

where the complete input and disturbance dynamics are described by Eqns. (5.88) and (5.89).

$$h_{\text{p}}(s) = M_{\text{s}}(s) \cdot h_1(L,s) \cdot M_{\text{obs}}(s) \tag{5.88}$$

$$h_{\text{v}}(s) = M_{\text{c}}(s) \cdot h_2(L,s) \cdot M_{\text{obs}}(s) \tag{5.89}$$

Figure 5.17 illustrates the control model with zero order hold elements. The variable u_{ref} is the reference control command complying with the variables of the pipeline model in Eqn. (5.34) which must be transformed to a supply mass flow reference using Eqn. (5.35). The disturbance variable v_{ref} is calculated from

Eqn. (5.36) given a mass flow customer reference W_{demand}.

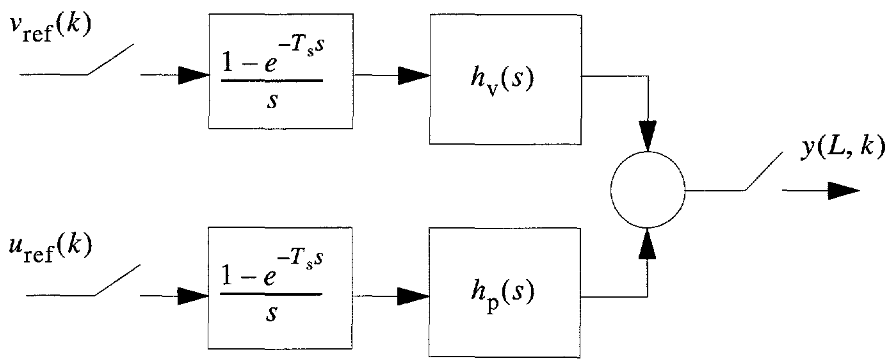


Figure 5.17: Control Model.

By including the zero order hold elements we obtain the discrete dynamic model in Eqn. (5.90).

$$y(L, z) = G(z) \cdot u_{\text{ref}}(z) + H(z) \cdot v_{\text{ref}}(z) \tag{5.90}$$

The input dynamics is given in Eqn. (5.91).

$$G(z) = \mathcal{Z}\left(\frac{1-e^{-T_s s}}{s} \cdot h_{\text{p}}(s)\right) \tag{5.91}$$

The disturbance dynamics is given in Eqn. (5.92).

$$H(z) = \mathcal{Z}\left(\frac{1-e^{-T_s s}}{s} \cdot h_{\text{v}}(s)\right) \tag{5.92}$$

The symbol $\mathcal{Z}$ denotes the $\mathcal{Z} = F(z)$ transform defined in Eqn. (5.93)

$$F(z) = \sum_{k=0}^{\infty} f(kT_{\text{s}})z^{-k} \tag{5.93}$$

for the discrete function $f(kT_{\text{s}})$.

5.5.3 Controller Design

As seen from the step responses, there is an extensive time delay between the control input and the pressure at the customer terminal for long distance pipelines. A control design using a traditional series compensator will then obtain an unacceptable low closed loop bandwidth if we are going to have satisfying gain and phase margins. If we are provided with good estimates of the process time delay, then the *Smith-Predictor* can be used to remove most of the effect of the time delay from the open loop transfer function. Other model predictive controllers where the length of the open loop horizon is chosen large enough to compensate for the process time delay will also remove this effect. The structure of the control system is shown in Figure 5.18. At time $t - \tau$, the predictor $h(s) \cdot e^{-\tau s}$ predicts $\hat{y}(L, t)$, the output value at the current time point as a result of the control trajectory up to time $(t - \tau)$. This prediction is subtracted from the observed current output value $y(L, t)$ and the deviation term $\varepsilon(t) = y(L, t) - \hat{y}(L, t)$ is summed with the predicted output value $\hat{y}(L, t + \tau)$ and this value is compared with the defined reference and then sent to the feedback controller. The prediction $\hat{y}(L, t + \tau)$ is a result of the control trajectory up to time t and the prediction model without time delay. So, this control policy has a feedback from the predicted output that can be controlled and a correction which is the error between the observed output value and the predicted current output value. The current output value is a result of the predictive feedback action up to time $t - \tau$. If the prediction model is perfect, then the current output value and the predicted output value are equal so that only the value of the predicted output $\hat{y}(t + \tau)$ is sent to the controller. The transfer function describing the dynamics is split into two parts. The transfer function $h(s)$ describes the system dynamics without time delay and the other part is the time delay. The series compensator denoted by $h_s(s)$, is designed on the basis of the process $h_p(s)$ without the time delay and can be designed using a frequency response diagram. Since the plant $h_p(s)$ contains an integrator term, a proportional controller can be used as a series compensator. Zero stationary deviation from a constant reference (equal to zero) is obtained when the open loop transfer function contains one integrator. Adding additional integrators into the controller should be avoided since this adds more negative phase and an increased negative slope of the open loop gain function. The closed loop transfer function for the Smith predictor is shown in Eqn. (5.94) and a scheme using only an ordinary series compensator $h_r(s)$ is shown in Eqn. (5.95). We see that the time delay is included in the open loop transfer function for the series compensator in Eqn. (5.95) but removed in Eqn. (5.94) opening the possibility for higher bandwidth. If a step is performed in the output reference, the output is of course delayed with the dead time τ.

$$\frac{y(s)}{r(s)} = M_{\text{Smith}}(s) = \frac{h_s(s)h(s)e^{-\tau s}}{1 + h_s(s)h(s)} \tag{5.94}$$

$$\frac{y(s)}{r(s)} = M(s) = \frac{h_r(s)h(s)e^{-\tau s}}{1 + h_r(s)h(s)e^{-\tau s}} \tag{5.95}$$

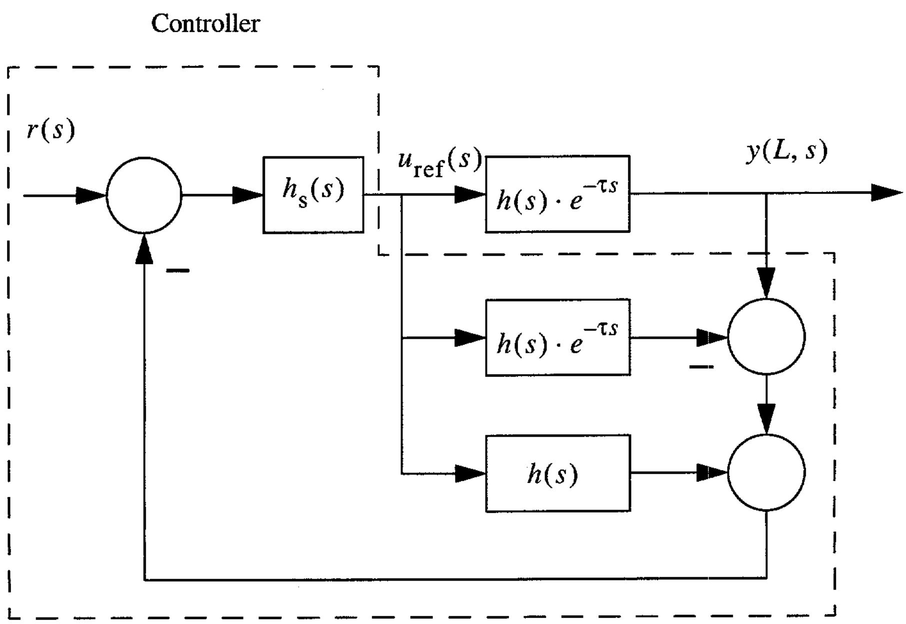


Figure 5.18: Smith predictor. Source: Finn Haugen (1992).

In addition to the Smith predictor, a feedforward from the predicted load trajectory is needed. The combined control law is then expressed in discrete form in Eqn. (5.96).

$$u_{\text{ref}}(k) = u_{\text{Smith}}(k) + u_{\text{ff}}(k) \tag{5.96}$$

5.5.4 Control System Example

A control system using the Smith predictive controller together with a feedforward from the predicted load will be designed. It is assumed that the future load pattern is known for the complete simulation horizon. The pipeline data are equal to the transmission line example in Section 5.4. The nonlinear creep flow model is used as the simulation model. The transfer function model describing the "inner-core" dynamics in the continuous form is given in Eqn. (5.87) with $h_{\mathrm{p}}(s)$ and $h_{\mathrm{v}}(s)$ as in Eqns. (5.85) and (5.86) respectively. The time constants of the denominators of these two transfer functions were set to be equal.

Feedforward term:

A feedforward term designed so that the demand $y(L, s) = 0$ is satisfied is then given in Eqn. (5.97).

$$\frac{u_{\mathrm{ff}}(s)}{v_{\mathrm{ref}}(s)} = -\frac{h_{\mathrm{v}}(s)}{h_{\mathrm{p}}(s)} = (T_2 s + 1) \cdot (T_3 s + 1) \cdot e^{\tau s} \tag{5.97}$$

This is an improper transfer function and cannot be implemented on a computer. To obtain a proper feedforward, we introduce the low pass filter $1/(T_{\mathrm{f}} s + 1)^2$ giving the modified feedforward in Eqn. (5.98).

$$h_{\mathrm{ff}}(s) = \frac{(T_2 s + 1)(T_3 s + 1)}{(T_{\mathrm{f}} s + 1)(T_{\mathrm{f}} s + 1)} \cdot e^{\tau s} \tag{5.98}$$

Adding the zero order hold element and z - transforming, we have the modified transfer function in discrete form given as

$$H_{\mathrm{ff}}(z) = Z\left\{\left(\frac{1 - e^{-T_s s}}{s}\right) \cdot \frac{(T_2 s + 1)(T_3 s + 1)}{(T_{\mathrm{f}} s + 1)(T_{\mathrm{f}} s + 1)}\right\} \cdot z^{N_{\mathrm{delay}}} \tag{5.99}$$

where N_{delay} denotes the number of sampling instants expressing the time delay. For a computerbased implementation, Eqn. (5.99) is transformed to a difference equation.

Smith predictor:

The prediction of the output at the future time $t + \tau$, which is a result of the current control input and the predicted feedforward is given by the transfer function system

$$\hat{y}(L, s) = h(s) \cdot u(s) + h_{\mathrm{v}}(s) \cdot e^{\tau s} \cdot v(s) \tag{5.100}$$

where

$$h(s) = -\frac{1}{s(T_1 s + 1)} \tag{5.101}$$

and

$$h_v(s) = \frac{(T_2 s + 1)(T_3 s + 1)}{s(T_4 s + 1)}. \tag{5.102}$$

The discrete transfer function system with zero order hold elements on the control input reference and the disturbance input reference yields

$$y(L, z) \cdot z^{N_{\text{delay}}} = G_1(z) \cdot u(z) + H(z) \cdot z^{N_{\text{delay}}} \cdot v(z) \tag{5.103}$$

where

$$G_1(z) = Z\left\{\left(\frac{1 - e^{-T_s s}}{s}\right) \cdot -\frac{1}{s(T_1 s + 1)}\right\} \tag{5.104}$$

and

$$H(z) = Z\left\{\left(\frac{1 - e^{-T_s s}}{s}\right) \cdot \frac{(T_2 s + 1)(T_3 s + 1)}{s(T_4 s + 1)}\right\}. \tag{5.105}$$

Equation (5.103) is reformulated to give a difference equation which is suitable for a computerbased implementation giving the output prediction $\hat{y}(L, k + N_{\text{delay}})$.

Prediction of current output:

The current output is predicted at time $k - N_{\text{delay}}$ using the transfer function

$$\hat{y}(L, z) = G(z) \cdot u(z) + H(z) \cdot v(z) \tag{5.106}$$

where

$$G(z) = G_1(z) \cdot z^{-N_{\text{delay}}} \tag{5.107}$$

Equation (5.106) is reformulated as a difference equation for a computer implementation giving the current output prediction $\hat{y}(L, k)$.

Series compensator:

With a perfect cancelling of the system time delay, the series compensator of the Smith predictor only controls an integrator with a time constant. Adding the zero order hold element, we have additionally a time delay. As a rule of thumb, this time delay is usually set to half of the sampling interval implying a time delay of half an hour for the system example. Due to the integrator, only a proportional controller is necessary for obtaining an output value which is equal to the reference when the load has settled down to a stationary value.

The plant to be used for the design of the proportional controller is then given in Eqn. (5.108).

$$h(s) = \frac{e^{-\frac{T_s}{2}s}}{s(T_1 s + 1)} \tag{5.108}$$

The task is to design a proportional controller with a gain K_p for the feedback loop illustrated in Figure 5.19 satisfying defined gain and phase margins and giving robustness of the control system.

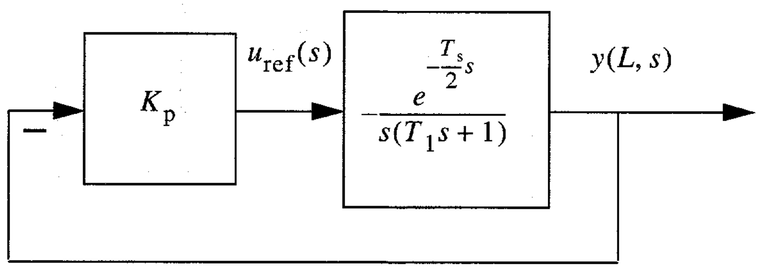


Figure 5.19: Control system for the series compensator included in the Smith predictor control algorithm.

The amplitude of the open loop frequency response of the system illustrated in

Figure 5.19 is given in Eqn. (5.109)

$$|h_0(j\omega)|_{\mathrm{dB}} = 20\log K_{\mathrm{p}} + 20\log\left(\frac{1}{\sqrt{(\omega T_1)^2 + 1}}\right) + 20\log\left(\frac{1}{\omega}\right) \quad (5.109)$$

and the phase is given in Eqn. (5.110).

$$\angle h_0(j\omega) = -90° - \mathrm{atan}(\omega T_1)° - \frac{180}{\pi}\cdot\left(\omega\frac{T_{\mathrm{s}}}{2}\right)° \quad (5.110)$$

The phase margin demand is specified in Eqn. (5.111)

$$\varphi = 180° - 90° - \mathrm{atan}(\omega_{\mathrm{c}} T_1)° - \frac{180}{\pi}\cdot\omega_{\mathrm{c}}\frac{T_{\mathrm{s}}}{2}° \geq \varphi_{\min} \quad (5.111)$$

where ω_{c} is the cross frequency. The gain margin demand is expressed in Eqn. (5.112)

$$\Delta K = -|h_0(j\omega_{180})|_{\mathrm{dB}} \geq \Delta K_{\min} \quad (5.112)$$

where ω_{180} is the phase cross frequency. A typical range of the stability margins for a control system design are $\varphi_{\min} \geq 45°$ and $\Delta K_{\min} \geq 6\mathrm{dB}$. Large stability margins give a slow but robust control system.

<u>Final control law:</u>

The final discrete control law for the "linear inner-core" is expressed in Eqn. (5.113)

$$\begin{aligned} &u_{\mathrm{ref}}(k) \\ &= -K_{\mathrm{p}}\cdot\{\hat{y}(L, k+N_{\mathrm{delay}}) + (y(L,k) - \hat{y}(L,k))\} + u_{\mathrm{ff}}(k) \end{aligned} \quad (5.113)$$

and the nonlinear control law for the supply mass flow reference yields

$$W_{\mathrm{ref}}(0, k) \tag{5.114}$$
$$= \sqrt{\frac{DA^2}{4fc^2} \cdot \{-K_{\mathrm{p}} \cdot \{\hat{y}(L, k + N_{\mathrm{delay}}) + (y(L, k) - \hat{y}(L, k))\} + u_{\mathrm{ff}}(k)\} + W^{*2}}$$

with $y(L, k)$ calculated as in Eqn. (5.115).

$$y(L, k) = p(L, k)^2 - p^*(L)^2 \tag{5.115}$$

The control system structure in discrete representation is illustrated in Figure 5.20.

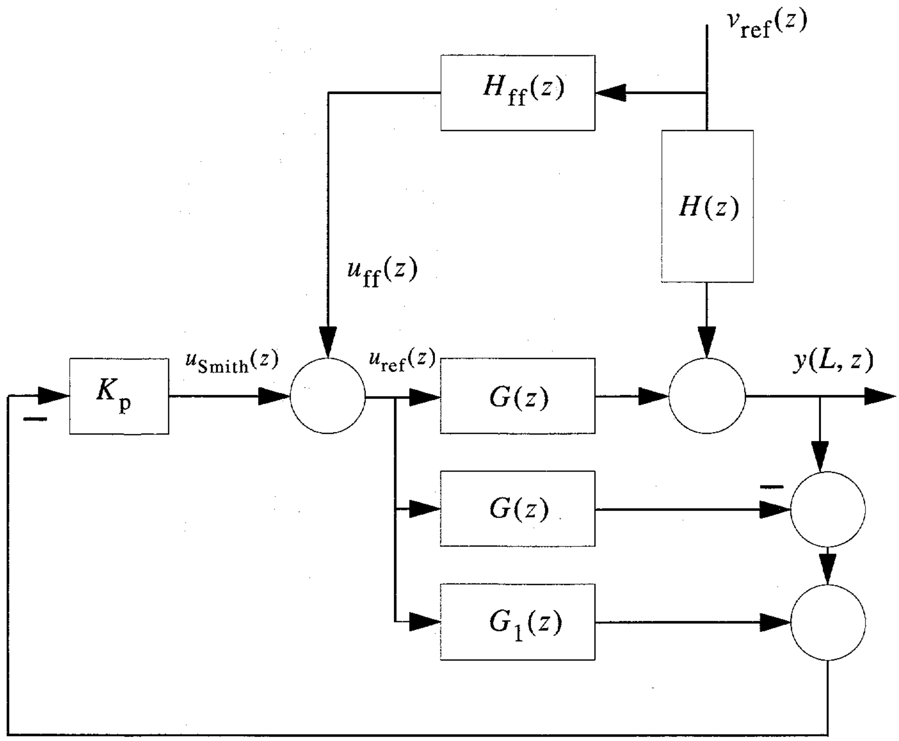


Figure 5.20: Discrete representation of the control system structure of the system example.

5.5.5 Simulation Results

The length of the transmission line used in the simulation was 750 kilometers. The diameter was set to 0.9 meters. Fannings friction factor was defined to be 0.0025. The speed of sound was 330 meters per second and the average compressibility factor was 0.9. The customer pressure reference was defined to be 60 bar. The average temperature was 285 Kelvin and the molecular weight was set to 90×10^{-3} kilograms per mole. The nonlinear creep flow model in Eqn. (3.5) was used as closed loop simulation model. Figure 5.21 and Figure 5.22 show the pressures and the flows at the supply and the customer terminal respectively for a control system where we use the Smith predictor combined with a feedforward. The control system is stable and with zero stationary deviation from the customer terminal reference pressure when the load settles down to the stationary value. The reason for that the customer terminal reference value is reached is because of the integrator term of the process. So, zero stationary deviation from the defined reference value is the case even if the controller is only a proportional controller. Of course, this holds only if the customer load is constant. Since the controller uses a feedback from the predicted output value, it certainly is a predictive controller. The maximum flow capacity was set to the value 1000 kilograms per second and the minimum flow capacity value was set to zero. A determined control signal outside these limits was set to the defined upper or lower limit respectively. The nonlinear creep flow model was used as the simulation model. The parameters of the control model was determined based on the step responses and after some tuning in closed loop with the nonlinear creep flow model. The time constants T_1 and T_4 were set to three hours. The filter constant T_f was given the value of one hour. A time delay of $\tau = 20$ hours was used. The time constants T_2 and T_3 of the disturbance transfer function providing the derivative effect description, was set to 6 and 0.2 hours respectively. A proportional gain equal to $K_p = -16 \cdot 10^{-6}$ was used.

5.5.6 Conclusions

The Smith controller is a very simple model predictive controller with a low computational effort. No open loop mathematical programs must be solved. The upper and lower control limits must be defined. The lacking ability of not handling constraints on the states and the control input is an obvious disadvantage. But, the control system is easy to dimension by the classical control methods. Since the process contains an integrator, we only need a proportional controller for the series compensator of the Smith predictor in order to obtain a zero deviation from the reference customer pressure when the load settles down to a stationary value. The feedforward term that was designed must be preconditioned to give a proper transfer function suitable for a computerbased implementation. Customer dynamics and measurement dynamics can introduce the dynamics that are required to define a proper model. The stability margins give robustness to the control system. Some tuning of the model parameters is necessary to obtain a satisfactory controller which is not unusual for control design in practise.

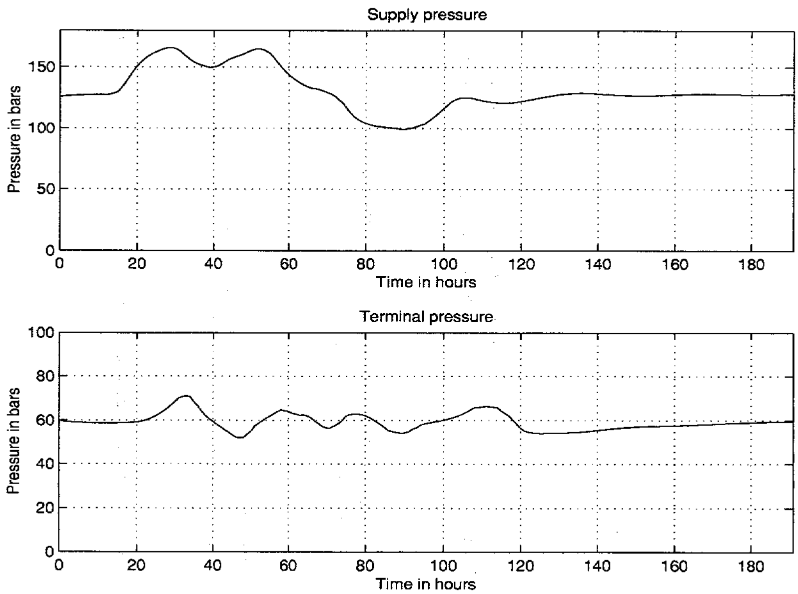


Figure 5.21: Pressure at supply point and customer point when using Smith predictor and feedforward.

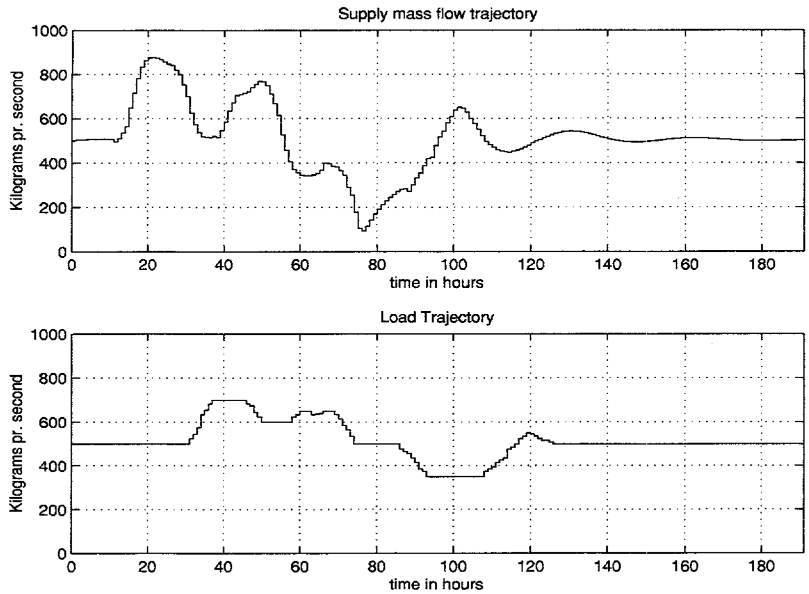


Figure 5.22: Supply and customer mass flow when using Smith predictor and feedforward.

5.6 Control Model with Distributed Supplies and Loads

Consider the transmission system illustrated in Figure 5.23. The control model with the control commands, the loads and an initial condition is represented in Eqn. (5.116). A distributed control command and a distributed load have been added to the model defined in Eqn. (5.34). It is assumed that the result from a stationary optimization specifies the stationary control flows and the pressure distribution along the pipeline given some defined constant load demands.

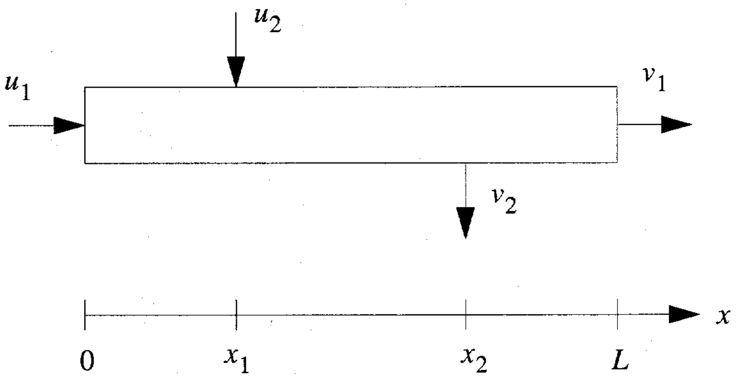


Figure 5.23: Transmission System with boundary and pointwise distributed control and demands.

