$$= \frac{1}{n\pi}\sin\frac{2n\pi}{3} - \frac{1}{n\pi}\int_0^1 \sin\frac{2n\pi x}{3}\mathrm{d}x + \frac{1}{n\pi}\sin\frac{4n\pi}{3}$$

$$- \frac{1}{n\pi}\sin\frac{2n\pi}{3} + \frac{1}{n\pi}(3-x)\sin\frac{2n\pi x}{3}\Big|_2^3 + \frac{1}{n\pi}\int_2^3 \sin\frac{2n\pi x}{3}\mathrm{d}x$$

$$= \frac{1}{n\pi}\sin\frac{4n\pi}{3} + \frac{3}{2n^2\pi^2}\cos\frac{2n\pi x}{3}\Big|_0^1 - \frac{1}{n\pi}\sin\frac{4n\pi}{3} - \frac{3}{2n^2\pi^2}\cos\frac{2n\pi x}{3}\Big|_2^3$$

$$= \frac{3}{2n^2\pi^2}\cos\frac{2n\pi}{3} - \frac{3}{2n^2\pi^2} - \frac{3}{2n^2\pi^2}\cos\frac{2n\pi}{3} + \frac{3}{2n^2\pi^2}\cos\frac{4n\pi}{3}$$

$$= \frac{3}{n^2\pi^2}\cos\frac{2n\pi}{3} - \frac{3}{n^2\pi^2}.$$

$$b_n = \frac{1}{l}\int_{-l}^{l} f(x)\sin\frac{n\pi}{l}\mathrm{d}x = 0\,.$$

故 $f(x) = \dfrac{2}{3} + \dfrac{3}{\pi^2}\sum\limits_{n=1}^{\infty}\left[\dfrac{-1}{n^2} + \dfrac{1}{n^2}\cos\dfrac{2n\pi}{3}\right]\cos\dfrac{n\pi x}{3}$，$x\in[0,3]$为所求.

A 类题

1. 将函数 $f(x)=2+|x|(-1\leqslant x\leqslant 1)$展开成以 2 为周期的傅里叶级数,并由此求级数 $\sum\limits_{n=1}^{\infty}\dfrac{1}{n^2}$ 的和.

2. 设 $f(x)$的周期为 $T=10$,且当 $-5\leqslant x<5$ 时,$f(x)=x$,将 $f(x)$展开成傅里叶级数.

3. 在$(0,\frac{1}{2})$内把 $f(x)=\cos\pi x$ 展开成以 1 为周期的正弦级数.

4. 将 $f(x)=x-1(0\leqslant x\leqslant 2)$展开成以 4 为周期的余弦级数.