Delivery Routing Optimization

MGSC662 Group Project

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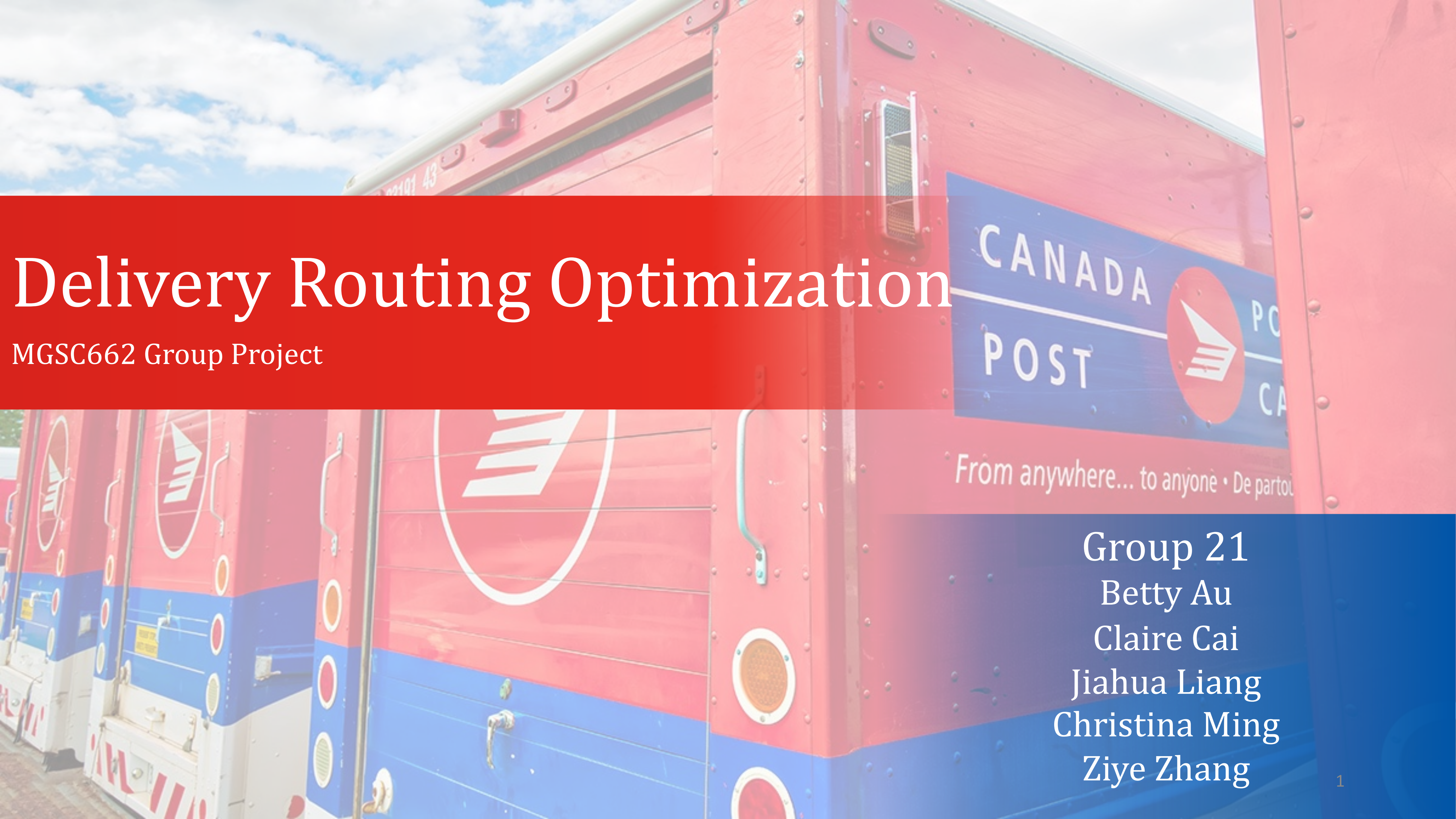

Agenda

- Introduction
- Problem Description
- Formulation
- Numerical Results
- Extensions
- Conclusion

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Canada Post – The Mission

Mission statement: to connect all Canadians in every community across Canada

Over 16.7 million addresses

Canada Post – Main Operations

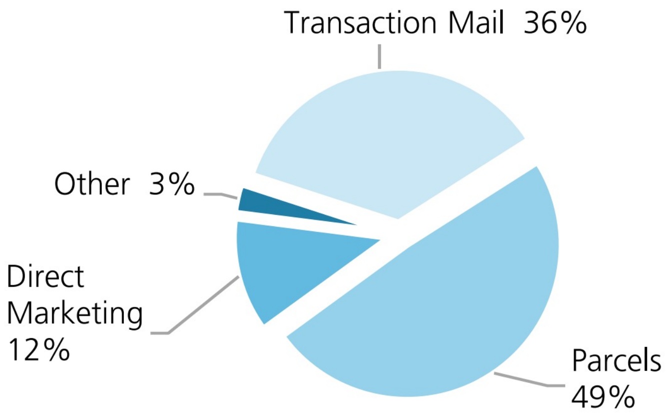
3 main lines of business: Parcels, Transaction Mail, and Direct Marketing

- A loss before tax of $779 million in 2020
- A decrease of $626 million in profitability compared with 2019

Revenue
(in millions of dollars)