Dynamic system

$$
\begin{aligned}
\frac{\partial}{\partial t}y(x,t) &= \alpha^2\cdot\frac{\partial^2}{\partial x^2}y(x,t)+\frac{\alpha^2}{2\varepsilon_1}\cdot 1_{[x_1-\varepsilon_1,\,x_1+\varepsilon_1]}(x)\cdot u_2(t)+\frac{\alpha^2}{2\varepsilon_2}\cdot 1_{[x_2-\varepsilon_2,\,x_2+\varepsilon_2]}(x)\cdot v_2(t)\\
\frac{\partial}{\partial x}y(0,t) &= u_1(t)\\
\frac{\partial}{\partial x}y(L,t) &= v_1(t)\\
y(x,0) &= y_0(x)
\end{aligned}
\tag{5.116}
$$

Mass is supplied into the control volume $V_1 = 2\varepsilon_1 A$ centred at position $x = x_1$ and mass is withdrawn from the transmission system from the control volume $V_2 = 2\varepsilon_2 A$ centred at position $x = x_2$.

The state variable is defined in Eqn. (5.117).

$$y(x,t) = p(x,t)^2 - p^*(x)^2 \tag{5.117}$$

The input vector is defined in Eqn. (5.118).

$$u = \begin{bmatrix} u_1 & u_2 \end{bmatrix}^T \tag{5.118}$$

The disturbance vector is defined in Eqn. (5.119).

$$v = \begin{bmatrix} v_1 & v_2 \end{bmatrix}^T \tag{5.119}$$

The control component u_1 satisfies

$$u_1(t) = -\frac{4fc^2}{DA^2} \cdot (W(0,t)^2 - W^*(0)^2) \tag{5.120}$$

and v_1 satisfies

$$v_1(t) = -\frac{4fc^2}{DA^2} \cdot (W(L,t)^2 - W^*(L)^2). \tag{5.121}$$

Define the variable W_1 to be the mass flow supplied into the control volume V_1. and W_2 to be the customer mass flow for the control volume V_2.

Locally, W_1 and u_2 are related as in Eqn. (5.122).

$$W_1(t) = \left(\frac{\alpha^2 \cdot A}{c^2 \cdot p_1^*}\right) \cdot u_2(t) + \left(\frac{W_1^*}{p_1^*}\right) \tag{5.122}$$

The variables W_2 and v_2 are related as in Eqn. (5.123).

$$W_2(t) = \left(\frac{\alpha^2 \cdot A}{c^2 \cdot p_2^*}\right) \cdot v_2(t) + \left(\frac{W_2^*}{p_2^*}\right) \tag{5.123}$$

Here, the state equation $p = c^2 \cdot \rho$ has been used together with the local approximation $\frac{\partial p^2}{\partial t} \approx \frac{\partial}{\partial t} p \cdot p^* = p^* \cdot \frac{\partial p}{\partial t}$.

The average stationary pressure values denoted by p_1^* and p_2^* respectively in the two control volumes V_1 and V_2 are calculated as in Eqn. (5.124)

$$\begin{aligned} p_1^* &= \int_0^L \frac{1}{2\varepsilon_1} \cdot 1_{[x_1 - \varepsilon_1,\, x_1 + \varepsilon_1]}(x) p^*(x) dx \\ p_2^* &= \int_0^L \frac{1}{2\varepsilon_2} \cdot 1_{[x_2 - \varepsilon_2,\, x_2 + \varepsilon_2]}(x) p^*(x) dx \end{aligned} \tag{5.124}$$

W_1^* and W_2^* are the defined distributed stationary mass flows. For a defined stationary customer offtake we have $W_2^* < 0$. So, with $v_2 < 0$ and $W_2^* < 0$, the customer offtake gets more negative.

The control model is abstractly expressed in Cauchy differential form in Eqn. (5.125)

$$\begin{aligned} \frac{dy}{dt} &= \mathcal{A}y + B_2 u_2 + C_2 v_2 \\ \mathcal{B}y &= u_1 \\ \mathcal{C}y &= v_1 \\ y(0) &= y_0 \end{aligned} \tag{5.125}$$

where $\mathcal{A} = \alpha^2 \cdot \frac{d^2}{dx^2}(\cdot)$, $\mathcal{B} = \frac{d}{dx}(\cdot)\Big|_{x=0}$, $\mathcal{C} = \frac{d}{dx}(\cdot)\Big|_{x=L}$,

$B_2 u_2 = b_2(x) \cdot u_2$ with $b_2(x) = \frac{\alpha^2}{2\varepsilon_1} \cdot 1_{[x_1 - \varepsilon_1,\, x_1 + \varepsilon_1]}(x)$ and

$C_2 v_2 = c_2(x) \cdot v_2$ with $c_2(x) = \frac{\alpha^2}{2\varepsilon_2} \cdot 1_{[x_2 - \varepsilon_2,\, x_2 + \varepsilon_2]}(x)$.

The operators $\mathcal{A}$, $\mathcal{B}$ and $\mathcal{C}$ form a boundary control system as in Eqn. (5.53).

Define the variable $z = y - Bu - Cv$. By using this variable and with the boundary control map given as

$$B_1 u_1 = b_1(x) \cdot u_1 = -\frac{1}{2L}(x-L)^2 \cdot u_1 \tag{5.126}$$

and the disturbance boundary map given as

$$C_1 v_1 = c_1(x) \cdot v_1 = \frac{1}{2L}x^2 \cdot v_1 \tag{5.127}$$

and inserting into Eqn. (5.125), we get the associated Cauchy differential system in Eqn. (5.128)

$$\begin{aligned} \frac{dz}{dt} &= \mathcal{A}z - B_1\dot{u}_1 + \mathcal{A}B_1u_1 + B_2u_2 - C_1\dot{v}_1 + \mathcal{A}C_1v_1 + C_2v_2 \\ z(0) &= z_0 \end{aligned} \tag{5.128}$$

where $D(\mathcal{A})$ is given by

$$\begin{aligned} D(\mathcal{A}) = \Big\{ & h \in L_2(0,L) | \\ & h \in C, \frac{dh}{dx} \in C, \frac{d^2h}{dx^2} \in L_2(0,L), \frac{d}{dx}h(0) = 0, \frac{d}{dx}h(L) = 0 \Big\} \end{aligned} \tag{5.129}$$

The complete system is described in state space form in Eqn. (5.130)

$$\begin{aligned} \frac{dy_e}{dt} &= \mathcal{A}_e y_e + \mathcal{B}_e u_e + C_e v_e \\ y_e(0) &= y_{0e} \end{aligned} \tag{5.130}$$

where we have $y_e = \begin{bmatrix} u & v & z \end{bmatrix}^T$, $y_{e0} = \begin{bmatrix} u_1(0) & v_1(0) & z_0 \end{bmatrix}^T$, $u_e = \begin{bmatrix} \dot{u}_1 & u_2 \end{bmatrix}^T$ and $v_e = \begin{bmatrix} \dot{v}_1 & v_2 \end{bmatrix}^T$. The system matrices are given in Eqn. (5.131).

$$
\begin{aligned}
\mathcal{A}_e &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \mathcal{A}B_1 & \mathcal{A}C_1 & \mathcal{A} \end{bmatrix} \\
\mathcal{B}_e &= \begin{bmatrix} I & 0 \\ 0 & 0 \\ -B_1 & B_2 \end{bmatrix} \\
\mathcal{C}_e &= \begin{bmatrix} 0 & 0 \\ I & 0 \\ -C_1 & C_2 \end{bmatrix}
\end{aligned} \tag{5.131}
$$

Defining $f = -B_1\dot{u}_1 + \mathcal{A}B_1u_1 + B_2u_2 - C_1\dot{v}_1 + \mathcal{A}C_1v_1 + C_2v_2$ from Eqn. (5.128), $z = y - B_1u_1 - C_1v_1$, and using Eqn. (5.63) then gives the mild analytic solution expressed in Eqn. (5.132).

$$
\begin{aligned}
y(t) = {} & T(t)y_0 + B_1u_1(t) - T(t)B_1u_1(0) - \int_0^t T(t-\eta)B_1\dot{u}_1(\eta) \\
& d\eta + \int_0^t T(t-\eta)\mathcal{A}B_1u_1(\eta)d\eta + \int_0^t T(t-\eta)B_2u_2(\eta)d\eta \\
& + C_1v_1(t) - T(t)C_1v_1(0) - \int_0^t T(t-\eta)C_1\dot{v}_1(\eta) \\
& d\eta + \int_0^t T(t-\eta)\mathcal{A}C_1v_1(\eta)d\eta + \int_0^t T(t-\eta)C_2v_2(\eta)d\eta
\end{aligned} \tag{5.132}
$$

Define the problems in Eqns. (5.133)-(5.137).

$$
\begin{aligned}
y_{1t} &= \alpha^2 \cdot y_{1xx} \\
y_{1x}(0) &= 0 \\
y_{1x}(L) &= 0 \\
y_1(x, 0) &= y_0(x)
\end{aligned} \tag{5.133}
$$

$$
\begin{aligned}
y_{2t} &= \alpha^2 \cdot y_{2xx} \\
y_{2x}(0) &= u_1(t) \\
y_{2x}(L) &= 0 \\
y_2(x, 0) &= 0
\end{aligned}
\tag{5.134}
$$

$$
\begin{aligned}
y_{3t} &= \alpha^2 \cdot y_{3xx} \\
y_{3x}(0) &= 0 \\
y_{3x}(L) &= v_1(t) \\
y_3(x, 0) &= 0
\end{aligned}
\tag{5.135}
$$

$$
\begin{aligned}
y_{4t} &= \alpha^2 \cdot y_{4xx} + \frac{\alpha^2}{2\varepsilon_1} \cdot 1_{[x_1 - \varepsilon_1, x_1 + \varepsilon_1]}(x) \cdot u_2(t) \\
y_{4x}(0) &= 0 \\
y_{4x}(L) &= 0 \\
y_4(x, 0) &= 0
\end{aligned}
\tag{5.136}
$$

$$
\begin{aligned}
y_{5t} &= \alpha^2 \cdot y_{5xx} + \frac{\alpha^2}{2\varepsilon_2} \cdot 1_{[x_2 - \varepsilon_2, x_2 + \varepsilon_2]}(x) \cdot v_2(t) \\
y_{5x}(0) &= 0 \\
y_{5x}(L) &= 0 \\
y_5(x, 0) &= 0
\end{aligned}
\tag{5.137}
$$

Using the superposition principle, see Greenberg (1988), the solution to the combined problem defined in Eqn. (5.116) is a linear combination of the individual solutions to the above problems expressed in Eqn. (5.138) and is equivalent to the solution given in Eqn. (5.132).

$$
y = y_1 + y_2 + y_3 + y_4 + y_5 \tag{5.138}
$$

The mild solution is given in Eqn. (5.139)

$$y(x,t) = \frac{1}{L}\cdot\int_0^L y_0(x)dx \tag{5.139}$$

$$+\sum_{n=1}^{\infty} e^{-\left(\frac{n\pi\alpha}{L}\right)^2\cdot t}\cdot\frac{2}{L}\cdot\int_0^L y_0(z)\cdot\cos\left(n\pi\cdot\frac{z}{L}\right)dz\cdot\cos\left(n\pi\cdot\frac{x}{L}\right)-\frac{\alpha^2}{L}$$

$$\cdot\int_0^t u_1(\eta)d\eta-\sum_{n=1}^{\infty}\frac{2\alpha^2}{L}\cdot\cos\left(n\pi\cdot\frac{x}{L}\right)\cdot\int_0^t e^{-\left(\frac{n\pi\alpha}{L}\right)^2\cdot(t-\eta)}u_1(\eta)d\eta+\frac{\alpha^2}{L}$$

$$\cdot\int_0^t u_2(\eta)d\eta+\sum_{n=1}^{\infty}\frac{2\alpha^2}{L}\cdot\cos\left(n\pi\cdot\frac{x_1}{L}\right)\cdot\frac{\sin\left(n\pi\cdot\frac{\varepsilon_1}{L}\right)}{\left(n\pi\cdot\frac{\varepsilon_1}{L}\right)}\cdot\cos\left(n\pi\cdot\frac{x}{L}\right)$$

$$\cdot\int_0^t e^{-\left(\frac{n\pi\alpha}{L}\right)^2\cdot(t-\eta)}u_2(\eta)d\eta+\frac{\alpha^2}{L}\cdot\int_0^t v_1(\eta)d\eta+\sum_{n=1}^{\infty}\frac{2\alpha^2}{L}\cdot(-1)^n$$

$$\cdot\cos\left(n\pi\cdot\frac{x}{L}\right)\cdot\int_0^t e^{-\left(\frac{n\pi\alpha}{L}\right)^2\cdot(t-\eta)}v_1(\eta)d\eta+\frac{\alpha^2}{L}\cdot\int_0^t v_2(\eta)d\eta$$

$$+\sum_{n=1}^{\infty}\frac{2\alpha^2}{L}\cdot\cos\left(n\pi\cdot\frac{x_2}{L}\right)\cdot\frac{\sin\left(n\pi\cdot\frac{\varepsilon_2}{L}\right)}{\left(n\pi\cdot\frac{\varepsilon_2}{L}\right)}\cdot\cos\left(n\pi\cdot\frac{x}{L}\right)$$

$$\cdot\int_0^t e^{-\left(\frac{n\pi\alpha}{L}\right)^2\cdot(t-\eta)}v_2(\eta)d\eta$$

where $0\le x\le L$ and $t\ge 0$.

Define the output vector in Eqn. (5.140) considering the pressures at the demand points.

$$y_{\mathrm{obs}}(t) = \left[y(x_2, t)\ y(L, t)\right]^{\mathrm{T}} \tag{5.140}$$

Laplace transformation of Eqn. (5.139) after setting the initial condition equal to zero (stationary operation point) and performing steps in the control and the disturbance variables yield the transfer function model in Eqn. (5.141)

$$y_{\mathrm{obs}}(s) = H_{\mathrm{p}}(s) \cdot u(s) + H_{\mathrm{v}}(s) \cdot v(s) \tag{5.141}$$

with the output vector y_{obs} defined in Eqn. (5.140). The transfer matrices between the input and the disturbance vector to the defined output vector are given in Eqns. (5.142) and (5.143).

$$H_{\mathrm{p}}(s) = \begin{bmatrix} h_{\mathrm{p}11}(x_2, s) & h_{\mathrm{p}12}(x_2, s) \\ h_{\mathrm{p}21}(L, s) & h_{\mathrm{p}22}(L, s) \end{bmatrix} \tag{5.142}$$

$$H_{\mathrm{v}}(s) = \begin{bmatrix} h_{\mathrm{v}11}(x_2, s) & h_{\mathrm{v}12}(x_2, s) \\ h_{\mathrm{v}21}(L, s) & h_{\mathrm{v}22}(L, s) \end{bmatrix} \tag{5.143}$$

The transfer functions between the input vector and the defined output vector are given in Eqns. (5.144)-(5.147).

$$h_{p11}(x_2, s) = -\frac{\alpha^2}{L} \cdot \frac{1}{s} - \sum_{n=1}^{\infty} \frac{K_1(n, x_2)}{T(n) \cdot s + 1} \tag{5.144}$$

$$h_{\mathrm{p}12}(x_2, s) = \frac{\alpha^2}{L} \cdot \frac{1}{s} + \sum_{n=1}^{\infty} \frac{K_2(n, x_2)}{T(n) \cdot s + 1} \tag{5.145}$$

$$h_{\mathrm{p}21}(L, s) = -\frac{\alpha^2}{L} \cdot \frac{1}{s} - \sum_{n=1}^{\infty} \frac{K_1(n, L)}{T(n) \cdot s + 1} \tag{5.146}$$

$$h_{\mathrm{p}22}(L, s) = \frac{\alpha^2}{L} \cdot \frac{1}{s} + \sum_{n=1}^{\infty} \frac{K_2(n, L)}{T(n) \cdot s + 1} \tag{5.147}$$

The transfer functions between the disturbance vector and the defined output vector are given in Eqns. (5.148)-(5.151).

$$h_{v11}(x_2, s) = \frac{\alpha^2}{L} \cdot \frac{1}{s} + \sum_{n=1}^{\infty} \frac{K_3(n, x_2)}{T(n) \cdot s + 1} \tag{5.148}$$

$$h_{v12}(x_2, s) = \frac{\alpha^2}{L} \cdot \frac{1}{s} + \sum_{n=1}^{\infty} \frac{K_4(n, x_2)}{T(n) \cdot s + 1} \tag{5.149}$$

$$h_{v21}(L, s) = \frac{\alpha^2}{L} \cdot \frac{1}{s} + \sum_{n=1}^{\infty} \frac{K_3(n, L)}{T(n) \cdot s + 1} \tag{5.150}$$

$$h_{v22}(L, s) = \frac{\alpha^2}{L} \cdot \frac{1}{s} + \sum_{n=1}^{\infty} \frac{K_4(n, L)}{T(n) \cdot s + 1} \tag{5.151}$$

The gain functions are given as

$$K_1(n, x) = \frac{2 \cdot \frac{\alpha^2}{L} \cdot \cos\left(n\pi \cdot \frac{x}{L}\right)}{\left(\frac{n\pi\alpha}{L}\right)^2} \tag{5.152}$$

$$K_2(n, x) = \frac{2 \cdot \frac{\alpha^2}{L} \cdot \cos\left(n\pi \cdot \frac{x_1}{L}\right) \cdot \frac{\sin\left(n\pi \cdot \frac{\varepsilon_1}{L}\right)}{\left(n\pi \cdot \frac{\varepsilon_1}{L}\right)} \cdot \cos\left(\frac{n\pi}{L} \cdot x\right)}{\left(\frac{n\pi\alpha}{L}\right)^2} \tag{5.153}$$

$$K_3(n, x) = \frac{2 \cdot \frac{\alpha^2}{L} \cdot (-1)^n \cdot \cos\left(n\pi \cdot \frac{x}{L}\right)}{\left(\frac{n\pi\alpha}{L}\right)^2} \tag{5.154}$$

$$K_4(n,x) = \frac{2 \cdot \frac{\alpha^2}{L} \cdot \cos\left(n\pi \cdot \frac{x_2}{L}\right) \cdot \frac{\sin\left(n\pi \cdot \frac{\varepsilon_2}{L}\right)}{\left(n\pi \cdot \frac{\varepsilon_2}{L}\right)} \cdot \cos\left(\frac{n\pi}{L} \cdot x\right)}{\left(\frac{n\pi\alpha}{L}\right)^2} \qquad (5.155)$$

The system time constants, are as before, given as

$$T(n) = \left(\frac{L}{n\pi\alpha}\right)^2, \; n \geq 1 \qquad (5.156)$$

where we insert the positions $x = x_2$ or $x = L$ into Eqns. (5.152)-(5.155) to get the gains between the inputs and the chosen output points.

<u>Conclusion:</u>

This section shows how to obtain an analytical solution to a defined distributed parameter control model for a gas transmission line with both Neuman boundary conditions and distributed control and load. Using the mild solution, this model has been converted to a transfer function model to describe the dynamics for a transmission line between a defined input and a defined output vector. The distributed parameter model and the transfer function model can be used for the design of a control system by the use of traditional or modern control methods.

5.7 Parameter Distributed Feedback and State Observer

5.7.1 Introduction

In this section, a distributed parameter controller and a distributed parameter state observer will be designed for the operation of a natural gas transmission line. The control command that is derived is a combination of a state feedback from estimated states and a feedforward term from the predicted customer load demand of natural gas. The feedforward term part was designed in Section 5.5 and will not be considered further in this section. So, in this section we want to design a nominally stable control system that brings an initial pressure distribution of the transmission line to a defined stationary pressure distribution with the customer mass flow equal to the stationary value of the defined operation point. The control model will be expressed in deviation variables from a defined stationary operation point.

The supply of mass at the defined inlet boundary of the transmission line is the physical control variable. The customer natural gas offtake takes place at the defined output boundary of the transmission line. It is assumed that the pressure is measured at the defined output end of the transmission line.

The boundary control model that is assumed to describe the transmission system will be approximated by a control model with a defined distributed control command and an average pressure measurement. It will be shown that the limit of the state solution of the approximated control model is equal to the state solution of the defined boundary control model. Furthermore, it will be shown that the limit of the defined average pressure measurement of the approximated control model is equal to the point measurement. The approximated control and measurement operators are bounded and of finite rank which simplifies the analysis and the control and state estimator design. So, the control design will be based on the approximated control model. But, the resulting control law is designed to be used by the boundary control model and therefore to support the operation of the defined gas transmission system.

It will be shown that the defined control model with the defined control command and observation is approximately controllable and approximately observable. Then, it will be shown that the control model is exponentially stabilizable and exponentially detectable. Furthermore, a nominally stable distributed parameter feedback law and a nominally stable distributed parameter state observer will be designed. Then, it will be shown that the coupled system with the designed linear feedback operator and with the estimated state feedback from the designed state estimator is nominally exponentially stable.

5.7.2 Control Model

Assume that the control model around a defined stationary operation point in the autonomous case is defined as in Section 5.4 and re-expressed in Eqn. (5.157)

$$\begin{aligned}
\frac{\partial y}{\partial t} &= \alpha^2 \cdot \frac{\partial^2 y}{\partial x^2} \\
\frac{\partial}{\partial x} y(0, t) &= u(t) \\
\frac{\partial}{\partial x} y(L, t) &= v(t) \\
y(0, x) &= y_0(x) \\
\xi(t) &= y(L, t)
\end{aligned} \tag{5.157}$$

where the state y, the control u and disturbance v are defined as in Section 5.4 and ξ is the observation. From Section 5.4, this control model has the state solution $y(x, \cdot)$, $0 \le x \le L$ given in Eqn. (5.74).

Now, define the control model in Eqn. (5.158)

$$\begin{aligned}
\frac{\partial y}{\partial t} &= \alpha^2 \cdot \frac{\partial^2 y}{\partial x^2} - \frac{\alpha^2}{\varepsilon_1} \cdot 1_{[0, \varepsilon_1]}(x) \cdot u(t) \\
\frac{\partial}{\partial x} y(0, t) &= 0 \\
\frac{\partial}{\partial x} y(L, t) &= v(t) \\
\xi(t) &= \int_0^L \frac{1}{\varepsilon_2} \cdot 1_{[L - \varepsilon_2, L]}(x) \cdot y(x, t) dx
\end{aligned} \tag{5.158}$$

where ξ is the observation, u is the control command and v is the disturbance. The control model defined in Eqn. (5.158) will be used as a basis for the analysis of controllability and observability, stabilizability and detectability and for the design of a control law and a Luenberger state estimator.

The mass flow at the supply point denoted by $W(0, t)$, is calculated using Eqn. (5.35) when the control command u is given and the stationary mass flow is defined. The value of the disturbance variable v is calculated using Eqn. (5.36) when the customer load is given and the stationary mass flow is defined.

Theorem 1: *Assume that the dynamics of a natural gas transmission line is described by the control model defined in Eqn. (5.157) and consider its approximation given in Eqn. (5.158). The limit of the state solution for the distributed control model defined in Eqn. (5.158) when* $\varepsilon_1 \to 0$, *is equal to the state solution of the boundary control model defined in Eqn. (5.157). The limit of the observation* ξ *defined in Eqn. (5.158) when* $\varepsilon_2 \to 0$, *is equal to the point observation* ξ *at the output boundary of the control model defined in Eqn. (5.157).*

Proof: Using the results from Section 5.4 and Eqn. (5.63), the state solution $y(x, \cdot)$ of the control model defined in Eqn. (5.158) is given in Eqn. (5.159)

$$y(x,t) = \frac{1}{L} \cdot \int_0^L y_0(x)dx + \sum_{n=1}^{\infty} e^{-\left(\frac{n\pi\alpha}{L}\right)^2 t} \cdot \frac{2}{L} \tag{5.159}$$

$$\cdot \int_0^L y_0(z) \cdot \cos\left(n\pi \cdot \frac{z}{L}\right)dz \cdot \cos\left(n\pi \cdot \frac{x}{L}\right) - \frac{\alpha^2}{L} \cdot \int_0^t u(\eta)d\eta - \sum_{n=1}^{\infty} \frac{2\alpha^2}{L}$$

$$\cdot \frac{\sin\left(n\pi \cdot \frac{\varepsilon_1}{L}\right)}{\left(n\pi \cdot \frac{\varepsilon_1}{L}\right)} \cdot \cos\left(n\pi \cdot \frac{x}{L}\right) \cdot \int_0^t e^{-\left(\frac{n\pi\alpha}{L}\right)^2 \cdot (t-\eta)} u(\eta)d\eta + \frac{\alpha^2}{L} \cdot \int_0^t v$$

$$(\eta)d\eta + \sum_{n=1}^{\infty} \frac{2\alpha^2}{L} \cdot (-1)^n \cdot \cos\left(n\pi \cdot \frac{x}{L}\right) \cdot \int_0^t e^{-\left(\frac{n\pi\alpha}{L}\right)^2 \cdot (t-\eta)} v(\eta)d\eta$$

where $0 \le x \le L$ and $t \ge 0$. Now, by letting $\varepsilon_1 \to 0$ in Eqn. (5.159), we obtain the state solution given in Eqn. (5.160)

$$\begin{aligned}\lim_{\varepsilon_1 \to 0} y(x, t) = {} & \frac{1}{L} \cdot \int_0^L y_0(x)dx + \sum_{n=1}^{\infty} e^{-\left(\frac{n\pi\alpha}{L}\right)^2 t} \cdot \frac{2}{L} \\ & \cdot \int_0^L y_0(z) \cdot \cos\left(n\pi \cdot \frac{z}{L}\right)dz \cdot \cos\left(n\pi \cdot \frac{x}{L}\right) - \frac{\alpha^2}{L} \cdot \int_0^t u(\eta)d\eta - \sum_{n=1}^{\infty} \frac{2\alpha^2}{L} \\ & \cdot \cos\left(n\pi \cdot \frac{x}{L}\right) \cdot \int_0^t e^{-\left(\frac{n\pi\alpha}{L}\right)^2 \cdot (t-\eta)} u(\eta)d\eta + \frac{\alpha^2}{L} \cdot \int_0^t v(\eta) \\ & d\eta + \sum_{n=1}^{\infty} \frac{2\alpha^2}{L} \cdot (-1)^n \cdot \cos\left(n\pi \cdot \frac{x}{L}\right) \cdot \int_0^t e^{-\left(\frac{n\pi\alpha}{L}\right)^2 \cdot (t-\eta)} v(\eta)d\eta\end{aligned} \tag{5.160}$$

Hence, we see that the state solution for the system defined in Eqn. (5.158) is in the limit equal to the state solution for the control model defined in Eqn. (5.157). Define $\bar{y}(\varepsilon_2, t)$ as in Eqn. (5.161)

$$\bar{y}(\varepsilon_2, t) = \sup_{x \in [L-\varepsilon_2, L]} y(x, t) \tag{5.161}$$

and $\underline{y}(\varepsilon_2, t)$ as in Eqn. (5.162).

$$\underline{y}(\varepsilon_2, t) = \inf_{x \in [L-\varepsilon_2, L]} y(x, t) \tag{5.162}$$

Then, we have satisfied Eqn. (5.163).

$$\underline{y}(\varepsilon_2, t) \le \int_{L-\varepsilon_2}^{L} \frac{1}{\varepsilon_2} \cdot y(x, t)dx \le \bar{y}(\varepsilon_2, t) \tag{5.163}$$

Since the solution $y(x, t)$ in Eqn. (5.159) is continuous we then have the fact that

$$\lim_{\varepsilon_2 \to 0} \underline{y}(\varepsilon_2, t) = \lim_{\varepsilon_2 \to 0} \bar{y}(\varepsilon_2, t) = y(L, t). \tag{5.164}$$

Hence, the limit of the average observation is given in Eqn (5.165).

$$\lim_{\varepsilon_2 \to 0} \xi(t) = y(L, t) \tag{5.165}$$

This completes the proof.

