

第五章 常微分方程

第一节 微分方程的基本概念

A 类题

1. $y = \frac{x^2}{2} + 4$. 2. 略. 3. 略. 4. $xy' + y = 0$.

B 类题

1. $mv'(t) = mg - kv(t)$, $v(0) = \sqrt{2gh}$. 2. $s'' = \frac{1}{6}g(1+s)$, $s(0) = 1$, $s'(0) = 0$.

第二节 一阶微分方程

A 类题

1. (1) $(1+x)e^y = 2x + C$; (2) $2^x + 2^{-y} = C$; (3) $(e^x - 1)(e^y - 1) = C$; (4) $\sin \frac{y}{x} = Cx$.

2. (1) $y^2 = x^2 + Cy$; (2) $y = \cos x(C + \tan x)$; (3) $x = \frac{C}{y} + y \ln y$; (4) $ye^{-x} - x^2 = C$.

3. (1) $(y-2x)^3 = C(y-x-1)^2$, $y = x+1$; (2) $y = Ce^{-f(x)} + f(x) - 1$;

(3) $\ln[(x+2)^2 + (y+3)^2] + 2\arctan \frac{y+3}{x+2} = C$; (4) $y = \frac{\frac{4}{3}x^3 + C}{x^2 + 1}$.

4. (1) $x^3 + y^3 + x^2 + x^2 e^{-y} = C$; (2) $x^2 y - \frac{y^3}{3} = C$.

5. (1) $2x^2 + 2xy + y^2 - 8x - 2y = C$; (2) $\tan(x-y+1) = x+c$;

(3) $y^{-5} = Cx^5 + \frac{5}{2}x^3$; (4) $y = \frac{1}{Ce^x - \sin x}$, $y=0$.

6. (1) $\mu(x) = x$, $x^2 + x^2 y^2 - x^2 y = C$; (2) $\mu(x) = e^x$, $e^x(xy + \sin y) = C$.

B 类题

1. $y^2 = C(x+1) - (x+1)\ln(x+1) - 1$. 2. $f(x) = -\frac{1}{2}e^x$.

3. $f(x) = \sqrt[9]{x}$. 4. $y = -4x \ln x + x$. 5. $y^2 = \frac{9}{2}x$.

C 类题

$y = e^x - e^{-x} + x - 1/2$.

