Size-based Momentum and Cross Momentum Strategies

Fine 435 Group Report

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INTRODUCTION

Investors are interested in factor investing that targets different attributes of returns. Momentum investing, one of the previously recognized market anomalies, has been exploited and tested profitable during the past decades. This paper first attempts to re-examine the momentum strategy at an aggregated level from January 1960 to June 2015 by replicating the formation of momentum portfolio using the methodology of Jegadeesh and Titman (1993), second to examine at different size levels and evaluate the performance of momentum strategies for small market capitalization stocks and large market capitalization stocks, and last to propose a new strategy called Cross Momentum Strategy during 2000-2010.

We begin by testing whether the momentum strategy can be explained to a great extent already by the Fama and French Three factor model. The test is conducted through means of measuring correlations, R^2, from regressing momentum on market, size and value.

Next, we compare the performance of momentum factor investing between portfolios of different sizes. Historical data and observations depict that portfolios consisting of small-cap stocks normally generate higher returns than portfolios consisting of large-cap stocks. Likewise, winner portfolios (portfolios that performed better in the past) generate higher returns than loser portfolios. Based on the above empirical observations, portfolios consisting of stocks that are winner over past years and have small market capitalization should yield higher returns.

Thus, we construct two short-long strategies, longing small-cap winner while shorting small-cap loser, and longing large-cap winner while shorting large-cap loser. In order to compare the performance of the two strategies, we compute the returns for both strategies and plot them over the same period. Hypothesis testing is applied to demonstrate whether there is a significant difference between the return of the two strategies. In this step, since the large-cap and small-cap can be defined differently for different cut-offs of the stocks' market capitalization. Thus, we also compute returns for different size cut-offs in order to determine an optimal one.

In the end, we propose a Cross Momentum Strategy which is a blend of the size portfolio and momentum portfolio. Since it is essential to test the assumption under various settings and time horizons

and small-cap stocks are more exposed to recession or bankruptcy risk, we are interested in the performance of the portfolios during years containing financial crises to verify the stability of the strategy.

DATA AND METHODOLOGY

We obtain our historical stock return data from the CRSP (Center for Research in Security Prices) database, The University of Chicago Booth School of Business. In this dataset, we consider all common stocks that are listed in NYSE, NASDAQ, and AMEX between January 1959 and June 2015. This dataset includes dates, stock tickers, exchanges on which individual stocks are traded, end-of-month returns including and excluding dividends, market capitalizations and end-of-month prices. One of the key advantages of this dataset is its completeness: all stocks in a month of a year are taken into consideration regardless of whether they are still being traded later on, which allows us to avoid the typical survival bias. In addition to CRSP, we also utilize the data on three common factors (rmrf, smb, and hml) and risk-free rate (rf) from French's webpage.

Our methodology of constructing momentum strategy for small and large market capitalization stocks is to group stocks into portfolios by size, followed by grouping by momentum using ex-ante market cap and price respectively, wait for one month, hold these portfolios for the next month and get monthly returns, reform portfolios in the next month and repeat the process. We estimate momentum by calculating the ratio of the current end-of-month price (P_t) over the price in month t-10 (P_{t-10}) and subtracting it by 1 (which is the total return over the past 10 months). In any given month, we determine the size cut-off criterion by ranking the market capitalizations of stocks in NYSE, setting the minimum to be the smallest market cap in all exchanges and the maximum to be the largest, then we allocate all stocks into these 10 size deciles. Hence, relatively larger-cap stocks are in size deciles 6,7,8,9 and 10, and smaller-cap stocks are in size deciles 1,2,3,4, and 5. We first attempt to define large-cap stocks to be top 10% (i.e., 10^{th} decile) and small-cap stocks to be bottom 10% (i.e., 1^{st} decile). We could redefine the grouping by including more

deciles of stocks to see if there is an optimal selection criterion that results in the most effective momentum strategy.

For these two size-based groups we implement the same ranking method to the momentum within each of the groups, but here we keep all 10 deciles of momentum portfolios instead of disregarding some of the deciles as in the previous procedure. After constructing size-momentum-weighted portfolios at end of month t, we wait for one month to alleviate microstructure effects, and calculate returns for month t+2. We repeat these steps month by month and record monthly returns. Therefore, we end up with 10 small-cap momentum portfolios and 10 large-cap momentum portfolios based on which comparison and analysis are performed.