As described in the introduction, the regulation problem, which is to bring the state $y \to 0$ for a given initial condition $y_0(x, 0)$ and a load equal to the stationary value implying $v = 0$, will be the basis for the design of the feedback part of the control law. Therefore, from now on we set $v = 0$. The control model defined in Eqn. (5.158) is given in Cauchy differential form in Eqn. (5.166)

$$\begin{aligned} \frac{d}{dt}y(t) &= \mathcal{A}y(t) + \mathcal{B}u(t) \\ y(0) &= y_0 \\ \xi(t) &= Dy(t) \end{aligned} \tag{5.166}$$

with state space Y given as $Y = L_2(0, L)$ and $\mathcal{A}h = \alpha^2 \cdot \frac{d^2 h}{dx^2}$. The domain of the operator $\mathcal{A}$ is given in Eqn. (5.167).

$$D(\mathcal{A}) = \left\{ h \in L_2(0, L) \middle| \; h \in C, \frac{dh}{dx} \in C, \frac{d^2 h}{dx^2} \in L_2(0, L), \frac{d}{dx}h(0) = \frac{d}{dx}h(L) = 0 \right\} \tag{5.167}$$

The state $y(t)$ is short notation for $y(\cdot, t)$. The initial condition function $y_0(\cdot)$ is assumed to satisfy $y_0(\cdot) \in Y$ and the trajectory state segment $y(\cdot, t)$ satisfies $y(\cdot, t) \in Y$ for all $t > 0$. The trajectory state segment is given by $y(\cdot, t) = \{y(x, t) | 0 \le x \le L\}$.

The control input map is given in Eqn. (5.168)

$$\mathcal{B}u = -\frac{\alpha^2}{\varepsilon_1} \cdot 1_{[0, \varepsilon_1]}(x) \cdot u \tag{5.168}$$

and the observation map is given in Eqn. (5.169).

$$Dh = \int_0^L \frac{1}{\varepsilon_2} \cdot 1_{[L-\varepsilon_2, L]}(x) \cdot h(x)dx \tag{5.169}$$

A distributed parameter controller will be defined which stabilizes the defined gas transmission model by a state feedback operator that uses the estimated states along the transmission line.

5.7.3 Background Results

Some necessary background results will now be presented. The definitions and theorems are taken from Curtain and Zwart (1995). The background results that will be used in this section assumes that one has a distributed parameter system description of the form defined in Eqn. (5.170). This description is in compliance with the description used for the defined control model of a gas transmission line used in this Section.

$$\begin{aligned} \frac{dy}{dt} &= \mathcal{A}y + \mathcal{B}u \\ y(0) &= y_0 \\ \xi &= Dy \end{aligned} \tag{5.170}$$

The operator $\mathcal{A}$ is the self-adjoint operator defined by $\mathcal{A}y = \sum_{n=1}^{\infty} \lambda_n \sum_{j=1}^{r_n} \langle y, \phi_{n_j} \rangle \phi_{n_j}$, where $\{\lambda_n, n \geq 1\}$ are distinct real numbers with $\lambda_1 > \lambda_2 > \ldots > \lambda_n > \ldots$, and $\{\phi_{n_j}, j = 1, \ldots, r_n, n \geq 1\}$ is an orthonormal basis in the state space Y. $\mathcal{B}$ and D are finite rank operators defined as $\mathcal{B}u = \sum_{i=1}^{m} b_i(x) \cdot u_i$ and $Dy = \left[\langle d_1, y\rangle\ \langle d_2, y\rangle\ \ldots\ \langle d_k, y\rangle\right]^{\mathrm{T}}$ where $b_i, d_i \in Y$. The linear system representation is written compactly as $\sum(\mathcal{A}, \mathcal{B}, D)$.

Definition of approximate controllability (Curtain and Zwart,1995):

The linear system denoted by $\sum(\mathcal{A}, \mathcal{B}, —)$ is approximately controllable on

$[0, \tau]$ for some finite $\tau > 0$, if given an arbitrary $\varepsilon > 0$, it is possible to steer the state from the origin to within a distance ε from all points in the state space at time τ, i.e. if the range of the controllability map defined as $B^{\tau} = \int_0^{\tau} T(\tau - \eta)\mathcal{B}u(\eta)d\eta$ satisfies

$$\overline{\text{range}(B^{\tau})} = Y \tag{5.171}$$

Here, Y denotes the state space for the control model.

Definition of approximate observability (Curtain and Zwart, 1995):

The linear system $\sum(\mathcal{A}, —, D)$ is approximately observable on $[0, \tau]$ for some finite $\tau > 0$ if knowledge of the output in $L_2([0, \tau];Z)$ determines the initial state uniquely, i.e. if

$$\text{kernel}(D^{\tau}) = \{0\} \tag{5.172}$$

for the observability map $D^{\tau} : Y \to L_2([0, \tau];Z)$ defined by

$$D^{\tau}y_0 = DT[0, \tau]y_0 \tag{5.173}$$

where $T[0, \tau]$ denotes the trajectory of the semigroup for the period $[0, \tau]$ given an initial condition $y_0 \in Y$. The map $DT[0, \tau]y_0$ denotes the output trajectory for the same period. The space Z denotes the output space. W can use duality to conclude approximate observability by checking approximate controllability. The system $\sum(\mathcal{A}, —, D)$ is approximately observable on $[0, \tau]$ if and only if the dual system $\sum(\mathcal{A}^*, D^*, —)$ is approximately controllable on $[0, \tau]$.

Theorem 4.2.1 in Curtain and Zwart (1995) provides tests for approximate controllability and approximate observability:

Theorem on approximate controllability and observability (Curtain and Zwart, 1995):

Consider the state linear system $\sum(\mathcal{A}, \mathcal{B}, D)$ *where* $\mathcal{A}$ *is a self-adjoint operator* *defined by* $\mathcal{A}y = \sum_{n=1}^{\infty} \lambda_n \sum_{j=1}^{r_n} \langle y, \phi_{n_j} \rangle \phi_{n_j}$, $\{\lambda_n, n \geq 1\}$ *are distinct real numbers with* $\lambda_1 > \lambda_2 > \ldots > \lambda_n > \ldots$ *and* $\{\phi_{n_j}, j = 1, \ldots, r_n, n \geq 1\}$ *is an orthonormal basis in* Y. $\mathcal{B}$ *and* D *are finite - rank operators defined by*

$$\mathcal{B}u = \sum_{i=1}^{m} b_i u_i \text{ where } b_i \in Y, \tag{5.174}$$

and

$$Dy = \left[\langle y, d_1 \rangle \ \langle y, d_2 \rangle \ \ldots \ \langle y, d_k \rangle\right]^{\mathrm{T}}, \text{ where } d_i, y \in Y. \tag{5.175}$$

$\sum(\mathcal{A}, \mathcal{B}, —)$ *is approximately controllable if and only if for all* n

$$\operatorname{rank}(B_n) = r_n \tag{5.176}$$

where

$$B_n = \begin{bmatrix} \langle b_1, \phi_{n_1} \rangle & \cdots & \langle b_m, \phi_{n_1} \rangle \\ \cdots & & \cdots \\ \langle b_1, \phi_{n_{r_n}} \rangle & \cdots & \langle b_m, \phi_{n_{r_n}} \rangle \end{bmatrix} \tag{5.177}$$

$\sum(A, —, D)$ *is approximately observable if and only if for all* n

$$\operatorname{rank}(D_n) = r_n \tag{5.178}$$

where

$$D_n = \begin{bmatrix} \langle \phi_{n_1}, d_1 \rangle & \cdots & \langle \phi_{n_1}, d_k \rangle \\ \cdots & & \cdots \\ \langle \phi_{n_{r_n}}, d_1 \rangle & \cdots & \langle \phi_{n_{r_n}}, d_k \rangle \end{bmatrix}. \tag{5.179}$$

The matrices B_n and D_n are interpreted as controllability and observability matrices for the distributed parameter system.

For the transmission line system example $\sum(\mathcal{A}, \mathcal{B}, D)$ we have that

$$\mathcal{A}y = \sum_{n=1}^{\infty} \lambda_n \langle y, \phi_n \rangle \phi_n \tag{5.180}$$

so that $r_n = 1$. The finite rank input is

$$\mathcal{B}u = b(x)u \tag{5.181}$$

and the finite rank output is

$$Dy = \langle d, y \rangle \tag{5.182}$$

For approximate controllability of the transmission system example, we have the demand

$$\operatorname{rank}(\langle b, \phi_n \rangle) = 1, n \geq 1 \tag{5.183}$$

and for approximate observability, we have the demand

$$\operatorname{rank}(\langle d, y \rangle) = 1, n \geq 1 \tag{5.184}$$

From Section 5.4, we have the operator $\mathcal{A}$ in Eqn. (5.166), which generates the strongly continuous semigroup $T(t)$ in Eqn. (5.46) on the Hilbert space Y. The bounded control input map $\mathcal{B} \in \mathcal{L}(U, Y)$ and the bounded observability map $D \in \mathcal{L}(Y, V)$ are defined in Eqn. (5.166). Introduce the following definition:

Definition on exponential stabilizability and stability (Curtain and Zwart,1995):

If there exists a linear operator $F \in \mathcal{L}(Y, U)$ such that $\mathcal{A} + \mathcal{B}F$ generates a semigroup $T_{\mathcal{B}F}(t)$ that is β_1-exponentially stable, then we say that the system $\sum(\mathcal{A}, \mathcal{B}, —)$ is β_1-exponentially stabilizable. The semigroup $T_{\mathcal{B}F}(t)$ is β_1-exponentially stable if there exist constants M_1 and α so that Eqn. (5.185) holds for $-\alpha < \beta_1$.

$$\|T_{\mathcal{B}F}(t)\| \le M_1 e^{-\alpha t} \text{ for } t \ge 0 \tag{5.185}$$

If there exists an operator $K \in \mathcal{L}(V, Y)$ such that $\mathcal{A} + KD$ generates an exponentially stable semigroup $T_{KD}(t)$, then we say that the system $\sum(\mathcal{A}, —, D)$ is β_2-exponentially detectable. The semigroup $T_{KD}(t)$ is β_2-exponentially stable if there exist constants M_2 and α so that Eqn. (5.186) holds for $-\alpha < \beta_2$.

$$\|T_{KD}(t)\| \le M_2 e^{-\alpha t} \text{ for } t \ge 0 \tag{5.186}$$

These two definitions are relevant for the stability of an β_1-exponentially stable feedback control system and a β_2-exponentially stable state observer.

5.7.4 Control System Design

Definition 1: Assume that the dynamics of a gas transmission line is described by Eqn. (5.166). Define the closed loop control system in Eqn. (5.187) with feedback controller defined by Eqn. (5.188).

$$\begin{aligned} \frac{dy}{dt} &= \mathcal{A}y + \mathcal{B}Fy \\ y(0) &= y_0 \end{aligned} \tag{5.187}$$

$$u(t) = Fy(t) = \kappa_1 \langle y, \phi_0 \rangle = \frac{\kappa_1}{L}\int_0^L y(x, t)dx \tag{5.188}$$

The constant κ_1 is chosen so that it satisfies $-\frac{\alpha^2}{L} \cdot \kappa_1 \le -\left(\frac{\pi\alpha}{L}\right)^2$. Further, define a constant β_1 as $\beta_1 = -\left(\frac{\pi\alpha}{L}\right)^2$.

Definition 2: Assume that the dynamics of a gas transmission line is described by Eqn. (5.166). Define the Luenberger state observer in Eqn. (5.189) with operator K defined in Eqn. (5.190).

$$\frac{d}{dt}\hat{y} = \mathcal{A}\hat{y} + \mathcal{B}u + KD(\hat{y} - y)$$
$$\hat{y}(0) = \hat{y}_0 \tag{5.189}$$

$$K\xi = \kappa_2 \xi \phi_0 \tag{5.190}$$

The constant κ_2 is chosen so that it satisfies $\frac{1}{L} \cdot \kappa_2 \le -\left(\frac{\pi\alpha}{L}\right)^2$. Further, define a constant β_2 as $\beta_2 = -\left(\frac{\pi\alpha}{L}\right)^2$.

Comment to Definition 1 and 2: Since the eigenvalues of the operator $\mathcal{A}$ are 0 and $\left\{-\left(\frac{n\pi\alpha}{L}\right)^2, n \ge 1\right\}$, the eigenvalue $-\left(\frac{\pi\alpha}{L}\right)^2$ is the slowest one for the stable part of $\mathcal{A}$. We choose the constants κ_1 and κ_2, so that the closed loop dynamics of the stabilized part of $\mathcal{A}$, is faster than the slowest mode of the stable part of $\mathcal{A}$. The eigenvalue for the unstable part being equal to 0, origins from the integrator effect of the system.

Definition 3: Assume that the gas transmission line is described by the control model in Eqn. (5.166). Define the closed loop control system in Eqn. (5.191) with the control command defined by $u(t) = F\hat{y}(t)$ where the operator F is given from Definition 1. Assume that the operator K is given by Definition 2.

$$\frac{dy}{dt} = \mathcal{A}y + \mathcal{B}F\hat{y} \qquad \frac{d}{dt}\hat{y} = \mathcal{A}\hat{y} + \mathcal{B}F\hat{y} + KD(\hat{y} - y)$$
$$y(0) = y_0 \qquad \hat{y}(0) = \hat{y}_0 \tag{5.191}$$

$$u(t) = F\hat{y}(t) \tag{5.192}$$

Now, the following theorems on stabilizability, detectability and stability of the considered gas transmission system defined in Eqn. (5.191) are formulated.

Theorem 2: *The gas transmission system defined in Eqn. (5.166) is β_1-exponentially stabilizable and β_2-exponentially detectable.*

Theorem 3: *The closed loop gas transmission system defined in Eqn. (5.191) is nominally exponentially stable.*

Proof of Theorem 2:

Since the bounded operator $\mathcal{B}$ of the defined control model satisfies

$$\langle b, \phi_n \rangle \phi_n = -\frac{2\alpha^2}{L} \cdot \frac{\sin\left(n\pi \cdot \frac{\varepsilon_1}{L}\right)}{\left(n\pi \cdot \frac{\varepsilon_1}{L}\right)} \cos\left(n\pi \cdot \frac{x}{L}\right) \neq 0, n \geq 1, \qquad (5.193)$$

this implies that $\mathcal{B}$ is of finite rank equal to one. Then, by Theorem 4.2.1 in Curtain and Zwart (1995), the defined gas transmission system is approximate controllable.

Since the defined bounded operator D of the control model satisfies

$$\langle d, \phi_n \rangle \phi_n = \frac{2}{L} \cdot (-1)^n \cdot \frac{\sin\left(n\pi \cdot \frac{\varepsilon_2}{L}\right)}{\left(n\pi \cdot \frac{\varepsilon_2}{L}\right)} \cdot \cos\left(n\pi \cdot \frac{x}{L}\right) \quad , \qquad (5.194)$$

$$= \frac{2}{L} \cdot (-1)^n \cdot \frac{\sin\left(n\pi \cdot \frac{\varepsilon_2}{L}\right)}{\left(n\pi \cdot \frac{\varepsilon_2}{L}\right)} \cdot \cos\left(n\pi \cdot \frac{x}{L}\right) \neq 0, n \geq 1$$

this implies that D is of finite rank equal to one. Then, by Theorem 4.2.1 in Curtain and Zwart (1995), the defined gas transmission system is approximate observable. The positive constants ε_1 and ε_2 must be chosen so that we have satisfied $\sin\left(n\pi \cdot \frac{\varepsilon_1}{L}\right) \neq 0$ and $\sin\left(n\pi \cdot \frac{\varepsilon_2}{L}\right) \neq 0$. If this is not the case, then the approximated control model is neither approximate controllable or approximate observable. The control command has then no effect on the differential equation. The output injection has also then no effect on the differential equation.

Theorem 5.2.6 in Curtain and Zwart (1995) states that if the system operator $\mathcal{A}$ satisfies the spectrum decomposition assumption at β_1, $Y^+_{\beta_1}$ is finite dimensional, $T^-_{\beta_1}(t)$ is β_1-exponentially stable, and $\sum(\mathcal{A}^+_{\beta_1}, \mathcal{B}^+_{\beta_1}, —)$ is controllable, then the system $\sum(\mathcal{A}, \mathcal{B}, —)$ is β_1-exponentially stabilizable. The theorem assumes that $\mathcal{B}$ is bounded and of finite rank.

Above, it was shown that the bounded operator $\mathcal{B}$ was of finite rank equal to one. On this basis, we can conclude that the linear system $\sum(\mathcal{A}, \mathcal{B}, —)$ is approximately controllable. This implies that the decomposed part $\sum(\mathcal{A}^{+}_{\beta_1}, \mathcal{B}^{+}_{\beta_1}, —) = \sum\left(0, -\frac{\alpha^2}{L}, —\right)$ is also controllable.

$$\frac{dy^{+}}{dt} = 0 \cdot y^{+} - \frac{\alpha^2}{L} \cdot u(t)$$

The operator $\mathcal{A}$ from Eqn. (5.166) has the spectral decomposition in Eqn. (5.195)

$$\mathcal{A}y = \sum_{n=0}^{\infty} \lambda_n \cdot \langle y, \phi_n \rangle \cdot \phi_n = \sum_{n=1}^{\infty} -\left(\frac{n\pi\alpha}{L}\right)^2 \cdot \frac{2}{L} \tag{5.195}$$
$$\cdot \int_0^L y(z, t) \cdot \cos\left(n\pi \cdot \frac{z}{L}\right) dz \cdot \cos\left(n\pi \cdot \frac{x}{L}\right) + 0 \cdot \frac{1}{L} \cdot \int_0^L y(z, t) dz$$

which generates the semigroup $T(t)$ from Eqn. (5.46) which is given as

$$T(t)y_0 = \sum_{n=0}^{\infty} e^{\lambda_n t} \cdot \langle y_0, \phi_n \rangle \cdot \phi_n = \frac{1}{L} \cdot \langle y_0, \phi_0 \rangle \quad .$$
$$+ \sum_{n=1}^{\infty} e^{-\left(\frac{n\pi\alpha}{L}\right)^2 \cdot t} \cdot \frac{2}{L} \cdot \int_0^L y(z, 0) \cdot \cos\left(n\pi \cdot \frac{z}{L}\right) dz \cdot \cos\left(n\pi \cdot \frac{x}{L}\right)$$

In Example 5.2.8 in Curtain and Zwart (1995), it is stated that the operator $\mathcal{A}$ in Eqn. (5.195) satisfies the spectrum decomposition property for any β_1. Further, the decomposed part $Y^{+}_{\beta_1}$ of the complete state space $Y = Y^{+} \oplus Y^{-}$ is of dimension one. Hence, the system $\sum(\mathcal{A}, \mathcal{B}, —)$ is β_1-exponentially stabilizable.

Theorem 5.2.7 in Curtain and Zwart (1995) states that if $\mathcal{A}$ satisfies the spectrum decomposition at β_2, Y^{+} is finite dimensional, $T_{\beta_2}(t)$ is β_2-exponentially stable, and $\sum(\mathcal{A}^{+}_{\beta_1}, —, D)$ is observable, then the system $\sum(\mathcal{A}, —, D)$ is β_2-

exponentially detectable.

Above, it was stated that the spectrum decomposition is satisfied for any β_2. Since it was shown that $\sum(\mathcal{A}, —, D)$ was approximately observable, then also $\sum(\mathcal{A}^+_{\beta_1}, —, D) = \sum\left(0, —, \frac{1}{L}\right)$ is observable. The decomposed part $Y^+_{\beta_2}$ is of dimension one. Hence, the system is β_2-exponentially detectable. This completes the proof.

Proof of Theorem 3:

Since the linear system $\sum(\mathcal{A}, \mathcal{B}, —)$ in Eqn. (5.166) by Theorem 2 is β_1-exponentially stabilizable, then by Theorem 5.2.6 and Example 5.2.8 in Curtain and Zwart (1995), a β_1-stabilizing feedback operator is given by $Fy = \kappa_1\langle y, \phi_0\rangle = \frac{\kappa_1}{L}\cdot\int_0^L y(x,t)dx$. The closed loop operator $\mathcal{A} + \mathcal{B}F$ has eigenvalues $\left\{-\frac{\alpha^2}{L}\cdot\kappa_1, -\left(\frac{n\pi\alpha}{L}\right)^2, n \geq 1\right\}$. The eigenvalue $-\frac{\alpha^2}{L}\cdot\kappa_1$ is due to the stabilized part of the finite dimensional unstable part of $\mathcal{A}$ with dimension one expressed in Eqn. (5.196)

$$\frac{dy^+}{dt} = 0\cdot y^+ - \frac{\alpha^2}{L}\cdot\kappa_1\cdot y^+ = -\frac{\alpha^2}{L}\cdot\kappa_1\cdot y^+ \tag{5.196}$$

where $y^+(t) = e^{-\frac{\alpha^2}{L}\cdot\kappa_1 t}y^+(0)$. Since κ_1 is chosen such that the stabilized part of $\mathcal{A}$ is faster than the slowest eigenvalue of the stable part of $\mathcal{A}$ and that β_1 is chosen equal to this slowest stable part, this implies that the control system in Eqn. (5.187) is β_1 - exponentially stable.

Hence, the growth bound of the closed loop semigroup $T_{\mathcal{B}F}(t)$ is given by

$$\sup(\mathrm{Re}(\lambda), \lambda \in \sigma(\mathcal{A} + \mathcal{B}F)) = \lim_{t\to\infty}\frac{\log\|T_{\mathcal{B}\mathrm{F}}(\mathrm{t})\|}{t} = \beta_1 \tag{5.197}$$

Since, by Theorem 2 the linear system $\sum(\mathcal{A}, —, D)$ is β_2 - exponentially detectable, then by Theorem 5.2.7 and Example 5.2.8 in Curtain and Zwart (1995), a β_2-stabilizing output injection operator K is given by $K\xi = \kappa_2\xi\phi_0 = \kappa_2\xi\frac{1}{L}$.

The closed loop operator $\mathcal{A} + KD$ has eigenvalues $\left\{\frac{1}{L}\cdot\kappa_2, -\left(\frac{n\pi\alpha}{L}\right)^2, n \geq 1\right\}$.

Just as explained earlier for the determination of κ_1, $\frac{1}{L}\cdot\kappa_2$ is chosen such that it is more negative than the slowest eigenvalue of the stable part of $\mathcal{A}$ and where β_2 is chosen such that it is equal to this slowest stable eigenvalue. This implies that the state estimator defined in Eqn. (5.189) is β_2 - exponentially stable.

Hence, the growth bound of $T_{KD}(t)$ is given by Eqn. (5.198)

$$\sup(\mathrm{Re}(\lambda), \lambda \in \sigma(\mathcal{A} + KD)) = \lim_{t \to \infty} \frac{\log\|T_{KD}(t)\|}{t} = \beta_2. \qquad (5.198)$$

The state estimation error defined as $e(t) = \hat{y}(t) - y(t)$ is given as $e(t) = T_{KD}(t)e_0$ where $e_0 = \hat{y}_0 - y_0$. Since $T_{KD}(t)$ is exponentially stable, we have that $\lim_{t \to \infty} e(t) = 0, \forall e_0$. From Eqn. (5.197) and Eqn. (5.198), we see that the constants β_1 and β_2 are decay constants of the closed loop semigroups $T_{\mathcal{B}F}(t)$ and $T_{KD}(t)$.

