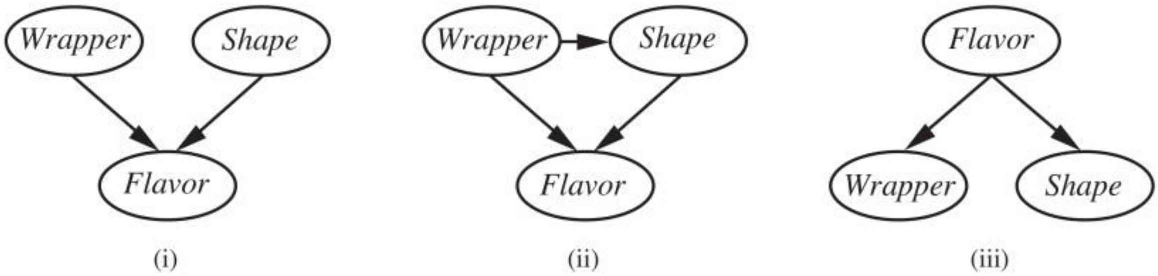
Artificial Intelligence Methods
Assignment 4
Kacper Multan

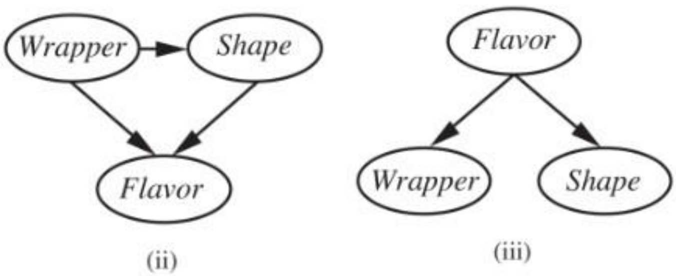
Exercise 1



	strawberry	anchovy
flavour	70%	30%
shape	80% round 20% square	10% round 90% square
wrapper	80% red 20% brown	10% red 90% brown

a. **Which network(s) can correctly represent P(Flavor, W rapper, Shape)? Consider if each network can represent all dependencies between the variables required to fit with the story?**

Networks (ii) and (iii) can correctly represent P(Flavor, W rapper, Shape).



Considering network (ii) - if we know the wrapper of the candy we can deduct the probability of the flavor. The same is true for the shape - if we know the shape we can calculate the probability of the flavor of the candy. Also, if we know the wrapper, we can get partial information about the shape of the candy. The network is fully connected, which makes every state dependent on every other state.

Network (iii) on the other hand gives us information about the probability of a certain wrapper or the probability of a certain shape, assuming that we know the flavor of the candy. This is the easiest way to represent the given problem.

b. Which network is the best representation for this problem? Consider the size of the representation, and how easily you can deduce the numbers required by the conditional probability tables in your chosen model.

Network (iii) is the best representation of the problem. Knowing the flavor of the candy we can easily deduct the probabilities of the wrapper and the shape of the candy.

c. Does network (i) assert that Wrapper is independent of Shape?

Yes it does. The wrapper of the candy does not give us any information about the shape of the candy. The same is true the other way around. Until we know the flavor of the candy, the wrapper and shape are independent.

d. What is the probability that your candy has a red wrapper?
59%

R - red wrapper
S - strawberry
A - anchovy

$$P(R) = P(R|S)\cdot P(S) + P(R|A)\cdot P(A) =$$
$$= 0{,}8\cdot 0{,}7 + 0{,}1\cdot 0{,}3 = 0{,}59$$

e. In the box is a round candy with a red wrapper. What is the probability that its flavor is strawberry?
99%

S - strawberry

RR - red and round

$$P(S|RR) = \frac{P(RR|S) \cdot P(S)}{P(RR)} = \frac{0,64 \cdot 0,7}{0,451} = \frac{0,448}{0,451} = 0,993$$

$$P(RR|S) = 0,8 \cdot 0,8 = 0,64$$

$$P(RR) = P(RR|S) \cdot P(S) + P(RR|A) \cdot P(A) = 0,64 \cdot 0,7 + 0,01 \cdot 0,3 =$$
$$= 0,448 + 0,003 = 0,451$$

f. An unwrapped strawberry candy is worth s on the open market and an unwrapped anchovy candy is worth a. Write an expression for the expected value of an unopened candy box.

0,7*s + 0,3*a

Exercise 2

a. Assume Mary has an exponential utility function with R = $500. Mary is given the choice between receiving $500 with certainty (probability 1) or participating in a lottery which has a 60% probability of winning $5000 and a 40% probability of winning nothing. Assuming Mary acts rationally, which option would she choose? Show how you derived your answer.

Mary should choose to receive $500.

$R = \$500$

$$EU(\text{receiving}) = U(\$500) = -e^{-\frac{500}{500}} = -e^{-1} = -\frac{1}{e} \approx -0,3679$$

$$EU(\text{lottery}) = 0,6 \cdot U(\$5000) + 0,4\, U(\$0) = 0,6 \cdot \left(-e^{\frac{-5000}{500}}\right) + 0,4 \cdot \left(-e^{\frac{-0}{500}}\right) =$$
$$= 0,6 \cdot \left(-e^{-10}\right) + 0,4 \cdot (-1) = 0,6 \cdot \left(-\frac{1}{e^{10}}\right) - 0,4 =$$
$$\approx -0,4$$

EU (receiving) > EU (lottery) so Mary should choose to receive $500.

b. Consider the choice between receiving $100 with certainty (probability 1) or participating in a lottery which has a 50% probability of winning $500 and a 50% probability of winning nothing. Approximate the value of R (to 3 significant digits) in an exponential utility function that would cause an individual to be indifferent to these two alternatives.

R = 152,384

$R = ?$

$EU(\text{receiving}) = U(\$100) = -e^{-\frac{100}{R}}$

$EU(\text{lottery}) = \frac{1}{2}U(\$500) + \frac{1}{2}U(\$0) = \frac{1}{2}\cdot(-e^{-\frac{500}{R}}) + \frac{1}{2}(-e^{-\frac{0}{R}})$

$$-e^{-\frac{100}{R}} = -\frac{1}{2}e^{-\frac{500}{R}} - \frac{1}{2}$$

$$\frac{1}{2}e^{-\frac{500}{R}} - e^{-\frac{100}{R}} + \frac{1}{2} = 0 \quad / \cdot 2$$

$$e^{-\frac{500}{R}} - 2e^{-\frac{100}{R}} + 1 = 0$$

$$k = e^{-\frac{100}{R}}$$

$$k^5 - 2k + 1 = 0$$

$k = 1 \quad \vee \quad k = 0{,}5188 \quad \vee \quad \cancel{k = -1{,}2906}$ false $k<0$

$e^{-\frac{100}{R}} = 1$

$0 = -\frac{100}{R}$

$\cancel{R \to \infty}$, cannot be

$e^{-\frac{100}{R}} = 0{,}5188$

$\ln(0{,}5188) = -\frac{100}{R}$

$R = \frac{-100}{\ln(0{,}5188)}$

$$\boxed{R \approx 152}$$