	2020	2019	Change	%[1]
Parcels				
Domestic Parcels	2,681	2,068	613	29.1
Outbound Parcels	302	243	59	24.2
Inbound Parcels	432	401	31	7.2
Other	19	23	(4)	(17.0)
Total Parcels	**3,434**	**2,735**	**699**	**25.0**

Revenue by line of business – 2020



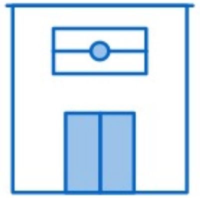
Summary of results

(in millions of dollars)	**2020**	2019
Revenue from operations	6,942	6,748
Cost of operations	7,740	6,935
Loss from operations	**(798)**	**(187)**
Investing and financing income (expense), net	19	34
Loss before tax	**(779)**	**(153)**

Canada Post – Costs and Delivery

With an increase of 70 million domestic parcels in 2020, an increase in delivery costs is expected

RETAIL POST OFFICES

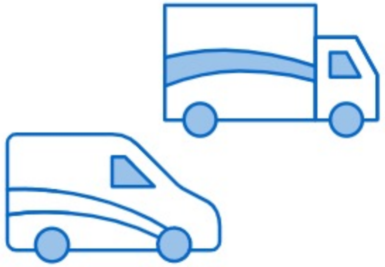


More than

6,100

retail post offices across Canada

FLEET

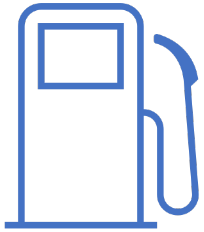


Almost

13,000

Canada Post vehicles

FUEL COSTS



Estimated to be

$0.15

per km

LABOR COSTS

Estimated to be

$18

per hour

Goals and Rationale

Focus: Vehicle routing problems within the domestic Parcels line of business

Technology and digital platform

Deployed telematics in over 1,200 vehicles and piloted telematics on motorized material handling equipment within the sort facilities.	Deploy telematics in vehicles of other facilities.
Piloted automated guided vehicles (AGVs) in our plants.	Deploy next generation of portable data terminals (PDTs).
Piloted drone delivery.	Pilot dynamic routing.
	Continue to test drones and robotics.

- Fastest-growing line of business domestically
- More demand expected due to consumer behaviors shifting towards online purchases
- To minimize delivery routing distance which in turn minimizes total costs incurred through fuel usage and labor

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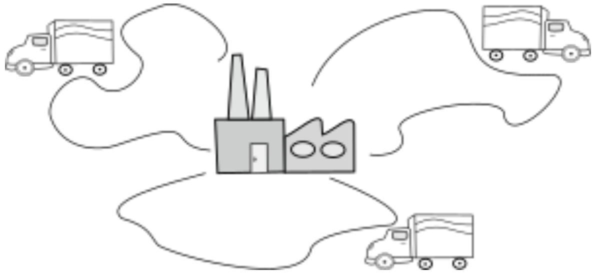
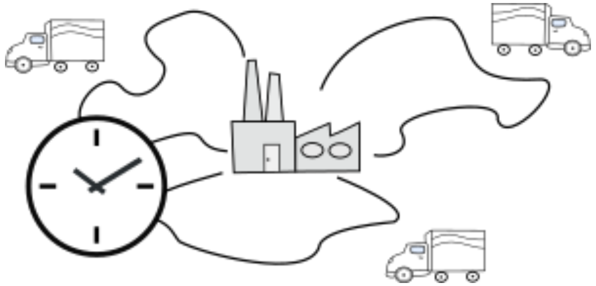
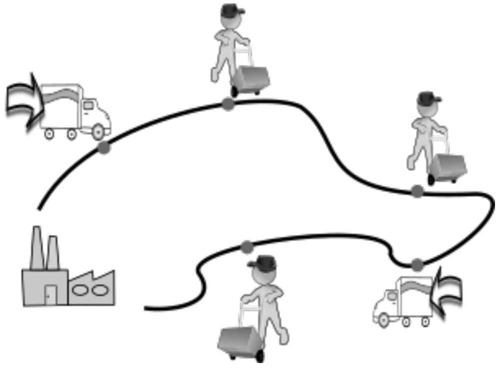
Problem Setup

- Travelling Salesman Problem (TSP)
- Capacitated Vehicle Routing Problem (CVRP)
- **Base Model:** Capacitated Vehicle Routing Problem with Time Windows (CVRPTW)
- **Extension:** Capacitated Vehicle Routing Problem with Simultaneous Delivery and Pickup and Time Windows (CVRPDPTW)







Problem Description

Business Scenario for Parcel Delivery:

- Parcels are **delivered** to a given set of customers
- Delivery vehicles operate with **limited carrying capacity**
- Customers can specify **time windows** within which the delivery will be made

Assumptions:

- Focus on one depot and one round of delivery
- All vehicles must depart from depot and return to the same depot
- All vehicles are homogeneous (same capacity, fuel consumption, etc.)
- Customers do not share location (each location has only one customer)
- The traffic is the same in both directions between each pair of locations
- No limit on the number of vehicles available at the depot

Objective:

- Optimize the vehicle routes for delivery efficiency and cost reduction

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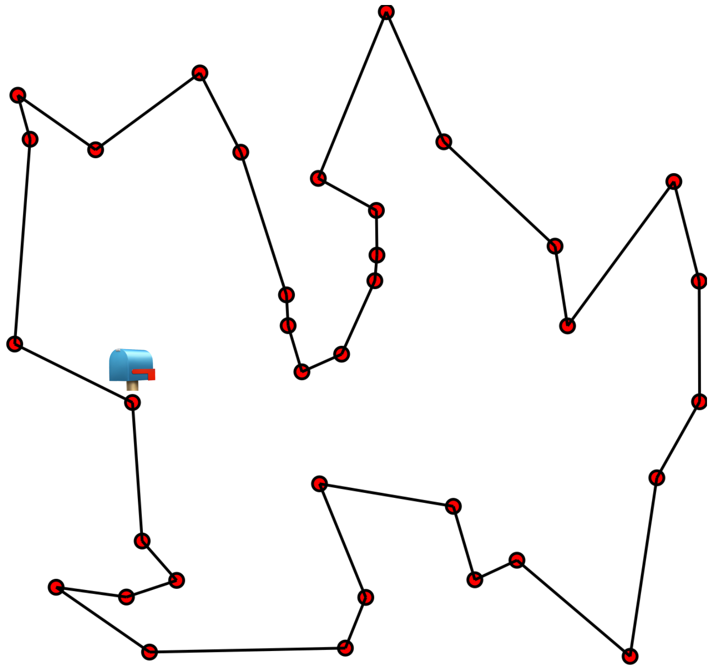
TSP – The Starting Point

Scenario:
- A mail carrier is assigned a certain number of parcels to be delivered
- No limit on capacity for vehicle
- No limit on time for delivery service
- Only one vehicle

Parameters:
d_{ij}: Manhattan distance from location i to j
$\{0\}$: depot
$\{1, \ldots n\}$: a set of customer locations

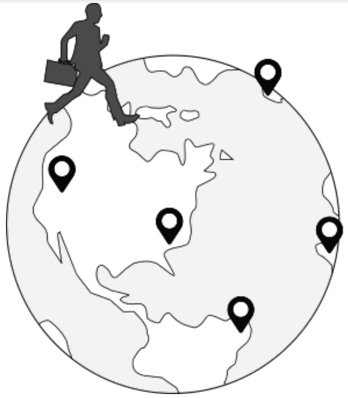
Decision variables:
x_{ij}: arc from location i to j
u_i: number of steps needed to reach location i



TSP – The Starting Point

Objective:

$$\text{Min} \sum_{i=0}^{n} \sum_{j=0}^{n} d_{ij} x_{ij}$$



Constraints:

$$\sum_{i=0}^{n} x_{ij} = 1, \qquad j \in \{0, \dots n\}$$

Must pass each location exactly once

$$\sum_{j=0}^{n} x_{ij} = 1, \qquad i \in \{0, \dots n\}$$

Must leave each location exactly once

$$\sum_{i=0}^{n} x_{ii} = 0, \qquad i \in \{0, \dots n\}$$

Avoid self-loops

$$u_i - u_j + (n+1) \cdot x_{ij} \leq n, \qquad i, j \in \{1, \dots n\}$$

Relate dummy variables and avoid subtours

$$1 \leq u_i \leq n, \qquad i \in \{1, \dots n\}$$

Range of u_i

$$x_{ij} = \{0, 1\}, \qquad i, j \in \{0, \dots, n\}$$

Binary constraint

Capacitated Vehicle Routing Problem (CVRP)

Scenario:

- A mail carrier is assigned a certain number of parcels to be delivered
- Limited vehicle capacity
- Multiple vehicles may be needed

Parameters:

d_{ij}: Manhattan distance from location i to j

q_i: delivery demand of customer i

Q: capacity of delivery vehicle(s)

$\{0\}$: depot

$\{1, \dots n\}$: a set of customer locations

Decision variables:

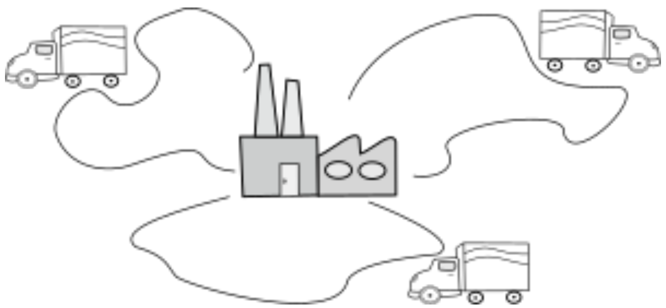
x_{ij}: arc from location i to j

y_i: cumulative amount of parcels that has been delivered after reaching location i

Capacitated Vehicle Routing Problem (CVRP)

Objective:

$$\text{Min} \sum_{i=0}^{n} \sum_{j=0}^{n} d_{ij} x_{ij}$$



Constraints:

$$\sum_{i=0}^{n} x_{ij} = 1, \qquad j \in \{1, \dots n\}$$

Must enter each customer location exactly once (excluding the depot)

$$\sum_{j=0}^{n} x_{ij} = 1, \qquad i \in \{1, \dots n\}$$

Must leave each customer location exactly once (excluding the depot)

$$\sum_{i=0}^{n} x_{ii} = 0, \qquad i \in \{0, \dots n\}$$

Avoid self-loops

Delivery demand

$$y_i + q_j \cdot x_{ij} - Q(1 - x_{ij}) \le y_j, \qquad i, j \in \{1, \dots, n\}$$