Since the closed loop operators $T_{\mathcal{B}F}(t)$ generated by $\mathcal{A} + \mathcal{B}F$ and $T_{KD}(t)$ generated by $\mathcal{A} + KD$ are β_1 and β_2 - exponentially stable respectively, then by the Separation Theorem 5.3.3 in Curtain and Zwart (1995), the closed loop gas transmission system defined in Eqn. (5.191) is nominally exponentially stable. This completes the proof.

5.7.5 The Regulation Problem formulated as an Optimal Control Problem

As for the above controller design, the control problem is to bring the state y to zero for any initial condition $y_0 \in D(\mathcal{A})$. The customer load is equal to the stationary mass flow through the transmission line for the defined stationary operation point. But now, we want to bring the state to zero at a minimum cost for a defined cost function. Since the defined control model was proved to be exponentially stabilizable, then it is also optimizable. See Curtain and Zwart (1995) for this implication. The solution of the linear quadratic optimal control problem for a distributed parameter system is an optimal exponentially stable controller. See, for example, the references Barbu (1994), Curtain and Zwart (1995) or Li and Young (1995).

The infinite time horizon optimal linear quadratic regulation problem for a gas transmission line described by the control model defined in Eqn. (5.158) is defined in Eqn. (5.199).

$$\begin{aligned} &\underset{u(\cdot) \in \mathcal{U}}{\text{minimize}} \int_0^{\infty} \frac{1}{2}\langle Qy, y\rangle + \frac{1}{2}Ru^2 dt \\ &\text{subject to} \\ &\qquad \frac{dy}{dt} = \mathcal{A}y + \mathcal{B}u \\ &\qquad y(0) = y_0 \end{aligned} \tag{5.199}$$

where $Q = Q(x), 0 \le x \le L$ is the state penalty function defined such that $\langle Qy, y\rangle > 0, \forall y \in D(\mathcal{A})$ and the scalar R satisfies $R > 0$. The solution to the optimal control problem for a given initial condition is given in Eqn. (5.200). See the references Curtain and Zwart (1995), Li and Young (1995) or Barbu (1994).

$$V(y_0) = \inf_{u(\cdot) \in \mathcal{U}} \int_0^{\infty} \frac{1}{2}\langle Qy, y\rangle + \frac{1}{2}Ru^2 dt = \langle Py_0, y_0\rangle \tag{5.200}$$

The initial condition satisfies $y_0 \in D(\mathcal{A})$. The unique positive operator P is given by the solution of the nonlinear Riccati equation in Eqn. (5.201).

$$\langle P y_1, \mathcal{A} y_2 \rangle + \langle \mathcal{A} y_1, P y_2 \rangle + \langle Q y_1, y_2 \rangle - \langle \mathcal{B}^* P y_1, R^{-1} \mathcal{B}^* P y_2 \rangle \qquad (5.201)$$
$$= 0, \forall y_1, y_2 \in D(\mathcal{A})$$

The control law in feedback form for the problem stated in Eqn. (5.199) yields

$$u(t) = F y(t) = -R^{-1} \mathcal{B}^* P y(t) \qquad (5.202)$$

where $\mathcal{B}^*$ is the adjoint of $\mathcal{B}$. From the definition of the adjoint (Young,1992), the operator $\mathcal{B} u = b(x) \cdot u(t)$ must satisfy

$$\langle \mathcal{B} u, y \rangle = \langle u, \mathcal{B}^* y \rangle \qquad (5.203)$$

Using Eqn. (5.203), we have

$$\langle u, \mathcal{B}^* y \rangle = \int_0^L u(t) \cdot \mathcal{B}^*(x) y dx = u(t) \cdot \int_0^L \mathcal{B}^* y dx = \int_0^L \mathcal{B} u(t) y dx \qquad (5.204)$$
$$= u(t) \cdot \int_0^L \mathcal{B} y dx$$

This implies that $\mathcal{B}^* = \mathcal{B}$.

The final control law with feedback from the estimated states using the distributed parameter Luenberger state observer defined in Eqns. (5.189) and (5.190) and the feedforward designed in Section 5.5 yields

$$u(t) = F \hat{y}(t) + u_{\mathrm{ff}}(t). \qquad (5.205)$$

If the designed optimal state feedback gain for the continuous control model is used for a discrete controller, we have

$$u(k) = F \hat{y}(k) + u_{\mathrm{ff}}(k). \qquad (5.206)$$

5.8 Transmission Network Formulation

In this section we will give a dynamic model description of the fluid dynamics of a natural gas transmission system in the neighbourhood of a defined stationary operation point. The stationary operation point can be defined from a stationary optimization. The subject of stationary optimization was considered in Chapter 3. We will illustrate the modelling procedure with an example.

In Section 5.7 it was shown how a boundary control model describing the fluid dynamics inside a transmission line could be approximated by a control model with a distributed control command and an average measurement. It was shown that the state solution of the approximate control model was equal in the limit to the state solution of the boundary control model. Furthermore, it was shown that the average measurement was equal in the limit to the point measurement.

Figure 5.24 illustrates a long distance natural gas transmission system. Mass is supplied at the defined inlet boundaries of transmission line one and two. At the defined outlet boundary of transmission line three there is a customer offtake of natural gas. A control volume has been defined for the intersection point between the three transmission lines. For each of the intersecting transmission lines to this control volume, we have boundary conditions of the third kind, that is, we have Robin boundary conditions. We will approximate these boundary conditions with operators included in the partial differential equations. Also, we will approximate the Neuman boundary conditions at the input boundaries of transmission line one and two and the Neuman boundary condition at the output boundary of transmission line three similarly as was done in Section 5.7. As a result, we obtain a gas transmission network description where the boundary conditions of the three transmission lines are zero. Furthermore, we obtain an explicit description of the linear operator that expresses the dynamics and the couplings of the transmission system. By separating this linear operator into a "diagonal part" and a "coupling" part, it will be shown how we can express the semigroup generated by this linear operator. Then, by using the solution of the inhomogenous Cauchy problem, we obtain the expression for the state solution of the derived model. Dirichlet boundary conditions will not be considered. So, we assume that all the control and disturbance variables are flow variables for the model derived in this section.

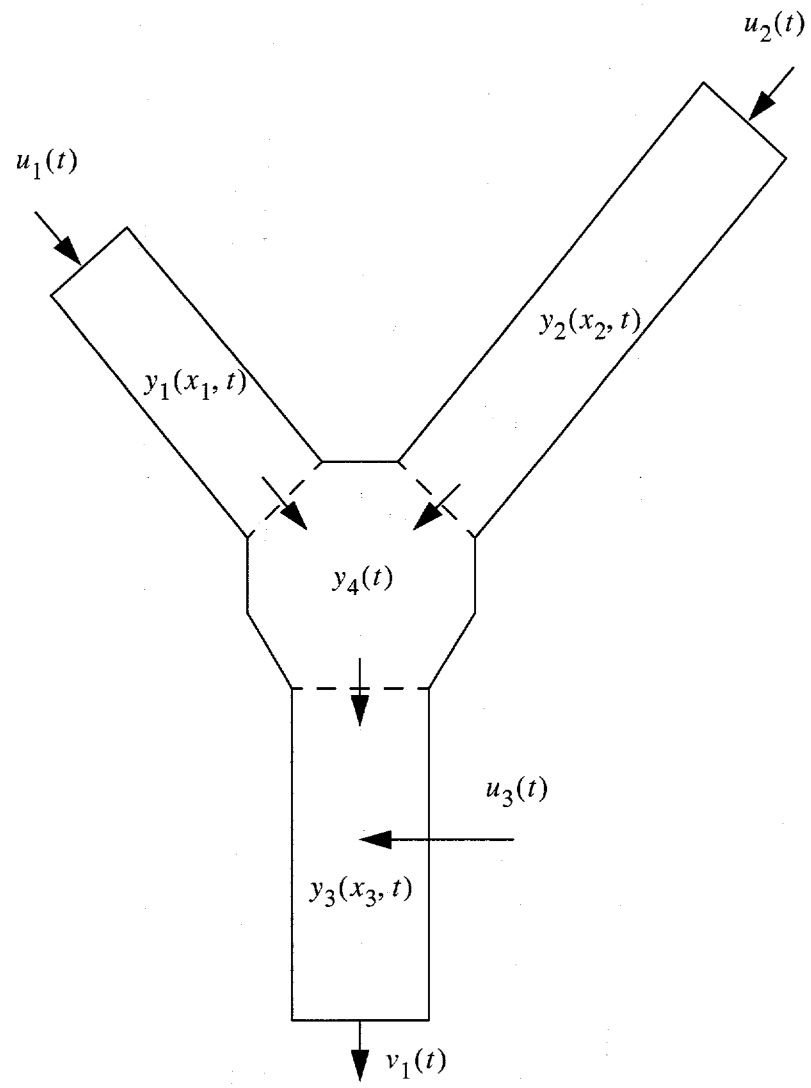


Figure 5.24: Transmission system example.

It was shown in Osiadacz, (1987) how the nonlinear creep flow model in Eqns. (5.1) and (5.2) could be reformulated to the model in Eqns. (5.207) and (5.208).

$$\begin{aligned} \frac{\partial p}{\partial t} &= \frac{c^2}{A\lambda} \cdot \frac{\partial^2 p}{\partial x^2} \\ W &= -\frac{1}{\lambda} \cdot \frac{\partial p}{\partial x} \end{aligned} \tag{5.207}$$

where

$$\lambda = \frac{2fc^2|W|}{DA^2 p}. \tag{5.208}$$

Define $p_1^*(\cdot)$, $p_2^*(\cdot)$ and $p_3^*(\cdot)$ to be the defined stationary pressure segment functions for the three transmission lines of the transmission system example and let p_4^* be the defined stationary pressure in the defined junction control volume. Define the state variables in Eqn. (5.209).

$$\begin{aligned} y_1(x_1, t) &= p_1(x_1, t) - p_1^*(x_1) \\ y_2(x_2, t) &= p_2(x_2, t) - p_2^*(x_2) \\ y_3(x_3, t) &= p_3(x_3, t) - p_3^*(x_3) \\ y_4(t) &= p_4(t) - p_4^* \end{aligned} \tag{5.209}$$

Define the average model coefficients in Eqn. (5.210)

$$\begin{aligned} \alpha_i^2 &= \frac{c^2}{A\lambda_{i,\,\mathrm{av}}}, \quad i = 1, 2, 3 \\ \beta_i &= \frac{1}{\lambda_{i,\,\mathrm{av}}} \end{aligned} \tag{5.210}$$

where $\lambda_{i,\,\mathrm{av}}$ is defined as

$$\lambda_{i,\,\mathrm{av}} = \frac{1}{L_i} \cdot \int_0^{L_i} \frac{2f_i c^2 |W_i^*(x_i)|}{D_i A_i^2 p_i^*(x_i)} dx_i. \tag{5.211}$$

The control variables u_1 and u_2 in Figure 5.24 are defined in Eqn. (5.212)

$$\begin{aligned} u_1(t) &= -\frac{1}{\beta_1}\cdot(W_1(0,t)-W_1{}^*(0)) = \frac{\partial}{\partial x_1}y_1(0,t) \\ u_2(t) &= -\frac{1}{\beta_2}\cdot(W_2(0,t)-W_2{}^*(0)) = \frac{\partial}{\partial x_2}y_2(0,t) \end{aligned} \tag{5.212}$$

and u_3 in Figure 5.24 is defined in Eqn. (5.213).

$$u_3(x,t) = \frac{\alpha_3^2}{2\varepsilon_{3d}}\cdot 1_{[x_{3d}-\varepsilon_{3d},\, x_{3d}+\varepsilon_{3d}]}(x_3)\cdot u_3(t) \tag{5.213}$$

The disturbance variable v_1 in Figure 5.24 is defined in Eqn. (5.214).

$$v_1(t) = -\frac{1}{\beta_3}\cdot(W_3(L_3,t)-W_3{}^*(L_3)) = \frac{\partial}{\partial x_3}y_3(L_3,t) \tag{5.214}$$

The mass flows at the boundary points are calculated as in Eqn. (5.215).

$$\begin{aligned} W_1(0,t) &= -\beta_1\cdot u_1 + W_1{}^*(0) \\ W_2(0,t) &= -\beta_2\cdot u_2 + W_2{}^*(0) \\ W_3(L_3,t) &= -\beta_3\cdot v_3 + W_3{}^*(L_3) \end{aligned} \tag{5.215}$$

The distributed mass flow denoted by W_d is calculated as in Eqn. (5.216).

$$W_d(t) = \frac{\alpha_3^2\cdot A_3}{c^2}\cdot u_3(t) + W_d{}^* \tag{5.216}$$

where $W_d{}^*$ is the defined distributed stationary mass flow, A_3 is the cross section area of transmission line three and c is the speed of sound. The state equation $p = c^2\cdot\rho$ was used to arrive at Eqn. (5.216).

The mass balance at the junction control volume yields

$$\frac{dm_4}{dt} = W_1(L_1, t) + W_2(L_2, t) - W_3(0, t) \tag{5.217}$$

where m_4 is the mass of natural gas inside the control volume.

Use of the state equation $p = c^2 \cdot \rho$ yields

$$\frac{dp_4}{dt} = \frac{c^2}{\varepsilon_{cv}} \cdot (W_1(L_1, t) + W_2(L_2, t) - W_3(0, t)) \tag{5.218}$$

where ε_{cv} is the volume of the defined junction control volume.

At the junction control volume, we define the following Robin boundary conditions when the transmission state is close to the defined stationary operation point.

$$\begin{aligned} W_1(L_1, t) &= h_1 \cdot (p_1(L_1, t) - p_4(t)) \\ W_2(L_2, t) &= h_2 \cdot (p_2(L_2, t) - p_4(t)) \\ W_3(0, t) &= h_3 \cdot (p_4(t) - p_3(0, t)) \end{aligned} \tag{5.219}$$

The value of the coefficients h_1, h_2 and h_3 are determined based on the stationary operation point. Subtracting the stationary operation point in Eqn. (5.219), use of the relations $W_1(L_1, t) = -\beta_1 \frac{\partial}{\partial x_1} p_1(L_1, t)$, $W_2(L_2, t) = -\beta_2 \frac{\partial}{\partial x_2} p_2(L_2, t)$, $W_3(0, t) = -\beta_3 \frac{\partial}{\partial x_3} p_3(0, t)$ and the state definitions in Eqn. (5.209) yields

$$\begin{aligned} \frac{\partial}{\partial x_1} y_1(L_1, t) &= -\frac{h_1}{\beta_1} \cdot (y_1(L_1, t) - y_4(t)) \\ \frac{\partial}{\partial x_2} y_2(L_2, t) &= -\frac{h_2}{\beta_2} \cdot (y_2(L_2, t) - y_4(t)). \\ \frac{\partial}{\partial x_3} y_3(0, t) &= -\frac{h_3}{\beta_3} \cdot (y_4(t) - y_3(0, t)) \end{aligned} \tag{5.220}$$

The boundary states $y_1(L_1, t)$, $y_2(L_2, t)$ and $y_3(0, t)$ are approximated similarly as for the defined average measurement in Section 5.7 so that Eqn. (5.220) is approximated by Eqns. (5.221)-(5.223).

$$\frac{\partial}{\partial x_1} y_1(L_1, t) \tag{5.221}$$

$$\approx -\frac{h_1}{\beta_1} \cdot \left(\int_0^{L_1} \frac{1}{\varepsilon_{1,\,\text{out}}} \cdot 1_{[L_1 - \varepsilon_{1,\,\text{out}},\, L_1]}(x_1) \cdot y_1(x_1, t) dx_1 - y_4(t) \right)$$

$$\frac{\partial}{\partial x_2} y_2(L_2, t) \tag{5.222}$$

$$\approx -\frac{h_2}{\beta_2} \cdot \left(\int_0^{L_2} \frac{1}{\varepsilon_{2,\,\text{out}}} \cdot 1_{[L_2 - \varepsilon_{2,\,\text{out}},\, L_2]}(x_2) \cdot y_2(x_2, t) dx_2 - y_4(t) \right)$$

$$\frac{\partial}{\partial x_3} y_3(0, t) \tag{5.223}$$

$$\approx -\frac{h_3}{\beta_3} \cdot \left(y_4(t) - \int_0^{L_3} \frac{1}{\varepsilon_{3,\,\text{in}}} \cdot 1_{[0,\, \varepsilon_{3,\,\text{in}}]}(x_3) \cdot y_3(x_3, t) dx_3 \right)$$

The boundary gradients in Eqns. (5.221)-(5.223) are approximated similarly as shown in Section 5.7 by distributed operators. The values of the gradients are determined by the right hand sides in Eqns. (5.221)-(5.223).

The control and disturbance variables are also approximated by distributed operators as shown in Section 5.7.

The transmission system is now written in Cauchy differential form in Eqn. (5.224)

$$\begin{aligned} \frac{dy}{dt} &= Ay + Bu + Cv \\ y(0) &= y_0 \end{aligned} \tag{5.224}$$

where the state is defined in Eqn. (5.225)

$$y(t) = \left[y_1(\cdot,t)\ y_2(\cdot,t)\ y_3(\cdot,t)\ y_4(t)\right]^{\mathrm{T}}, \tag{5.225}$$

the control vector is defined in Eqn. (5.226)

$$u(t) = \left[u_1(t)\ u_2(t)\ u_3(t)\right]^{\mathrm{T}} \tag{5.226}$$

and the disturbance vector is defined in Eqn. (5.227).

$$v(t) = \left[v_1(t)\right] \tag{5.227}$$

We have that $y_{i0}(\cdot) \in L_2(0, L_i)$ and $y_i(\cdot, t) \in L_2(0, L_1)$ for $t > 0$ where $i = 1, 2, 3$. The junction control volume satisfies $y_{40}, y_4(t) \in R$ for $t > 0$. The operator A is represented by dividing it in a "diagonal" part which is denoted by A_1 and a "coupling" part which is denoted by A_2 as expressed in Eqn. (5.228).

$$Az = A_1 z + A_2 z \tag{5.228}$$

The "diagonal" operator $A_1 z$ is given in Eqn. (5.229)

$$A_1 z = \begin{bmatrix} \alpha_1^2 \cdot \dfrac{d^2 z_1}{dx_1^2} \\ \alpha_2^2 \cdot \dfrac{d^2 z_2}{dx_2^2} \\ \alpha_3^2 \cdot \dfrac{d^2 z_3}{dx_3^2} \\ -\left(\dfrac{c^2}{\varepsilon_{\mathrm{cv}}} \cdot h_1 + \dfrac{c^2}{\varepsilon_{\mathrm{cv}}} \cdot h_2 + \dfrac{c^2}{\varepsilon_{\mathrm{cv}}} \cdot h_3\right) \cdot z_4 \end{bmatrix} \tag{5.229}$$

where $D(A_{1,ii})$, $i = 1, 2, 3$ is given in Eqn. (5.230).

$$D(A_{1,ii}) = \left\{ z_i \in L_2(0, L_i) \right| \tag{5.230}$$

$$\left. z_i \in C, \frac{dz_i}{dx_i} \in C, \frac{d^2 z_i}{dx_i^2} \in L_2(0, L_i), \frac{d}{dx_i} z_i(0) = 0, \frac{d}{dx_i} z_i(L_i) = 0 \right\}$$

and $D(A_{1,44}) = \{z_4 \in R\}$.

The "coupling" operator $A_2 z$ is given in Eqn. (5.231)

$$A_2 z = \begin{bmatrix} A_{2,11} z_1 + A_{2,12} z_2 + A_{2,13} z_3 + A_{2,14} z_4 \\ A_{2,21} z_1 + A_{2,22} z_2 + A_{2,23} z_3 + A_{2,24} z_4 \\ A_{2,31} z_1 + A_{2,32} z_2 + A_{2,33} z_3 + A_{2,34} z_4 \\ A_{2,41} z_1 + A_{2,42} z_2 + A_{2,43} z_3 + A_{2,44} z_4 \end{bmatrix} \tag{5.231}$$

with components given in Eqns. (5.232)-(5.247).

$$A_{2,11} z_1 = -\frac{\alpha_1^2 \cdot h_1}{\varepsilon_{1,\text{out}} \cdot \beta_1} \tag{5.232}$$

$$\cdot \mathbf{1}_{[L_1 - \varepsilon_{1,\text{out}}, L_1]}(x_1) \cdot \int_0^{L_1} \frac{1}{\varepsilon_{1,\text{out}}} \cdot \mathbf{1}_{[L_1 - \varepsilon_{1,\text{out}}, L_1]}(x_1) \cdot z_1(x_1) dx_1$$

$$A_{2,12} z_2 = 0 \tag{5.233}$$

$$A_{2,13} z_3 = 0 \tag{5.234}$$

$$A_{2,14} z_4 = \frac{\alpha_1^2 \cdot h_1}{\varepsilon_{1,\text{out}} \cdot \beta_1} \cdot \mathbf{1}_{[L_1 - \varepsilon_{1,\text{out}}, L_1]}(x_1) \cdot z_4 \tag{5.235}$$

$$A_{2,21} z_1 = 0 \tag{5.236}$$

$$A_{2,22}z_2 = -\frac{\alpha_2^2 \cdot h_2}{\varepsilon_{2,\text{out}} \cdot \beta_2} \tag{5.237}$$

$$\cdot 1_{[L_2-\varepsilon_{2,\text{out}}, L_2]}(x_2) \cdot \int_0^{L_2} \frac{1}{\varepsilon_{2,\text{out}}} \cdot 1_{[L_2-\varepsilon_{2,\text{out}}, L_2]}(x_2) \cdot z_2(x_2)dx_2$$

$$A_{2,23}z_3 = 0 \tag{5.238}$$

$$A_{2,24}z_4 = \frac{\alpha_2^2 \cdot h_2}{\varepsilon_{2,\text{out}} \cdot \beta_2} \cdot 1_{[L_2-\varepsilon_{2,\text{out}}, L_2]}(x_2) \cdot z_4 \tag{5.239}$$

$$A_{2,31}z_1 = 0 \tag{5.240}$$

$$A_{2,32}z_2 = 0 \tag{5.241}$$

$$A_{2,33}z_3 \tag{5.242}$$

$$= -\frac{\alpha_3^2 \cdot h_3}{\varepsilon_{3,\text{in}} \cdot \beta_3} \cdot 1_{[0, \varepsilon_{3,\text{in}}]}(x_3) \cdot \int_0^{L_3} \frac{1}{\varepsilon_{3,\text{in}}} \cdot 1_{[0, \varepsilon_{3,\text{in}}]}(x_3) \cdot z_3(x_3)dx_3$$

$$A_{2,34}z_4 = \frac{\alpha_3^2 \cdot h_3}{\varepsilon_{3,\text{in}} \cdot \beta_3} \cdot 1_{[0, \varepsilon_{3,\text{in}}]}(x_3) \cdot z_4 \tag{5.243}$$

$$A_{2,41}z_1 = \frac{c^2}{\varepsilon_{cv}} \cdot h_1 \cdot \int_0^{L_1} \frac{1}{\varepsilon_{1,\text{out}}} \cdot 1_{[L_1-\varepsilon_{1,\text{out}}, L_1]}(x_1) \cdot z_1(x_1)dx_1 \tag{5.244}$$

$$A_{2,42}z_2 = \frac{c^2}{\varepsilon_{cv}} \cdot h_2 \cdot \int_0^{L_2} \frac{1}{\varepsilon_{2,\text{out}}} \cdot 1_{[L_2-\varepsilon_{2,\text{out}}, L_2]}(x_2) \cdot z_2(x_2)dx_2 \tag{5.245}$$

$$A_{2,43}z_3 = \frac{c^2}{\varepsilon_{cv}} \cdot h_3 \cdot \int_0^{L_3} \frac{1}{\varepsilon_{3,\mathrm{in}}} \cdot 1_{[0,\,\varepsilon_{3,\mathrm{in}}]}(x_3) \cdot z_3(x_3)dx_3 \tag{5.246}$$

$$A_{2,44}z_4 = 0 \tag{5.247}$$

The control input map Bu is given in Eqn. (5.248)

$$Bu \tag{5.248}$$

$$= \begin{bmatrix} -\frac{\alpha_1^2}{\varepsilon_{1,\mathrm{in}}} \cdot 1_{[0,\,\varepsilon_{1,\mathrm{in}}]}(x_1) & 0 & 0 \\ 0 & -\frac{\alpha_2^2}{\varepsilon_{2,\mathrm{in}}} \cdot 1_{[0,\,\varepsilon_{2,\mathrm{in}}]}(x_2) & 0 \\ 0 & 0 & \frac{\alpha_3^2}{2\varepsilon_{3d}} \cdot 1_{[x_{3d}-\varepsilon_{3d},\,x_{3d}+\varepsilon_{3d}]}(x_3) \\ 0 & 0 & 0 \end{bmatrix}$$

$$\cdot \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

and the disturbance input map Cv is given in Eqn. (5.249).

$$Cv = \begin{bmatrix} 0 \\ 0 \\ \dfrac{\alpha_3^2}{\varepsilon_{3,\,\mathrm{out}}} \cdot 1_{[L_3 - \varepsilon_{3,\,\mathrm{out}},\, L_3]}(x_3) \\ 0 \end{bmatrix} \cdot \begin{bmatrix} v_1 \end{bmatrix} \tag{5.249}$$

The operator A_1 is the infinitesimal generator of the strongly continuous semigroup $T(t)$ given in Eqn. (5.250)

$$T(t)y_0 = \begin{bmatrix} \sum_{n=0}^{\infty} e^{\lambda_{1n}t} \cdot \langle y_{10}, \phi_{1n} \rangle \cdot \phi_{1n} \\ \sum_{n=0}^{\infty} e^{\lambda_{2n}t} \cdot \langle y_{20}, \phi_{2n} \rangle \cdot \phi_{2n} \\ \sum_{n=0}^{\infty} e^{\lambda_{3n}t} \cdot \langle y_{30}, \phi_{3n} \rangle \cdot \phi_{3n} \\ e^{-\left(\frac{c^2}{\varepsilon_{cv}} \cdot h_1 + \frac{c^2}{\varepsilon_{cv}} \cdot h_2 + \frac{c^2}{\varepsilon_{cv}} \cdot h_3\right) \cdot t} \cdot y_{40} \end{bmatrix} \tag{5.250}$$

where $\phi_{in}(x_i) = \sqrt{\dfrac{2}{L_i}} \cdot \cos\left(n\pi \cdot \dfrac{x_i}{L_i}\right)$, $n \geq 1$, $\phi_{i0}(x_i) = \sqrt{\dfrac{1}{L_i}}$, and

$\lambda_{in} = -\left(\dfrac{n\pi\alpha_i}{L_i}\right)^2$, $i = 1, 2, 3$.

