6.1

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第六章

6.1

由题知，利用 Monte Carlo 法求出选择概率

$$P_{si} = \frac{f_i}{\sum_{i=1}^{M} f_i} \quad (M=10)$$

$\therefore \sum_{i=1}^{M} f_i = 2.5+1.0+3.0+1.2+2.1+0.8+2.3+1.5+0.9+1.8=17.1$

同样，通过幂函数变换后

$f' = f^k \quad \because k=2 \quad \therefore f' = f^2$

即 $f_1^2 = 6.25 \quad f_2^2 = 1 \quad f_3^2 = 9 \quad \ldots\ldots$

$\therefore \sum_{i=1}^{M} f_i' = 6.25+1+9+1.44+4.41+0.64+5.29+2.25+0.81+3.24=34.33$

故可通过公式得出个体适应度及选择概率：

个体编号	原适应度	调整后适应度	原选择概率	调整后选择概率
1	2.5	6.25	0.146	0.182
2	1.0	1.0	0.058	0.029
3	3.0	9.0	0.175	0.262
4	1.2	1.44	0.070	0.042
5	2.1	4.41	0.123	0.128
6	0.8	0.64	0.047	0.037
7	2.3	5.29	0.135	0.154
8	1.5	2.25	0.088	0.066
9	0.9	0.81	0.053	0.024
10	1.8	3.24	0.105	0.094

6.1 已知10个个体的适应度如表6.6所示，用幂函数变换法求出调整后的适应度值（k=2），然后采用适应度比例法求出调整前后各个个体的选择概率.

个体编号	原适应度	调整后的适应度	原选择概率	调整后的选择概率
1	2.5	6.25	25/171 (0.15)	625/2833 (0.18)
2	1.0	1.0	10/171 (0.058)	100/2833 (0.029)
3	3.0	9.0	10/57 (0.18)	900/2833 (0.36)
4	1.2	1.44	4/57 (0.07)	144/2833 (0.042)
5	2.1	4.41	7/57 (0.12)	441/2833 (0.13)
6	0.8	0.64	8/171 (0.047)	64/2833 (0.019)
7	2.3	5.29	23/171 (0.13)	529/2833 (0.15)
8	1.5	2.25	5/57 (0.088)	225/2833 (0.066)
9	0.9	0.81	1/19 (0.053)	81/2833 (0.024)
10	1.8	3.24	2/19 (0.11)	324/2833 (0.094

解：幂函数变换法变换公式 $f'=f^K$

由题知 $K=2$ $\therefore f'=f^2$

$f_1'=f_1^2=6.25$ $f_2'=f_2^2=1.0$ $f_3'=f_3^2=9.0$ $f_4'=f_4^2=1.44$

$f_5'=f_5^2=4.41$ $f_6'=f_6^2=0.64$ $f_7'=f_7^2=5.29$ $f_8'=f_8^2=2.25$

$f_9'=f_9^2=0.81$ $f_{10}'=f_{10}^2=3.24$

适应度比例方法：个体被选择概率为 $P_{si}=\dfrac{f_i}{\sum_{i=1}^{n} f_i}$

原来：$\sum_{i=1}^{10} f_i = 2.5+1.0+3.0+1.2+2.1+0.8+2.3+1.5+0.9+1.8=17.1$

$P_{s1}=\frac{2.5}{17.1}=\frac{25}{171}$ $P_{s2}=\frac{10}{171}$ $P_{s3}=\frac{30}{171}=\frac{10}{57}$ $P_{s4}=\frac{12}{171}=\frac{4}{57}$

$P_{s5}=\frac{21}{171}=\frac{7}{57}$ $P_{s6}=\frac{8}{171}$ $P_{s7}=\frac{23}{171}$ $P_{s8}=\frac{15}{171}=\frac{5}{57}$