Vehicle capacity

Relate dummy variables and avoid subtours

$$q_i \le y_i \le Q, \qquad i \in \{1, \dots, n\}$$

Constraints on the cumulative delivery

$$x_{ij} = \{0, 1\}, \qquad i, j \in \{0, \dots, n\}$$

Binary constraint

Base Problem – CVRP with Time Windows (CVRPTW)

Scenario:

- A mail carrier is assigned a certain number of parcels to be delivered
- Limited vehicle capacity
- Parcels will be delivered within customer-specified time windows
- Number of vehicles will be determined by optimization

Parameters:

d_{ij}: Manhattan distance from i to j

q_i: delivery demand of customer i

Q: capacity of delivery vehicle(s)

w_i^a: start of time window for customer i

w_i^b: end of time window for customer i

s_i: service time needed for customer i

t_{ij}: travelling time from location i to j

$\{0\}$: depot

$\{1, \ldots n\}$: a set of customer locations

Decision variables:

x_{ij}: arc from location i to j

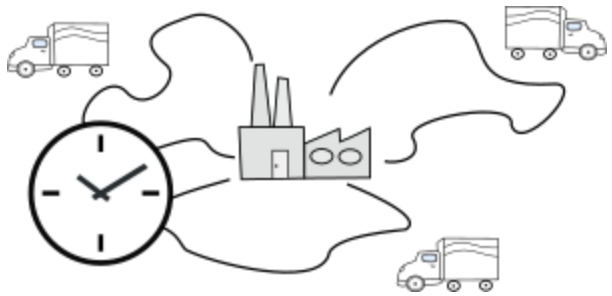
y_i: cumulative amount of parcels that has been delivered after reaching location i

w_i: service start time for customer i (the time instant when the carrier arrives at the customer's), 9am, 10:30am etc.

Base Problem – CVRP with Time Windows (CVRPTW)

Objective:

$$\text{Min} \quad \sum_{i=0}^{n}\sum_{j=0}^{n} d_{ij}x_{ij}$$



Constraints:

$$\sum_{i=0}^{n} x_{ij} = 1, \qquad j \in \{1, \dots n\}$$

Enter each customer location exactly once (excluding the depot)

$$\sum_{j=0}^{n} x_{ij} = 1, \qquad i \in \{1, \dots n\}$$

Leave each customer location exactly once (excluding the depot)

$$\sum_{i=0}^{n} x_{ii} = 0, \qquad i \in \{0, \dots n\}$$

Avoid self-loops

$$y_i + q_j \cdot x_{ij} - Q(1 - x_{ij}) \leq y_j, \qquad i, j \in \{1, \dots n\}$$

Avoid subtours (from delivery demand perspective)

$$q_i \leq y_i \leq Q, \qquad i \in \{1, \dots n\}$$

Constraints on the cumulative delivery

Service time

travelling time

$$w_i + (s_i + t_{ij}) \cdot x_{ij} - M(1 - x_{ij}) \leq w_j, \qquad i \in \{0, \dots n\}, j \in \{1, \dots n\}$$

Large number

Relate dummy variable and avoid subtours (from time perspective)

$$w_i^a \leq w_i \leq w_i^b, \qquad i \in \{1, \dots n\}$$

Service start time must be within time window

$$x_{ij} = \{0,1\}, \qquad i, j \in \{0, \dots, n\}$$

Binary constraint

MTZ Formulation

- Subtour elimination from **geometry** perspective (Basic TSP)

$$u_i - u_j + (n+1) \cdot x_{ij} \le n, \qquad i, j \in \{1, \dots n\}$$

$$\Rightarrow u_i + 1 \cdot x_{ij} - n \cdot (1 - x_{ij}) \le u_j$$

One step

Total number of steps needed to reach location j

$$1 \le u_i \le n, \qquad i \in \{1, \dots n\}$$

- Subtour elimination from **demand** perspective

$$y_i + q_j \cdot x_{ij} - Q \cdot (1 - x_{ij}) \le y_j, \qquad i, j \in \{1, \dots n\}$$

Delivery demand of customer j

Total amount of parcels being delivered at location j

$$q_i \le y_i \le Q, \qquad i \in \{1, \dots n\}$$

- Subtour elimination from **time** perspective

$$w_i + (s_i + t_{ij}) \cdot x_{ij} - M \cdot (1 - x_{ij}) \le w_j, \qquad i \in \{0, \dots n\}, j \in \{1, \dots n\}$$

Total time elapsed between two customers

The time when carrier arrives at customer j

$$w_i^a \le w_i \le w_i^b, \qquad i \in \{1, \dots n\}$$

Commonalities:

- Dummy variable represents "something" cumulative
- Dummy variables for two connected points (x_{ij} =1) are related by adding additional amount of "something"
- Use a (relatively) large number and $(1 - x_{ij})$ to switch between two cases (x_{ij} = 1 or x_{ij} = 0)
- Associate with lower and upper bound constraint of the dummy variable

Miller-Tucker-Zemlin

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Numerical Implementation

Data source:

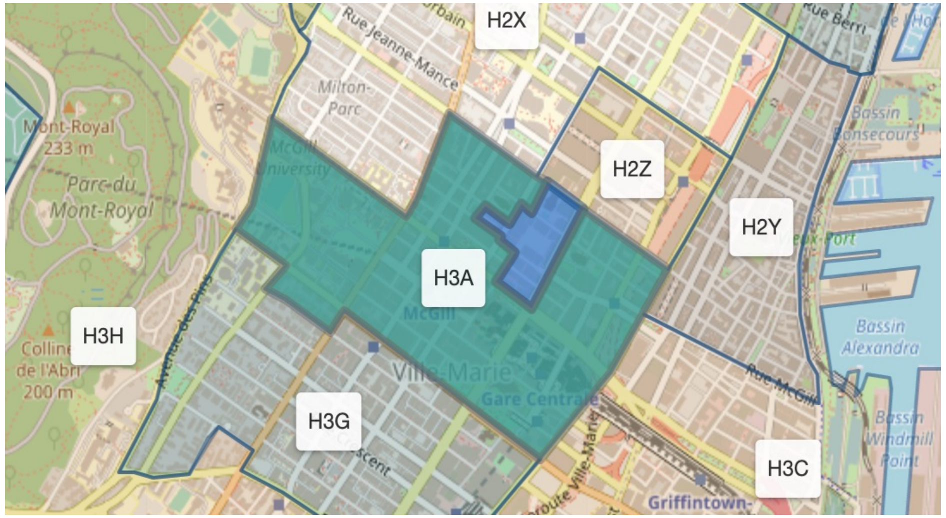
- The Open Database of Addresses, dataset for QC (Statistics Canada)
- Depot location coordinates (Canada Post)

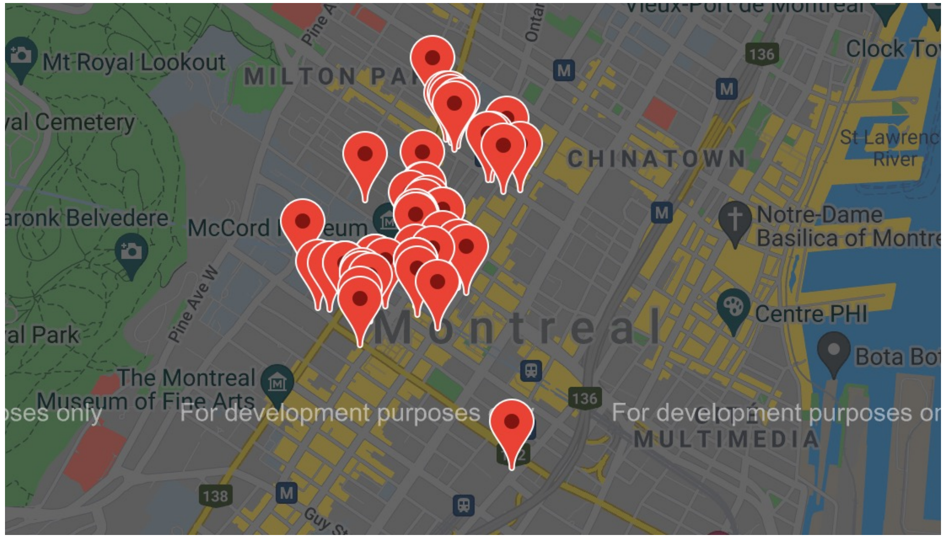
Data processing:

- Extract “Address”, “City”, “Postal Code”, “Latitude”, “Longitude” Columns
- Filter for a particular region in downtown Montreal, with postal code starting with H3A
- Randomly select 50 customer locations in this region
- Find the nearest Canada Post depot

Tool:

- Python + Gurobi





Numerical Implementation

Numerical Parameters:

- Average vehicle travelling speed: $V = 30$ km/h
- Vehicle capacity: $Q = 50$

Estimated

- Fuel cost per vehicle: $C_f = 0.15$ \$/km
- Delivery driver cost: $C_d = 18$ \$/h
- Average Parcel Price: 27.5 \$/Parcel

External Source

- Customer delivery demand: $q_i \in \{1, 2, .., 8\}$ parcels
- Start of time windows: $w_i^a \in \{9am, 10am, 11am\}$
- End of time windows: $w_i^b \in \{12pm, 13pm, 14pm\}$
- Service time at each customer: $S_i \in \{5min, 10min, 15min\}$

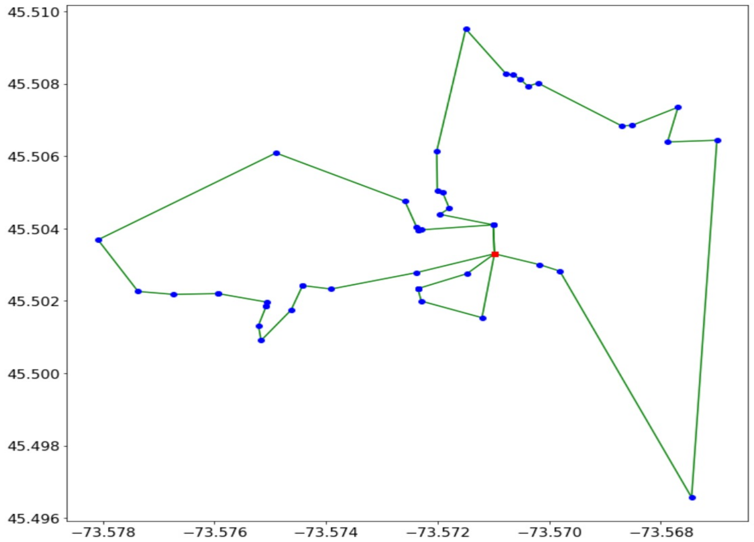
Simulated

Base Model Results

Optimal Objective Function Value (total Distance): D = 7.73 km for all vehicles

Optimal Routes:

```
The optimal delivery route for vehicle 1
0 > 15 > 16 > 18 > 24 > 27 > 26 > 28 > 29 > 25 > 31 > 32 > 33 > 30 > 39 > 38 > 3 > 4 > 21 > 22 > 43 > 45 > 0
The optimal delivery route for vehicle 2
0 > 40 > 49 > 46 > 48 > 47 > 50 > 41 > 0
The optimal delivery route for vehicle 3
0 > 44 > 17 > 19 > 20 > 42 > 2 > 23 > 11 > 6 > 5 > 35 > 34 > 10 > 7 > 8 > 1 > 36 > 9 > 13 > 12 > 37 > 14 > 0
```