The semigroup $S(t)$ generated by the operator A can now be expressed using the operator integral equation as in Eqn. (5.251)

$$S(t)y_0 = T(t)y_0 + \int_0^t T(t-\eta)A_2 y(\eta)d\eta \,. \tag{5.251}$$

See the references Barbu (1994) and Curtain and Zwart (1995) for theory on the operator integral equation.

By the use of the state solution in Eqn. (5.63) of the inhomogenous Cauchy problem defined in Eqn. (5.62) and defining $f(t) = Bu(t) + Cv(t)$, the state solution is given in Eqn. (5.252).

$$y(t) = S(t)y_0 + \int_0^t S(t-\eta)Bu(\eta)d\eta + \int_0^t S(t-\eta)Cv(\eta)d\eta \tag{5.252}$$

Assuming that u and v are constant for an entire sampling interval T_s yield the discrete model in Eqn. (5.253)

$$y(k+1) = \Phi y(k) + \Gamma u(k) + \Psi v(k) \tag{5.253}$$

where

$$\begin{aligned} \Phi y(k) &= S(T_s)y(k) \\ \Gamma u(k) &= \int_0^{T_s} S(\eta)Bu(k)d\eta \\ \Psi v(k) &= \int_0^{T_s} S(\eta)Cv(k)d\eta \end{aligned} \tag{5.254}$$

and the time value at the discrete sampling instant k is given by $t_k = k \cdot T_s$.

If we approximate $S(T_s)y(k)$ as done in Eqn. (5.255)

$$S(T_s)y(k) \approx T(T_s)y(k) + \int_0^{T_s} T(\eta)A_2 y(k)d\eta, \tag{5.255}$$

we obtain a more convenient expression for practical simulation purposes. In addition the infinite sums in $T(t)$ must be truncated by finite sums of terms. The approximation error is a function of the length of the sampling interval. Alternatively, the integral in $S(T_s)y(k)$ could be approximated by the first order method in Eqn. (5.256).

$$S(T_s)y(k) = T(T_s)y(k) + A_2y(k) + e(T_s) \tag{5.256}$$

A third alternative using the trapezoid rule for integration yield the implicit scheme given in Eqn. (5.257).

$$\begin{aligned} & S(T_s)y(k) \\ & = T(T_s)y(k) + \frac{T_s}{2} \cdot \{A_2y(k) + T(T_s)A_2y(k+1)\} + e(T_s^2) \end{aligned} \tag{5.257}$$

Under the assumption that $T(t)A_2y(t)$ is continuous and $\frac{d^2}{dt^2}T(t)A_2y(t)$ exists on $[0, T_s]$, the trapezoid rule is of second order accuracy. See Cheney and Kincaid (1999) for a proof.

<u>Approximation of a semilinear partial differential equation:</u>

From the model in Eqns. (5.207) and (5.208) we see that the factor λ is a function of x and t. The solution from a defined stationary optimization problem will determine the stationary pressure and flow distributions. Using the optimization result then gives a λ that is a function of x. Hence, we obtain a semilinear diffusion equation. We can approximate this model by a set of linear partial differential equations and linear ordinary differential equations. Between two linear partial differential equations, we put a linear ordinary differential equations expressing mass balance for the defined control volume as shown earlier. The linear partial differential equations and the linear ordinary differential equation are connected by Robin boundary conditions. The boundary gradients are approximated by distributed operators as shown earlier so that the boundary conditions of each partial differential equation are zero. A linear partial differential equation defined to describe the dynamics for a certain region of the transmission line gets its constant α^2 parameter value by taking the average of the $\alpha^{*2}(x)$ function on the defined domain for the linear partial differential equation. Hence, using this approximation method of the semilinear partial differential equations, we can still describe the dynamics of natural gas transmission systems with the *linear* system representations in Eqn. (5.224) and (5.253).

In this section it has been shown a method of how to model the gas dynamics of natural gas transmission networks. The state solution is given as the solution to the inhomogenous Cauchy problem.

5.9 Formulation of a Model Predictive Control Scheme

In this Section, we will define an open loop problem of a linear model predictive control scheme that uses the linear distributed parameter gas transmission network model that was derived in Section 5.8. The notation for the dynamic model used in this section is similar to the notation for the model formulated in Section 5.8.

Define the quadratic cost function in Eqn. (5.258)

$$
\begin{aligned}
& J(y_k, \pi_{\mathrm{ol}}, \vartheta_{\mathrm{ol}}) \\
& = \sum_{i=1}^{M_{\mathrm{ol}}} \left\{ \sum_{j=1}^{n} \frac{1}{2} \langle Q_j y_{j,\,k+i|k}, y_{j,\,k+i|k} \rangle + \frac{1}{2} u_{k+i-1|k}^{\mathrm{T}} R u_{k+i-1|k} \right\}
\end{aligned}
\tag{5.258}
$$

where π_{ol} is the open loop control sequence in Eqn. (5.259)

$$
\pi_{\mathrm{ol}} = \{ u_{k+i-1|k} | i = 1, 2, \ldots, M_{\mathrm{ol}} \} \tag{5.259}
$$

and ϑ_{ol} is the disturbance sequence given in Eqn. (5.260)

$$
\vartheta_{\mathrm{ol}} = \{ v_{k+i-1|k} | i = 1, 2, \ldots, M_{\mathrm{ol}} \} \tag{5.260}
$$

The penalty function Q_j is a positive function or a positive scalar. It is a positive scalar if the state component y_j is the state of a defined junction control volume. It is a positive continuous function or a positive scalar if the state component y_j is a state segment function.

Assume that the model in Eqn. (5.253) is used as prediction model for the open loop horizon where the calculations are done at time t_k. This model, used as an open loop prediction model, is compactly written as in Eqn. (5.261)

$$
\chi(y_{k|k}, \pi_{\mathrm{ol}}, \vartheta_{\mathrm{ol}}) = 0 \tag{5.261}
$$

where

$$\chi(y_{k|k}, \pi_{\mathrm{ol}}, \vartheta_{\mathrm{ol}}) \tag{5.262}$$

$$= \begin{bmatrix} y_{k+1|k} - (\Phi y_{k|k} + \Gamma u_{k|k} + \Psi v_{k|k}) \\ y_{k+2|k} - (\Phi y_{k+1|k} + \Gamma u_{k+1|k} + \Psi v_{k+1|k}) \\ \cdots \\ y_{k+M_{\mathrm{ol}}|k} - (\Phi y_{k+M_{\mathrm{ol}}-1|k} + \Gamma u_{k+M_{\mathrm{ol}}-1|k} + \Psi v_{k+M_{\mathrm{ol}}-1|k}) \end{bmatrix}$$

The open loop state sequence $\mathcal{Y}_{\mathrm{ol}}$ is given in Eqn. (5.263).

$$\mathcal{Y}_{\mathrm{ol}} = \{y_{k+i|k} \big| i = 1, 2, \ldots, M_{\mathrm{ol}}\} \tag{5.263}$$

Define the state space in Eqn. (5.264)

$$Y = \{y_{k+i|k} \big| \underline{y} \le y_{k+i|k} \le \bar{y}, i = 1, 2, \ldots, M_{\mathrm{ol}}\} \tag{5.264}$$

where $\underline{y}_j$ and $\bar{y}_j$ are the lower and upper limit scalars or continuous constraint segment functions respectively for the state component j with the state y_j depending on if the state component describes the fluid state in a transmission line or the state in a junction control volume.

Define the open loop control space in Eqn. (5.265).

$$\begin{aligned} &\mathcal{U}_{\mathrm{ol}}(k) \\ &= \{u_{k+i-1|k} \big| \underline{u}_{k+i-1|k} \le u_{k+i-1|k} \le \bar{u}_{k+i-1|k}, i = 1, 2, \ldots, M_{\mathrm{ol}}\} \end{aligned} \tag{5.265}$$

We have the predicted open loop disturbance sequence in Eqn. (5.266)

$$\mathcal{V}_{\mathrm{ol}}(k) = \{\hat{v}_{k+i-1|k} \big| i = 1, 2, \ldots, M_{\mathrm{ol}}\} \tag{5.266}$$

which is formulated based on the natural gas open loop load forecast. With the above definitions, we define the constrained finite time linear quadratic open loop problem in Eqn. (5.267).

$$\pi_{\text{ol}}^{\text{opt}} = \inf_{\pi_{\text{ol}}} J(y_{k|k}, \pi_{\text{ol}}, \vartheta_{\text{ol}})$$

subject to

$$\begin{aligned} &\chi(y_{k|k}, \pi_{\text{ol}}, \vartheta_{\text{ol}}) = 0 \\ &\mathcal{Y}_{\text{ol}} \in Y \\ &\pi_{\text{ol}} \in \mathcal{U}_{\text{ol}}(k) \\ &\vartheta_{\text{ol}} = \mathcal{V}_{\text{ol}}(k) \\ &y_{k|k} = y_k \end{aligned} \tag{5.267}$$

The optimal open loop control input sequence $\pi_{\text{ol}}^{\text{opt}} = \pi_{\text{ol}}^{\text{opt}}(k)$ determined at time t_k is a function of the initial condition, the available control space and the load forecast as expressed in Eqn. (5.268).

$$\pi_{\text{ol}}^{\text{opt}}(k) = \pi_{\text{ol}}^{\text{opt}}(y_k, \mathcal{U}_{\text{ol}}(k), \mathcal{V}_{\text{ol}}(k)) \tag{5.268}$$

Even if the notation of the dynamic model as a set of equality conditions implies an implicit prediction model, the dynamic model is to be used as an explicit predictor as in Eqn. (5.253). The prediction model can be written in augmented form for the open loop horizon.

In this section, we have suggested how the open loop problem of a linear model predictive control scheme can be formulated using the transmission network model formulated in Section 5.8.

5.10 Summary and Discussion

A control and state constrained model predictive controller using an optimization model that was linear in squared pressures and flows formulated around a stationary operation point was designed for a single gas transmission line. The control law is nonlinear due to the optimization model. A linear model predictive controller using an open loop optimization model based on the value of the system state at start of the open loop horizon was also designed. The state and the control references are determined from a stationary operation point. The purpose with these two model predictive controllers was to show that the creep flow model used in this thesis can be reformulated to different adequate control models. The control models can be generalized to network descriptions.

The mild solution was given for a distributed parameter model describing the fluid dynamics of a natural gas transmission line with boundary control and distributed control together with boundary disturbance and distributed disturbance. This was possible through the formulation of the control model as an inhomogenous Cauchy formulation. A transfer function description was then derived.

A control scheme being a combination of a feedforward from a predicted load demand together with the use of a Smith predictor was designed for a transmission system using a transfer function control model. The transfer functions were constructed from observation of the qualitative behaviour of the step responses from the analytical solution of a distributed parameter model for the state variables at the linear inner core and then adjustments of the model parameters against the nonlinear simulation model. Simulation results of the closed loop behaviour was shown. This model predictive controller has its large advantage in its simplicity without major on-line calculations.

A distributed parameter controller was designed for a natural gas transmission line. A boundary control model was approximated by a control model with a distributed control command. It was shown that the state solution of the approximated model was in the limit equal to the state solution of the defined boundary control model. Furthermore, it was shown that the defined control model was approximately controllable and approximately observable. Also, it was further shown that the control model was exponentially stabilizable and exponentially detectable. Then, an exponentially stable controller was designed. An exponentially stable Luenberger state observer was also defined. Then, it was shown that the coupled control system with linear feedback from estimated states was exponentially stable. The linear quadratic control problem was also defined.

Furthermore, a distributed parameter network formulation was derived where all the boundary conditions of the transmission lines were zero. The boundary conditions of the third kind (Robin type) originating at the junction points of the defined network formulation are approximated by distributed operators that are part of the partial differential equations. The linear operator expressing the dynamics and the couplings of the transmission system was explicitly obtained. An expression for the strongly continuous semigroup generated by this linear operator was also derived. The state solution is given as the solution to the inhomogenous Cauchy problem. A formulation of a linear model predictive control scheme was defined in Section 5.9 assuming use of the transmission system model derived in Section 5.8.

Chapter 6

Conclusions and Future Work

6.1 Conclusions

Different optimization and control strategies using different optimization and control models for the operation support of gas transmission systems, have been studied in this thesis. The security of the supply with regards to meeting the customer demands, the control of important quality parameters and the minimization of transportation costs are the main control objectives. The emphasis has been directed on long distance transmission systems with continuous control variables. It has been assumed that the network configuration and the configuration of each compressor station are predetermined.

Stationary Optimization and Linear Model Predictive Control:

The result from the stationary optimization determines the parameters of a linear control and estimation model and the state and control references. The main goal for the model predictive controller is to bring or drive the state of the transmission system towards the state reference optimally, and at the same time to meet the defined system constraints.

1. Security of supply: This is handled by the linear model predictive controller by increasing the penalty weight when approaching the customer points for the state deviation from the currently defined optimal stationary pressure distribution of the transmission system. In the stationary optimization, the physical and the contractual constraints such as the quality parameter limits and the minimum customer pressures are satisfied on a stationary basis. Latest updates on the transient load forecasts and the generally transient maximum and minimum source capacity forecasts are inserted into the current quadratic open loop nonlinear programming problem. In this way, one or several sources with good production capacity can increase their production to compensate for the sources that face reduced production capacity. Hence, the customer demands together with meet-

ing the contractual minimum customer pressures can still be satisfied. The accuracy of the linear model predictive controller may be less than that of the nonlinear predictive control scheme presented in this thesis due to the linear approximation of the dynamics. Therefore, the pressures at the customer terminals may face larger variance around the terminal pressure references using the linear model predictive controller compared to using the nonlinear control scheme. Setting the customer terminal reference pressure and the minimum pressure limit above the defined minimum contractual limit can compensate for this reduced accuracy. The increased linepack that results when using this procedure will, of course, give increased compression costs. But, the security of the supply at the customer terminals has a higher priority than the transportation costs.

2. Control of quality parameters: The quality parameters are controlled implicitly by the model predictive controller since it calculates the control commands with the objective to bring the state of the system towards the current stationary optimal operation point. Quality parameter constraints are satisfied on a stationary basis for the optimal stationary operation point that is defined.

3. Transportation costs: The objective function of the stationary optimization problem expresses the costs in connection with the transportation of the natural gas from the supply points to the defined customer delivery points. The objective of the model predictive controller is to bring the gas transmission network to the currently determined optimal economical stationary state. The dynamic transition towards this optimal state shall also be performed at the lowest cost. Each open loop problem of the model predictive control algorithm uses a quadratic cost function to penalize the deviation of the state and the control vector from their respective optimal references.

4. Computational feasibility: A major advantage of the proposed control strategy is that each open loop problem is a strictly convex programming problem. Commercial quadratic programming algorithms find a global minimum at a high convergence rate implying few iterations. Also, only one mathematical programming problem is solved at each sampling instant. An update on the stationary optimization can be done at a regular basis or when the system is expected to change the operation point significantly. All these factors imply that the proposed control strategy has low computational effort. Of course the computational effort increases with the size of the transmission network.

5. Complex thermodynamics: For critical operational reasons or for the interest of a more accurate description of stationary behaviour to gain increased economical benefits, it is possible to include the energy equation and a complex thermodynamic state equation. The result from the stationary equation will then give optimal stationary flows through the system transmission lines and an optimal pressure distribution. In addition, the temperature distribution and the compressibility factor distribution will be given. This optimization result then specifies the

system matrices of the control and estimation model used by the linear model predictive controller. In this way, complex thermodynamics on a stationary basis can be included implicitly. It is important to notice that the number of optimization variables increases for the stationary optimization.

7. Local or global stationary optimization: The stationary optimization can be solved to a local minimum if a local optimization algorithm is used. Use of a global optimization algorithm, such as the Alpha Branch and Bound algorithm, finds the global minimum inside a defined termination criteria. The main drawback of using a global optimization algorithm is the increased computational effort.

8. Infeasibility handling: A simple way of handling infeasibility is to include only the control constraints in the open loop problems when the transmission system is in an infeasibility mode. Another approach is to relax the state constraints. The relaxation of the state constraints is penalized quadratically in the objective function of the open loop problem when the system is in the infeasibility mode. Optimal relaxation based on a defined priority list of the state constraints is another more complex alternative which increases the computational burden.

9. Adaptive properties: With the update of the stationary optimization when expecting the average customer load level to change significantly gives a new set of parameters to the control and estimation model used by the model predictive controller.

10. Maximization of transportation capacity: The objective function in the stationary problem can be replaced with the objective of maximizing transportation capacity through the transmission system. Other alternative objective functions can of course also be defined.

Nonlinear Model Predictive Control:

Each open loop problem is a nonlinear programming problem that is solved to a local minimum inside a defined termination criteria using a structured sequential quadratic programming procedure of Newton type. The purpose with the model predictive controller is to bring or drive a defined output vector towards the output vector reference optimally and, at the same time, to meet the defined system constraints.

1. Security of the supply: The deviation from a defined customer terminal pressure reference is penalized quadratically. The latest update on the transient load forecasts and the generally transient maximum and minimum source capacity forecasts are inserted into the current quadratic open loop nonlinear programming problem as for the linear model predictive controller. Choosing different penalty weights at the different customer terminals can put the main priority on to the terminals that are considered to be the most important ones. Based on the defined priority, the available gas resources inside the network and the available source capacity can then be directed towards the most important terminals. Each iteration

of the open loop problem uses a linear time variant prediction model developed from the nonlinear creep flow model around the nominal state and the control trajectories from the previous iteration. Therefore, the fluid dynamics can be more accurately described by this control scheme compared to the fluid dynamics description by the linear model predictive control scheme. This model can also include complex stationary thermodynamics as explained below. As a result, the customer pressures may be kept closer to the customer terminal reference pressures implying improved security of the supply.

2. Control of quality parameters: Important quality parameters are included in the defined output vector and are therefore controlled transiently. If the source compositions are time varying and/or the system has many internal mixing stations and junction points, the use of a quality tracker may be needed to predict the value of the quality parameters as they enter a mixing or a junction point.

3. Transportation costs: Transportation costs are minimized implicitly by minimizing the pressure level of the transmission system. The transition towards this low pressure level is also performed at a minimum cost. A quadratic cost function is defined for each open loop problem for the deviation of the defined control vector and the defined output vector from their respective references. The pressure level between the different compressor stations, the pipelines and the control valves can be differentiated through the choice of the penalty weights in the quadratic cost function. Different pressure levels between the different compressor stations, the pipelines and the control valves can then take an account of the different compression costs. Due to the fact that the nonlinear model predictive controller may describe the dynamics more accurately, the variance of the customer terminal pressures around the respective reference values may be smaller compared to that of the linear model predictive controller. Therefore, one may set the terminal pressure references and the lower operational pressure limits for the controller closer to the minimum contractual limits. This reduces the line pack of natural gas in the system and therefore the transportation costs. But, for supply security reasons, the transmission system should in any case have some operational flexibility by the addition of some extra line pack of natural gas.

4. Computational feasibility: Since the nonlinear programming problem of the current open loop problem is solved to a local minimum by a sequence of quadratic programs, the computational effort is increased considerably compared to a linear model predictive control scheme with a stationary optimization which possess only a single quadratic programming problem at each open loop problem and at each sampling instant. Therefore, the stationary optimization with the linear model predictive controller may be preferred against the nonlinear model predictive control scheme for large transmission systems if the computer capacity that is available is low.

5. Complex stationary fluid flow description and thermodynamics: This control

scheme can also use a stationary optimization where the energy equation is included in addition to the mass and the momentum equation to describe the stationary fluid flow through a pipeline. In addition, a complex thermodynamic state equation can be included. The control objective may be the minimization of transportation cost. Stationary complex thermodynamics is included by using the available stationary compressibility factor and temperature distributions in the expression for the speed of sound along the transmission lines for the linear time variant prediction model. The calculated stationary optimal control and state references can also be used.

6. Infeasibility handling: Similar procedures as for the control strategy with stationary optimization and the linear model predictive control can be used. But the prioritized infeasibility handling algorithm suggested for the linear model predictive controller has so far been developed only for linear model predictive controllers.

7. Maximization of transportation capacity: The proposed control algorithm can be used with the objective to maximize the transportation capacity. Increased transmission capacity through the pipelines is obtained by increasing the pressure difference reference between the input and the output of each transmission line. The pressure differences over the control valves must be large enough such that their flow capacities correspond with the pipeline capacities. With a control policy that has an objective to maximize the transportation capacity, it should be possible to reach the maximum delivery capacity at each customer terminal. This capacity is strongly dependent on the offtake capacity of the downstream transmission networks, the consumers offtake capacity and of course the market need. The interruptible customers can be helpful in reaching the maximized demand volumes. The components of the load vector will, in the capacity maximization case, be defined to be a part of the control vector. The reason for this is that the customer offtakes are now variables that we want to maximize. For the customer terminals where the load is specified, this can be handled by setting the lower and the upper control capacity limits to the value of the customer demand. Time varying reference for the control vector will now be determined by the maximum source availability at each supply point and the maximum offtake possibility at each customer point. The upstream and downstream networks of a considered transmission system in a third party access market will be obliged to have a minimum transportation availability. Quality demands are handled as in point two.

8. State estimator: Operators at the gas dispatch centre usually have access to an on-line simulation model. This model can be used to provide the initial state estimate of each open loop problem. A state estimator equivalent to the linear model predictive controller can also be used. Other state estimators referred to in this thesis can also be used.

To check the controller performance for a real transmission network, some preliminary investigations should take place. Closed loop simulations using historic load patterns and the use of the complex network simulator available for dispatchers as a simulation model can indicate a lot about the controller performance. In these simulations, it will also be advantageous to insert the maximum available production capacity as the upper source space limits for the model predictive controller. Tuning of the penalty weights, model tuning, the use of advanced thermodynamic equations, choice of the control parametrization dimension and the length of the open loop horizon must also be evaluated in these simulations for a satisfactory controller performance. Penalizing the change in the control command between each discrete time instant of the open loop horizon produces smooth control commands. Chapter 3 also showed that for long distance transportation systems without recompression, which is the case for some off-shore systems, the time delay effect between the sources and the customer points is considerable. Therefore, an open loop horizon up to several days may be necessary to meet transient customer loads depending on the pipeline lengths. The increased length of the transmission system makes it more difficult to meet the time varying customer demands and still have a low pressure level of the network so that transportation costs are kept at a low level. One of the reasons is that this calls for longer prediction horizons. The uncertainties of the load forecasts will of course increase as we move in time into the future from the current time. Recompression along the long distance transmission pipelines will generally yield better controllability. As a consequence of an increased controllability, the line pack can be reduced which implies a reduction in the running transportation costs. A drawback is of course the higher investment costs because of the additional compressor station installations.

The proposed model predictive controllers must be interpreted as operator support tools. The controllers can provide control command suggestions to the gas dispatching personnel. The dispatchers at the gas control centre have the final decision on the values of the control commands that are to be implemented on the transmission system. The same experienced personnel will also be able to detect obvious unsatisfactory system behaviour. The suggested current open loop control trajectory may work as a nominal control suggestion that can be accepted, modified or completely rejected. Look ahead simulations using the complex simulators available to the dispatch personnel can take part in the final decision making process. The experience of the dispatch personnel must not be forgotten in the decision process.

The performance of the above two operational support strategies depends highly on the quality of the load forecasts. Underestimation of the load can lead to infeasibility. As a consequence, the gas transporter may fail to satisfy the contractual obligations upon the customer. This case can, for example, occur if the system has a low degree of extra line pack and the system is exposed to an unexpected extra load demand. In the case of an overestimation of the future load demand, the system will face an increased pressure level by the above control policies. This situation implies extra transportation costs.

