## Summary

Concurrent objects are convenient tools for programmers. With concurrent objects, programmers can write code with multiple threads as if they are writing a single-threaded code. However, it is crucial to know the correctness of concurrent objects. In this thesis, we study a technique to justify the correctness of synchronization objects. We first present CSP models for common concurrent primitive according to their behaviours. We then systematically build several concurrent objects from their Scala sources. We make assertions of these synchronization objects with a technique derived from linearizability testing. We find these assertions can effectively find bugs in a concurrent datatype and provide a history to give more context for the developer.

## 1 Introduction

Concurrent objects are convenient tools for programmers. With concurrent datatypes, programmers can write code with multiple threads as if they are writing a single-threaded code. However, it is crucial to know the correctness of concurrent objects. If the implementation of a concurrent object is wrong, then code using the concurrent object is very likely to be faulty.

The Channel object is one synchronization object commonly used in Go. The channel object can be used to share data from one process to another process. A process can send data to another process by calling send with the data to share. Likewise, a process can receive data from other processes by calling receive function. In Go and Communicating Scala Object package, the channels are unbuffered by default. If there is no process to receive the data, the sending process blocks until a process is willing to receive its data. Similarly, a receiving process blocks until a process sends some data.

```
trait Channel{
  def send(data: Int): Int
  def receive (): Int
}
```

Figure 1: Interface of a MenWomen object

In this thesis, we shall study the correctness of synchronization objects. Each synchronization in a synchronization object involves multiple processes, whereas synchronization in concurrent datatypes like concurrent queue and concurrent only involves a single process.

There are two main properties to check for a synchronization object, the safety property and the liveness property. The safety property states that the history of the synchronization object should satisfy some conditions. For example, if one process sends 1 when no other process is sending, then a process calling receive should only receive 1. The liveness property states that the concurrent object should not refuse to synchronize when synchronization is possible between one or more processes. For example, if a process calls send and a process receive, the system should be able to synchronize and should not deadlock.

## 1.1 Thesis Overview

In the remaining part of Section 1, we describe the correctness condition for a synchronization object and abstractly how to test these conditions in CSP using the linearization test technique.

In section 2, we build CSP modules for common concurrent primitives such as shared variables, monitors and semaphores.

Starting from section 3, we use the linearization test technique to distinguish between correct and faulty implementation for several synchronization objects. We first implement the synchronization object in CSP according to its Scala

source code. Then we write specifications for a system using the synchronization object and carry out the tests.

## 1.2 Synchronization linearizability test

To verify the correctness of a concurrent datatype, one can carry out the linearizability test described in the paper Testing for Linearizability [1]. The linearizability testing framework logs the orders of each function call and function return. Then for the observed history, the testing framework attempt to find a series of synchronization point that obeys the safety property. The concurrent datatype implementation is considered faulty if the framework can not find a valid synchronization point series.

In this remaining section, we shall look at a few examples of histories of systems using the Channel object. Figure 2 visualizes the history of a system with two processes. Process T1 calls send with argument 1 and returns. Process T2 calls receive and returns with 1. Each long horizontal line in the timeline represents a function call made by the corresponding process. The short vertical bars at the two ends of the long horizontal line indicate the function call's starting time and ending time. And the long vertical line between T1 and T2 represents the synchronization between the two processes.

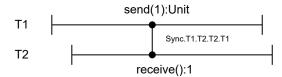


Figure 2: Visualized history of T1 calling send(1) and T2 calling receive()

Figure 3 shows a timeline similar to Figure 2, but T2 returns 2 instead of 1. In this case, the linearizability test framework can not justify the return of process T2's receive, and suggests the trace is generated by a faulty channel implementation.

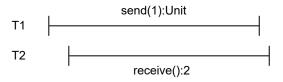


Figure 3: Visualized history of T1 sends 1 but T2 receives 2

In Figure 4, both processes calls send, and no synchronization is possible. Note that the liveness condition is not invalidated even if the system deadlocks in this case.

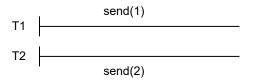


Figure 4: Visualized history of both T1 and T2 calling send

Scheduling is one of the reasons validating a history can be complicated. In Figure 5, process T3 calls send(3) first but gets descheduled. Then T1 calls send(1) and synchronizes with T2 which later calls receive. The linearization framework usually needs to search a large state to find a valid series of synchronization points.

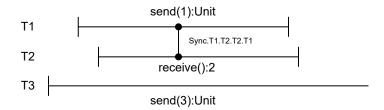


Figure 5: Visualized history of T3 get descheduled

## 1.3 Checking safety property using CSP

The history observed by the linearizability framework can be captured as a trace in CSP. A Call event in CSP represents the start of a function call in the observed history. A Return event represents the returning of a function call. For the safety property, we check that set of all possible histories of a testing system is a subset of all correct histories. In CSP, this corresponds to an assertion that a testing system trace refines a specification of systems using the synchronization object.

A generic and scalable system is used as the testing system. Each process in the testing system can call any function from the concurrent object with any arguments allowed. Each process must be allowed to terminate. Otherwise, the testing system only models a system that runs forever, given that there is no deadlock. We shall see how this affects bug finding in a concurrent datatype in later objects.

The specification process is constructed using the linearization technique. On the high level, the specification process for the system internally uses Sync events to represent synchronization between processes. Inside the specification process, some sub-processes generate corresponding Call and Return event for every synchronization point. When all sub-processes are placed in parallel, the Sync event agrees. So the resulting specification system generates all possible

histories.

We shall see a concrete implementation of a testing system and a specification process in the MenWomen section.