Our first concern is whether the total momentum strategy can be explained to a great extent by the Fama and French Three-Factor Model. This question is a prerequisite and is essential to our entire analysis in the sense that if the momentum factor can be explained by the three factors, incorporating it into the model is meaningless. We test the relationship by measuring alpha and R^2 from regressing momentum on market, size, and value:

$$UMD_t = \alpha + \beta_1 RMRF_t + \beta_2 SMB_t + \beta_3 HML_t + \epsilon_t$$

where UMD_t is the monthly premium on winners minus losers in momentum strategy in month t, $RMRF_t$ is the monthly excess market return, SMB_t is the monthly premium on small firms minus big firms, and HML_t is the monthly premium on high book-to-market firms minus low book-to-market firms.

We compare visually the winners-minus-losers strategy for large-cap and small-cap momentum portfolios. The monthly return on the loser portfolio is subtracted from the return on the winner for the group of large-cap stocks (denoted by UMD_B) and the group of small-cap stocks (denoted by UMD_S). The one-dollar evolution of returns is traced for each of them.

For the purpose of testing statistical significance, the t test is conducted. First of all, we conjecture both UMD_B and UMD_S have average monthly returns that are statistically different from (more specifically, greater than) zero rate of return. That is:

$$H_{a1}: \overline{UMD_B} > 0 \qquad and \qquad H_{a2}: \overline{UMD_S} > 0$$

and t statistic is calculated for each of them:

$$t_{UMD_B} = \frac{\overline{UMD_B} - 0}{\sigma_{UMD_B}/\sqrt{n}} \qquad and \qquad t_{UMD_S} = \frac{\overline{UMD_S} - 0}{\sigma_{UMD_S}/\sqrt{n}}$$

We further compare the strength of momentum effect between the group of large-cap stocks and the group of small-cap stocks. We conjecture that the momentum effect is stronger for small-cap stocks than large cap stocks:

$$H_{a3}: \overline{UMD_{SMB}} > 0$$

where $\overline{UMD_{SMB}}$ is the average monthly returns on momentum strategy for small stocks minus the average monthly returns on momentum strategy for large stocks.

Knowing small-cap momentum strategy is superior to large-cap, how can we leverage this information to achieve better results? In this last step, we address this question by proposing a better strategy called Cross Momentum Strategy: Buy Small Up and Sell Big Down (denoted by $U_S MD_B$). We prove this factor works quite well since the 2000s.

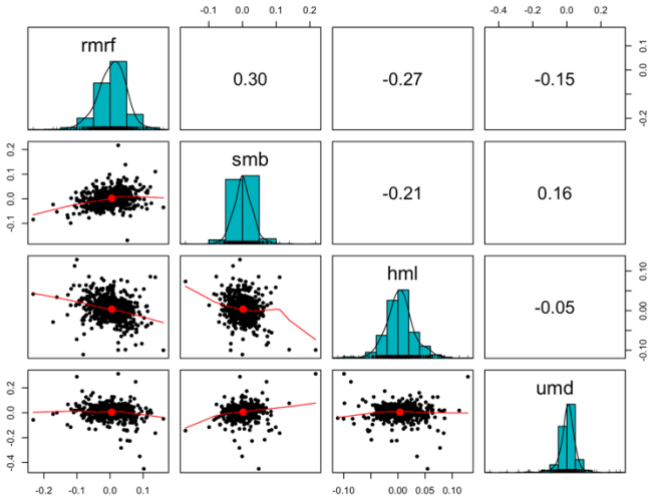
In cross momentum strategy, the stocks are first sorted into groups as regards to their market capitalization by using 20% cut-off. Then, within the small-cap and large-cap portfolios respectively, we rank the stocks according to their momentum. The "Buy Small Up and Sell Big Down" is buying the winner portfolio within the small-cap group and shorting the loser portfolio within the large-cap group.

In order to evaluate the performance of our strategy, the Cross Momentum portfolio is compared with both the Market-effect portfolio and Size-effect portfolio. The latter two portfolios' returns are obtained from the French's website. To compare it with the original momentum strategy, we also examine its performance difference as regards to small-cap and large-cap momentum portfolios constructed previously. We plot one-dollar evolution for every one of these portfolios and calculate annualized returns, volatilities and Sharpe ratios.