Creep Flow Model used as basis for Control Models and Simulation Models:

In this work, different reformulations of the nonlinear creep flow model have been used. As a result, different control, simulation and optimization models have been obtained. A reformulation to a linear model is used for the linear model predictive controller. It is reformulated to a linear time variant prediction model for the sequential quadratic programming procedure used for the proposed nonlinear model predictive control scheme. The creep flow model is used to represent the nonlinear dynamics inside the long distance transmission lines in all the simulations of this work. By using a stationary momentum equation, the stiffness of the fluid equations is considerably reduced. Ordinary differential equation solvers for stiff systems were used for the nonlinear simulation and the nonlinear prediction models used in this work. The thesis explains how the model can include complex stationary thermodynamics. Two different model predictive control strategies are shown for a single gas transmission line. First, the creep flow model is reformulated to a control model that is linearized based on an instant operation point. Second, the creep flow model is reformulated to a control model that is linear in squared pressures and squared mass flows. The creep flow model is also reformulated to a distributed parameter control and simulation model for a single transmission line. A distributed parameter network model is also formulated from this model basis.

Distributed Parameter Control Model for a Single Gas Transmission Line and for the Description of a Transmission Network:

The nonlinear creep flow model was formulated to a partial differential equation that was linear in squared pressures and squared mass flows. It has been shown how supplies and loads at the boundaries and along the transmission line can be included. The distributed parameter model was reformulated to an associated form where boundary conditions are zero. Given initial condition, the mild analytical solution was then given as the solution of the inhomogenous Cauchy problem.

Furthermore, a distributed parameter gas network model was formulated. The model gives a description of the fluid dynamics in the neighbourhood of a defined stationary operation point. The operator expressing the dynamics and the couplings in the transmission network was explicitly obtained. An expression for the strongly continuous semigroup generated by the operator expressing the dynamics and couplings of the transmission network was also shown. The network state trajectory was expressed as the solution of the inhomogenous Cauchy problem.

It is the intention that the distributed parameter control model can be used for control system design and for the design of state estimators. Alternatively, it can be used as a transient simulation model.

As a special case, the biquadratic model used in Section 5.4 - 5.7 can replace the local linear diffusion model used in Section 5.8 to describe the pipeline dynamics. This is the case if the transmission system has no junction points where transmission lines intersect. Then, the state trajectory of each transmission line is only affected by its initial condition together with the values of the control and disturbance vector and not through the interaction between the pipelines.

Transfer Function Control Model:

A defined distributed parameter control model is Laplace transformed to yield a distributed parameter input-output transfer function model for a single gas transmission line. The transmission line may have supplies and loads along the transmission line in addition to supply and offtake at the boundaries. This control model may be suitable for designing control systems for a transmission line with local distribution companies, gas storages and connections to other transmission lines along the pipeline.

The transfer function model can be used as a basis for controller design using multivariable frequency analysis.

Distributed Parameter Controller for a Single Gas Transmission Line:

First, a parabolic boundary control model was formulated to describe the dynamics of a natural gas transmission line with supply at the input boundary and customer offtake at the defined output boundary. Then, another control model was designed to give an approximate description of the dynamics of the boundary control model. It was shown that the state solution of the approximated control model was equal in the limit of the state solution of the boundary control model. It was also shown that a defined average observation defined for the approximate control model was in the limit equal to the defined point observation of the boundary control model. The approximated control model was used for the controller and state estimator design. A distributed parameter control system was then designed for a single gas transmission line. It was a coupled system of a linear feedback and a Luenberger state estimator combined with a feedforward from the predicted load forecast. It was shown that the defined distributed control model was approximately controllable and approximately observable. Then, it was shown that it was exponentially stabilizable and exponentially detectable. Finally, it was shown that the control system was nominally exponentially stable. The final control law in the physical variables was nonlinear.

Smith Predictor and Feedforward for the Control of a Natural Gas Transmission Line:

A control law that is a combination of a Smith predictor and a feedforward from predicted load demand for a long distance single gas transmission line has been proposed in this thesis. Based on the qualitative behaviour from the step responses of a developed distributed parameter control model in this thesis, simple transfer function control models are formulated. These intuitive simple models are used for the controller design. The parameters of these models are determined from simulation experiments. The transfer function models give increased insight into the dynamics of a transmission line which can also be valuable for natural gas networks.

A main advantage of this control scheme is that it is a very simple, unconstrained model predictive controller with low computational effort and it uses classical control theory that is well known to every control engineer. The final control law for the physical control variable mass flow is nonlinear.

The control system can be used to determine the mass flow through the compressor station that is connected to the pipeline that transports the natural gas to the customer terminal.

Quality Tracker:

A simple time delay quality tracker has been proposed in this thesis. An approximate time delay is determined for the transportation time for the molecules to move from the inlet to the outlet of a defined pipeline segment. This time is calculated based on a defined stationary operation point of the transmission system. This procedure is used to calculate the different time delays for all the pipeline segments of the network. It is assumed that the transmission system operates transiently in the neighbourhood of this operation point. This tracker can be used in combination with the proposed nonlinear model predictive controller for the control of important quality parameters.

Thermodynamic State Equation:

This thesis explains how a thermodynamic state equation is defined and where the parameters of this model are determined by the use of the least squares identification and observation data. The model is supposed to describe behaviour adequately for a predetermined defined expected pressure and temperature range. It should be possible to develop an automated procedure by taking a gas sample at regular intervals and by performing an experiment for expected pressure and temperature range at each sampling interval. As a result, an update on the coefficients of the given model structure is obtained. This state equation can then be used in connection with the stationary optimization of a transmission line. For each natural gas source, a corresponding equation of state can be determined by the use of the above method. As an approximation, one may assume an average gas composition for a natural gas that leaves a mixing point and then enters a transportation line. The average gas composition can then be used as the basis in the identification procedure for the determination of the parameters of the state equation.

6.2 Suggestions for Future Work

Nominal and Robust Stability:

Even if the simulation results presented in this thesis is encouraging, the difficult task of proving closed loop stability both in feasible and infeasible mode, nominally and robustly, still remains for the proposed model predictive controllers of Chapter 3 and 4. Factors such as, nonlinear dynamics, transient load demands, load forecast errors, time varying production capacity and state estimation errors are some of the reasons that makes it difficult to prove stability. Since the gas resources are limited, the time for the system operation is limited. Usually, stability is proved on an infinite time basis. It seems that the best way to evaluate stability, infeasibility handling and controller performance is by evaluating simulations of different practical scenarios.

Large Scale Optimization with both Continuous and Discrete Variables:

With the fast increasing computational power, larger computational loads can be solved on a time basis suitable for on-line operation. Global optimizers handling problems with both continuous and discrete variables, is then likely to be possible to use in an on-line application. Discrete variables defining network grid and station configuration will usually be updated on a different time basis to that of continuous variables. A dynamic optimization using mixed integer optimization can be performed, let us say, once in a gas day to find the "optimal" value of the discrete optimization variables. To reduce the dimension of the problem, a low dimension of the parametrized control trajectory of the continuous variables is chosen. The current available load forecasts are used in the optimization. A linear optimization model determined from a defined current stationary operation point can be used to simplify the problem. A closed loop control for the continuous variables is then performed, let us say, with a sampling interval of one hour using the continuously updated load forecasts. In the future, we can expect that a global optimization algorithm also solves the open loop problems with both continuous and discrete variables fast enough for on-line operation with a large dimension of the parametrized control trajectory of the continuous variables.

Linear Model Predictive Control using Distributed Parameter Model:

In Section 5.9 it was suggested how a finite time horizon open loop mathematical programming problem could be formulated to be used in a linear model predictive control scheme. It was assumed that the linear distributed parameter model formulated in Section 5.8 described the dynamics. Further research is necessary to solve this type of mathematical programming problem.

Distributed Parameter Model with Spatially Varying Coefficients:

It was explained in Section 5.8 how a *semilinear* partial differential equation could be approximated by a coupled set of linear partial differential equations so that the *linear* network model description derived in Section 5.8 could still be used. Further study to find the analytical solutions of the semilinear partial differential equations of the types given in Sections 5.4 and 5.8 is still of interest.

Optimal Control of Gas Transmission Systems:

Optimal control using the distributed parameter model in Section 5.8 can also be studied in the future.

Load Forecasts:

The proposed operator support strategies all depend on good quality load forecasts. Further research in this area should be of interest in the gas transportation community. Large supply source diversity together with a highly transient third party access market with spot sales, increases this challenge.

References

Adjiman, C.S., Dallwig S., Floudas, C.A. & Neumaier, A. 1998a,'A Global Optimization Method, aBB, for General Twice-Differentiable Constrained NLPs - I. Theoretical Advances', *Computers and Chemical Engineering,* vol. 22, no. 9, pp. 1137-58.

Adjiman, C.S., Androulakis, I.P. & Floudas, C.A. 1988b, 'A Global Optimization Method, aBB, for General Twice-Differentiable Constrained NLPs - II. Implementation and Computational Results', *Computers and Chemical Engineering*, vol. 22, no. 9, pp. 1159-79.

Androulakis, I.P., Maranas, C.D. & Floudas, C.A. 1995, 'aBB: A Global Optimization Method for General Constrained Nonconvex Problems', *Journal of Global Optimization*, vol.7, no. 4, pp. 337-63.

Ashouri, F. 1986, 'An Expert System for Predicting Gas Demands', *Omega: The International Journal of Management Science*, vol. 21, no. 3, pp. 307-17.

Avery, W., Brown, G.G., Rosenkranz, J.A. & Wood, R.K. 1992, 'Optimization of Purchase, Storage and Transmission Contracts for Natural Gas Utilities', *Operations Research Society of America*, vol. 40, no 3. pp. 446-62.

Balchen, J.G. & Mumme, K.J. 1988, '*Process Control*', Van Nostrand Reinhold, New York.

Barbu, V. 1994, '*Mathematical Methods in Optimization of Differential Systems*', Kluvwer Academic Publishers.

Bejan, A. 1993, '*Heat Transfer*', John Wiley & Sons, Inc.

Bertsekas, D. 1995, '*Nonlinear Programming*', Athena Scientific.

Bhaduri, S. & Talachi, R.K. 1988, 'Optimization of Natural Gas Pipeline Design', In *Technical Economics, Synfuels and Coal Energy Symposium. American Society of Mechanical Engineers*, New York, USA.

Biegler, L. 1998, 'Efficient Solution of Dynamic Optimization and NMPC Problems', *Proceedings of the International Symposium on Nonlinear Model*

Predictive Control - Assessment and Future Directions, June 1998, Ascona, Switzerland.

Brown, R.G. & Hwang, P.Y.C. 1992, '*Introduction to Random Signals and applied Kalman Filtering*', JohnWiley&Sons, Inc.

Camacho, E.F. & Bordons, C. 1999, '*Model Predictive Control*', Springer-Verlag, London Limited.

Canuto, C. & Quarteroni, A. 1987, 'On the Boundary Treatment in Spectral Methods for Hyperbolic Systems', *Journal of Computational Physics*, vol. 71, no. 1, pp. 100-10.

Carmichael, N., Pritchard, A.J. & Fincham, A.E. 1981, 'Observers, and Application to Gas Distribution', *Proceedings of the International Conference on Control and its Applications IEE,* March 1981, University of Warwick, Coventry, UK.

Chapman, M.J., Pearson, D.W. & Shields, D.N. 1985, 'Modelling and Observation of the Pressures in a Gas Supply Network', *Systems Science*, vol.11, no. 1, pp. 75-83.

Cheney, W. & Kincaid, D. 1999, '*Numerical Mathematics and Computing*', Brooks/Cole Publishing Company.

Choudary, P., Tye, C., Wynne, R.J. & Clark, A.S. 1985, 'State Estimation as an Aid to Gas Distribution Control', In *IFAC Identification and System Parameter Estimation 1985*, pp. 1883-88, York. UK.

Curtain, R.F., Zwart, H.J. 1995, '*An Introduction to Infinite Dimensional Linear Systems Theory*', Springer-Verlag.

Domican, E.H. 1990, 'Opportunities for Skill: In the Management and Operation of a major Gas Transmission System', *Gas Engineering & Management*, vol. 30, no. 6, pp. 185-88.

Estrada, J., Moe A. & Martinsen, K.D. 1995, '*The Development of European Gas Markets: Environmental, Economic and Political Perspectives*', John Wiley&Sons Ltd.

Ewers, H.J. & Stackelberg, F. 1993, 'Regional and Local Distribution Systems for Natural Gas', in *Natural Gas in the Internal Market,* ed. E.J. Mestmäcker, Graham & Trotman, pp. 243-269.

Fasold, H.G. & Wahle, H.N. 1992a, Berucksichtigung des Realgasverhaltens im Zusammenhang mit der Planung und Berechnung von Erdgasversorgungssystemen. (Takin account of the Real Gas Behaviour in connection with Planning and Calculation of Gas Distribution Networks.), *gwf-Gas/Erdgas*, vol. 133, no. 6, pp. 265-76.

Fasold, H.G. & Wahle H.N. 1992b, Der Antriebsgasverbrauch in Turboverdicterstationen fur den Erdgastransport: Methoden der planerischen Berechnung und Ergebnisse. (Gas Consumption in Turbo Compressors used in Natural Gas Transport: Methods for Planning and Results.), *gwf-Gas/Erdgas*, vol. 133, no. 7, pp. 303-12.

Fasold, H.G. & Wahle, H.N. 1992c, Physikalische Grundlagen des Transports von Fluiden in Rohrleitungen mit Folgerungen fur die Leitungsplanung. (Physical Properties for the Transport of Fluids in Pipelines with Consequenses for the Planning of Distribution Pipeline Systems.), *3R international*, vol. 31, no. 10/11, pp. 725-29.

Fasold, H.G. & Wahle, H.N. 1993, Die Berechnung des Antriebsgasverbrauches fur Erdgasferntransportsysteme: Metoden und Ergebnisse. (The Calculation of Gas Consumption in connection with Natural Gas Transport Systems: Methods and Results.), *gwf-Gas/Erdgas*, vol. 134, no. 7, pp. 321-32.

Fasold, H.G. 1994, Planung, Berechnung und Steuerung von Gasversorgungssystemen.(Planning, Calculation and Operation of Gas Distribution Systems.), Compendium used at Rheinisch-Westfalische-Technische Hochschule Aachen.

Fasold, H.G. & Wahle, H.N. 1996a, Einflus der Rohrrauhigkeit und der Rohrrebungszahl auf die Transportkapazität und die spezifischen Kosten von Gasrohrleitungen. (The impact of Pipeline Roughness and Friction Factor for the Transportation Capacity and Costs for Gas Pipelines.), *gwf-Gas/Erdgas*, vol. 137, no. 3, pp. 109-18.

Fasold, H.G., Wahle, H.N. & Korb, W. 1996b, Ein Thermodynamisches Modell zur Berechnung der Verdichtungsarbeit von Turboverdichtern beim Transport von Erdgas. (A Thermodynamic Model for the Calculation of Compression Work of Turbo Compressors in Systems for Transport of Natural Gas), *gwf-Gas/Erdgas*, vol. 137, no. 11, pp. 639-50.

Flanigan, O. 1995, '*Underground Gas Storage Facilities*', Gulf Publishing Company.

Franklin, G.F., Powel, J.P. & Workman, M.L. 1990, '*Digital Control of Dynamic Systems*', Addison-Wesley Publishing Company, Inc.

Fukuda, K., Okamato, K., Fukui, E. & Nishitani, H. 1989, 'Modelling and Control System Design of Natural Gas Supply Systems', *Computers Chemical Engineering*, vol. 13, no. 11/12, pp. 1299-1308.

Furey, B.P. 1993, 'A sequential Quadratic Programming-based Algorithm for Optimization of Gas Networks', *Automatica*, vol. 29, no. 6, pp. 1439-50.

Garcia, C.E. & Prett, D.M. 1989, 'Model Predictive Control: Theory and Practice - a Survey', *Automatica*, vol. 25, no. 3, pp. 335-48.

Goldwater, M.H. & Fincham, A.E. 1981, 'Modelling of gas supply systems', *IEE*

Control Engineering Series 13, vol.2, pp. 150-77.

Gopal, V. & Biegler, L.T. 1998, 'Large Scale Inequality Constrained Optimization and Control', *IEEE Control Systems*, December 1998, pp. 59-68.

Goslinga, J., Kaulback, M., Witczak, K. & McNeill, B. 1994, 'A Method for Pipeline Network Optimization', *Proceedings of the 13th International Conference on Offshore Mechanics and Arctic Engineering. Pipeline Technology ASME 1994*, February/March 1994, Houston, Texas, USA.

Gravdahl, J.T. 1998, Modeling and control of surge and rotating stall in compressors, Dr. Ing. thesis, Norwegian University of Science and Technology.

Greenberg, M.D. 1988, '*Advanced Engineering Mathematics*', Prentice-Hall, Inc.

Guy, J.J. 1967, 'Computation of Unsteady Gas Flow in Pipe Networks', *International Journal of Chemical Engineering - Symposium Series*, no. 23.

Hansen, T.C. 1988, Optimization of large Networks for natural Gas, PhD thesis, Danish University of Technology.

Holschumacher, W. & Wolf, H. 1991, Dispatching als Technisch-Gaswirtschaftliche Aufgabe (Dispatching for the Technical Operation of Gas Transportation), *gwf-Gas/Erdgas*, vol. 30, no. 10/11, pp. 470-76.

IEA 1992, '*Gas Control*', International Energy Agency, Paris.

IEA 1994, '*Natural Gas Transportation: Organisation and Regulation*', International Energy Agency, Paris.

Ioannou, P. A. & Sun, J. 1996, '*Robust Adaptive Control*', Prentice-Hall.

Jabbour, K. & Meyer, W. 1989, 'GAuLF: Gas Automated Load Forecaster', *Proceedings of the 32nd Midwest Symposium on Circuits and Systems*, August 1989, Champagne, Illinois, USA.

Jenicek, T., Kralik, J., Sterba, J., Vostry, Z. & Zaworka, J. 1992, 'Design and Control for Gas Quality - Quality Tracking', *Oil Gas - European Magazine*, no. 4, pp. 40-4

Kalman, R.E. 1960, 'A New Approach to Linear Filtering and Prediction Problems', *Transactions of the ASME Journal of Basic Engineering*, pp. 35-45.

Kalman, R.E. & Bertram, J.E. 1960, 'Control System Analysis and Design via the "Second Method" of Lyapunov, 1: Continuous-Time Systems', *Journal of Basic Engineering*, no. 59, pp. 371-393.

Kalman, R.E., Ho, Y.C. & Narendra, K.S. 1961, 'Controllability of Linear Dynamical Systems', *Contributions to Differential Equations*, vol. 1, no. 2, Wiley, New York.

Kiuchi, T., Izumi, H. & Huke, T. 1995, 'An Operation Support System of Large

City Gas Networks based on Fluid Transient Model', *Transactions of the ASME. Journal of Energy Resources Technology*, vol. 117, no. 4, pp. 324-8.

Kralik, J., Stiegler, P., Vostry, Z. & Zavorka, J. 1988, '*Dynamic Modeling of Large Scale Networks with Application to Gas Distribution*', Elsevier.

Kwon, W.H. & Pearson, A.E. 1977, 'A Modified Quadratic Cost Problem and Feedback Stabilization of a Linear System', *IEEE Transactions on Automation and Control*, vol. 22, pp. 838-42.

Larson, R.E. & Wismer, D.A. 1971, 'Hierarchical Control of Transient Flow in Natural Gas Pipeline Networks', *IFAC Symposium on Distributed Parameter Systems*, Banff, Alberta Canada.

Lewandrowski, A., Gosiewski, A. & Firla, A. 1974, 'The Optimization of Gas Transmission Systems', In *Third IFAC Symposium on Multivariable Control Systems*, Manchester, UK.

Li, W.C. & Biegler, L.T. 1989, 'Multistep, Newton-Type Control Strategies for Constrained Nonlinear Processes', Institution of Chemical Engineers, vol. 67. pp. 562-77.

Li, X. & Yong, J. 1995, '*Optimal Control Theory for Infinite Dimensional Systems*', Birkhauser.

Lo, K.L. 1984, 'Optimization and Analysis of Gas Supply Networks', *Trans Inst MC*, vol. 6, no. 5, pp. 253-60.

Luenberger, D.G. 1984, '*Linear and Nonlinear Programming*', 2nd edn, Addison-Wesley Publishing Company.

Luongo, C. A., Yeung, W. C. & Gilmour B.J. 1991, 'Optimizing the Operation of Gas Transmission Networks', *Computers in Engineering*, vol. 1, pp. 663-66.

Maier, R. & Schmidt, G. 1986, 'A Completely Parallel Scheme for Simulation of Transients in Large Scale Gas Transmission Networks', *Proceedings of the first European Workshop in Parallel Processing Techniques for Simulation.*, October 1985, Manchester, UK.

Mallinson, J. & Fincham, A.E. 1993, 'Methods for optimizing gas transmission networks', *Annals of Operation Research*, vol. 43, no. 1-4, pp. 443-54.

Mayne, D.Q., Rawlings, J.B., Rao, C.V. & Scokaert, P.O.M. 2000, 'Constrained Model Predictive Control: Stability and Optimality', *Automatica*, vol. 36, no. 6, pp. 789-814.

Manojlovic, V., Arsenovic, M. & Pajovic, V. 1994, 'Optimized Design of a Gas-Distribution Pipeline Network', *Applied Energy*, vol. 48, no. 3, pp. 217-24.

Mantri, V.B., Preston, L.B. & Pringle, C.S. 1986, 'Computer Program Optimizes Natural Gas Pipeline Operation', *Pipe Line Industry - Houston, Texas: Gulf Pub-*

lishing Company, vol. 65, no. 1, pp. 37-38.

Marques, D. 1985, On-line Optimization of Large Dynamic Systems, PhD thesis, University of Wisconsin-Madison.

Marques, D. & Morari, M. 1986, 'Model Predictive Control of Gas Pipeline Networks', *Proceedings of the 1986 American Control Conference IEEE*, June 1986, Seattle, Washington, USA.

Mestmäcker, E.J. (ed.) 1993, *Natural Gas In The Internal Market: A Review of Energy Policy*, Graham & Trotman Limited, London.

Muske, K.R. & Rawlings, J.B. 1993, 'Model Predictive Control with Linear Models', *Process Systems Engineering - American Institute of Chemical Engineers Journal*, vol.38, no. 10, pp. 1512-16.

Muske, K.R. 1995, Linear Model Predictive Control of Chemical Processes, PhD thesis, University of Texas at Austin.

Nichols, N.K. & Stringer, S.M. 1994, 'The State Estimation of Flow Demands in Gas Networks from Sparse Pressure Telemetry', *Trans Inst MC*, vol. 16, no. 2, pp. 66-74.

Nieplocha, J. 1988, 'Discrete Time Quadratic Optimal Control of Gas Network Systems', *Foundations of Control Engineering*, vol.13, no.4, pp. 175-86.

Oliveira, N.M.C. 1994, Newton-Type Algorithms for Nonlinear Constrained Chemical Process Control, PhD thesis, Carnegie Mellon University.

Oliveira, N.M.C. & Biegler, L.T. 1994, 'Constraint Handling and Stability Properties of Model Predictive Control', *Journal of American Institute of Chemical Engineers*, vol. 40, no.7, pp. 1138-55.

Oliveira, S.L. & Morari, M. 1994, 'Robust Model Predictive Control for Nonlinear Systems', *Proceedings of the 33rd Conference on Decision and Control*, December 1994, Lake Buena Vista, USA.

Oliveira N.M.C. & Biegler, L.T. 1995, 'An Extension of Newton-type Algorithms for Nonlinear Process Control', *Automatica*, vol. 31, no. 2, pp. 281-86.

Osiadacz, A.J. 1980, 'Nonlinear Programming Applied to the Optimum Control of a Gas Compressor Station', *International Journal for Numerical Methods in Engineering*, vol. 15, pp. 1287-01.

Osiadacz, A.J. & Bell, D.J. 1981, 'A Simplified Algorithm for Large Scale Gas Networks', *Proceedings of the 8th IFAC World Congress*, Kyoto, Japan.

Osiadacz, A.J. & Bell, D.J 1986, 'A simplified Algorithm for Optimization of Large Scale Gas Networks', *Optimal Control Applications and Methods*, vol. 7, no.1, pp. 95-104.

Osiadacz, A.J. 1987, '*Simulation and Analysis of Gas Networks*', Gulf Publishing Company, USA.

Osiadacz, A.J. & Rudowski, K. 1987, 'Analysis of Loop Methods for Simulating Gas Networks', *Computer Methods in Applied Mechanics and Engineering*, vol. 65, no. 3, pp. 201-13.

Osiadacz, A.J. 1988a, 'Dynamic Optimization of Large Gas Networks using Hierarchical Methods', in *IEE Colloquium on 'Large Scale and Hierarchical Systems'*, March 1988, London, UK.

Osiadacz, A.J. & Pienkosz, K. 1988b, 'Methods of Steady State Simulation for Gas Networks', *International Journal of Systems Science*, vol. 19, no. 7, pp. 1311-21.

Osiadacz, A.J. & Bell, D.J. 1988c, 'Optimization Techniques for Large Networks: Gas and Water', in *Simulation and Optimization of Large Scale Systems*, ed. A.J. Osiadacz, Clarendon Press, Oxford.

Osiadacz, A.J. & Salimi, M.A. 1988, 'Comparison between Sequential and Hierarchical Simulation of Gas Networks. Part 2: Dynamic Simulation of Flow of Gas in Networks', *Information and Decision Technologies*, vol. 14, no. 2, pp. 99-123.

Osiadacz, A.J. 1990, 'Numerical Analysis of a Method of Transient Simulation for Gas Networks', *International Journal in Systems Science*, vol. 21, no. 5, pp. 961-75.