## 1.4 Checking liveness property using CSP

For liveness property, we check the same generic and scalable testing system refines the same specification process, but in the failure model. One could use a datatype-specific specification process that does not explicitly use any synchronization points. However, reusing the linearizer process is easier.

## 1.5 Related work

Testing for Linearizability [1] presents a framework to test concurrent datatypes. However, because the testing framework uses observations of histories, it is unlikely to exhaust all possible histories of a system.

In Chapter 19 of Understanding Concurrent Systems [2], the author describes a CSP model for shared variables and provides a tool to analyze shared variable programs. But the tool lacks support for objects frequently used in concurrent programming, such as monitors and semaphores.

There are also runtime programming tools to detect race conditions and deadlocks in concurrent code. Thread Sanitizer [3] detects race conditions and deadlocks in C++ and Go.

# 2 Common Objects

#### 2.1 Shared Variable

The usage of shared variables is common in concurrent datatypes. For example, some concurrent datatypes may temporarily store the identity of a waiting process. However, CSP is more like a functional programming language and does not support mutable variables.

A recursive process in CSP can capture the behaviour of a shared variable. The recursive process holds the value of the variable in its parameter. At any time, the variable process is willing to answer a query for the variable value in channel getValue. Alternatively, the process can receive an update on the variable value in channel getValue, after which the function recurses with the new variable value.

Because it is natural for a concurrent datatype to use multiple shared variables, the global variable is implemented as a CSP module in Figure 6 to allow better code reuse. The module requires two parameters. TypeValue is the set of possible values for the variable, and initialValue is the value before any process modifies the variable. An uninitialized variable module is also available in the same Figure 6, with the only difference that the variable non-deterministically chooses an initial value from TypeValue at start time. runWith is a convenient helper function to run a given process P with the Var process. If the parameter

hide is true, runWith function hides all events introduced by the shared variable. In later chapters, we will see how the runWith function helps reduce the code complexity of the synchronization object implementation.

Figure 7 is an example of two processes using a shared variable. The first line in the example creates a shared variable VarA with value ranging from 0 to 2 and initialized with 0. Process P increments VarA modulo 3 forever and process Q reads VarA forever. Process P interleaves with process Q, and the combined process is further synchronized with the variable VarA process. In the resulting process System, changes to VarA made by process P is visible to process Q.

```
\label{eq:instance} \begin{array}{l} \mathsf{instance} \  \, \mathsf{VarA} = \mathsf{ModuleVariable}(\{0 \, . \, .2\}, 0) \\ \mathsf{P} = \mathsf{VarA} :: \mathsf{getValue}? \, \mathsf{a} \, \to \, \mathsf{VarA} :: \mathsf{setValue}! \, ((\mathsf{a}+1)\%3) \, \to \, \mathsf{P} \\ \mathsf{Q} = \mathsf{VarA} :: \mathsf{getValue}? \, \mathsf{a} \, \to \, \mathsf{Q} \\ \mathsf{System} = \mathsf{VarA} :: \mathsf{runWith}(\mathsf{false}, \mathsf{P}|||\mathsf{Q}) \end{array}
```

Figure 7: Example of two processes using a shared variable

## 2.2 Semaphore

A Semaphore is a simple but powerful concurrent primitive. This thesis shall describe and use a simplified binary semaphore from [TODO: Reference], which removes interrupts and timeout operations.

A binary semaphore can either be raised or lowered. A up function call raises the semaphore regardless of the semaphore state. If a process calls the down method when the semaphore is raised, the semaphore becomes lowered. However, if the semaphore is unraised, the process waits until another process calls up and proceeds to put down the semaphore. Depending on the initial state of the semaphore, a binary semaphore can be further categorized as a mutex semaphore or a signalling semaphore.

Modelling a semaphore is straightforward in CSP. Figure 8 is the CSP semaphore module. A process may call up function or down function via channel upChan or channel downChan respectively. The semaphore is modelled by a process implemented by two mutually recursive functions Semaphore(True) and Semaphore(False). The semaphore process representing an unraised state accepts a upChan event by any process and proceeds to the raised process. The semaphore process representing a raised state can either accept a upChan event and recurse to the raised process, or accept a downChan event and proceed to the unraised process.

Like the shared variable in the earlier subsection, the semaphore is encapsulated in a CSP module. To create a semaphore, one needs to supply two arguments. TypeThreadID is the set of identities of processes that use this semaphore. initialState is a boolean value indicating the starting state of the semaphore. If initialState is true, the semaphore is raised initially. Otherwise, the semaphore is lowered.

```
—set of possible value for the variable
--inital value for the variable
module ModuleVariable(TypeValue, initialValue)
  Var(value) = getValue!value \rightarrow Var(value)
              \square setValue? value \rightarrow Var(value)
  chanset = {|getValue, setValue|}
exports
  --(Bool, Proc) \rightarrow Proc
  runWith(hide,P) = if hide then (Var(initialValue) [|chanset|] P) \setminus chanset
                               else Var(initialValue) [|chanset|] P
  channel getValue, setValue: TypeValue
endmodule
module ModuleUninitVariable(TypeValue)
  Var(value) = getValue!value \rightarrow Var(value)
             \square setValue? value \rightarrow Var(value)
  chanset = {|getValue, setValue|}
exports
  runWith(hide,P) =
    if hide then (( | \sim | \times : TypeValue \bullet Var(x) ) [| chanset |] P) \ chanset
    else (|~| x:TypeValue • Var(x)) [| chanset |] P
  channel getValue, setValue: TypeValue
endmodule
```

