Artificial Intelligence Methods
Assignment 2
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The Monty Hall Problem

You are confronted with three doors A, B, and C. Behind exactly one of the doors there is \$10 000. The money is yours if you choose the correct door. After you have made your first choice of door but still not opened it, an official comes in. He works according to two rules:

1. He starts by opening a door. He knows where the prize is, and he is not allowed to open that door. Furthermore, he cannot open the door you have chosen. Hence, he opens a door with nothing behind.
2. Now there are two closed doors, one of which contains the prize. The official will ask you if you want to alter your choice (i.e., to trade your door for the other one that is not open).

Should you do that?

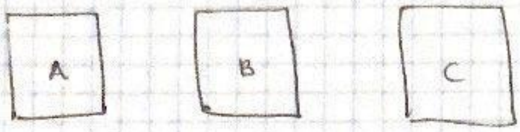
Yes. At the beginning the contestant chooses one of the doors, where the probability that the prize is going to be behind the door equals ⅓ for every door.

This means that the probability that the prize is behind the door chosen by the contestant is ⅓ and the probability that the prize is behind one of the two doors left is ⅔.

After the official opens one of the doors not chosen by the contestant and reveals that there is nothing behind the door, we know that the probability of the prize being behind the second door not chosen by the contestant equals ⅔.

In conclusion, the chance of the prize being behind the door chosen by the contestant equals ⅓ and the chance that it is behind the door neither chosen by the contestant nor opened by the official equals ⅔. Therefore it is always better to alter the choice and switch to the other door.

This is supported by the following calculations:



The contestant chooses door A

The probability of the prize beeing behind the doors is

$P(A) = \frac{1}{3}$

$P(B) + P(C) = \frac{2}{3}$

The offical chooses one of the doors not chosen by the contestant – for example door B and reveals that it isn't the one with the prize