Base Model Results

Revenue Estimation:

Average Price per Parcel

Total Number of Parcels Delivered

$$Revenue = 27.5\ \$/\text{Parcel} \times 101\text{Parcels} = \$\ 2777.5$$

Operation Cost Estimation:

Total Travel Distance [km]

Total Travel Time [h]

Total Service Time [h]

$$Operation\ cost = D \times \mathrm{C_f} + \mathrm{C_d} \times \left(\frac{\mathrm{D}}{\mathrm{V}} + \sum_{i=1}^{n} s_i \right)$$

Fuel Cost [$/km]

Driver Salary [$/h]

$$= 7.73\text{km} \times 0.15\$/\text{km} + 18\$/\text{h} \times \left(\frac{7.73}{\frac{30\text{km}}{\text{h}}} + 9.08h \right)$$

$$= \$\ 169.2$$

Profit = Revenue – Operation Cost = 2777.5 – 169.2 = $ 2608.3 for serving this particular set of customers

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Extension – CVRPTW with Delivery and Pickup (CVRPTWDP)

Scenario:

- Simultaneous delivery and pick up
- No direct mailing between customers (vehicles will collect all pickups and return to depot)
- Parcels either come from or arrive at the depot

Parameters:

d_{ij}: the Manhattan distance between locations i and j

q_i: the delivery demand of customer i

b_i: the pickup demand of customer i

Q: vehicle capacity

w_i^a: start of time window for customer i

w_i^b: end of time window for customer i

s_i: service time for customer i

t_{ij}: travel time from location i to j

Extension – CVRPTW with Delivery and Pickup (CVRPTWDP)

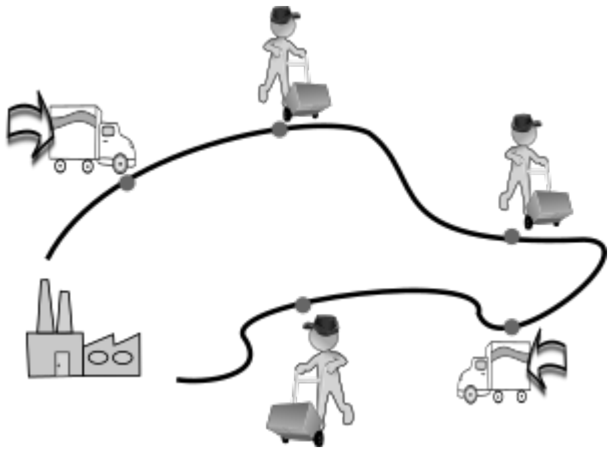
Decision Variables:

x_{ij}: whether the vehicle travels from location i to j (arc)

R_{ij}: the total delivery amount on vehicle on arc (i, j)

P_{ij}: the total pickup amount on vehicle on arc (i, j)

w_i: time dummy variables (service start time for customer i)



Extension – CVRPTW with Delivery and Pickup (CVRPTWDP)

Objective:

$$\boldsymbol{Min \sum_{i=0}^{n} \sum_{j=0}^{n} d_{ij} x_{ij}}$$

Constraints:

$$\sum_{i=0}^{n} x_{ij} = 1, j \in \{1, \ldots, n\}, (4.1)$$

$$\sum_{j=0}^{n} x_{ij} = 1, i \in \{1, \ldots, n\}, (4.2)$$

$$\sum_{i=0}^{n} x_{ii} = 0, i \in \{0, \ldots, n\}, (4.3)$$

$$\sum_{i=0}^{n} R_{ij} - q_j = \sum_{i=0}^{n} R_{ji}, j \in \{1, \ldots, n\}, (4.4)$$

$$\sum_{i=0}^{n} P_{ij} + b_j = \sum_{i=0}^{n} P_{ji}, j \in \{1, \ldots, n\}, (4.5)$$

$$\sum_{i=1}^{n} P_{0i} = 0, (4.6)$$

$$\sum_{i=1}^{n} R_{i0} = 0, (4.7)$$

$$R_{ij} + P_{ij} \leq Q x_{ij}, i, j \in \{0, \ldots, n\}, (4.8)$$

$$w_j \geq w_i + (s_i + t_{ij}) x_{ij} - M(1 - x_{ij}), i \in \{0, \ldots, n\}, j \in \{1, \ldots, n\} (4.9)$$