Figure 6: The shared variable module in CSP

```
 \begin{tabular}{ll} module Module Semaphore (Type Thread ID, initial State) \\ --Raised \\ Semaphore (True) &= down Chan? id $\to$ Semaphore (False) \\ &= up Chan? id $\to$ Semaphore (True) \\ --Unraised \\ Semaphore (False) &= up Chan? id $\to$ Semaphore (True) \\ \\ chanset &= \{ \{ up Chan, down Chan \} \} \\ exports \\ &--run With :: (Bool, Proc) $\to$ Proc \\ run With (hide, P) &= (Semaphore [| chanset || P) $\setminus$ \\ &= (if \ hide \ then \ chanset \ else \ \{ \} ) \\ channel \ up Chan, \ down Chan: Type Thread ID \\ end module \\ \end{tabular}
```

Figure 8: The binary semaphore module in CSP

#### 2.3 Monitor

#### 2.3.1 JVM Monitor

A Monitor is another powerful concurrent primitive. This thesis will also use a simplified monitor from [TODO:reference]. JVM Monitor provides two key features, mutual exclusion and waiting.

Monitors can be used to prevent race conditions. At any time, only one process can run code inside a synchronized block that belongs to one monitor. The function op1 in Figure 9 uses synchronized block to prevent race condition on variable a.

Inside a synchronized block, the process can also perform wait, notify, and notifyAll. When a process inside the synchronized block calls wait, the process suspends and waits for notification from other processes. Since a waiting process may be spurious waked up, so a wait call is used with a while loop and a condition. In the MoniorExample of Figure 9, op2 waits until there is 10 op1 calls. In op1, a process calls notifyAll after incrementing the shared variable a. When there aren't 10 op1 calls, process waiting in op2 first wakes, finds the condition a < 10 true, and returns to sleep.

```
class MonitorExample {
  private var a = 0;

  def op1():Unit = synchronized{
     a=(a+1)%20;
     notifyAll()
  }
  def op2():Unit = synchronized{
     while(a<10) wait();
  }
}</pre>
```

Figure 9: A simple Scala class that uses a monitor internally

## 2.3.2 Monitor Module

The monitor process has two states and behaves differently in two states, captured by two processes in Figure 11.

When there is no running process, the behaviour of the monitor is captured by the CSP process inactive, with parameter waiting being the set of waiting processes. The monitor can allow a process to run synchronized code with a waitEnter event. Or, the monitor can spuriously wake a process with a SpuriousWake event, and the spuriously waken process behaves like a normally waken process.

When there is a running process, the behaviour of the monitor is captured by the CSP process active, with parameter cur being the identity of the process running synchronized block, and waiting being the set of the waiting process. The monitor process should respond to method calls from the running process cur, and the monitor should not allow another process to obtain the monitor lock. In active state, the monitor can also spuriously wake a waiting process.

On the client process side, most functions are implemented by simply synchronizing with the monitor on an event. For example, before entering the synchronized block, the process sends a WaitEnter event. The only exception is the wait function, as after being notified, the process needs to resume execution. The process first sends a wait event to tell the monitor that it is waiting and release the monitor lock. The monitor then receives a waitNotify or spuiousWake event, for being notified. Then the process reobtains the monitor lock with a waitEnter event.

The monitor module also provides a few useful macros. The synchronized function wraps a CSP code to run under the protection of the monitor. The whilewait function implements the common Scala pattern 'while(cond) wait()'. The implementation uses a functional replacement for while statement in imperative programming languages. The cond parameter is a CSP function of type '(Proc, Proc) -> Proc'. The return process of cond first performs some events to check the condition of the while statement. If the condition is true, the return process continues to run the process in the first parameter, or the return process runs the process in the second parameter.

With these process side functions and macros, the Scala MonitorExample class in Figure 9 can be converted into the CSP code in Figure 10.

```
\label{eq:instance_vara} \begin{split} & \text{instance VarA} = \mathsf{ModuleVariable}(\{0\mathinner{.\,.}20\},0) \\ & \text{instance Monitor} = \mathsf{ModuleMonitor}(\mathsf{TypeThreadID},\,\mathsf{False}) \\ & op1(\mathsf{me}) = \mathsf{synchronized}(\mathsf{me},\\ & --a = (a+1)\%20;\\ & \mathsf{VarA} :: \mathsf{getValue}?x \to \mathsf{VarA} :: \mathsf{setValue}! ((x+1)\%20) \to \\ & --\mathsf{notifyAll}();\\ & \mathsf{Monitor} :: \mathsf{notifyAll}(\mathsf{me}) \\ ) \\ & op2(\mathsf{me}) = \mathsf{synchronized}(\mathsf{me},\\ & --\mathsf{while}(a < 10) \; \mathsf{wait}()\\ & \mathsf{Monitor} :: \mathsf{whileWait}(\mathsf{me}, \; \setminus \; \mathsf{ktrue}, \mathsf{kfalse} \bullet \\ & \mathsf{VarA} :: \mathsf{getValue}?x \to \mathsf{if} \; \mathsf{x} < 10 \; \mathsf{then} \; \mathsf{ktrue} \; \mathsf{else} \; \mathsf{kfalse} \\ & )\\ ) \\ ) \\ \end{split}
```

Figure 10: The CSP implementation of MonitorExample

There are two design choices worth mentioning in the implementation of the monitor.

First, note that both WaitNotify and SpuriousWake events come from the monitor process instead of directly synchronizing with the currently running process. When a process calls notify or notifyAll, it needs to synchronize with the monitor

process. This is because the running process does not know how many processes are waiting. If the monitor is implemented the other way, the notifying process will block if there is no waiting process. Similarly, a process calling notifyAll does not know how many processes it should wake up.