$P_{s9}=\frac{9}{171}=\frac{1}{19}$ $P_{s10}=\frac{18}{171}=\frac{2}{19}$

调整后 $\sum_{i=1}^{10} f_i' = 6.25+1.0+9.0+1.44+4.41+0.64+5.29+2.25+0.81+3.24=28.33$

$P_{s1}'=\frac{6.25}{28.33}=\frac{625}{2833}$ $P_{s2}'=\frac{100}{2833}$ $P_{s3}'=\frac{900}{2833}$ $P_{s4}'=\frac{144}{2833}$

$P_{s5}'=\frac{441}{2833}$ $P_{s6}'=\frac{64}{2833}$ $P_{s7}'=\frac{529}{2833}$ $P_{s8}'=\frac{225}{2833}$

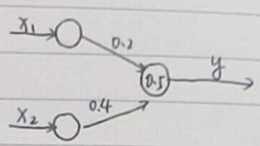
$P_{s9}'=\frac{81}{2833}$ $P_{s10}'=\frac{324}{2833}$

8.1、8.2

第八章

8.1

$$y=\begin{cases}1, & x\geq\theta\\ 0, & x<\theta\end{cases}$$



$u=\sum w_{ij}y_i^{k-1}$，通过枚举法可得:

① $(x_1,x_2)=(0,0)$

$\therefore x_1\cdot w_1+x_2\cdot w_2=0+0=0$ $\therefore y=0$

② $(x_1,x_2)=(1,0)$

$\therefore x_1\cdot w_1+x_2\cdot w_2=0.2+0\times0.4=0.2<\theta$ $\therefore y=0$

③ $(x_1,x_2)=(0,1)$

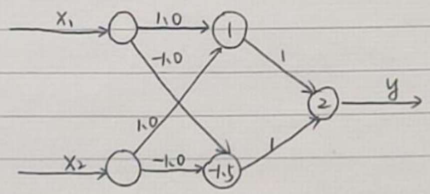
$\therefore x_1\cdot w_1+x_2\cdot w_2=0+1\times0.4=0.4<\theta$ $\therefore y=0$

④ $(x_1,x_2)=(1,1)$

$\therefore x_1w_1+x_2\cdot w_2=0.2+0.4=0.6>\theta$ $\therefore y=1$

综上可证，该神经网络描述与逻辑功能

8.2



通过枚举法验证:

① $(x_1,x_2)=(0,0)$

$u_1=1.0\times0+1.0\times0=0<\theta_1$ $y_1=0$

$u_2=-1.0\times0+(-1.0\times0)=0>\theta_2$ $y_2=1$

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$\therefore u_2=1\times0+1\times(1)=1<2$ $y=0$

② $(x_1,x_2)=(1,0)$

$u_1=1.0\times1+(1.0)\times0=1.0\geq\theta_1$ $y_1=1$

$u_2=(-1.0)\times1+(-1.0)\times0=-1.0\geq\theta_2$ $y_2=1$

$\therefore u=1\times1+1\times1=2\geq2$ $y=1$

③ $(x_1,x_2)=(0,1)$

$u_1=1.0\times0+1.0\times1=1.0\geq\theta_1$ $y_1=1$

$u_2=(-1.0)\times0+(-1.0\times1)=-1.0\geq\theta_2$ $y_2=1$

$\therefore u=1\times1+1\times1=2\geq2$ $y=1$

④ $(x_1,x_2)=(1,1)$

$u_1=1\times1.0+1\times(1.0)=2.0\geq\theta_1$ $y_1=1$

$u_2=(-1.0)\times1+(-1.0)\times1=-2.0\leq\theta_2$ $y_2=0$

$\therefore u=1\times1+1\times(-1)=0\leq2$ $y=0$

综上可证，该神经网络描述"异或"逻辑功能.