$$w_i^a \leq w_i \leq w_i^b, i \in \{0, \ldots, n\}, (4.10)$$

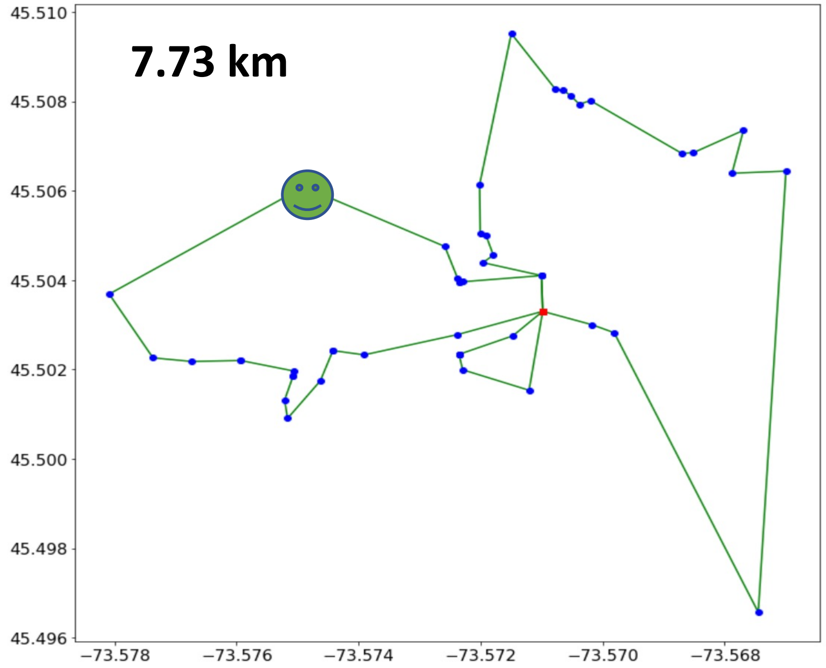
$$x_{ij} = \{0, 1\}, i, j \in \{0, \ldots n\}, (4.11)$$

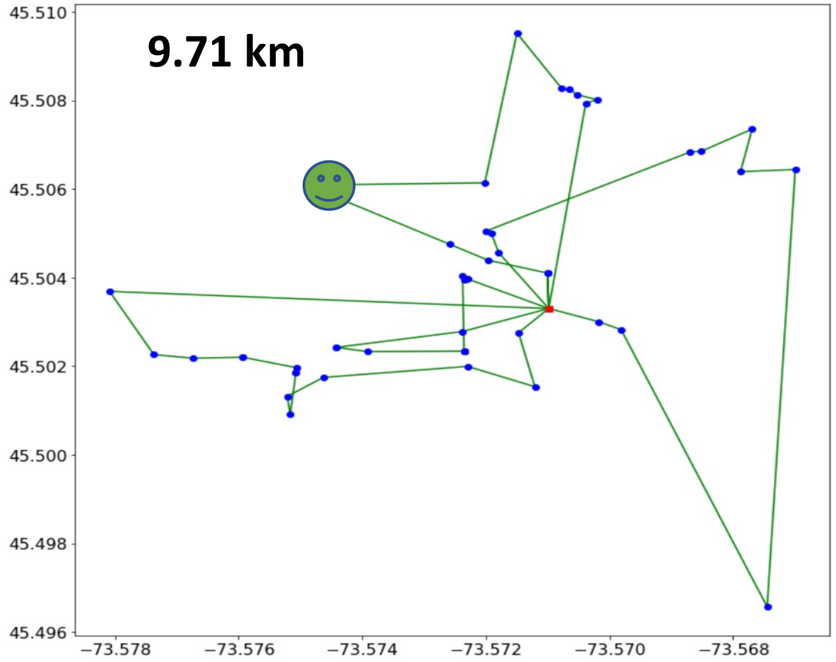
$$R_{ij}, P_{ij} \geq 0, i, j \in \{0, \ldots, n\}, (4.12, 4.13)$$

Extension – CVRPTW with Delivery and Pickup (CVRPTWDP)

Results:

```
The optimal delivery route for vehicle 1
0 > 12 > 7 > 6 > 36 > 35 > 11 > 9 > 8 > 2 > 37 > 1 > 10 > 41 > 0
The optimal delivery route for vehicle 2
0 > 15 > 47 > 49 > 48 > 50 > 38 > 14 > 13 > 24 > 39 > 4 > 5 > 0
The optimal delivery route for vehicle 3
0 > 40 > 31 > 34 > 33 > 32 > 26 > 27 > 25 > 28 > 29 > 30 > 0
The optimal delivery route for vehicle 4
0 > 42 > 19 > 17 > 16 > 0
The optimal delivery route for vehicle 5
0 > 45 > 46 > 44 > 22 > 23 > 3 > 43 > 20 > 21 > 18 > 0
```





Extension – CVRPTW with Delivery and Pickup (CVRPTWDP)

Revenue Estimation:

Service Fee for Pickup   Total Number of Parcels Picked up

$$Revenue = \frac{27.5\$}{\text{Parcel}} \times 101 \text{ Parcels} + \frac{3\$}{\text{Parcel}} \times 195 \text{ Parcels} = \$\ 3362.5$$

where 3$ is the service fee per parcel picked up

Operation Cost Estimation:

$$Operation\ cost = D \times \mathrm{C_f} + \mathrm{C_d} \times \left(\frac{\mathrm{D}}{\mathrm{V}} + \sum_{i=1}^{n} s_i \right)$$

$$= 9.71\text{km} \times 0.15\$/\text{km} + 18\$/\text{h} \times \left(\frac{9.71\text{km}}{30km/h} + 9.08h\right)$$

$$= \$\ 171$$

Profit = Revenue – Operation Cost = 3362.5 – 170.78 = $ 3192   for serving the same 50 customers

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Conclusion

Overall, the total variable costs could be reduced to increase profits through routing optimization

1000 seconds (about 16 and a half minutes) as runtime limit

Near-optimal solution with long runtime

Non-simultaneous pickup and delivery

Direct delivery between customers

Thank you!

