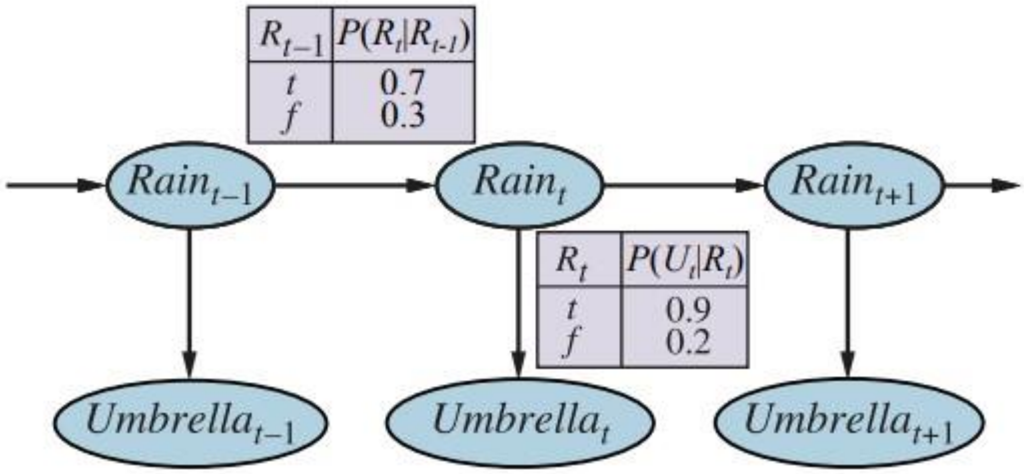
Artificial Intelligence Methods
Assignment 3
Kacper Multan

Exercise 1

Describe the umbrella world as an HMM:



- **What is the set of unobserved variable(s) for a given time-slice t (denoted Xt in the book)?**

The unobserved variables are rain and sunshine.
Xt = 1 - rain
Xt = 0 - sunshine

- **What is the set of observable variable(s) for a given time-slice t (denoted Et in the book)?**

The observed variables are the presence or lack of the umbrella.
Et = 1 - umbrella
Et = 0 - no umbrella

- **Present the dynamic model P(Xt|Xt−1) and the observation model P(Et|Xt) as matrices.**

$$\begin{array}{c} \\ YRT \\ YRF \end{array} \begin{array}{c} \begin{matrix} RT & RF \end{matrix} \\ \begin{bmatrix} 0,7 & 0,3 \\ 0,3 & 0,7 \end{bmatrix} \end{array} \qquad \begin{array}{c} \\ RT \\ RF \end{array} \begin{array}{c} \begin{matrix} UT & UF \end{matrix} \\ \begin{bmatrix} 0,9 & 0,1 \\ 0,2 & 0,8 \end{bmatrix} \end{array}$$

YRT - yesterday rain true
YRF - yesterday rain false
RT - rain true
RF - rain false

UT - umbrella true
UF - umbrella false

- **Which assumptions are encoded in this model?Are the assumptions reasonable for this particular domain? (See 14.1 Time and Uncertainty on Page 479).**

  - the world is viewed as a series of snapshots or time slices
  - each time slice in a discrete-time probability model contains a set of random variables, some observable and some not (X_t - unobservable, E_t observable)
  - Markov assumption that the current state depends on only a finite fixed number of previous states
  - Assumption that changes in the world state are caused by a process of change that is governed by laws that do not themselves change over time.

    It is a reasonable assumption to make in the umbrella world, as the conditional probability of rain, $P(R_t | R_{t-1})$, is the same for all t, and we only need to specify one conditional probability table. Although this assumption might not be the best in the real world.

  - The first-order Markov assumption - the state variables contain all the information needed to characterize the probability distribution for the next time slice. (Where the number of important previous states equals 1)

    Although this assumption works well for the umbrella world, it could be less useful in the real world, where the weather depends not only on the previous day but on the days before as well.

  - the sensor Markov assumption ($P(E_t | X_{0:t}, E_{1:t-1}) = P(E_t | X_t)$) - the evidence variables may be influenced by both preceding variables and the existing state variables. However, a competent state should be sufficient for generating the present sensor readings.

    This is a reasonable assumption, because the director's decision to take the umbrella doesn't have any reasons to depend on whether it rained or not on the previous days.

Exercise 2

Implement filtering using the Forward operation (see Equation 14.5 on Page 485 and Equation 14.12 on Page 492) by programming. The forward operation can be done with matrix operations in the HMM.

- Verify your implementation by calculating $P(X_2|e_{1:2})$, where $e_{1:2}$ is the evidence that the umbrella was used both on day 1 and day 2. The desired result is that the probability of rain at day 2 (after the observations) is 0.883.
- Use your program to calculate the probability of rain at day 5 given the following sequence of observations:

$e_{1:5}$ = {Umbrella1 = true, Umbrella2 = true, Umbrella3 = false, Umbrella4 = true, Umbrella5 = true}.

Document your answer by showing all normalized forward messages (in the book, the un-normalized forward messages are denoted $f_{1:k}$ for k = 1, 2, . . . , 5) in the PDF report.