Secondly, a monitor can introduce divergence by repetitively spuriously waking up a waiting process, whose condition keeps unsatisfied and never changes. This is an unwanted behaviour in failure testing. So the monitor module has an extra parameter disableSpurious to disable spurious wakeups.

//Should I split some functions into respective paragraphs here. Or use line number.

```
module ModuleMonitor(TypeThreadID, disableSpurious)
 channel Notify, NotifyAll, Exit, Wait,
          WaitNotify, WaitEnter, SpuriousWake: TypeThreadID
 chanset = {| Notify, NotifyAll, Exit, Wait, WaitNotify, WaitEnter, SpuriousWake|}
 --A list of event for every event e in s
 repeat(ch, s) = if s={} then SKIP else ch?a:s \rightarrow repeat(ch, diff(s, {a}))
 --cur is current active running thread
  -- waiting is a set of threads waiting to be notified
 active(cur, waiting) =
   --current running thread notify
   Notify . cur \rightarrow (
     --do nothing if no thread is waiting
      if waiting = {} then active(cur, {})
      --wakeup a process
     else WaitNotify? a:waiting →
           active(cur, diff(waiting, {a}))
   ) \Box —current running thread notify
All
    NotifyAll . cur \rightarrow (
     repeat(WaitNotify, waiting);
      active(cur, {})
   ) \Box —current running thread exit
   Exit . cur \rightarrow (
      inactive (waiting)
   ) \Box —current running thread wait
   Wait.cur \rightarrow (
      inactive (union(waiting, {cur}))
   ) □ −–spurious wakeup
   waiting \neq {} & SpuriousWake? a:waiting \rightarrow (
      active(cur, diff(waiting, {a}))
 --when no active thread is running
  inactive(waiting) =
   --pick a thread that is ready to enter
   WaitEnter? a \rightarrow (
      active(a, waiting)
   ) 🗆
   --spurious wakeup
   waiting \neq {} & SpuriousWake? a:waiting \rightarrow (
      inactive ( diff (waiting, {a}))
```

Figure 11: The CSP Monitor Module - Part 1 - the monitor process

```
exports
  --Given a process that uses the monitor
  --Return the process synchronized with the monitor server process
  --If hide is true, monitor channels are hidden
  runWith(hideSpurious, hideInternal, P) =
     let hideset0 = if hideInternal then chanset else {} within
    let hideset1 = if hideSpurious then hideset0 else diff (hideset0, {|SpuriousWake|}) within
    (inactive({}) [|chanset|] P) \setminus hideset1
  --java-like synchronized function
  \mathsf{synchronized}(\mathsf{me},\,\mathsf{P}) = \mathsf{WaitEnter}.\,\mathsf{me} \to \mathsf{P};\,\mathsf{Exit}.\,\mathsf{me} \to \mathsf{SKIP}
  enter(me) = WaitEnter.me \rightarrow SKIP
  exit(me) = Exit.me \rightarrow SKIP
  --notify()
  \mathsf{notify}(\mathsf{me}) = \mathsf{Notify}.\mathsf{me} \to \mathsf{SKIP}
  --notifyAll()
  \mathsf{notifyAll}\,(\mathsf{me}) = \mathsf{NotifyAll}\,.\,\mathsf{me} \to \mathsf{SKIP}
  --wait()
  wait(me) =
    \mathsf{Wait}\,.\,\mathsf{me}\,\to\,
     if disableSpurious then (
       (\mathsf{WaitNotify}.\,\mathsf{me} \to \mathsf{WaitEnter}.\,\mathsf{me} \to \mathsf{SKIP})
   \square (SpuriousWake.me \rightarrow WaitEnter.me \rightarrow SKIP)
       ({\sf WaitNotify.me} \to {\sf WaitEnter.me} \to {\sf SKIP})
  whileWait(me,cond) = while(cond)(wait(me);SKIP)
endmodule
```

Figure 12: The CSP Monitor Module - Part 2 - client process side functions

## 3 MenWomen

For simplicity, a process calling ManSync is called a man process, and a process calling WomanSync is called a woman process.

## 3.1 Implementation

One way to implement the MenWomen object is to use a monitor and a shared variable indicating the stage of synchronization. Figure 13 is a Scala implementation of the MenWomen object with monitor.

- A man process enters the synchronization and waits until the current stage is 0. Then in stage 0, the man process sets the global variable him inside the MenWomen object to its identity. Then the man process notifies all processes so that a waiting woman process can continue. Finally, the man process waits for stage 2.
- A women process enters the synchronization and waits until the current stage is 1. The woman process sets the global variable her to its identity and returns the value of the global variable him.
- In stage 2, the waiting man process in stage 0 is wakened up by the woman process in stage 1. The man process notifies all waiting processes and returns the value of her.

The code snippet in Figure 13 is a Scala implementation of the MenWomen process using a monitor by Gavin Lowe. With the shared variable and monitor module, the Scala code is further translated to a CSP code in Figure 14. With the convention described in the introduction section, every function call begins with a Call event containing all parameters. And every function call ends with a Return event containing the return value.

#### 3.2 Linearization Test

Recall that in the testing system, each process can call any function provided by the concurrent datatype or choose to terminate. In the CSP implementation, a helper function is used. chaosP(P) runs the process P that terminate with SKIP any number of time. With the helper function, the process P only needs to non-deterministically choose to perform manSync or womanSync with its identity.