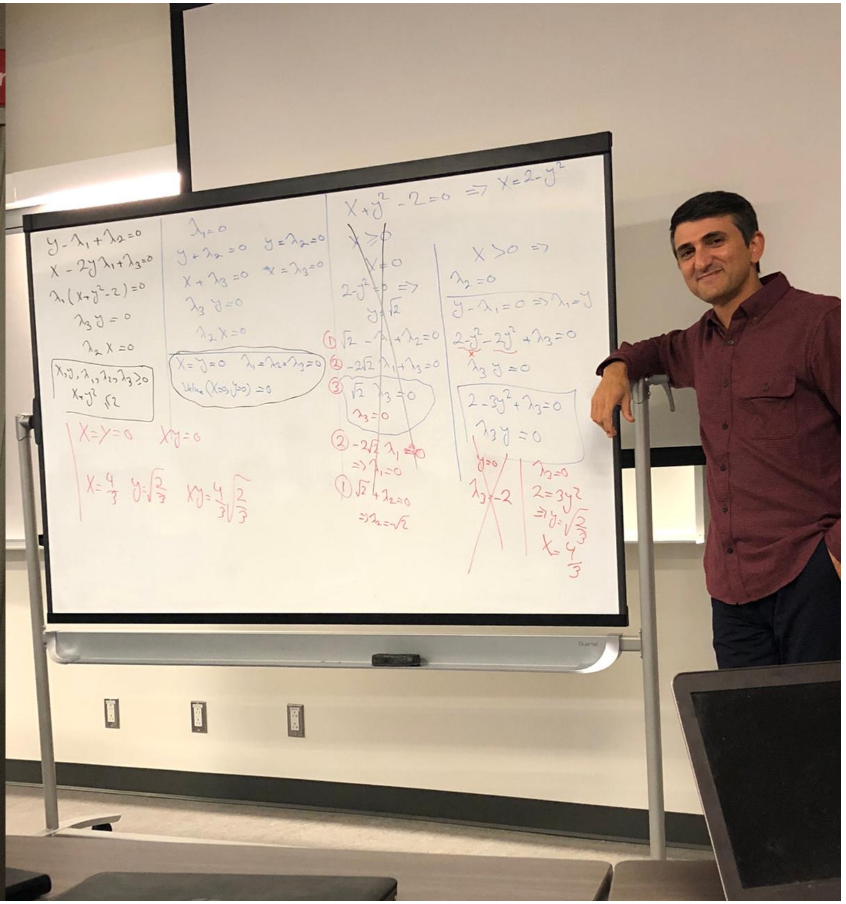

Photo credit to Ali

Q&A

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Appendix

- Table 1: Revenue & Volume by Business Line
- Table 2: Costs

Table 1: Revenue & Volume by Business Line

	Revenue (in millions of dollars)				Volume (in millions of pieces)			
	2020	2019	Change	%[1]	**2020**	2019	Change	%[1]
Parcels								
Domestic Parcels	2,681	2,068	613	29.1	292	222	70	30.9
Outbound Parcels	302	243	59	24.2	13	10	3	31.2
Inbound Parcels	432	401	31	7.2	84	88	(4)	(5.0)
Other	19	23	(4)	(17.0)	–	–	–	–
Total Parcels	**3,434**	**2,735**	**699**	**25.0**	**389**	**320**	**69**	**21.0**
Transaction Mail								
Domestic Lettermail	2,335	2,540	(205)	(8.5)	2,432	2,683	(251)	(9.7)
Outbound Letter-post	83	96	(13)	(13.8)	39	45	(6)	(15.2)
Inbound Letter-post	66	78	(12)	(16.2)	69	98	(29)	(29.6)
Total Transaction Mail	**2,484**	**2,714**	**(230)**	**(8.9)**	**2,540**	**2,826**	**(286)**	**(10.5)**
Direct Marketing								
Personalized Mail™	365	485	(120)	(25.1)	648	886	(238)	(27.1)
Neighbourhood Mail™	283	401	(118)	(29.7)	2,474	3,461	(987)	(28.8)
Total Smartmail Marketing™	**648**	**886**	**(238)**	**(27.2)**	**3,122**	**4,347**	**(1,225)**	**(28.5)**
Publications Mail™	129	146	(17)	(12.0)	187	215	(28)	(13.5)
Business Reply Mail™ and Other Mail	19	20	(1)	(5.9)	14	16	(2)	(9.3)
Other	13	14	(1)	0.4	–	–	–	–
Total Direct Marketing	**809**	**1,066**	**(257)**	**(24.3)**	**3,323**	**4,578**	**(1,255)**	**(27.7)**
Other Revenue	215	233	(18)	(8.2)	–	–	–	–
Total	**6,942**	**6,748**	**194**	**2.5**	**6,252**	**7,724**	**(1,472)**	**(19.4)**

Table 2: Costs

Costs of Operations - Canada Post Segment	
	2,020
Average Fuel Price of 2020 ($/liter)	1.01
Grumman LLV Fuel Efficiency (liter/100 km)	15
Fuel cost per vehicle per km ($/km)	**0.15**
Delivery driver hourly wage (Payscale)	18
Delivery driver hourly wage (Indeed)	18
Hourly labor cost ($/employee)	**18.00**