RESULTS

Preliminary Analysis

Before moving to our size-based momentum analysis, we first reconstructed the total momentum strategy in the period 1960-2015. We respond to the crucial question raised at the beginning: To what extent can the returns of the momentum strategy be explained by the Fama-French Three-Factor Model? If the momentum strategy is a valid stand-alone strategy, then the returns of momentum strategy should *not* align with predictions by the Fama-French Model. To verify this, we test the correlations between the four factors, and perform a regression of UMD on the other three and see the R^2.



The correlations of the four risk factors including market risk factor, size effect, value effect as well as the momentum risk factor were evaluated by pairs shown above. It suggests that there is a small negative correlation between HML and momentum risk factor and both market risk factor and SMB are slightly positively associated with momentum risk factor. However, the nearly zero correlation shows the relationships are not strong enough for HML being represented by the three factors.

The regression results are as following:

$$UMD_t = \underbrace{0.59\%}_{(p\ value=0.005)} - 0.29 * RMRF_t + 0.39 * SMB_t + 0.14 * HML_t + \epsilon_t$$

$$Adjusted\ R^2 = \ 0.069 \ll 1$$

All three parameters are significant at 0.1 level. The estimated alpha is 0.59% and is highly significant, which indicates the overall momentum strategy performs better than the Fama-French Three-Factor Model predicts. Besides, the R^2 is rather small. This means that only 6.9 % of the variation in momentum returns can be explained by the variation in these three factors.

From the preliminary analysis, we draw the conclusion that the power of the Fama-French Three-Factor Model in explaining the returns of the momentum strategy is rather limited, and momentum strategy by itself is a valid investment strategy.

Determine Optimal Size Cut-off

Having confirmed the validity of the momentum strategy, we can now focus on how to implement this strategy so that we maximize our returns. As mentioned above, we divide stocks into two groups, one is the large-cap group and the other is the small-cap group, and within each of the two groups we long the winner and short the loser. Now the question is, what is the best definition of large-cap group and small-cap group? Top 10% and bottom 10%, or Top 20% and bottom 20%, or else?

Table 1: Annualized Summary Statistics for Different Size Cut-offs

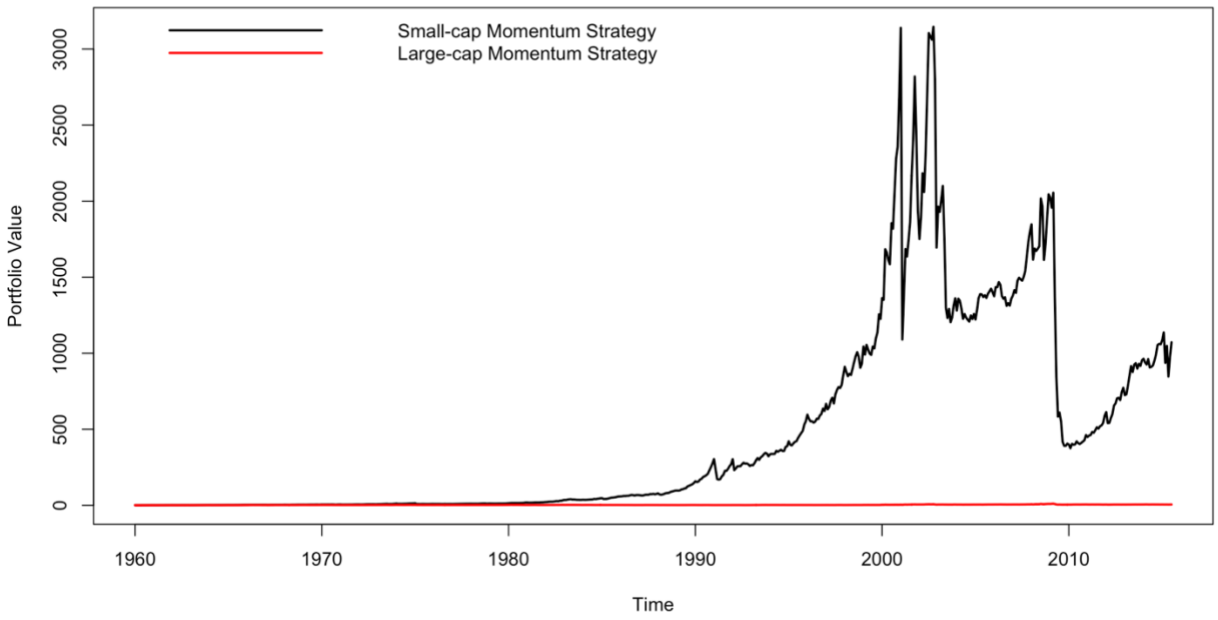
Cut-off	10% by Size		20% by Size (***)		30% by Size		40% by Size		50% by Size	
Group	Large cap	Small cap	Large cap	Small cap	Large cap	Small cap	Large cap	Small cap	Large cap	Small cap
Return	5.85%	13.65%	4.59%	16.24%	5.35%	15.51%	5.44%	14.11%	5.86%	12.42%
Volatility	17.85%	31.39%	17.18%	24.56%	16.96%	23.41%	16.98%	22.55%	17.02%	22.93%
Sharpe Ratio	0.065	0.285	-0.006	0.470	0.039	0.462	0.044	0.418	0.069	0.337