All processes in the testing system interleave with other processes and synchronize with the processes of shared variables and the process of the monitor. Since the testing system should only include Call and Return events, all other events are hidden using the first two boolean flags in runWith defined in earlier sections.

```
\label{eq:Thread} Thread(me) = chaosP(manSync(me) \; \sqcap \; womanSync(me)) \\ System(All) = runWith(True, True, ||| \; me:All \; \bullet \; Thread(me)) \\
```

Figure 15: Definition of CSP processes in the testing system

```
class MenWomen extends MenWomenT{
    private var stage = 0
    private var him = -1
    private var her = -1

def manSync(me: Int): Int = synchronized{
    while(stage != 0) wait()
    him = me; stage = 1; notifyAll()
    while(stage != 2) wait()
    stage = 0; notifyAll(); her
}

def womanSync(me: Int): Int = synchronized{
    while(stage != 1) wait()
    her = me; stage = 2; notifyAll();
}
```

Figure 13: A correct MenWomen object implementation in Scala

```
instance VarStage = ModuleVariable({0,1,2},0)
instance VarHim = ModuleUninitVariable(TypeThreadID)
instance VarHer = ModuleUninitVariable(TypeThreadID)
instance Monitor = ModuleMonitor(TypeThreadID)
manSync(me) =
   Call ! me! ManSync \rightarrow
  Monitor::enter(me);
     Monitor::whileWait(me, \ ktrue,kfalse •
        \mathsf{VarStage} {::} \mathsf{getValue} \, ? \, \mathsf{x} \to
        if x \neq 0 then ktrue else kfalse
     VarHim::setValue!me \rightarrow
     \mathsf{VarStage} {::} \mathsf{setValue} \ ! \ 1 \rightarrow
     Monitor:: notifyAll (me);
     Monitor::whileWait(me, \ ktrue,kfalse •
        \mathsf{VarStage} {::} \mathsf{getValue} \, ? \mathsf{x} \to
        if x \neq 2 then ktrue else kfalse
     \mathsf{VarStage} {::} \mathsf{setValue} \; ! \; \mathsf{0} \; \to \;
     Monitor:: notifyAll (me);
     VarHer::getValue?ans \rightarrow (
  Monitor:: exit (me);
  \mathsf{Return}\,!\,\mathsf{me}\,!\,\mathsf{ManSync}\,!\,\mathsf{ans}\!\to
  SKIP
  )
womanSync(me)=
   Call ! me!WomanSync \rightarrow
  Monitor::enter(me);
     Monitor::whileWait(me, \ ktrue,kfalse •
        \mathsf{VarStage} {::} \mathsf{getValue} \, ? \times \to
        if x \neq 1 then ktrue else kfalse
     \mathsf{VarHer} {::} \mathsf{setValue} \; ! \; \mathsf{me} \; \rightarrow \;
     \mathsf{VarStage} {::} \mathsf{setValue} \; ! \; 2 \to
     Monitor:: notifyAll (me);
     VarHim::getValue?ans \rightarrow (
  Monitor:: exit (me);
  Return! me! WomanSync! ans→
  SKIP
  )
```

Figure 14: Translated CSP code for the correct MenWomen object immplementation

Similarly, a linearizer process can non-deterministically choose to perform manSync, synchronize with another linearizer process calling womanSync, and return with the identity it received from the Sync event. Also, the linearizer can choose to call womanSync or terminate. Linearizers(All) puts all linearizers processes in parallel.

The Linearizers function creates a combined linearizer process in three steps. First, it put all linearizer processes in parallel using Replicated Generalized Parallel in CSPM. Specifically, General Parallel uses three arguments, the linearizer identity set, the linearizer process, and a synchronization alphabet set. The synchronization alphabet set for a process identity is the set of Sync events where the process identity appears on the first or the third argument. If a linearizer process wants to send an event in its synchronization alphabet set, it must synchronize with all other linearizer processes whose synchronization alphabet sets include this event. Then Linearizers runs the paralleled process with the specification process of Sync event. For MenWomen object, the synchronization is stateless and only requires the return value to be the identity of the other process. Finally, like the testing system, all Sync events are hidden to provide all valid histories.

Figure 17 visualizes how the linearizer process can generate the trace that corresponds to the history described in Figure 2, where one process calls manSync and another process calls womanSync. And 18 visualizes the linearizer deadlocks as required if both processes calls manSync.

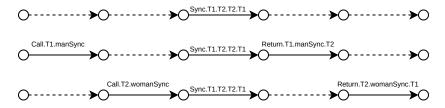


Figure 17: Linearizers generating the history where one process calls manSync and another process calling womenSync

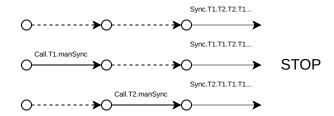


Figure 18: Linearizers generating the history where one process calls manSync and another process calling womenSync

Finally, we perform the test using trace refinement for safety property and

```
Lin(All, me)= (
  Call ! me! ManSync\rightarrow
  Sync ! me ? mereturn ? other ? otherreturn \rightarrow
  Return ! me ! ManSync ! mereturn \rightarrow
  Lin(All, me)
)⊓(
  Call ! me! WomanSync \rightarrow
  Sync ? other ? otherreturn ! me ? mereturn \rightarrow
  \mathsf{Return}\,!\,\mathsf{me}\,!\,\mathsf{WomanSync}\,!\,\mathsf{mereturn}\,\rightarrow\,
  Lin(All, me)
)⊓STOP
LinEvents(All, me)=union({
  ev \mid ev \leftarrow \{ |Sync| \},
  let Sync.t1.a.t2.b=ev within
     countList(me, <t1, t2>)=1 and
     member(t1, All) and
     member(t2, All)
\},\{|Call.me,Return.me|\})
Linearizers (All)=((|| me: All \bullet [LinEvents(All,me)] Lin(All,me)) [|{|Sync|}|| Spec)
                       \setminus \{|Sync|\}
```

Figure 16: Definition of linearizer process in CSP

failure refinement for liveness. As expected, the correct implementation passes all tests.