Osiadacz, A.J. & Swierczewski, S. 1994, 'Optimal Control of Gas Transportation Systems', *Proceedings of the Third IEEE Conference on Control Applications*, August 1994, Glasgow, U.K.

Ostromuhov, L.A. 1998, 'Economic and Operational Pressure and Flow Gas Network Optimization', *Erdgas-Transport*, vol. 114, no. 7/8, pp. 373-7.

Parkinson, J.S. 1984, Mathematical Modelling and the Application of State Estimation Techniques to Gas Distribution Networks, MSc Thesis, University of Manchester.

Parkinson, J.S. & Wynne, R.J. 1986, 'Reduced-Order Modelling and State Estimation Applied to Gas Distribution Systems', *IEEE Transactions on Automatic Control*, vol. AC-31, no. 8, pp. 701-9.

Parkinson, J.S. & Wynne, R.J. 1992, 'Systems Modelling and Control Applied to a Low Pressure Gas Distribution Network', *Journal of Proceedings of the Institution of Mechanical Engineers*, vol. 206, no. 1, pp. 35-44.

Percell, P.B. & Van Reet, J.D. 1986, 'A Compressor Station Optimizer for Planning Gas Pipeline Operation', *Proceedings of the Pipeline Engineering Symposium 1990*, January 1990, New Orleans, USA.

Pratt, K.F. & Wilson, J.G. 1984, 'Optimization of the Operation of Gas Transmission Systems', *Trans Inst MC*, vol. 6, no. 5, pp. 261-69.

Ramchandani, N.L. & Gray, J.O. 1993, 'Economic Optimization of Gas Networks', *Proceedings of the 12th Triennial World Congress of the International Federation of Automatic Control*, July 1993, Sydney, Australia.

Rawlings, J.B. & Muske, K.R. 1993, 'The Stability of Constrained Receding Horizon Control', *IEEE Transactions on Automatic Control*, vol. 38, no. 10, pp. 1512-16.

Renardy, M. & Rogers, R. 1993, '*An Introduction to Partial Differential Equations*', Springer-Verlag, New York.

Rist, D. 1996, Dynamic Realer Gase - Grundlagen, Berechnungen und Daten fur Thermogasdynamic: Stromungsmechanic und Gastechnik. (Dynamic Real Gases - Basics, Calculations and Data for Thermodynamics of Natural Gas: Fluid mechanics and Gas Technology.), Springer-Verlag, Berlin.

Schiesser, W.E. 1991, '*The Numerical Method of Lines: Integration of Partial Differential Equations*', Academic Press, Inc.

Schröder, P., Unterseer, K. & Scheibe, D. 1990, 'On-Line Simulation in Gas Transmission Networks with Variable Gas Qualities and Gas Temperatures, *Proceedings of the 1990 European Simulation Multiconference*, SCS Europe, Ghent, Belgium.

Sjøen, K. 1997, Gas Transport (In Norwegian), Student Literature at the Norwegian University of Science and Technology.

Sood, A.K., Funk, G.L. & Delmastro, A.C. 1971, 'Dynamic Optimization of a natural Gas Pipeline Using Gradient Search Technique', *International Journal of Control*, vol. 14, no. 6. pp. 1149-57.

Stelter, R. 1987, 'Two Stage Singular Perturbation Model Reduction for Gas Transmission Networks', *Proceedings of the Automatic Control - World Congress 1987*, July 1987, Munich, West Germany.

Stelter, R. & Nour-Eldin, H.A. 1987, 'Gas Network Operation Optimization using Singular Perturbation Model Reduction', *Proceedings of the IMACS/IFAC International Symposium on Modelling and Simulation of Distributed Parameter Systems*, October 1987, Hiroshima Institute of Technology, Hiroshima, Japan.

Stelter, R. 1988a, 'On the Dynamic Simulation of Gas Transmission Networks', *Journal of Systems Analysis Modelling and Simulation*, vol. 5, no. 5, pp. 423-433.

Stelter, R. 1988b, 'Control of Gas Transport Systems - a Multilevel Approach', *Proceedings of the IFAC Distributed Intelligence Systems*, 27 June - 1 July 1988, Varna, Bulgaria.

Stern, J.P. 1993, 'The Prospects for Third Party Access in European Gas Markets', in *Natural Gas in the Internal Market,* ed. E.J. Mestmäcker, Graham&Trotman, pp. 183-201.

Søderstrøm, T. & Stoica, P. 1989, '*System Identification*', Prentice Hall.

EL-Tayeb, S.E., Sayyouh, M.H., Mahgoub, I. & EL-Salally, M. 1997, 'Optimal Configuration of Natural Gas Gathering Pipeline Networks', *OIL GAS - European Magazine*, no. 3, pp. 22-24.

Vada, J. 2000, Prioritized Infeasibility Handling in Linear Model Predictive Control: Optimality and Efficiency, Dr. thesis, Norwegian University of Science and Technology, Norway.

Weimann, A., Scheibe, D. & Schröder, P. 1990, 'Online Simulation of Gas Networks with Changing Gas Qualities, In *Advanced Information Processing in Automatic Control. Selected Papers from the IFAC/IMACS/IFORS Symposium (AIPAC '89)*, July 1989, Nancy, France.

White, F.M. 1994, '*Fluid Mechanics*', McGraw-Hill,inc.

Wilson, J.G. & Furey, B.P. 1988, 'Steady-State Optimization of Large Gas Transmission Systems in Simulation and Optimization of Large Systems', ed. Osiadacz, A.J., Oxford University Press.

Wolf, D.D. & Smeers, Y. 1996, 'Optimal Dimensioning of Pipe Networks with Applications to Gas Transmission Networks', *Operation Research*, vol. 44, no. 4, pp. 596-608.

Wong, P.J. & Larson, R.E. 1968a, 'Optimization of Natural Gas Pipeline Systems via Dynamic Programming', *IEEE Trans Automatic Control*, vol. AC-13, no.5, pp. 475-481.

Wong, P.J. & Larson, R.E. 1968b, 'Optimization of Tree-Structured Natural-Gas Transmission Networks', *Journal of Mathematical Analysis and Applications*, vol. 24, pp. 613-26.

Wong, M.L. 1988, The Application of Constrained Optimization to Gas Transmission Networks, PhD thesis, University of Oxford.

Wynne, R.J, Parkinson, J.S. & Lees, A. 1988, 'The Design and Implementation of a Control System for a Multifeed Gas Network', *Proceedings of the International Conference on Control 88. IEE*, April 1988, Oxford, UK.

Wynne, R.J. 1991, 'Modelling and Simulation for Control Systems Design: a Study of a Large Gas Distribution System', *Proceedings of the 1991 Summer Computer Simulation Conference*, San Diego, California, USA.

Young, N. 1992, '*An Introduction to Hilbert Space*', Cambridge University Press.

Appendix A

Nomenclature

Nomenclature Chapter 2

Latin symbols

a constant in relation to determine minimum offtake

A cross section

ACQ annual contractual quantity

AMO yearly take or pay volume

b constant in relation to determine maximum possible offtake

BRO buyers real offtake

c speed of sound or constant used in relation to determine take or pay volume

C, m, n empirical constants used in the formula for Nusselt number

C_{p} heat capacity of the natural gas under constant pressure

D diameter

D_{c} correction factor for yearly take or pay volume

DCQ daily contractual quantity

DD average number of degree days per day for the period

D_{g} number of days in the time period

D_{h} hydraulic diameter

D_n is the inner diameter in the n - layer

D_{o} diameter of the outer most layer against surroundings

f Fanning's friction factor

g gravitational constant

h entalphy or used as thermal conductivity

h_i inner film coefficient

h_o outer film coefficient

h_t total enthalpy

k thermal conductivity of the fluid

k_h heat transfer coefficient between surroundings and fluid

k_n heat transfer coefficient in the n - layer

k_υ constant for inner fluid resistance

K^* constant

L length of a pipeline segment

M_g monthly variation in the number of customers

MOP maximum possible offtake

n layer number of the pipe

N number of layers or number of days of maintenance

N_{in} number of pipelines into junction point

N_g annual average number of customers

$N_{sources}$ number of sources supplying junction or mix point

Nu Nusselt number

p pressure

p^* stationary pressure distribution

Pr Prandtl number

Re Reynolds number

$(\mathrm{Re})_{cr}$ critical Reynolds number

T fluid temperature

t time

T_{amb} ambient temperature

$T_{av}{}^*$ average temperature of stationary operation point

T_c correction factor for yearly take or pay volume

u internal energy of fluid

U gas usage per customer per degree day per day

W mass flow

W^* stationary mass flow through a pipeline

W_{gas} daily gas use forecast in mass flow

$W_j(x_j = L_j, t)$ are mass flows out from pipeline j into junction or mix point

$W_{j,\,source}(t)$ source mass flow connected to mix point

x position

x_0 initial position

x_j position along pipeline j connected to mix or junction point

$y_{i,\,mix}$ value of quality parameter i out from mix point

$y_i(x_j = L_j, t)$ value of the quality parameter i out from pipeline j

y_i weight percent of component i

$y_{ji,\,source}(t)$ composition of component i from source j

Z compressibility factor

$Z_{av}{}^*$ average compressibility factor

Greek symbols

α angle for change in cross section

ε absolute wall roughness

θ pipeline angle

θ_1 parameter of time delay model

θ_2 parameter of time delay model

λ Darcy's friction factor

μ dynamic viscosity

υ velocity

υ^* stationary velocity distribution

ρ density

τ time delay

$\tilde{\tau}$ approximated time delay

Operators

∇ gradient operator

Δ Laplacian operator

Nomenclature Chapter 3

Latin symbols

a vector in the linear inequality constraint in open loop nonlinear programming problem

A cross section, system matrix or matrix for linear inequalities in open loop nonlinear programming problem

B input matrix used in continuous state space model

c speed of sound

C load matrix used in continuous state space model

$\mathcal{C}$ controllability matrix

$\overline{CO_2}$ upper limit of accepted concentration of carbon dioxide

$\underline{CO_2}$ lower limit of accepted concentration of carbon dioxide

$\overline{CO_{2_1}}$ carbon dioxide concentration from processing facility 1

CO_{2_2} carbon dioxide concentration from processing facility 2

CS_1 compressor station one

CS_2 compressor station two

C_v heat capacity under constant volume

C_p heat capacity of the natural gas under constant pressure

D diameter

D_i diameter surrounding node i

f Fanning's friction factor or value objective function in nonlinear programming problem

f vector in the linear inequality expression of open loop quadratic programming problem formulated only as function of parametrized control trajectory

F piston force

F Matrix in the linear inequality expression of open loop quadratic programming problem formulated only as function of parametrized control trajectory

g gravitational constant or vector of inequality constraints in nonlinear programming problem

$\underline{GCV}$ lower limit of accepted region for the gross calorific value

$\overline{GCV}$ upper limit of accepted region for the gross calorific value

GCV_1 gross calorific value from processing facility 1

GCV_2 gross calorific value from processing facility 2

h vector of equality constraints of nonlinear programming problem

H measurement matrix for both continuous and discrete system

h_t total enthalpy

h is the internal energy plus the pressure energy

I identity matrix

k number of surrounding nodes to node j

k gradient vector in cost function of open loop quadratic programming problem formulated only as function of parametrized control trajectory

K constant used in AGA equation

K Hessian matrix in cost function of open loop quadratic programming problem formulated only as function of parametrized control trajectory

k_h heat transfer coefficient

K_i fluid constant for surrounding node i

K_{cost} energy cost due to compression

K_k Kalman filter gain at time t_k

M number of spatial points along considered pipeline or dimension of parametrized control sequence

M_j number of centrifugal compressors in parallel module j

MW molecular weight

N number of discrete time instants of open loop

N_e number of pairs

N_s number of parallel modules in series connection

O observability matrix

p pressure

$\bar{p}$ upper pressure limit

$\underline{p}$ lower pressure limit

$p^*(x)$ optimal pressure from stationary optimization at position x

p_{av} average pipeline pressure

p_{in} inlet pressure of a compressor unit

p_i pressure at discrete spatial position i used in the stationary optimization formulation

p_{out} outlet temperature of a compressor unit

p_{ref} reference pressure

P_{driver} power supplied from driver to compressor

P_{gas} power increase of the natural gas

P_{station} compression power including mechanical losses for considered compressor station

P_{ji} power of unit i inside parallel module j

P_{mec} mechanical energy supplied to driver shaft of compressor unit

P_{cal} calorific amount of energy bought and necessary for providing the mechanical energy to drive including energy loss

P_{electric} electric energy supplied including losses to provide necessary momentum energy to compressor unit

$\tilde{P}_k$ state estimate uncertainty matrix after event

$\hat{P}_k$ one step prediction uncertainty matrix for state estimate

P_{loss} energy loss

$p_{\mathrm{suction},\, i}$ suction pressure of compressor station i

P_0 initial uncertainty in state estimate

$\dot{Q}$ heat flow

$Q_3(x)$ state penalty matrix for pipeline three at position x of system example

$Q_1(x)$ state penalty matrix for pipeline one at position x of system example

Q_k uncertainty matrix for process noise at time t_k or used as matrix for penalizing state vector from it's reference

Q_n volumetric flow at standard conditions

R' gas constant including molecular weight of fluid

R gas constant

R_k uncertainty matrix for measurement noise at time t_k or used as a matrix for penalizing control vector from it's reference

$R_{\Delta u}$ weight matrix penalizing change in control command between each time instant of open loop horizon

$S_{1,k+i+1}, S_{2,k+i+1}, S_{3,k+i+1}$ matrices for penalizing constraint relaxation of the relaxation vectors $\delta_{1,k+i+1|k}, \delta_{2,k+i+1|k}, \delta_{3,k+i+1|k}$

S_1 source one

S_2 source two

S_{ij} constant intersecting node i of node j

$S_{ij}{}^*$ value of constant intersecting node i of node j determined from stationary optimization

t time

$\overline{T}$ upper temperature limit

$\underline{T}$ lower temperature limit

T temperature

T_{amb} ambient temperature

T_{av} average fluid temperature

T_i temperature at discrete spatial position i used in the stationary optimization formulation

T_{in} pipeline inlet temperature or inlet temperature of a compressor unit

T_{out} pipeline outlet temperature or temperature of gas out from a compressor unit

T_{ref} reference temperature

u control vector of continuous variables or used for internal energy of a fluid

u^* control vector reference from stationary optimization

$\mathcal{u}$ parametrized open loop control trajectory in augmented form

$\mathcal{u}^*$ open loop control reference in augmented form

$\underline{\mathcal{u}}$ predicted lower limit trajectory in augmented form of open loop control trajectory

$\overline{\mathcal{u}}$ predicted upper limit trajectory in augmented form of open loop control trajectory

$\mathcal{u}_{\mathrm{ol}}$ open loop control trajectory in augmented form

u_{c} control vector of continuous variables if one have both continuous and discrete control variables

u_c^* reference for u_c from stationary optimization

u_d control vector of discrete variables

u_d^* reference for u_d from stationary optimization

$u_{i,s}$ stationary mass flow through station i

$u_{k+i|k}$ open loop control command at discrete time instant i of open loop horizon starting and calculated at time t_k

$\underline{u}_{k+i|k}$ predicted lower limit of continuous control vector at time instant t_{k+i} of current open loop problem

$\bar{u}_{k+i|k}$ predicted upper limit of continuous control vector at time instant t_{k+i} of current open loop problem

v load demand vector

v^* stationary current load demand vector

$\hat{\upsilon}$ predicted open loop load trajectory

$v_{k+i|k}$ Predicted load at discrete time instant i of open loop horizon starting and calculated at time t_k

$V(\theta)$ sum of squared errors for a given value of parameter vector θ

$\hat{v}(t_{k+i})$ predicted load demand for future time point t_{k+i}

V_j volume at node j

W mass flow

W^* optimal calculated stationary mass flow through considered pipeline

W_j net mass flow offtake at node j

w_k process noise for discrete state space model

x state vector, position variable or optimization variables in a nonlinear programming problem

x Open loop state trajectory in augmented form

x^* open loop state reference in augmented form

$\underline{x}$ lower limit of optimization variables of nonlinear programming problem or lower limit of state vector

$\bar{x}$ upper limit of optimization variables of nonlinear programming problem or upper limit of state vector

x^* stationary pressure reference

$\tilde{x}$ estimated pressures

$\tilde{x}_k$ state estimate after event

$\tilde{x}_{k|k}$ Estimated initial condition of open loop horizon starting and calculated at time t_k

$\hat{x}_k$ one step state prediction

x_0 initial condition for state vector

$\hat{x}_0$ initial state estimate guess

y measurement vector

y_i data vector for experiment i

Y data vector in augmented form for all experiments

y_{CO_2} concentration of carbon dioxide at customer point

y_{GCV} gross calorific value at customer point

$y_{source, GCV}$ vector of gross calorific value of source one and two of transmission system example

y_{source, CO_2} vector of concentration of carbon dioxide at the two sources

Z compressibility factor

Z_{av} average compressibility factor

Z_{in} compressibility factor at inlet of a compressor unit

Z_{out} compressibility factor of fluid out from compressor unit

Greek symbols

β_{JT} Joule-Thomson coefficient

Γ discrete input matrix used in discrete state space model

Δx_i discrete spatial length node i

$\Delta \rho_M$ modelled density deviation

Δx spatial step length

$\delta_{k+i+1|k}$ total constraint relaxation vector at time t_{k+i+1} of open loop starting at t_k

$\delta_{k+i+1|k}{}^*$ optimal relaxations at considered time instant of open loop

ε model error or compression ratio

η_m mechanical efficiency factor

η_s isotropic efficiency factor

η_k measurement noise for discrete state space model

θ vector of unknown coefficients for state equation or pipeline angle

$\hat{\theta}$ identified value of parameter vector θ

κ constant used in speed of sound expression

λ Darcy's friction factor

λ^* fluid model constant

ξ_{cost} cost pr. energy unit of energy supplied to driver unit

ξ_{ji} cost per energy unit for driver i of parallel module j

ρ fluid density

ρ_n density at standard conditions

Υ isotropic exponent

υ fluid velocity

Φ system matrix for discrete system or regression matrix

φ_1 function used for the difference equation for temperature along pipeline

φ_2 function used for the difference equation for pressure along pipeline

Ψ discrete load matrix used in discrete state space model

ω_y measurement noise

ω_v noise in measured load demand

Nomenclature Chapter 4

Latin symbols

a_{c} vector in linear inequality state constraint

A system matrix for continuous state space model (constant or time variant) or cross section

$A_{[t_{k+i}, t_{k+i+1}]}$ approximated constant value of the time variant system matrix $A(t)$ for the time interval $t \in [t_{k+i}, t_{k+i+1}]$

A_{c} matrix in linear inequality state constraint

$A_{\alpha}(t)$ matrix of parameter coefficients

A_{diff} differentiation matrix for second order partial derivatives

A_{QP}, a_{QP} matrix and vector specifying linear inequality constraint for quadratic programming problem

B matrix relating input vector to continuous state space model

B_{in} matrix relating input mass flow of each pipeline to state space model for example

B_{out} matrix relating output mass flow of each pipeline to state space model for transmission example

c speed of sound

C matrix relating customer load to continuous state space model

c_1 coefficient in expression of compressor work

c_2 coefficient in expression of compressor work

CS_i compressor station i

D diameter or measurement operator

$\tilde{D}$ approximated bounded measurement operator

f Fanning's friction factor or used as the right hand side function of nonlinear simulation model

F matrix relating the effect of estimated or actual initial condition of open loop horizon to open loop state trajectory

g function relating output vector to state and input vector

$\mathbf{g}$ augmented vector function that relates parametrized control trajectory and open loop state trajectory to the open loop output trajectory

G matrix relating the effect of open loop control trajectory to open loop state trajectory

G_{ol} control parametrization matrix

$\mathbf{h}$ nonlinear dynamics in equality form for entire open loop horizon in augmented form

$\mathbf{h}_{\text{s}}$ scaled nonlinear dynamics in equality form for entire open loop horizon in augmented form

h dynamics written in implicit form or used as vector function of equality

H matrix relating the effect of predicted open loop load trajectory to open loop state trajectory

J objective function value for line search

$\mathbf{K}, \mathbf{k}$ matrix and vector defining discrete linear time variant prediction model in augmented form for the entire open loop at a considered iteration

K_{cost} compressor cost

M dimension of parametrized control sequence or number of spatial discretization points along a pipeline

M_y output vector scaling matrix

M_u input vector scaling matrix

MW molecular weight

N number of discrete time instants of open loop horizon

p pressure

$\tilde{p}$ determined pressure of p at a considered position and time of open loop horizon in the sequential quadratic programming procedure

p_{d} discharge pressure

p_{s} suction pressure

p_{T_i} customer terminal pressure at terminal i

Q weight matrix penalizing output vector

R weight matrix penalizing input vector or gas constant

S_{relax} penalty relaxation matrix

t time

t_k time at start of current open loop horizon

T temperature

T_1 matrix relating control vector to input mass flow vector of transmission system example

T_2 matrix relating control vector to output mass flow vector of transmission system example

T_3 matrix relating load demand vector to output mass flow vector of transmission system example

u control (input) vector of continuous variables

$\bar{u}$ upper limit of input vector

$\underline{u}$ lower limit of input vector

$u^*(t)$ time varying input reference

$\mathcal{u}_{\mathrm{ol}}$ open loop control trajectory in augmented form

$\mathcal{u}$ parametrized open loop control trajectory in augmented form

$\mathcal{u}_{\mathrm{s}}$ scaled parametrized control trajectory in augmented form

u_s^* scaled parametrized control trajectory reference in augmented form

$\underline{u}_s$ scaled lower open loop control trajectory limit in augmented form

$\overline{u}_s$ scaled upper open loop control trajectory limit in augmented form

$u_{[t_k, t_k + \Delta t_{ol}]}$ open loop control trajectory

$u_{k+i|k}$ open loop control command at discrete time instant i of open loop horizon starting and calculated at time t_k

$\underline{u}_{k+i|k}$ predicted lower control vector limit at discrete time instant i for open loop horizon starting at time t_k and predicted at time t_k

$\bar{u}_{k+i|k}$ predicted upper control vector limit at discrete time instant i for open loop horizon starting at time t_k and predicted at time t_k

u_0 parametrized control trajectory after line search of the sequential quadratic programming procedure from previous iteration

u_{s0} scaled parametrized control trajectory from previous iteration of the sequential quadratic programming procedure

$u_{s,QP}^*$ scaled solution from quadratic program

v load demand vector

υ predicted open loop load trajectory

$v(t_k)$ load demand at time t_k

$\hat{v}_{[t_k, t_k + \Delta t_{ol}]}$ predicted load trajectory of open loop

$\hat{v}_{k+i|k}$ predicted load at discrete time instant i of open loop horizon starting and calculated at time t_k

W mass flow

$\overline{W}$ maximum mass flow through compressor station

$\underline{W}$ minimum mass flow through compressor station

$\tilde{W}$ determined mass flow value of W at a considered position and time of open loop horizon in the sequential quadratic programming procedure

W_{CS_i} mass flow through compressor station i

W_{in} input mass flow vector

W_{out} output mass flow vector

W_{ij} mass flow through control valve bringing gas from pipeline i to pipeline j

x state vector or position

$\boldsymbol{x}$ open loop state trajectory in augmented form

$\bar{x}$ upper limit for state vector

$\underline{x}$ lower limit for state vector

$x(t_k)$ state value at time t_k

$\tilde{x}$ estimated state

x_0 initial condition of current open loop

$\boldsymbol{x}_0$ optimal state trajectory solution in absolute values from previous iteration of the sequential quadratic programming procedure

$\tilde{x}_{k|k}$ Estimated initial condition of open loop horizon starting and calculated at time t_k

y output vector

$\boldsymbol{y}$ open loop output trajectory in augmented form

$y_{[t_k, t_k + \Delta t_{\mathrm{ol}}]}$ open loop output trajectory

$y_{k+i+1|k}$ predicted value of output vector at discrete time instant $i+1$ of open loop horizon starting and calculated at time t_k

$\bar{y}$ upper limit of output vector

$\underline{y}$ lower limit of output vector

y^* output reference

$\boldsymbol{y}_{\mathrm{s}}$ scaled open loop output trajectory in augmented form

y_{GCV_j} gross calorific value out from mix point j

$y_{\mathrm{CO}_{2_j}}$ concentration of carbon dioxide out from mix point j

$\boldsymbol{y}_{\mathrm{s}}^*$ scaled open loop output trajectory reference in augmented form

$\underline{\boldsymbol{y}}_{\mathrm{s}}$ scaled lower output trajectory limit in augmented form

$\bar{\boldsymbol{y}}_{\mathrm{s}}$ scaled upper open loop output trajectory limit in augmented form

Z compressibility factor

Greek symbols

α constant used in optimization model

$\tilde{\alpha}$ determined value of α used in sequential quadratic programming procedure

β constant used in optimization model

$\tilde{\beta}$ determined value of β used in sequential quadratic programming procedure

$\Gamma_{k+i|k}$ input matrix at time instant i of open loop horizon starting and calculated at t_k for linear time variant prediction model