8.3

8.3

$X=[1,-1,1]$，$Y=[1,1]$，选 $\varepsilon=0.1$，

$$W_1=\begin{bmatrix}1&1&1\\2&0&2\\3&3&-3\end{bmatrix}\qquad W_2=\begin{bmatrix}1&-1&1\\0&1&1\end{bmatrix}$$

$$u^2=\sum w_{ij}y_1=\begin{bmatrix}1&1&1\\2&0&2\\3&3&-3\end{bmatrix}\begin{bmatrix}1\\-1\\1\end{bmatrix}=\begin{bmatrix}1\\4\\-3\end{bmatrix}\Rightarrow f(u_i^2)=\begin{bmatrix}f(1)\\f(4)\\f(-3)\end{bmatrix}=\begin{bmatrix}0.4621\\0.9640\\-0.9051\end{bmatrix}=y_2$$

$$u^3=\sum w_{ij}y_2=\begin{bmatrix}1&-1&1\\0&1&1\end{bmatrix}\begin{bmatrix}f(1)\\f(4)\\f(-3)\end{bmatrix}=\begin{bmatrix}f(1)-f(4)+f(-3)\\f(4)+f(-3)\end{bmatrix}$$

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$$y_3=f(u_3)=\begin{bmatrix}f(f(1)-f(4)+f(-3))\\f(f(4)+f(-3))\end{bmatrix}=\begin{bmatrix}0.1989\\0.0294\end{bmatrix}$$

$$d_i^m=d_i^3=y_i^3(1-y_i^3)(y_i^3-y_{si})$$

$$f'(u_i^k)=\frac{2e^{-u_i^k}}{(1+e^{-u_i^k})^2}=\frac{e^{-u_i^k}}{(1+e^{-u_i^k})}\cdot\frac{2}{1+e^{-u_i^k}}\Rightarrow d_i^k=\frac{\partial y_i^k}{\partial u_i^k}=(y_i^k+1)(1-2y_i^k)$$

$$\therefore d_1^3=(y_1^3+1)(1-2y_1^3)(y_1^3-y_{si})=(0.1989+1)(1-2\times0.1989)(0.1989-1)=-0.5784$$

$$d_2^3=(y_2^3+1)(1-2y_2^3)(y_2^3-y_{si})=(0.0294+1)(1-2\times0.0294)(0.0294-1)=-0.9404$$

$$\Delta W_{11}^2=-\varepsilon d_1^3y_1^2=-0.1\times(-0.5784)\times0.4621=0.0267\quad \Delta W_{21}^2=-\varepsilon d_2^3y_1^2=0.0435$$

$$\Delta W_{12}^2=-\varepsilon d_1^3y_2^2=-0.1\times(-0.5784)\times0.9640=0.0558\quad \Delta W_{22}^2=-\varepsilon d_2^3y_2^2=0.0907$$

$$\Delta W_{13}^2=-\varepsilon d_1^3y_3^2=-0.1\times(-0.5784)\times(-0.9051)=0.0524\quad \Delta W_{23}^2=-\varepsilon d_2^3y_3^2=-0.0851$$

故可计算隐藏层梯度：

$$d_1^2=(y_1^2+1)(1-2y_1^2)\sum_i d_i^3W_{i1}^2=(0.4621+1)(1-2\times0.4621)[-0.5784\times1+(-0.9404)\times0]=-0.0641$$

$$d_2^2=(y_2^2+1)(1-2y_2^2)\sum_i d_i^3W_{i2}^2=(0.9640+1)(1-2\times0.9640)[-0.5784\times(-1)+(-0.9404)\times1]=0.6600$$

$$d_3^2=(y_3^2+1)(1-2y_3^2)\sum_i d_i^3W_{i3}^2=(-0.9051+1)(1+2\times0.9051)[-0.5784\times1+(-0.9404)\times1]=-0.4050$$

通过隐藏层梯度可求输入层权值变化量：

$$\Delta W_{11}^1=-\varepsilon d_1^2y_1^1=0.0064\quad \Delta W_{21}^1=-\varepsilon d_2^2y_1^1=\boxed{0.0660}\quad \Delta W_{31}^1=-\varepsilon d_3^2y_1^1=0.0405$$

$$\Delta W_{12}^1=-\varepsilon d_1^2y_2^1=-0.0064\quad \Delta W_{22}^1=-\varepsilon d_2^2y_2^1=0.0660\quad \Delta W_{32}^1=-\varepsilon d_3^2y_2^1=-0.0405$$

$$\Delta W_{13}^1=-\varepsilon d_1^2y_3^1=0.0064\quad \Delta W_{23}^1=-\varepsilon d_2^2y_3^1=-0.0660\quad \Delta W_{33}^1=-\varepsilon d_3^2y_3^1=0.0405$$

$$W_1'=W_1+\Delta W_1=\begin{bmatrix}1&1&1\\2&0&2\\3&0&-3\end{bmatrix}+\begin{bmatrix}0.0064&-0.0064&0.0064\\-0.0660&0.0660&-0.0660\\0.0405&-0.0405&0.0405\end{bmatrix}$$

$$W_2'=W_2+\Delta W_2=\begin{bmatrix}1&-1&1\\0&1&1\end{bmatrix}+\begin{bmatrix}0.0267&0.0558&-0.0524\\0.0435&0.0907&-0.0851\end{bmatrix}$$

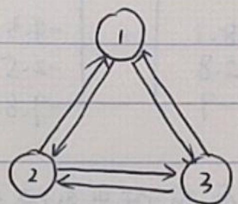
8.7

8.7

$$W=\begin{bmatrix}0 & 1 & -2\\ 1 & 0 & 3\\ -2 & 3 & 0\end{bmatrix},\quad \theta=\begin{bmatrix}-1\\ 2\\ 1\end{bmatrix}\qquad V(0)=\{1,0,1\}$$

其中，能量函数

$$E=-\frac{1}{2}\sum_{i=1}^{N}\sum_{j\neq i}^{N} W_{ij}V_iV_j+\sum_{i=1}^{N}\theta_i V_i$$



（采用同步工作方式）

$$E=-\frac{1}{2}\begin{bmatrix}1 & 0 & 1\end{bmatrix}\begin{bmatrix}0 & 1 & -2\\ 1 & 0 & 3\\ -2 & 3 & 0\end{bmatrix}\begin{bmatrix}1\\ 0\\ 1\end{bmatrix}+\begin{bmatrix}1 & 0 & 1\end{bmatrix}\begin{bmatrix}-1\\ 2\\ 1\end{bmatrix}$$

$$=-\frac{1}{2}\begin{bmatrix}1 & 0 & 1\end{bmatrix}\begin{bmatrix}-2\\ 4\\ -2\end{bmatrix}+0=-\frac{1}{2}\times(-4)=2$$

8.10

8.10

$$W=\begin{bmatrix} 0 & 3.4 & 2.8 & -3.1 \\ 3.4 & 0 & 4.7 & -1.2 \\ 2.8 & 4.7 & 0 & -5.9 \\ -3.1 & -1.2 & -5.9 & 0 \end{bmatrix} \qquad \theta=[6.3\ \ -4.3\ \ -2.5\ \ -9.6]$$

$$E=-\frac{1}{2}[1,0,1,0]\begin{bmatrix} 0 & 3.4 & 2.8 & -3.1 \\ 3.4 & 0 & 4.7 & -1.2 \\ 2.8 & 4.7 & 0 & -5.9 \\ -3.1 & -1.2 & -5.9 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}+[1\ \ 0\ \ 1\ \ 0]\begin{bmatrix} 6.3 \\ -4.3 \\ -2.5 \\ -9.6 \end{bmatrix}=1$$

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$$u'=\begin{bmatrix} 0 & 3.4 & 2.8 & -3.1 \\ 3.4 & 0 & 4.7 & -1.2 \\ 2.8 & 4.7 & 0 & -5.9 \\ -3.1 & -1.2 & -5.9 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}-\begin{bmatrix} 6.3 \\ -4.3 \\ -2.5 \\ -9.6 \end{bmatrix}$$

$$=\begin{bmatrix} 2.8 \\ 8.1 \\ 2.8 \\ -9 \end{bmatrix}-\begin{bmatrix} 6.3 \\ -4.3 \\ -2.5 \\ -9.6 \end{bmatrix}=\begin{bmatrix} -3.5 \\ 12.4 \\ 5.3 \\ 0.6 \end{bmatrix}$$

由题状态可推出系统演化律为二值硬限器，即

$$v_i(k+1)=\begin{cases} 1 & u_i(k)\geq 0 \\ 0 & u_i(k)<0 \end{cases}$$

$\therefore v(1)=[0\ \ 1\ \ 1\ \ 1]^T$

$$u''=\begin{bmatrix} 0 & 3.4 & 2.8 & -3.1 \\ 3.4 & 0 & 4.7 & -1.2 \\ 2.8 & 4.7 & 0 & -5.9 \\ -3.1 & -1.2 & -5.9 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}-\begin{bmatrix} 6.3 \\ -4.3 \\ -2.5 \\ -9.6 \end{bmatrix}$$

$$=\begin{bmatrix} 3.1 \\ 3.5 \\ -1.2 \\ -7.1 \end{bmatrix}-\begin{bmatrix} 6.3 \\ -4.3 \\ -2.5 \\ -9.6 \end{bmatrix}=\begin{bmatrix} -3.2 \\ 7.8 \\ 1.3 \\ 2.5 \end{bmatrix}$$

$\therefore v(2)=[0,1,1,1]^T$

$\therefore$ 稳定状态即为 $[0\ \ 1\ \ 1\ \ 1]$

8.10.

$$E = -\frac{1}{2}\sum_{i=1}^{4}\sum_{j=1}^{4} W_{ij} V_i V_j + \sum_{i=1}^{4} \theta_i V_i$$

$$= 1$$

神经元调整次序： $1 \to 2 \to 3 \to 4$.

$K=0$ 时　$V(0) = \{1, 0, 1, 0\}$

$K=1$ 时　$V_1(1) = f(\sum_{j=1}^{4} W_{1j} V_j(0) - \theta_1) = f(W_{13} V_3(0) - \theta_1) = f(-3.5) = 0$.

$V_2(1) = V_2(0) = 0$.　$V_3(1) = V_3(0) = 1$　$V_4(1) = V_4(0) = 0$

$\therefore V(1) = \{0, 0, 1, 0\}$

$K=2$ 时.　$V_2(2) = f(\sum_{j=1}^{4} W_{2j} V_j(1) - \theta_2) = f(9) = 1$.

$V_1(2) = V_1(1) = 0$　$V_3(2) = V_3(1) = 1$　$V_4(2) = V_4(1) = 0$

$\therefore V(2) = \{0, 1, 1, 0\}$.

$K=3$ 时.　$V_3(3) = f(\sum_{j=1}^{4} W_{3j} V_j(2) - \theta_3) = f(7.2) = 1$.

$V_1(3) = V_1(2) = 0$　$V_2(3) = V_2(2) = 1$　$V_4(3) = V_4(2) = 0$

$V(3) = \{0, 1, 1, 0\}$.

$K=4$ 时　$V_4(4) = f(\sum_{j=1}^{4} W_{4j} \cdot V_j(3) - \theta_4) = f(2.5) = 1$.

故 $V(4) = \{0, 1, 1, 1\}$

$K=5$ 时　$V_1(5) = f(\sum_{j=1}^{4} W_{1j} V_j(4) - \theta_1) = f(-3.2) = 0$

$V(5) = \{0, 1, 1, 1\}$

故稳定状态为 $\{0, 1, 1, 1\}$.