Figure 19: Part of liveness and safetyness test in CSP for MenWomen object

## 3.3 A faulty version

We shall examine another MenWomen implementation in Figure 20. One key difference in this faulty MenWomen object is that it uses Option data in the shared variables to store the identity of the process calling manSync and the process calling womanSync.

The safeness property test shows that this implementation handles scheduling carelessly. This implementation fails the test Spec2Thread  $\sqsubseteq_T$  System2, and FDR provides a trace that violates the safeness specification, shown in Figure 21. FDR further allows the user to expand the hidden  $\tau$  events in the testing system, which are normally processes' interactions with shared variables and the monitor. By understanding the expanded trace in CSP, we can find a equivalent way to trigger the bug in Scala.

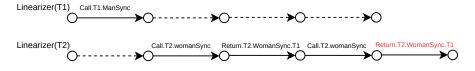


Figure 21: Trace

- A process T1 calls manSync. On first line, since the shared variable him is initially None, T1 skips the wait, set him to Some(T1) and waits for a process calling womanSync.
- A process T2 calls womanSync and returns T1. At this stage, there is no
  waiting process waiting to run womanSync and the shared variable is not
  None. So T2 does not wait at any point, notifies all waiting process, and
  returns.
- Before T1 reenters the synchronized block, process T2 calls womanSync again. him has not been reset by T1 yet. So T2 pairs with T1 again, which should not be allowed.

```
class FaultyMenWomen extends MenWomenT{
 private var him: Option[Int] = None
 private var her: Option[Int] = None
 def manSync(me: Int): Int = synchronized{
   while(him.nonEmpty) wait()
   him = Some(me); notifyAll()
   while(her.isEmpty) wait()
   \mathbf{val} \ \mathrm{Some}(\mathrm{res}) = \mathrm{her}
   her = None; notifyAll()
   res
 }
 \mathbf{def} womanSync(me: Int): Int = synchronized{
   \mathbf{while}(\text{her.nonEmpty}) \text{ wait}()
   her = Some(me); notifyAll()
   while(him.isEmpty) wait()
   val Some(res) = him
   him = None; notifyAll()
    res
```

Figure 20: A Faulty Scala implementation of MenWomen object

## 4 ABC

With an ABC object, three processes can exchange data with each other two processes. More specifically, one process calling aSync, one process calling bSync, and one process calling cSync synchronizes. Then each of the three processes returns with the arguments of two other processes. For simplicity, we shall call a process calling syncA as an A-process, a process calling syncB as a B-process, and a process calling syncC as a C-process.

One of the challenges to check an ABC object is the huge number of states in the CSP model. In this section, we shall see how the linearizer process can be optimized and the efficiency of the explicit linearization point test.

## 4.1 Implementation

Figure 22 is a Scala implementation of a ABC object with semaphore. In each round of synchronization,

- Initially semaphore aClear is raised. An A-process acquires semaphore
  aClear, sets the shared variable a to its parameter, raises semaphore bClear
  and waits to acquire semaphore aSignal. A B-process and a C-process
  behaves similarly in turn, except they use different semaphores and shared
  variables.
- After a C-process raises semaphore a Signal, the A-process is able to continue. The A-process reads the shared variable b and c, raises the semaphore b Signal, and returns b and c. Likewise, B and C also take the value of two other shared variable and raise respective semaphores in turn.

Using the shared variable and semaphore module, it is easy to translate the Scala implementation to a CSP implementation.

## 4.2 Testing

Similar for the MenWomen object, a testing system is defined through any number of working processes and a specification is built from a Sync channel, linearizer processes, and a synchronization alphabet set.

One key difference of the ABC object is that processes can call with any argument from the set TypeData, whereas in MenWomen object, processes can only call with their identity. As shown in Figure 24 So inside chaosP, the processes also choose an argument with General Non-Deterministic Choice.

```
Thread(me)=chaosP( □ x:TypeData • (
    SyncA(me,x)
    □ SyncB(me,x)
    □ SyncC(me,x)
)
```

Figure 24: Definition of processes in the testing system.

```
class ABC[A,B,C] extends ABCT[A,B,C]{
  // The identities of the current (or previous) threads.
 private var a: A =
 private var b: B =
 private var c: C = 
  // Semaphores to signal that threads can write their identities.
  private val aClear = MutexSemaphore()
 private val bClear, cClear = SignallingSemaphore()
  // Semaphores to signal that threads can collect their results.
  private val aSignal, bSignal, cSignal = SignallingSemaphore()
  \mathbf{def} \operatorname{syncA}(\operatorname{me: A}) = \{
                          //(A1)
    aClear.down
    a = me; bClear.up // signal to b at (B1)
    aSignal.down
                         //(A2)
    val result = (b,c)
    bSignal.up
                         // signal to b at (B2)
    result
  }
  \mathbf{def} \operatorname{syncB}(\operatorname{me: B}) = \{
    {\bf bClear.down}
                          // (B1)
    b = me; cClear.up // signal to C at (C1)
    bSignal.down
                         // (B2)
    val result = (a,c)
    cSignal.up
                         // signal to c at (C2)
    result
  \mathbf{def} \operatorname{syncC}(\operatorname{me: C}) = \{
                          // (C1)
    cClear.down
    c = me; aSignal.up // signal to A at (A2)
                         // (C2)
    cSignal.down
    val result = (a,b)
    aClear.up
                         // signal to an A on the next round at (A1)
    result
```