Δt_{ol} length of open loop horizon

$\Delta p_{\mathrm{valve},\, i}$ differential pressure over control valve i

$\Delta p_{\mathrm{pipe},\, j}$ input minus output pressure of pipe j

Δx spatial step length

δ_{relax} relaxation vector

$\delta_{\mathrm{relax}}{}^*$ optimal relaxation

$\bar{\delta}_{\mathrm{relax}}$ upper limit for constraint relaxation vector

$\delta_{\mathrm{Termination}}$ line search termination criteria

ε_i compression ratio over compressor station i

$\bar{\varepsilon}$ maximum compression ratio over compressor station

$\underline{\varepsilon}$ minimum compression ratio over compressor station

$\varepsilon_{\mathrm{line}}$ tolerance for finding a local minimum along line of search

κ constant in speed of sound expression

λ constant used in optimization model

ξ observation vector

ξ_{cost} energy price

υ fluid velocity

Φ_y open loop output trajectory penalty matrix

Φ_u Open loop input trajectory penalty matrix

$\Phi_{k+i|k}$ system matrix at time instant i of open loop horizon starting and calculated at t_k for linear time variant prediction model

Φ_{QP} Hessian matrix of objective function in specified quadratic programming problem

φ_{QP} Gradient vector of objective function in specified quadratic programming problem

$\Psi_{k+i|k}$ load matrix at time instant i of open loop horizon starting and calculated at t_k for linear time variant prediction model

Nomenclature Chapter 5

Latin symbols

A cross section, system matrix or linear operator for parameter distributed system

$\mathcal{A}$ linear system operator

$\mathcal{A}_e$ extended system operator

a_{ij} constant

b_{ji} constant

b function of boundary control input operator

$b_{\mathrm{N,\,in},\,j}$ Neuman input operator

$b_{\mathrm{N,\,out},\,j}$ Neuman output operator

$b_{\mathrm{D,\,in},\,j}$ Dirichlet input operator

$b_{\mathrm{D,\,out},\,j}$ Dirichlet output operator

B supply matrix, input operator of parameter distributed control model

B_{b} matrix operator

B_{d} matrix operator

B_2 distributed control input operator

$\mathcal{B}$ boundary control input operator or just boundary operator

$\mathcal{B}_{\mathrm{e}}$ extended control input operator

B^* adjoint operator of B

B^{τ} controllability map

$\overline{\mathrm{range}(B^{\tau})}$ closure of the range of the controllability map B^{τ}

B_n approximate controllability matrix

c speed of sound or function for disturbance input operator

C offtake matrix in continuous state space model or used as disturbance operator in parameter distributed control model

C_{b} matrix operator

C_{d} matrix operator

C_2 distributed disturbance operator

$\mathcal{C}$ boundary disturbance operator

C_e extended disturbance operator

$C_r(x, i)$ step response coefficient vector

CS compressor station

d number of disturbance variables

D diameter or used as measurement operator

$D(A)$ domain of operator A

$D(\mathcal{A})$ domain of operator $\mathcal{A}$

$D(\mathcal{A}_i)$ domain of operator $\mathcal{A}_i$

D^τ observability map

kernel(D^τ) kernel of the observability map D^τ

D_n approximate observability matrix

e estimation error or used to denote approximation error

e_r estimation error for pipeline r

f Fanning's friction factor or time function in abstract differential system (Cauchy formulation)

F Linear feedback operator

$G(z)$ transfer function between input reference and output for discrete model

$G_1(z)$ transfer function $G(z)$ without time delay

$H(z)$ transfer function between disturbance reference and output for discrete model

$h(s)$ process dynamics without time delay

$h_0(s)$ open loop transfer function

$h_r(s)$ transfer function of an ordinary series compensator

$h_s(s)$ transfer function for series compensator of Smith predictor

$h_1(x, s)$ transfer function between control input and state

$h_2(x, s)$ transfer function between disturbance and state

h_1, h_2, h_3 mass transfer coefficients used in the Robin boundary conditions

$h_p(s)$ transfer function between input reference and output

$h_{pij}(s)$ transfer function between control input j and output i

$h_v(s)$ transfer function between disturbance reference and output

$h_{vij}(s)$ transfer function between disturbance input j and output i

$\mathcal{H}$ Operator for network couplings

$H_p(s)$ transfer matrix between input vector and output vector

$H_v(s)$ transfer matrix between disturbance vector and output vector

$H_{ff}(z)$ feedforward transfer function for discrete model

$J(y_0;u_{[0,\infty]})$ cost for infinite optimization horizon given an initial condition and control trajectory

$J(y_k, \pi_{ol}, \mathcal{V}_{ol})$ cost function

$J_{opt}(y_0)$ minimum cost for the infinite optimization horizon given initial condition and optimal control trajectory

K output injection operator

K_p controller gain

$K_i(n, x)$ gain number i

L length of pipeline or used as linear operator for Luenberger observer

$L_2(\dot{0}, L)$ Hilbert space

$L_2(\dot{0}, L_i)$ Hilbert space corresponding with transmission line i

L_r length of pipeline r

m number of control variables

M dimension of parametrized control sequence or closed loop transfer function

M_c transfer function approximating customer dynamics

M_n the number of pipelines connected to junction point

M_{obs} transfer function approximating measurement dynamics

M_{ol} Number of discrete time steps of open loop horizon

M_s transfer function approximating source dynamics

M_{Smith} closed loop transfer function of Smith predictor

MW molecular weight

M_1 positive integer

M_2 positive integer

N number of time instants in open loop

N_{delay} number of sampling instants of time delay

N_n number of junction points in gas network

n number of states variables of a transmission system

n_r number of measurements for pipeline r

n_u dimension of control vector

n_v dimension of disturbance vector

n_y dimension of output vector

p pressure

p^* optimal stationary pressure at a given position along the pipeline

p_{av} average pressure of a control volume

p_1 average pressure of a control volume where mass is injected

p_2 average pressure of a control volume where mass is injected

p_{ji} average pressure value for a defined control volume

p_n pressure at standard condition

p_{obs} observed pressure

P operator used in Riccati equation for parameter distributed optimal controller

q state penalty constant

Q volumetric flow, penalty matrix or state penalty function

Q^* optimal stationary volumetric flow at a given position along the pipeline or adjoint operator of Q

Q_n volumetric flow at standard conditions

$r(s)$ output reference

R gas constant, control penalty matrix or control penalty constant for parameter distributed controller

$R_{\Delta \mathrm{u}}$ weight matrix penalizing change in control command between each time instant of open loop horizon

$S(t)$ semigroup for gas transmission system generated by linear operator Φ

s number of disturbance variables

s_{b} number of boundary disturbance variables

s_{d} number of pointwise distributed disturbance variables

$S_{u,\,\mathrm{in},\,j}$ control input boundary index set

$S_{v,\,\text{in},\,j}$ disturbance input boundary index set

$S_{u,\,\text{out},\,j}$ control output boundary index set

$S_{v,\,\text{out},\,j}$ disturbance output boundary index set

sup supremum

t time

$T(t)$ C_0 semigroup

T time constant or temperature

$T(t)$ semigroup

$T[0, \tau]$ trajectory of the semigroup for the period $[0, \tau]$

$T(n)$ time constant number n

T_{s} length of sampling interval

T_1 time constant in approximated transfer function between disturbance and output

T_2 time constant in approximated transfer function between disturbance and output

T_3 time constant in approximated transfer function between disturbance and output

$T_{\mathcal{B}\text{F}}$ closed loop control system semigroup

T_{KD} closed loop state estimator semigroup

T_{cs} time constant for compressor station dynamics

T_{c} time constant for customer dynamics

T_{obs} time constant of measurement dynamics

T_{p} time constant for natural gas processing

T_n temperature at standard conditions

u control input of scalar type for a distributed parameter model or vector for a system of distributed parameter models

u_{b} boundary control vector

u_{d} pointwise distributed control vector

u_{phy} control vector of physical control commands

u_1 boundary control input

u_2 control command at a specified position along transmission line

U step size in input or disturbance input space

U_1 boundary control input space

U_2 control space for control input at a point along transmission line

$\mathcal{U}_{\rm ol}(k)$ open loop control space

$\tilde{u}$ deviation from control reference for biquadratic optimization model

u^* control vector reference

$\tilde{\underline{u}}, \tilde{\bar{u}}$ upper and lower control constraint limits in deviation variables from control reference for model predictive controller using biquadratic optimization model

$u_{k|k}^{\rm opt}$ optimal control command in absolute values applied to at time t_k for model predictive controller using biquadratic optimization model

$\tilde{u}_{k|k}^{\rm opt}$ optimal control command in deviation variables of squared mass flow calculated by model predictive controller using biquadratic optimization model

$u_{\rm ff}$ control command from feedforward term

$u_{\rm Smith}$ control command from Smith predictor

$u_{\rm ref}$ input reference for input variable of parameter distributed control model

$\tilde{v}$ deviation from defined average load correspondingly for biquadratic optimization model

v disturbance variable scalar or vector for distributed parameter model or system of distributed parameter models or measurement variable

$v_{\rm b}$ boundary disturbance vector

$v_{\rm d}$ pointwise disturbance vector

$v_{\rm phy}$ vector of physical disturbance values

v_1 boundary disturbance

v_2 load disturbance at a specified position of transmission line

$\vartheta_{\rm ol}$ open loop disturbance sequence

V step size of disturbance, volume or used as disturbance space

$\mathcal{V}_{\rm ol}(k)$ predicted open loop disturbance sequence

V_1 boundary disturbance control space or used as volume where mass is injected

V_2 distributed disturbance control space or used as volume where mass is withdrawn

$\mathcal{V}_{\text{ol}}$ open loop disturbance sequence

V_{ji} volume around position x_{ji} where mass is supplied or withdrawn

v_{ref} reference for disturbance variable

W mass flow

W^* optimal stationary mass flow

W_{demand} customer demand reference

W_1 mass flow at distributed supply point of pipeline example

W_2 mass flow at distributed offtake point of pipeline example

$W_{\text{sup}, i}$ distributed supply mass flow corresponding with $u_{d, i}$

$W_{\text{off}, i}$ distributed offtake mass flow corresponding with $v_{d, i}$

W_{offtake} offtake mass flow

W_{ref} supply mass flow reference received from dispatch centre

$W_{\text{d, supply}}$ supply mass flow

x position or state vector

x_i position along pipeline i

$\tilde{x}$ deviation from stationary state reference for biquadratic optimization model

$\underline{\tilde{x}}, \bar{\tilde{x}}$ upper and lower state constraint limits in deviation variables from state vector reference for model predictive controller using biquadratic optimization model

y state variable or distributed parameter state vector

$\underline{y}$ lower limit constraint

$\bar{y}$ upper limit constraint

$\tilde{y}$ estimated state variable

Y State space, function space and constrained state space

$\mathcal{Y}_{\text{ol}}$ open loop state sequence

y_t partial derivative of y with respect to time

y_0 initial condition for parameter distributed optimal controller

y_i state variable for pipeline i

y_{e} extended state variable for a single pipeline or extended state vector for a gas network

y_{obs} observation vector

Z compressibility factor or observation space

$\mathcal{Z}$ z - transform

z observed states or change of variable in associated form of distributed parameter system or variable used in Cauchy differential form in Section 5.8

z_0 initial condition of the new variable in associated form

$\tilde{z}$ estimated observed states

Greek symbols

α model constant or parameter vector for all pipelines

α_i model constant for pipeline i

β constant

β_i constant used in junction point model

β_1 constant

β_2 constant

Γ supply matrix in discrete state space model

ΔK gain margin

ε positive real number

ε_1 positive real number

ε_2 positive real number

ε_{ji} positive real number

$\varepsilon_{1,\text{ in}}$, $\varepsilon_{2,\text{ in}}$, $\varepsilon_{3,\text{ in}}$, $\varepsilon_{1,\text{ out}}$, $\varepsilon_{2,\text{ out}}$, $\varepsilon_{3,\text{ out}}$, ε_{cv}, $\varepsilon_{3\text{d}}$ positive constants

κ_1 gain

κ_2 gain

λ_n eigenvalue

λ_{in} eigenvalue n for transmission line i

ξ observation variable

π_{ol} is the open loop control sequence

$\pi_{\text{ol}}^{\text{opt}}$ open loop optimal control sequence

ρ density

τ time delay

τ_c time delay for customer dynamics

τ_{obs} measurement time delay

τ_s time delay at supply facility

τ_ε time delay uncertainty

υ velocity

Φ system matrix in discrete state space model or linear network operator

φ phase margin

ϕ_n eigenfunction (Riesz basis)

ϕ_{in} eigenfunction n for transmission line i

$\chi(y_{k|k}, \pi_{ol}, \vartheta_{ol})$ model equations for open loop horizon in compact form

Ψ offtake matrix in discrete state space model

ω_c cross frequency

ω_{180} phase cross frequency

Ω domain

Appendix B

Detailed Derivations

B.1 Long Derivations used in Section 5.4

$$T(t)Bu(0) = \sum_{n=0}^{\infty} e^{-\left(\frac{n\pi\alpha}{L}\right)^2 \cdot t} \langle Bu(0), \phi_n \rangle \phi_n \tag{B.1}$$

$$= \sum_{n=1}^{\infty} e^{-\left(\frac{n\pi\alpha}{L}\right)^2 \cdot t} \cdot \frac{2}{L} \cdot \int_0^L -\frac{1}{2L} \cdot (z-L)^2 \cdot u(0) \cdot \cos\left(n\pi \cdot \frac{z}{L}\right)$$

$$dz \cdot \cos\left(n\pi \cdot \frac{x}{L}\right) + e^{0 \cdot t} \cdot \frac{1}{L} \cdot \int_0^L -\frac{1}{2L} \cdot (x-L)^2 \cdot u(0) dx$$

$$= -\frac{L}{6} \cdot u(0) - \sum_{n=1}^{\infty} e^{-\left(\frac{n\pi\alpha}{L}\right)^2 \cdot t} \cdot \frac{2}{L} \cdot \frac{1}{\left(\frac{n\pi}{L}\right)^2} \cdot \cos\left(n\pi \cdot \frac{x}{L}\right) \cdot u(0)$$

$$T(t)Cv(0) = \sum_{n=0}^{\infty} e^{-\left(\frac{n\pi\alpha}{L}\right)^2 \cdot t} \langle Cv(0), \phi_n \rangle \phi_n \tag{B.2}$$

$$= \sum_{n=1}^{\infty} e^{-\left(\frac{n\pi\alpha}{L}\right)^2 \cdot t} \cdot \frac{2}{L} \cdot \int_0^L \frac{1}{2L} \cdot z^2 \cdot v(0) \cdot \cos\left(n\pi \cdot \frac{z}{L}\right)$$

$$dz \cdot \cos\left(n\pi \cdot \frac{x}{L}\right) + e^{0 \cdot t} \cdot \frac{1}{L} \cdot \int_0^L \frac{1}{2L} \cdot x^2 \cdot v(0) dx$$

$$= \frac{L}{6} \cdot v(0) + \sum_{n=1}^{\infty} e^{-\left(\frac{n\pi\alpha}{L}\right)^2 \cdot t} \cdot \frac{2}{L} \cdot \frac{(-1)^n}{\left(\frac{n\pi}{L}\right)^2} \cdot \cos\left(n\pi \cdot \frac{x}{L}\right) \cdot v(0)$$

$$\int_0^t T(t-\eta) \mathcal{A} B u(\eta) d\eta \tag{B.3}$$

$$= \int_0^t T(t-\eta) \left(\alpha^2 \cdot \frac{d^2}{dx^2}\left(-\frac{1}{2L} \cdot (x-L)^2\right)\right) u(\eta) d\eta$$

$$= -\frac{\alpha^2}{L} \cdot \int_0^t T(t-\eta) u(\eta) d\eta = \sum_{n=1}^{\infty} -\frac{\alpha^2}{L} \cdot \int_0^t e^{-\left(\frac{n\pi\alpha}{L}\right)^2 \cdot (t-\eta)} \cdot \frac{2}{L}$$

$$\cdot \int_0^L \cos\left(n\pi \cdot \frac{z}{L}\right) dz \cdot \cos\left(n\pi \cdot \frac{x}{L}\right) \cdot u(\eta) d\eta - \frac{\alpha^2}{L} \cdot \int_0^t e^{0 \cdot (t-\eta)} \cdot u(\eta) d\eta$$

$$= -\frac{\alpha^2}{L} \cdot \int_0^t u(\eta) d\eta$$

$$\int_0^t T(t-\eta)\mathcal{A}Cv(\eta)d\eta = \int_0^t T(t-\eta)\left(\alpha^2 \cdot \frac{d^2}{dx^2}\left(\frac{1}{2L} \cdot x^2\right)\right)v(\eta)d\eta \tag{B.4}$$

$$= \frac{\alpha^2}{L} \cdot \int_0^t T(t-\eta)v(\eta)d\eta = \sum_{n=1}^{\infty} \frac{\alpha^2}{L} \cdot \int_0^t e^{-\left(\frac{n\pi\alpha}{L}\right)^2 \cdot (t-\eta)} \cdot \frac{2}{L}$$

$$\cdot \int_0^L \cos\left(n\pi \cdot \frac{z}{L}\right)dz \cdot \cos\left(n\pi \cdot \frac{x}{L}\right) \cdot v(\eta)$$

$$d\eta + \frac{\alpha^2}{L} \cdot \int_0^t e^{0 \cdot (t-\eta)} \cdot v(\eta)d\eta = \frac{\alpha^2}{L} \cdot \int_0^t v(\eta)d\eta$$

$$\int_0^t T(t-\eta)B\dot{u}(\eta)d\eta = \int_0^t \sum_{n=0}^{\infty} e^{-\left(\frac{n\pi\alpha}{L}\right)^2 \cdot (t-\eta)} \langle B\dot{u}(\eta), \phi_n \rangle \phi_n \tag{B.5}$$

$$= \sum_{n=1}^{\infty} \frac{2}{L} \cdot \int_0^t e^{-\left(\frac{n\pi\alpha}{L}\right)^2 \cdot (t-\eta)}$$

$$\int_0^L -\frac{1}{2L} \cdot (z-L)^2 \cdot \cos\left(n\pi \cdot \frac{z}{L}\right) \cdot dz \cdot \cos\left(n\pi \cdot \frac{x}{L}\right) \cdot \dot{u}(\eta)d\eta$$

$$+ \int_0^t e^{0 \cdot (t-\eta)} \cdot \frac{1}{L} \cdot \int_0^L -\frac{1}{2L} \cdot (x-L)^2 dx \cdot \dot{u}(\eta) \cdot d\eta = -\frac{L}{6}$$

$$\int_0^t \dot{u}(\eta)d\eta - \sum_{n=1}^{\infty} \frac{2}{L} \cdot \int_0^t e^{-\left(\frac{n\pi\alpha}{L}\right)^2 \cdot (t-\eta)} \cdot \frac{1}{\left(\frac{n\pi}{L}\right)^2} \cdot \cos\left(n\pi \cdot \frac{x}{L}\right) \cdot \dot{u}(\eta)d\eta$$

$$= -\frac{L}{6}(u(t) - u(0))$$

$$- \sum_{n=1}^{\infty} \frac{2}{L} \cdot \frac{1}{\left(\frac{n\pi}{L}\right)^2} \cdot \cos\left(n\pi \cdot \frac{x}{L}\right) \cdot \int_0^t e^{-\left(\frac{n\pi\alpha}{L}\right)^2 \cdot (t-\eta)} \cdot \dot{u}(\eta)d\eta$$

$$= -\frac{L}{6}(u(t) - u(0)) - \sum_{n=1}^{\infty} \frac{2}{L} \cdot \frac{1}{\left(\frac{n\pi}{L}\right)^2} \cdot \cos\left(n\pi \cdot \frac{x}{L}\right) \cdot e^{-\left(\frac{n\pi\alpha}{L}\right)^2 \cdot t}$$

$$\cdot \left\{ \left[e^{\left(\frac{n\pi\alpha}{L}\right) \cdot \eta} u(\eta) \right]_0^t - \left(\frac{n\pi\alpha}{L}\right)^2 \cdot \int_0^t e^{\left(\frac{n\pi\alpha}{L}\right)^2 \cdot \eta} \cdot u(\eta)d\eta \right\} = \ldots$$

$$\ldots = -\frac{L}{6} \cdot u(t) - \sum_{n=1}^{\infty} \frac{2}{L} \cdot \frac{1}{\left(\frac{n\pi}{L}\right)^2} \cdot \cos\left(n\pi \cdot \frac{x}{L}\right) \cdot u(t) + \frac{L}{6} \cdot u(0)$$

$$+ \sum_{n=1}^{\infty} e^{-\left(\frac{n\pi\alpha}{L}\right)^2 \cdot t} \cdot \frac{2}{L} \cdot \frac{1}{\left(\frac{n\pi}{L}\right)^2} \cdot \cos\left(n\pi \cdot \frac{x}{L}\right)$$

$$\cdot u(0) + \sum_{n=1}^{\infty} 2 \cdot \frac{\alpha^2}{L} \cdot \cos\left(n\pi \cdot \frac{x}{L}\right) \cdot \int_0^t e^{-\left(\frac{n\pi\alpha}{L}\right)^2 \cdot (t-\eta)} u(\eta) d\eta = Bu(t)$$

$$- T(t)Bu(0) + \sum_{n=1}^{\infty} 2 \cdot \frac{\alpha^2}{L} \cdot \cos\left(n\pi \cdot \frac{x}{L}\right) \cdot \int_0^t e^{-\left(\frac{n\pi\alpha}{L}\right)^2 \cdot (t-\eta)} u(\eta) d\eta$$

$$\int_0^t T(t-\eta)C\dot{v}(\eta)d\eta = \int_0^t \sum_{n=0}^{\infty} e^{-\left(\frac{n\pi\alpha}{L}\right)^2 \cdot (t-\eta)} \langle C\dot{v}(\eta), \phi_n \rangle \phi_n \tag{B.6}$$

$$= \sum_{n=1}^{\infty} \frac{2}{L}$$

$$\cdot \int_0^t e^{-\left(\frac{n\pi\alpha}{L}\right)^2 \cdot (t-\eta)} \int_0^L \frac{1}{2L} \cdot z^2 \cdot \cos\left(n\pi \cdot \frac{z}{L}\right) \cdot dz \cdot \cos\left(n\pi \cdot \frac{x}{L}\right) \cdot \dot{v}(\eta)d\eta$$

$$+ \int_0^t e^{0 \cdot (t-\eta)} \cdot \frac{1}{L} \cdot \int_0^L \frac{1}{2L} \cdot x^2 dx \cdot \dot{v}(\eta) \cdot d\eta = \frac{L}{6}$$

$$\int_0^t \dot{v}(\eta)d\eta - \sum_{n=1}^{\infty} \frac{2}{L} \cdot \int_0^t e^{-\left(\frac{n\pi\alpha}{L}\right)^2 \cdot (t-\eta)} \cdot \frac{(-1)^n}{\left(\frac{n\pi}{L}\right)^2} \cdot \cos\left(n\pi \cdot \frac{x}{L}\right) \cdot \dot{v}(\eta)d\eta$$

$$= \frac{L}{6}(v(t) - v(0))$$

$$- \sum_{n=1}^{\infty} \frac{2}{L} \cdot \frac{(-1)^n}{\left(\frac{n\pi}{L}\right)^2} \cdot \cos\left(n\pi \cdot \frac{x}{L}\right) \cdot \int_0^t e^{-\left(\frac{n\pi\alpha}{L}\right)^2 \cdot (t-\eta)} \cdot \dot{v}(\eta)d\eta$$

$$= \frac{L}{6}(v(t) - v(0)) + \sum_{n=1}^{\infty} \frac{2}{L} \cdot \frac{(-1)^n}{\left(\frac{n\pi}{L}\right)^2} \cdot \cos\left(n\pi \cdot \frac{x}{L}\right) \cdot e^{-\left(\frac{n\pi\alpha}{L}\right)^2 \cdot t}$$

$$\cdot \left\{ \left[e^{\left(\frac{n\pi\alpha}{L}\right)^2 \cdot \eta} v(\eta) \right]_0^t - \left(\frac{n\pi\alpha}{L}\right)^2 \cdot \int_0^t e^{\left(\frac{n\pi\alpha}{L}\right)^2 \cdot \eta} \cdot v(\eta)d\eta \right\} = \ldots$$

$$\dots = \frac{L}{6}\cdot v(t) + \sum_{n=1}^{\infty}\frac{2}{L}\cdot\frac{(-1)^n}{\left(\frac{n\pi}{L}\right)^2}\cdot\cos\left(n\pi\cdot\frac{x}{L}\right)\cdot v(t) - \frac{L}{6}\cdot v(0)$$

$$-\sum_{n=1}^{\infty} e^{-\left(\frac{n\pi\alpha}{L}\right)^2\cdot t}\cdot\frac{2}{L}\cdot\frac{(-1)^n}{\left(\frac{n\pi}{L}\right)^2}\cdot\cos\left(n\pi\cdot\frac{x}{L}\right)$$

$$\cdot v(0) - \sum_{n=1}^{\infty} 2\cdot\frac{\alpha^2}{L}\cdot(-1)^n\cdot\cos\left(n\pi\cdot\frac{x}{L}\right)\cdot\int_0^t e^{-\left(\frac{n\pi\alpha}{L}\right)^2\cdot(t-\eta)} v(\eta)d\eta$$

$$= Cv(t) - T(t)Cv(0)$$

$$-\sum_{n=1}^{\infty} 2\cdot\frac{\alpha^2}{L}\cdot(-1)^n\cdot\cos\left(n\pi\cdot\frac{x}{L}\right)\cdot\int_0^t e^{-\left(\frac{n\pi\alpha}{L}\right)^2\cdot(t-\eta)} v(\eta)d\eta$$