3. (1) 利用莱布尼茨定理中的余项估计，$|r_n(x)|\leqslant\frac{1}{n^2}$；（2）该级数不收敛.

B 类题

1. 和函数 $S(x)=\frac{x^2}{e^x-1}$，估计利用 $|S_n(x)-S(x)|$.　　2. 略.

第二章　多元函数的微分学

第一节　多元函数的极限与连续

A 类题

1. (1) $\frac{x^2-y^2}{2x}$；（2）0；（3）$\frac{xy}{x^2+y^2}$.　　2. (1) C；（2）A；（3）C.　　3. 略.

4. $f(x)=\sqrt{1+x^2}$.

B 类题

1. (1) $\frac{1}{2}$；（2）0；（3）1.　　2. 略.

第二节　偏导数和全微分

A 类题

1. (1) $3\cos 5$；（2）必要;充分；（3）$ye^{xy}dx+xe^{xy}dy$；（4）$dx+dy$.

2. (1) D；（2）C；（3）D；（4）D.

3. (1) $z_x=2x\ln(x^2+y^2)+\frac{2x^3}{x^2+y^2}$，　$z_y=\frac{2yx^2}{x^2+y^2}$；

(2) $z_x=\frac{x-y}{x^2+y^2}$，　$z_y=\frac{x+y}{x^2+y^2}$；

(3) $z_x=y^x\ln y\cdot\ln xy+\frac{1}{x}y^x$，　$z_y=xy^{x-1}\ln(xy)+\frac{1}{y}y^x$.

4. $du=dx+\left(\frac{1}{2}\cos\frac{y}{2}+ze^{yz}\right)dy+ye^{yz}dz$.　　5. 略.

B 类题

略.

C 类题

略.

第三节　复合函数的微分化

A 类题

1. (1) $\frac{(t-2)e^t}{t^3}\cos\frac{e^t}{t^2}$；（2）$4x\cos(x^2-2y)$；（3）$\frac{u-v}{u^2+v^2}$.

2. $\frac{\partial z}{\partial x}=\frac{\partial z}{\partial u}\frac{\partial u}{\partial x}+\frac{\partial z}{\partial v}\frac{\partial v}{\partial x}=\frac{1}{2}\sqrt{\frac{y}{x}}\frac{\partial z}{\partial u}+\frac{1}{y}\frac{\partial z}{\partial v}$；

$\frac{\partial z}{\partial y}=\frac{\partial z}{\partial u}\frac{\partial u}{\partial y}+\frac{\partial z}{\partial v}\frac{\partial v}{\partial y}=\frac{1}{2}\sqrt{\frac{x}{y}}\frac{\partial z}{\partial u}-\frac{x}{y^2}\frac{\partial z}{\partial v}$；

$$\mathrm{d}z=\frac{\partial z}{\partial x}\mathrm{d}x+\frac{\partial z}{\partial y}\mathrm{d}y=\left[\frac{1}{2}\sqrt{\frac{y}{x}}\frac{\partial z}{\partial u}+\frac{1}{y}\frac{\partial z}{\partial v}\right]\mathrm{d}x+\left[\frac{1}{2}\sqrt{\frac{x}{y}}\frac{\partial z}{\partial u}-\frac{x}{y^2}\frac{\partial z}{\partial v}\right]\mathrm{d}y.$$

3. $\frac{\partial z}{\partial \xi}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial \xi}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial \xi}+\frac{\partial z}{\partial w}\cdot\frac{\partial w}{\partial \xi}=-\frac{\partial z}{\partial v}+\frac{\partial z}{\partial w}$,

$\frac{\partial z}{\partial \eta}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial \eta}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial \eta}+\frac{\partial z}{\partial w}\cdot\frac{\partial w}{\partial \eta}=\frac{\partial z}{\partial u}-\frac{\partial z}{\partial w}$,

$\frac{\partial z}{\partial \zeta}=\frac{\partial z}{\partial u}\cdot\frac{\partial u}{\partial \zeta}+\frac{\partial z}{\partial v}\cdot\frac{\partial v}{\partial \zeta}+\frac{\partial z}{\partial w}\cdot\frac{\partial w}{\partial \zeta}=-\frac{\partial z}{\partial u}+\frac{\partial z}{\partial v}$.

4. $\frac{\partial z}{\partial x}=\mathrm{e}^{xy}[y\sin(x+y)+\cos(x+y)]+\mathrm{e}^{x+y}(\cos xy-y\sin xy)$

$\frac{\partial z}{\partial y}=\mathrm{e}^{xy}[x\sin(x+y)+\cos(x+y)]+\mathrm{e}^{x+y}(\cos xy-x\sin xy)$

5. $\frac{\partial w}{\partial x}=\frac{\partial f}{\partial u}\frac{\partial u}{\partial x}+\frac{\partial f}{\partial v}\frac{\partial v}{\partial x}=f'_1+yzf'_2$

$\frac{\partial^2 w}{\partial x\partial z}=\frac{\partial}{\partial z}(f'_1+yzf'_2)=f''_{11}+xyf''_{12}+yf'_2+yz(f''_{21}+xyf''_{22})$

$=yf'_2+f''_{11}+y(x+z)f''_{12}+xy^2zf''_{22}$.

6. (1) $\frac{\partial^2 z}{\partial x^2}=-\sin(x+2y)$，$\frac{\partial^2 z}{\partial y^2}=-4\sin(x+2y)$，$\frac{\partial^2 z}{\partial x\partial y}=-2\sin(x+2y)$；

(2) $\frac{\partial^2 z}{\partial x^2}=-\frac{1}{x^2}$，$\frac{\partial^2 z}{\partial y^2}=-\frac{1}{y^2}$，$\frac{\partial^2 z}{\partial x\partial y}=0$.

7. $\mathrm{d}z=\frac{\partial z}{\partial u}\mathrm{d}u+\frac{\partial z}{\partial v}\mathrm{d}v=\mathrm{e}^{xy}[y\sin(x+y)+\cos(x+y)]\mathrm{d}x+\mathrm{e}^{xy}[x\sin(x+y)+\cos(x+y)]\mathrm{d}y$.

B 类题

$\frac{\partial z}{\partial x}=\frac{yz+y}{\mathrm{e}^z-xy}$，$\frac{\partial z}{\partial y}=\frac{xz+x}{\mathrm{e}^z-xy}$，$\frac{\partial x}{\partial y}=-\frac{xz+x}{yz+y}$

C 类题

略.

第四节　方向导数与梯度

1. (1) $\left(\frac{1}{3},\frac{-1}{3},\frac{2}{3}\right)$；　(2) $-\frac{9\sqrt{3}}{2}$；　(3) $-\frac{1}{\sqrt{2}}$.　　**2.** (1) B；　(2) D；　(3) B.

3. 0.　　**4.** -1.　　**5.** $\frac{1}{6}$.　　**6.** $\frac{16}{\sqrt{6}}$.

B 类题

1. $\pm\frac{\sqrt{2}}{2}$.　　**2.** 0.　　**3.** $\frac{18}{\sqrt{14}}$.