Figure 22: A semaphore-based Scala implementation of the ABC object

```
instance VarA = ModuleUninitVariable(TypeData)
instance VarB = ModuleUninitVariable(TypeData)
instance\ Var C = Module Uninit Variable (Type Data)
instance aClear = ModuleMutexSemaphore(TypeThreadID)
instance bClear = ModuleSignallingSemaphore(TypeThreadID)
instance cClear = ModuleSignallingSemaphore(TypeThreadID)
instance aSignal = ModuleSignallingSemaphore(TypeThreadID)
instance bSignal = ModuleSignallingSemaphore(TypeThreadID)
instance cSignal = ModuleSignallingSemaphore(TypeThreadID)
runWith(hide,p)=
  VarA::runWith(hide,
  VarB::runWith(hide,
  VarC::runWith(hide,
  aClear::runWith(hide,
  bClear::runWith(hide,
  cClear :: runWith(hide,
  aSignal::runWith(hide,
  bSignal :: runWith(hide,
  cSignal :: runWith(hide,
  ))))))))))
SyncA(me,avalue) =
  Call ! me! ASync! avalue \rightarrow
  --aClear.down
  aClear:: downChan! me \rightarrow
  --a = me
  VarA::setValue ! avalue \rightarrow
  --bClear.up
  bClear:: upChan! me \rightarrow
  --aSignal.down
  aSignal :: downChan ! me \rightarrow
  --(b,c)
  \mathsf{VarB} {::} \mathsf{getValue} \, ? \, \mathsf{b} \, \to \,
  VarC::getValue?c \rightarrow
  −−bSignal.up
  bSignal :: upChan! me \rightarrow
  --\mathrm{result} \, \rightarrow \,
  Return! me! ASync! (b.c) \rightarrow
  SKIP
```

Figure 23: Translated CSP Code for the correct ABC implementation

The testing specification uses the same component, the Sync channel, linearizer processes, and linEventns. The definition of Sync channel is shown in Figure 25. The event  $Sync.t_1.a.b.c.t_2.d.e.f.t_3.g.h.i$  represents the synchronizations between three threads,  $t_1, t_2, t_3$ . Process  $t_1$  calls aSync with a and returns (b, c). The second process  $t_2$  calls bSync with d and returns (e, f). And the last process  $t_3$  calls cSync with g and returns (h, i). The Sync specification process, shown in the same figure, checks that in each Sync events, the return value of each process is the pair of arguments of the two other function call.

Figure 26 is the definition of a linearizer process, written a similar format with the MenWomen object.

```
--thread identity calling ASync. ASync parameter a. ASync return pair (b,c)
--thread identity calling BSync. BSync parameter b. BSync return pair (a,c)
--thread identity calling CSync. CSync parameter c. CSync return pair (a,b)
channel Sync: TypeThreadID.TypeData.TypeData.TypeData.
TypeThreadID.TypeData.TypeData.TypeData.
TypeThreadID.TypeData.TypeData.TypeData

Spec = Sync?aid?a?b?c
?bid:diff(TypeThreadID,{aid})!b!a!c
?cid:diff(TypeThreadID,{aid,bid})!c!a!b

→ Spec
```

Figure 25: Definition of Sync channel and specification of Sync event

```
\label{eq:System3} System(\{T1,T2,T3\}) \\ Spec3Thread=Linearizers(\{T1,T2,T3\}) \\ assert Spec3Thread <math>\sqsubseteq_T System3 \\ assert Spec3Thread \subseteq_F System3 \\
```

Figure 27: Part of test for ABC object. This tests a system with three processes.

#### 4.2.1 Speeding up model compilation

Consider the specification process with three processes. Let M be the size of the set of all possible arguments. Consider the trace in Figure 28, where process T1 calls aSync with A, T2 calls bSync with B, T3 calls cSync with C. Then they synchronization.

```
--Linearizer for a process
Lin(All, me)=(
  --me synchronizes as thread A
  Call ! me! ASync? a \rightarrow
  Sync! me!a?b?c?t2:diff(All,{me})?t2b?t2a?t2c?t3:diff(All,{me,t2})?t3c?t3a?t3b \rightarrow
  Return!me!ASync!b!c \rightarrow
  Lin(All, me)
) \sqcap (
  ——me synchronizes as thread B
  Call ! me!BSync?b \rightarrow
  Sync?t2:diff(All,{me})?t2b?t2a?t2c!me!b?a?c?t3:diff(All,{me,t2})?t3c?t3a?t3b \rightarrow
  Return!me!BSync!a!c \rightarrow
  Lin(All, me)
) □ (
  --me synchronizes as thread C
  Call ! me! CSync? c \rightarrow
  Sync?t2:diff(All,{me})?t2b?t2a?t2c?t3:diff(All,{me,t2})?t3a?t3b!me!c?a?b \rightarrow
  Return\,!\,me\,!\,CSync\,!\,a\,!\,b\,\rightarrow\,
  Lin(All,me)
```

Figure 26: Definition of linearizer process

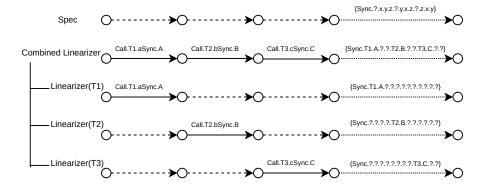


Figure 28: The set of possible Sync event for each CSP process

The last transition in the diagram is the Sync event between three processes. Above the edge is the set of possible Sync event that every process accepts. Each linearizer accepts  $3^2 * M^8$  possible Sync event. The combined linearizer accepts  $M^6$  sync event. However, according to the sync specification, only one Sync event is valid.

