3. Comparing Generated Sets

4. EM Derivation

E step:

M step:

We derive the M step by taking the derivative of the expectation with respect to each of the parameters.

Let $E = \sum_{t=1}^{N} \sum_{C_t} \{\pi_t^{(i)}(log(P(C_t)) + log(P(X_t^1|C_t, X_{t-1}^1) + log(P(X_t^2|C_t, X_{t-1}^2))\}$

Solving for π:

$$\frac{\partial}{\partial \pi} E = \frac{\partial}{\partial \pi} \sum_{t=1}^{N} \{\pi_t^1(log(\pi_t^1) + log(N(X_{t-1}^1, \sigma I))) + (1 - \pi_t^1)(log(1 - \pi_t^1) + log(N(X_{t-1}^2, \sigma I)))\}$$

$$= \sum_{t=1}^{N} log(\pi_t^1) + 1 + log(1 - \pi_t^2) - 1 + log(N(X_{t-1}^1, \sigma I)) + log(N(X_{t-1}^2, \sigma I)) = 0$$

$$\sum_{t=1}^{N} log(\frac{\pi_t^1}{1 - \pi_t^1}) = -\sum_{t=1}^{N} log(N(X_{t-1}^1, \sigma I)) + log(N(X_{t-1}^2, \sigma I)) = log(\prod_{t=1}^{N} \frac{\pi_t^1}{1 - \pi_t^1})$$

$$\prod_{t=1}^{N} \frac{\pi_t^1}{1 - \pi_t^1} = e^{-\sum_{t=1}^{N} log(N(X_{t-1}^1, \sigma I)) + log(N(X_{t-1}^2, \sigma I))}$$

Solving for μ:

$$\frac{\partial}{\partial \mu} E = \frac{\partial}{\partial \mu} \sum_{t=1}^{N} \pi_t^{(i)} log(N(X_{t-1}^1, \sigma I)) + \pi_t^{(i)} log(N(X_{t-1}^2, \sigma I))$$

$$= \frac{\partial}{\partial \mu} \sum_{t=1}^{N} log(\frac{e^{\frac{-(X_t^1)^2)}{2\sigma^2}}}{\sigma\sqrt{2\pi}}) + log(\frac{e^{\frac{-(X_t^2)^2)}{2\sigma^2}}}{\sigma\sqrt{2\pi}})$$

$$= \sum_{t=1}^{N} \frac{X_t^1}{\sigma^2} + \frac{X_t^2}{\sigma^2}$$

Solving for σ: