习题 9.11 答案与提示

(A)

1. 切线方程 $\frac{x-6}{5}=\frac{y-2}{3}=\frac{z-4}{4}$，法平面方程 $5x+3y+4z=52$.

2. 切线方程 $\begin{cases}x-R=0\\ vy-R\omega z=0\end{cases}$，法平面方程 $R\omega y+vz=0$.

3. 切线方程 $\frac{x-1}{1}=\frac{y-1}{1}=\frac{z-1}{2}$，法平面方程 $x+y+2z=4$.

4. 切线方程 $\sqrt{2}x-R=-\sqrt{2}y+R=-\sqrt{2}z+R$，法平面方程 $x-y-z+R/\sqrt{2}=0$.

5. 切线方程 $\frac{x-1}{16}=\frac{y-1}{9}=\frac{z-1}{-1}$，法平面方程 $16x+9y-z-24=0$.

6. 切平面方程 $x+2y+3z-14=0$，法线方程 $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z-3}{3}$.

7. 切平面方程 $\frac{x_0x}{a^2}+\frac{y_0y}{b^2}+\frac{z_0z}{c}=1$；法线方程为 $\frac{a^2(x-x_0)}{x_0}=\frac{b^2(y-y_0)}{y_0}=\frac{c^2(z-z_0)}{z_0}$.

8. 切平面方程 $4x+2y-z-6=0$，法线方程 $\frac{x-2}{4}=\frac{y-1}{2}=\frac{z-4}{-1}$.

9. 切平面方程 $z=\frac{\pi}{4}-\frac{1}{2}(x-y)$，法线方程 $\frac{x-1}{1}=\frac{y-1}{-1}=\frac{z-\pi/4}{2}$.

(B)

1. $\{1,\quad \tan\alpha,\quad f_x(x_0,y_0)+f_y(x_0,y_0)\tan\alpha\}$.

2. 略.

3. 略.

4. 略.

总习题 9 答案与提示

1. 0.

2. 不存在.

3. (1)充分，必要； (2)必要，充分； (3)充分； (4)充分.

4. (1)连续； (2)存在且 $f_x(0,0)=f_y(0,0)=0$； (3)不连续； (4)可微.

5. $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{y^2(\ln x-1)}{x^2(\ln y-1)}$， $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=\left[\frac{y^2}{x^3}+\frac{2y^2(x-y)(\ln x-1)}{x^4(\ln y-1)}-\frac{y^3(\ln x-1)^2}{x^4(\ln y-1)^2}\right]\frac{1}{\ln y-1}$.

6. $\frac{\partial z}{\partial x}=yf'_1+\frac{1}{y}f'_2-\frac{y}{x^2}g'$，$\frac{\partial^2 z}{\partial x\partial y}=f'_1-\frac{1}{y^2}f'_2+xyf''_{11}-\frac{x}{y^3}f''_{22}-\frac{1}{x^2}g'-\frac{y}{x^3}g''$.

7. $\frac{\partial u}{\partial x}=\frac{uf'_1(1-2yvg'_2)-f'_2g'_1}{(xf'_1-1)(2yvg'_2-1)-f'_2g'_1}$， $\frac{\partial v}{\partial x}=\frac{g'_1(xf'_1+uf'_1-1)}{(xf'_1-1)(2yvg'_2-1)-f'_2g'_1}$.

8. 略.

9. (1) $\frac{\partial z}{\partial x}=\frac{\partial z}{\partial y}=\frac{1}{2\ln 2}=\frac{1}{\ln 4}$； (2) $\cos\alpha=\cos\beta=\frac{1}{\sqrt{2+\ln^2 4}}$， $\cos\gamma=\frac{-\ln 4}{\sqrt{2+\ln^2 4}}$.

10. $\frac{1}{5}\{3,4\},\frac{1}{5}\{4,3\}$. **11.** $x+2z=7, x+4y+6z=21$. **12.** 略.

13. $x+y+z=\sqrt{3}$. **14.** 最高点$(0,0,4)$,最低点$\left(\frac{8}{3},\frac{8}{3},-\frac{4}{3}\right)$. **15.** $x_0+y_0+z_0$.

习题 10.1 答案与提示

(A)

1. $Q=\iint\limits_D \rho(x,y)\mathrm{d}\sigma$.

2. 略.

3. (1) $\iint\limits_D (x+y)^2\mathrm{d}\sigma \geqslant \iint\limits_D (x+y)^3\mathrm{d}\sigma$; (2) $\iint\limits_D (x+y)^2\mathrm{d}\sigma \leqslant \iint\limits_D (x+y)^3\mathrm{d}\sigma$.

4. (1)等于零; (2)大于零; (3)等于零; (4)大于零.

5. $\frac{1}{4}$.

(B)

1. 根据二重积分的性质,比较下列积分的大小:

(1) $\iint\limits_D \ln(x+y)\mathrm{d}\sigma \geqslant \iint\limits_D [\ln(x+y)]^2\mathrm{d}\sigma$; (2) $\iint\limits_D \ln(x+y)\mathrm{d}\sigma \leqslant \iint\limits_D [\ln(x+y)]^2\mathrm{d}\sigma$.

2. (1) 大于零; (2) 小于零.

3. $\frac{100}{51}\leqslant I\leqslant 2$.

4. 利用反证法.

5. 9.876.

6. $a=\frac{1}{2}, b=\frac{1}{2}, c=\frac{1}{2}$,积分值为 0.402.

习题 10.2 答案与提示

(A)

1. (1) 1; (2) $\frac{20}{3}$; (3) $-\frac{3\pi}{2}$.

2. (1) $\frac{6}{55}$; (2) $\frac{64}{15}$; (3) $\frac{13}{6}$.

3. (1) $\int_0^4 \mathrm{d}x\int_x^{2\sqrt{x}} f(x,y)\mathrm{d}y$ 或 $\int_0^4 \mathrm{d}y\int_{\frac{y^2}{4}}^{y} f(x,y)\mathrm{d}x$;

(2) $\int_{-r}^{r} \mathrm{d}x\int_0^{\sqrt{r^2-x^2}} f(x,y)\mathrm{d}y$ 或 $\int_0^r \mathrm{d}y\int_{-\sqrt{r^2-x^2}}^{\sqrt{r^2-x^2}} f(x,y)\mathrm{d}x$;

(3) $\int_1^2 \mathrm{d}x\int_{\frac{1}{x}}^{x} f(x,y)\mathrm{d}y$ 或 $\int_{\frac{1}{2}}^{1} \mathrm{d}y\int_{\frac{1}{y}}^{2} f(x,y)\mathrm{d}x+\int_1^2 \mathrm{d}y\int_y^2 f(x,y)\mathrm{d}x$.

4. (1) $\int_0^1 \mathrm{d}y\int_{e^y}^{e} f(x,y)\mathrm{d}x$; (2) $\int_0^1 \mathrm{d}y\int_{2-y}^{1+\sqrt{1-y^2}} f(x,y)\mathrm{d}x$;