With the above analysis, it is tempting to reduce the redundancy in Sync event. Optimize the linearizer process by using the information from the specification process. Instead of choosing all possible remaining arguments, the individual linearizer could choose correct arguments according to the specification process. Figure 29 includes part of the simplified code. This change does not reduce the number of transitions in the resulting specification, but it helps FDR build the process faster.

With this optimization, the testing for less than 5 processes finishes quickly.

```
 \begin{array}{c} \textbf{channel Sync: TypeThreadID . TypeThreadID . TypeData} \\ \textbf{TypeData . TypeData . TypeData} \\ \textbf{Lin(All,me)} = (\\ \textbf{Call ! me! ASync? a} \rightarrow \\ \textbf{Sync! me? t2:diff(All,\{me\})? t3:diff(All,\{me,t2\})! a?b?c} \rightarrow \\ \textbf{Return! me! ASync!b!c} \rightarrow \\ \textbf{Lin(All,me)} \\ \textbf{)} & \dots \end{array}
```

Figure 29: Simplified definition of Sync channel and part of simplified linearizer

## 4.3 Faulty version

Recall that in Java and Scala, raising a semaphore immediately allows another thread waiting to acquire the semaphore to continue. So it is essential to take a copy of the two other arguments before raising the semaphore.

On the other hand, what if the implementation of syncA does not take a copy of the argument? It turns out that the faulty ABC object passes tests for three processes but fails the linearisation test with at least four threads.

#### 4.3.1 Explanation of the error case

For the test Spec4Thread  $\sqsubseteq_T$  System4, FDR displays a trace of the testing system that violates the specification. From the trace, it seems that process T1 synchronizes with T2 and T3 in the first round, and should return (B,C), but (E,F), the argument in the second round is returned. Expanding the  $\tau$  event and translating CSP traces into program traces makes it possible to see what goes wrong in the faulty version when there are four threads.

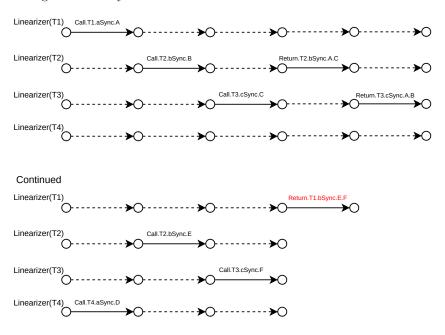


Figure 30: Trace that violates the specification

- In the first round of synchronization, process  $T_A$ ,  $T_B$ ,  $T_C$  call aSync, bSync or cSync respectively, and put down its argument in turn.
- Process  $T_A$  raises bSignal. Before  $T_A$  exits, the other two processes  $T_B$ ,  $T_C$  returns. Now  $T_A$  should return argument of  $T_B$  and  $T_C$ .
- Another round of synchronization starts. Thread  $T_D$ ,  $T_B$ ,  $T_C$  call aSync, bSync or cSync respectively, and overwites the shared variable a,b,c in turn.
- Now  $T_A$  returns with (b,c) from the second round, which may not be the argument of bSync and cSync in the first round.

## 5 Barrier Counter

Barrier synchronization is commonly used in concurrent programming ... A barrier object takes an argument p. A process calling sync blocks until all p processes call sync.

This section shall examine a barrier object with an internal counter. When a sync call exits, it returns the number of times barrier synchronization has happened.

## 5.1 Implementation

In this section, we shall use a monitor based barrier counter. Additionaly, this implementation uses three shared variables. seqNumber is the number of synchronization that happened. count is the number of processes participating in the current round of synchronization and is waiting to return. leaving indicates if processes are leaving.

There are two stages in each round of synchronization.

- In the first stage, a new process comes and waits for all p processes. Each
  process increments count.
- If a process finds the value of count is *p* after incrementing count, the process knows it is the last process in this round. The *p*-th process sets leaving to true and notifies all waiting processes to allow other processes in this round to leave and blocks any new coming processes.
- In the second stage, all p processes, including the p-th process, decrements count to and returns with the value of the shared variable seqNumber.
- If a process finds the value of count equal to 0, then the process knows it is the last process to leave in this round. Before exiting, it increments seqNumber and sets leaving to true, to allow a new round to start.

## 5.2 A stateful concurrent datatypes

```
class BarrierCounter(n: Int) extends BarrierCounterT{
 private\ var\ seqNumber = 0\ //\ the\ current\ sequence\ number
 private var count = 0 // The number of waiting threads.
 private var leaving = false // Are we in the leaving phase?
 \mathbf{def} sync = synchronized{
   while(leaving) wait() // Wait for previous round to finish
   count += 1
   if(count == n){
     leaving = true;
     count -= 1;
     notifyAll();
     {\rm seqNumber}
   } else{
     while(!leaving) wait()
     count = 1
     if(count == 0){
       // Allow next round to continue
       leaving = false;
       notifyAll()
       // Increment sequence number for next round
       seqNumber += 1;
       seqNumber-1
     else seqNumber
```

Figure 31: Implementation of Barrier Counter in Scala

# 6 Terminating Queue

# 7 Reference

# References

- [1] Lowe, G. Testing for linearizability. Concurrency and Computation: Practice and Experience 29, 4 (2017).
- [2] ROSCOE, A. Understanding Concurrent Systems. 01 2011.
- [3] SEREBRYANY, K., AND ISKHODZHANOV, T. Threadsanitizer: data race detection in practice. In *Proceedings of the workshop on binary instrumentation and applications* (2009), pp. 62–71.