Table 1 above compares different cut-offs. Grouping stocks that are in top 20% largest market cap gives the worst momentum effect and the group of 20% smallest stocks has the best momentum effect, with

a Sharpe Ratio of -0.006 and 0.470 respectively. This suggests if we conduct the momentum strategy separately within large-cap stocks and small-cap stocks. The difference between the performance of the two groups is most manifest with a cut-off at 20%. Based on this observation, we define the cut-off criterion to be the top 20% largest and bottom 20% smallest of all stocks and continue our analysis using this criterion.

In Figure 1, we track the values of these portfolios, assuming they all start at $1 at the beginning of the sample period (1965-01). The small-cap momentum strategy is so dominated that the large-cap one is indistinguishable from 0. This suggests that the return of the short-long strategy is much higher in small-cap portfolios than large-cap portfolios.

Figure 1: One-Dollar Evolution over 1965-2015



Hypothesis Testing

We now conduct the t tests correspondingly to what we have proposed in Methodology. All hypotheses are tested against the null that the average return on the former is smaller or equal to the latter.

Hypothesis 1: The average monthly return on the large-cap momentum strategy is greater than zero rate of return.

The one-tailed test has $t = 1.9913$ and $p\ value\ =\ 0.02343$, which is significant at a 5% level. Despite this, the average monthly return is as low as 0.38%, even less than the average risk-free rate of return of 0.39%. The one-dollar evolution ends up at 5.55$. While it is statistically significant, its effect is too low to be profitable during the sample period.

Hypothesis 2: The average monthly return on the small-cap momentum strategy is greater than zero rate of return.

The one-tailed test has $t = 4.9259$ and $p\ value\ =\ 5.305e-07$. The average monthly return on small-cap momentum strategy over the sample period is 1.35%. The one-dollar evolution has grown to 1027.24$ in June 2015. In contrast with the large-cap momentum, the small-cap momentum is rather profitable and has a strong compound effect.

Hypothesis 3: The difference between the average monthly return on the small-cap momentum strategy and large-cap momentum strategy is greater than zero rate of return.

We expect a significant result drawing from the last two hypotheses. After subtracting the monthly return on large cap from small cap, we again conduct a one-sided t test. The resulting statistics are: $t = 4.1575;\ p\ value\ =\ 1.819e-05$. In addition to that, we have a monthly premium of 0.97%, which means that we can certainly reject the null hypothesis and confirm again that the difference in the market sizes of the stocks that consist of momentum portfolios can indeed lead to different results in portfolio returns.