The probability that the prize is behind B

$P(B) = 0$, so

$P(B) + P(C) = \frac{2}{3}$

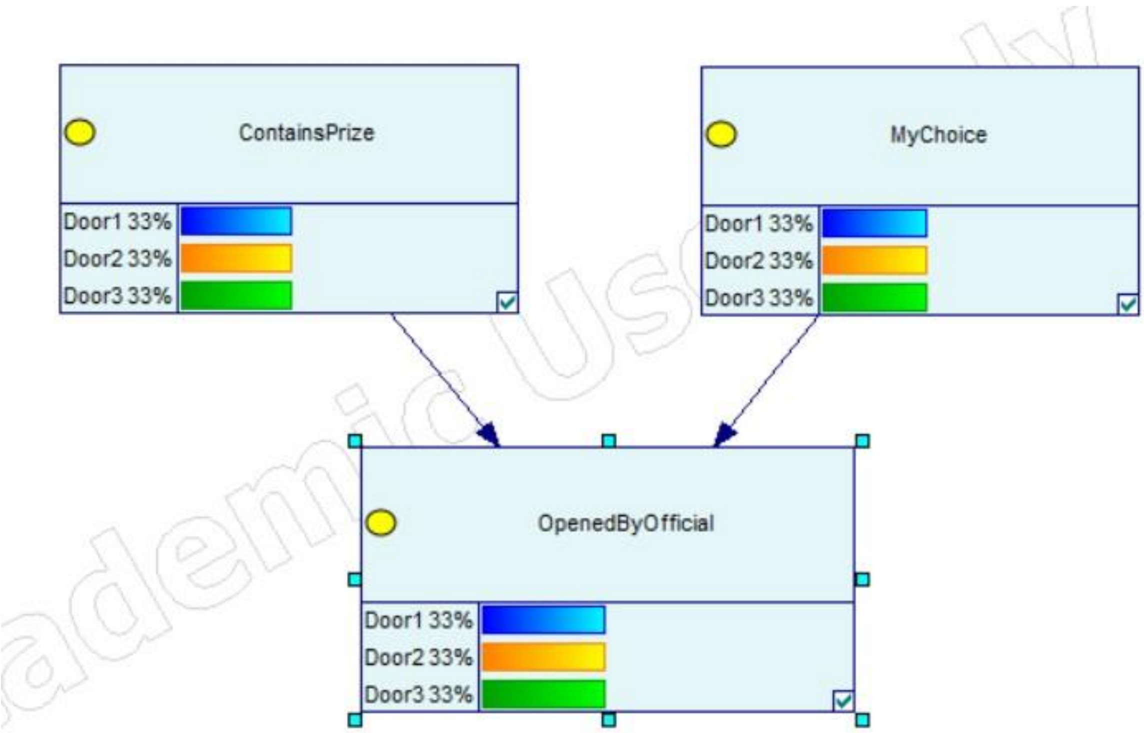
$0 + P(C) = \frac{2}{3}$

$P(C) = \frac{2}{3}$

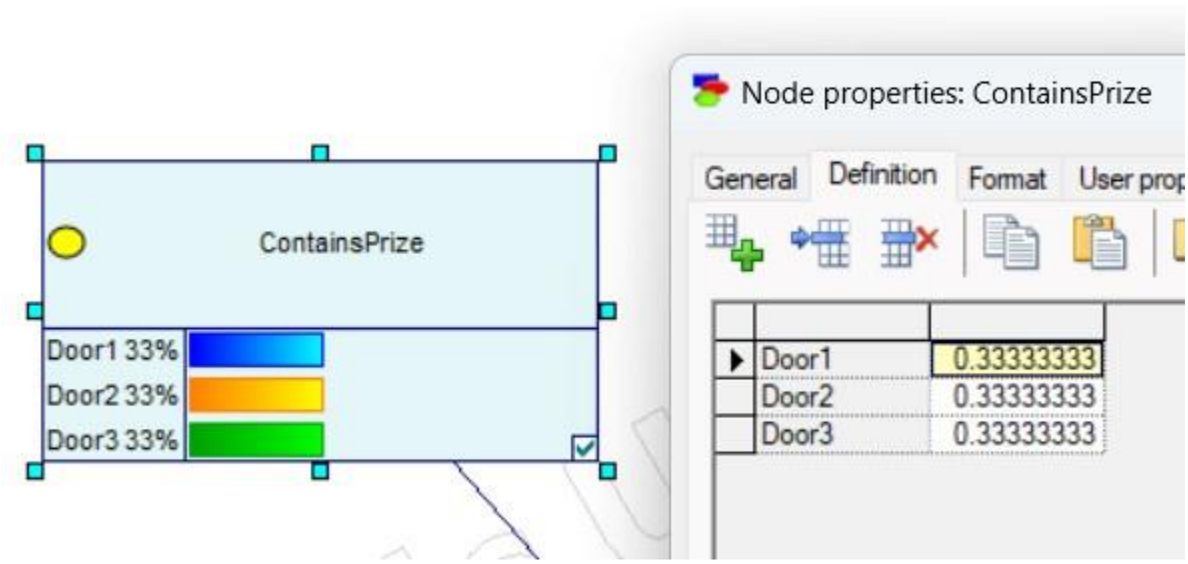
The probability that the prize is behind door C is higher then the probability of it beeing behind door A. Therefor should the contestant alter his choice and switch to door C.

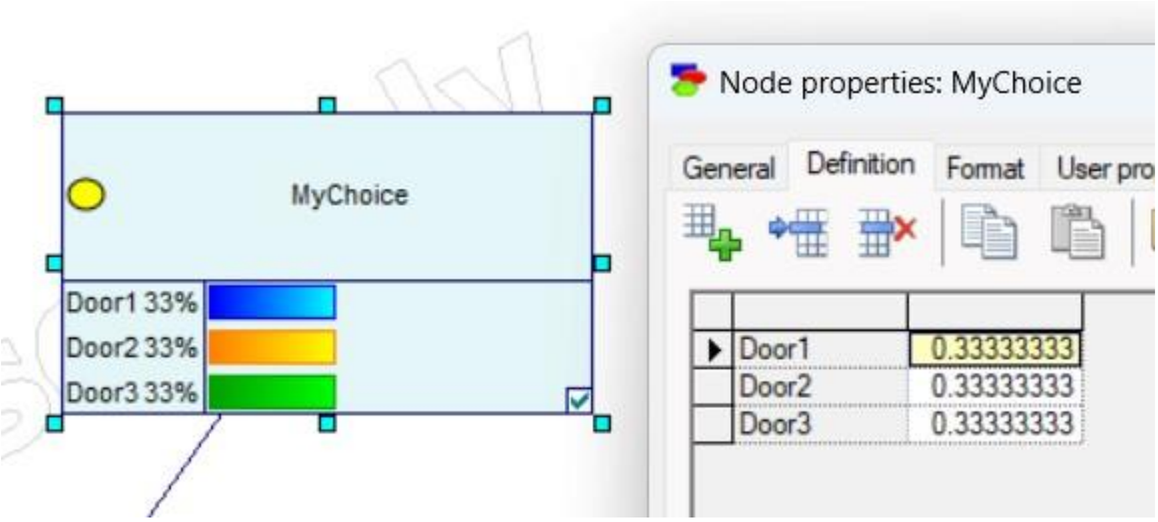
To analyze this problem I used GeNIe.

I draw a Bayesian network that represents this problem.

