《Event-triggered Luenburger observer》

Notation

$$y(t) \in \mathbb{R}^m = \begin{bmatrix} y_1(t) \\ y_2(t) \\ \vdots \\ y_m(t) \end{bmatrix}$$

$$Y(t) \in \mathbb{R}^{Tm} = \begin{bmatrix} y_1(t-T+1) \\ \vdots \\ y_1(t) \\ \hline \vdots \\ \hline y_m(t-T+1) \\ \vdots \\ y_m(t) \end{bmatrix}$$

Be careful!

The time-horizon model in this paper is defined a little different from the usual form.

$$Y_i(t) \in \mathbb{R}^m = \begin{bmatrix} y_i(t-T+1) \\ \vdots \\ y_i(t-1) \\ y_i(t) \end{bmatrix}$$

System model development

model (1):

$$X(t) = AX(t-1)$$
$$y(t) = CX(t) + e(t)$$

$X(t) \in \mathbb{R}^n$, $y(t), e(t) \in \mathbb{R}^m$

model (2): $Y(t) = HX(t-T+1) + E(t)$

$Y(t), E(t) \in \mathbb{R}^{Tm}$

$$H = \begin{bmatrix} C_1 \\ C_1 A \\ \vdots \\ C_1 A^T \\ \hline \vdots \\ \hline C_m \\ C_m A \\ \vdots \\ C_m A^T \end{bmatrix}$$

Model (3): $Y(t) = \begin{bmatrix} H & I \end{bmatrix} \begin{bmatrix} X(t-T+1) \\ E(t) \end{bmatrix} \triangleq Q Z(t)$

$$Z(t) = \begin{bmatrix} X(t-T+1) \\ E(t) \end{bmatrix} \in \mathbb{R}^{n+Tm}$$

Model (4): Given shift matrices S, G_i, N_i

$$S = \begin{bmatrix} 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 \\ 0 & 0 & \cdots & 0 \end{bmatrix} \quad G_i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ -C_i A^T \end{bmatrix} \quad N_i = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ b_i^T \end{bmatrix}$$

(b_i is ith natural basis vector)

$$\begin{bmatrix} X(t-T+1) \\ E_1(t) \\ \vdots \\ E_m(t) \end{bmatrix} = \begin{bmatrix} A & 0 & \cdots & 0 \\ G_1 & S & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ G_p & 0 & \cdots & S \end{bmatrix} \begin{bmatrix} X(t-\tau) \\ E_1(t-1) \\ \vdots \\ E_m(t-1) \end{bmatrix} + \begin{bmatrix} 0 \\ N_1 \\ \vdots \\ N_m \end{bmatrix} y(t)$$

Model (5): → This is what we will use for estimation scheme.

$$Z(t) = \bar{A} Z(t-1) + \bar{B} y(t)$$

$$Y(t) = Q Z(t)$$

Where, $Z(t) = \begin{bmatrix} X(t-T+1) \\ E(t) \end{bmatrix}$ $\bar{A} = \left[\begin{array}{cccc} A & 0 & \cdots & 0 \\ \hline G_1 & S & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ G_m & 0 & \cdots & S \end{array}\right]$ $\bar{B} = \begin{bmatrix} 0 \\ N_1 \\ \vdots \\ N_m \end{bmatrix}$

$$Q = [H \quad I]$$

projection operator

$\boxed{\text{Def}}$ The closest sparse solution measured by 2-norm

Π is the projection operator such that:

$$\| \Pi(z) - z \|_2 \leqslant \| z' - z \|_2$$

$$z = \begin{bmatrix} X \\ E \end{bmatrix} \in \mathbb{R}^{n+Tm}, \quad \Pi(z), z' \in \mathbb{R}^{n+Tk} \qquad k \ll m$$

$\boxed{\text{Calculation}}$

$$\Pi(z) = \Pi(X, E) = (X, \Pi(E))$$

$E \in \mathbb{R}^{Tm} \xrightarrow{\text{Reshape}} \tilde{E} \in \mathbb{R}^{m \times T}$ (1)

Then, $\bar{E}_i = \| \tilde{E}_i \|_2$ ($\tilde{E}_i$ means ith row of $\tilde{E}$)

Next, set $(m-k)$ smallest entries of $\bar{E}_i$ as zero,

which leads the corresponding entries in $\Pi(E)$ be zero.

(please refer to Example 4.2 in paper)

Algorithm

① Initialization: $\hat{z}_0 = 0$, $\bar{A}$, $\bar{B}$, Q, k (largest number of attacks), T (horizon time)

② Time update: $\hat{z}_T(t) = \bar{A}\hat{z}(t-1) + \bar{B}y(t)$

③ solve projection operator:

$$\hat{z}_\pi(t) = \Pi(\hat{z}_T(t)) \begin{cases} 1^\circ \ \hat{E}_T(t) = \hat{z}_T(t)[n+1 : end] \in \mathbb{R}^{Tm} \\ 2^\circ \ \text{Reshape } \hat{E}_T(t) \text{ to } \tilde{E}_T(t)\ \mathbb{R}^{m\times T} \\ 3^\circ \ \text{Calculate 2-norm of each row of } \tilde{E}_T(t) \to \bar{E}_T(t) \in \mathbb{R}^m \\ 4^\circ \ \text{set } (m-k) \text{ smallest entries of } \bar{E}_T(t) \text{ be zero,} \\ \quad \text{Collect the indexes in set ``idx''} \\ 5^\circ \ \Pi(\hat{E}_T(t)) = \hat{E}_T(t)[\sim idx] \\ \quad [\text{set idx of } \hat{E}_T(t) \text{ be zero, then return } \hat{E}_T(t) \text{ to } \Pi(\hat{E}_T(t))] \\ 6^\circ \ \Pi(\hat{z}_T(t)) = \begin{bmatrix} \hat{z}_T(t)[1:n] \\ \Pi(\hat{E}_T(t)) \end{bmatrix} \end{cases}$$

$$\hat{z}(t) = \hat{z}_\pi(t)$$

④ Lyapunov-based Event trigger:

Define $V(\hat{z}(t)) = \frac{1}{2}\|Y(t) - Q\hat{z}(t)\|_2^2$

while $V(\hat{z}_\pi(t)) \geq (1-a)V(\hat{z}_\pi(t-1))$ do: (set $a \in [0,1]$, the author did not talk about it, let's try $a = 0.01$ firstly)

$m = 0$

$\hat{z}_\pi(t) = \Pi(\hat{z}(t))$ (Do the ③ process again)

$\hat{z}^{(0)}(t) = \hat{z}_\pi(t)$

$V_{temp} = V(\hat{z}_\pi(t))$

while $V_{temp} \geq (1-a)V(\hat{z}_\pi(t))$ do:

$\hat{z}^{(m+1)}(t) = \hat{z}^{(m)}(t) + L(Y(t) - Q\hat{z}^{(m)}(t))$

(where, $L = Q^T\Sigma$, choose a positive definite matrix Σ)

$V_{temp} = V[\Pi(\hat{z}^{(m+1)}(t))]$ (Do ③ again and calculate V)

$m = m+1$

end while

$\hat{z}(t) = \hat{z}^{(m)}(t)$

end while