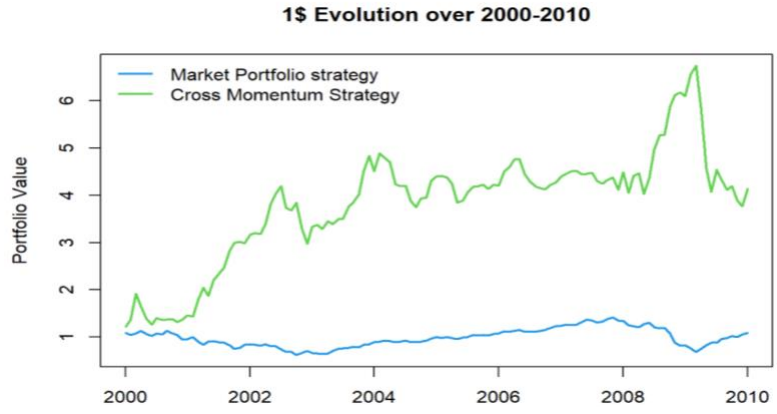
At this point, we have shown that the momentum strategies for large-cap stocks and small-cap stocks are both greater than zero, and the momentum strategy performs better in small-cap portfolios.

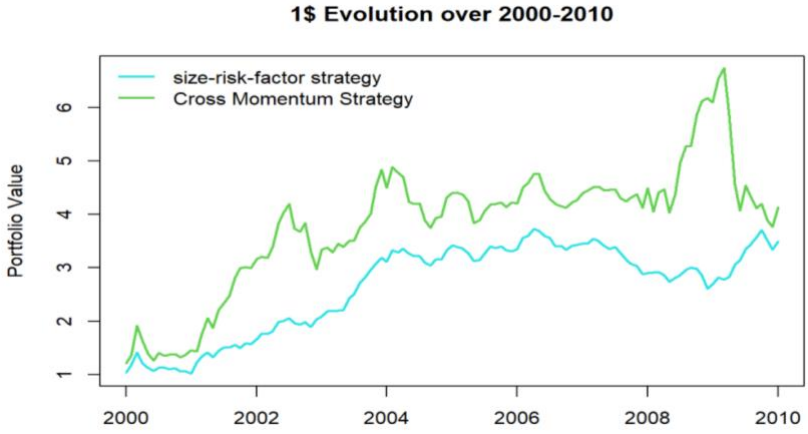
Cross Momentum Strategy

Does this mean that the size-based momentum strategies perform consistently well during bad times as it does in good times? Is there a better strategy to adopt during periods that are more volatile, such as 2000-2010 in which two recessions have occurred? In this section, we suggest an effective way in

implementing it, which we call the Cross Momentum Strategy: Long the winner portfolio within the small-cap group and short the loser portfolio within the large-cap group. Here, both size risk factor in Fama-French Three factor model and momentum risk factor are taken into consideration.

To evaluate the Cross Momentum portfolio's performance, it is compared with the Market-effect portfolio and then Size-effect portfolio. In the first two charts, with the same initial value invested, we can see that the green line of cross momentum strategy portfolio value is always higher than the market portfolio and size portfolio lines from 2000 to 2010. It is suggested that the Cross Momentum strategy always gains higher than the market and size portfolio during the whole period. Especially compared with the market portfolio, it gains much higher returns and reaches a high peak in 2009.

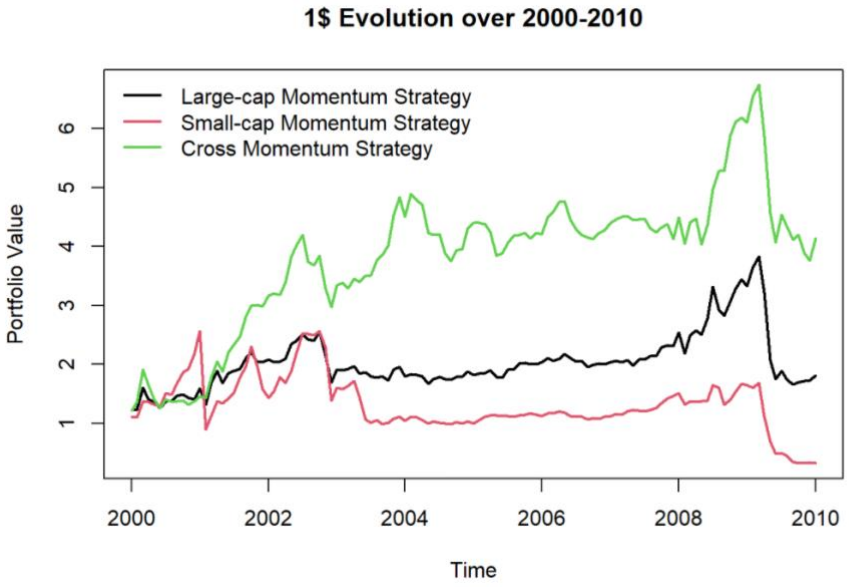




The outperformance of the strategy and the high spike after the recession may due partially to the size effect of small caps stocks. Since the small and midsize companies are more flexible to adjust their work forces and ramp up their production, they can be more quickly to catch up the recovery of the whole

big economic environment than the large company. Also, during recovery merger and acquisition activities between small companies increase, which may also benefit the small firm a lot to expand their business and thus outperform the larger firms.