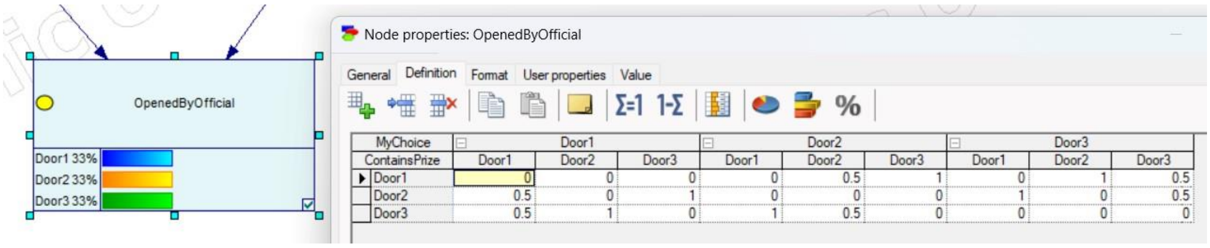


I also created conditional probability tables for each node in the network.





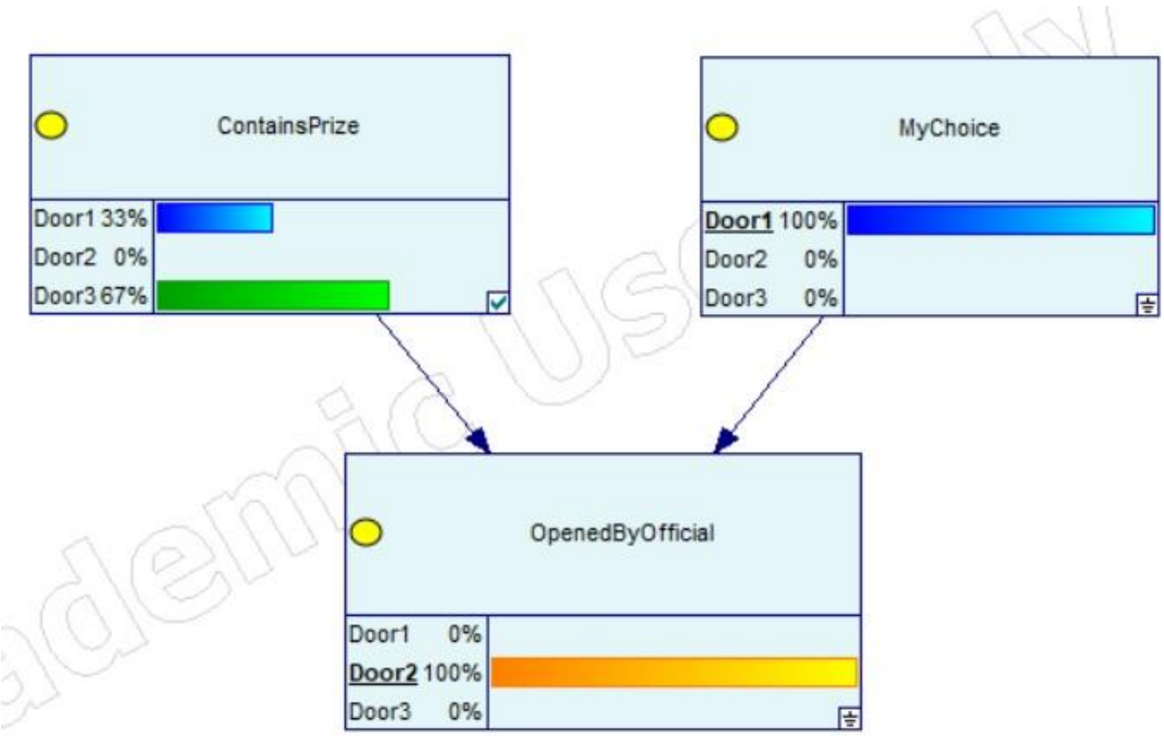
Depending on which choice the contestant makes (if he chooses the door with or without the prize behind it) the probabilities in the officials choices may vary.



Finally I also created a table showing the probability of the prize being behind each door depending on the choice the contestant made and the door the official opened.

		My Choice		
		Door1	**Door2**	**Door3**
	Door1	chosenDoor: 0 | otherDoor: 1	chosenDoor: 0,33 | otherDoor: 0,66	chosenDoor: 0,33 | otherDoor: 0,66
Opened by Oficial	**Door2**	chosenDoor: 0,33 | otherDoor: 0,66	chosenDoor: 0 | otherDoor: 1	chosenDoor: 0,33 | otherDoor: 0,66
	Door3	chosenDoor: 0,33 | otherDoor: 0,66	chosenDoor: 0,33 | otherDoor: 0,66	chosenDoor: 0 | otherDoor: 1

This example represents the situation proven previously by the calculations:


ContainsPrize
Door1 33%
Door2 0%
Door3 67%
MyChoice
Door1 100%
Door2 0%
Door3 0%
OpenedByOfficial
Door1 0%
Door2 100%
Door3 0%