Following, compared to the small-cap momentum strategy and the large-cap momentum strategy which simply take momentum effect within specific market capitalization stock groups, the Cross Momentum strategy performs better through 2000-2010 shown in the figure below. It is shown in the graph, if we invest 1 dollar in 2000 into the portfolio constructed by the Cross-Momentum Strategy, and then track the value evolution, we will gain much higher growth of the value than the other two strategies. This shows how exploiting size effect helps the momentum strategy perform better in the market.



The results of comparison are shown in Table 2. By implementing Cross Momentum strategy, we can earn on average an annualized return of 18.05% from December in 1999 to December in 2009 which is much higher than any other four strategies in the table. Also, the financial crises cause small-cap momentum to be very volatile, being equal to 43.21%. By contrast, the annualized volatility of the Cross Momentum strategy is 28.56%, less than the other two momentum portfolios. The reduction may be caused by the lower correlation between the stocks in the Cross Momentum portfolio.

Table 2: Comparison Between Different Strategies During 2000-2010

Key Statistics (annualized)	Market	Size Factor	Small-cap Momentum	Large-cap Momentum	Cross Momentum
Return (%)	2.31	12.53	0.85	10.33	18.05
Volatility (%)	17.21	17.13	43.21	29.43	28.56
Sharpe Ratio	-0.025	0.65	-0.044	0.26	0.54

The low volatilities of the market portfolio and the size factor portfolio might be due to the diversification benefits generated by the number of stocks it contains. We include almost all stocks in market portfolios so that it is almost only subject to systematic risk. However, considering the transaction cost may rise along with an increase in the number of stocks held, Cross Momentum strategy may also benefit from its reduced number of stocks in its portfolio.

The Sharpe ratio adjusts the past performance of a portfolio for its excess risk. It helps to evaluate whether the strategy still outperforms after considering both its returns and volatility. As the result shown, Cross Momentum strategies do gain a higher risk-adjusted return than the market portfolio as well as the previous two momentum strategies. However, suggested in the table that its Sharpe ratio is only the second highest and is slightly lower than size effect strategy. This is because of its much higher volatility generated than the size-effect strategy. There is always a trade-off between the number of stocks in the portfolio which reduce the volatility and the transaction cost investors may pay. Also, the Sharpe ratio has several weaknesses. It assumes that investment returns are normally distributed, and it treats all volatility the same. Nonetheless, sharp volatility to the upside is a big good thing to our Cross Momentum strategy but may be regarded as a bad risk by the Sharpe ratio.

CONCLUSION

Our analysis shows that adding a fourth factor, the momentum factor, is a good practice. It helps the Fama-French Three factor model better explain the behavior of stock returns because the momentum effect has extremely low correlations with size, value and market, and is poorly explained by the three factors. The considerably large and significant alpha suggests the momentum strategy by itself is a valid and profitable strategy. We then analyze the momentum strategies for large-cap stocks and small-cap stocks separately and use graphical and statistical tools to prove that although both of them have greater-than-zero average returns, the small-cap momentum strategy performs much better than the large-cap momentum strategy; the optimal cut-off that further separates them apart is smallest 20% of stocks versus largest 20% of stocks. Besides that, we observe the behaviors of market, size, small-cap momentum and large-cap momentum strategies and confirm that historically, small stocks outperform and recover quickly from economic downturns. To exploit this, we design the Cross Momentum strategy (longing small-cap winner portfolio and shorting large-cap loser portfolio) which generates a superior average return to all of the above strategies during Jan 2000 and Dec 2009. Although we come up with some explanations of why this happens, our finding of the Cross Momentum strategy suffers from data snooping problems and is not guaranteed to work in future financial crises. Future research could test this strategy on later recessions, such as the Coronavirus Crash.

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