Measuring the Capacitance of a Small Capacitor Using Discharge Decay and Phase Shift

Ziyao Xiong

***Abstract*--- In this experiment, I used two methods to measure the capacitance of the same capacitor and compared their results. In the first method, I used an oscilloscope to generate a 1.1kHz square wave and measured the voltage across the capacitor over time. By fitting the curve during the discharge of the capacitor with an exponential decay function, I calculated its capacitance to be 555.73 +/- 27.46 pF. In the second method, I considered the internal capacitance of the oscilloscope. By supplying a 50kHz sine wave across the circuit, I measured the phase shift between the output voltage and the input voltage. From there I calculated the circuit's total reactance and capacitance. Knowing the capacitance of the oscilloscope, the capacitance of the capacitor under investigation was determined to be 533.17 +/- 14.56 pF.**

I. Introduction

Capacitors are devices that can store electrical energy in an electric field across negatively and positively charged metal plates. A capacitor's capacity to store electric charge is referred to as capacitance [1]. It is crucial to know the accurate value of the capacitance of a capacitor under use, as this indicates the maximum amount of charge and energy that can be stored in the capacitor for a given voltage.

Capacitors are ubiquitous in everyday life since they exist in practically all electrical components. Their application varies with the size of their capacitance. While large capacitor banks can be used as energy sources for nuclear weapons and other speciality weapons, small capacitors also play a critical role in electric filters and tuned circuits [2]. Determining the exact value of capacitance for a small capacitor can be strenuous, as the measuring device often has its own capacitance comparable to that of the capacitor, which may affect the measured value.

In this experiment, I will present two methods to measure the capacitance of a relatively small capacitor, one with consideration of the internal capacitance of the measuring device (an oscilloscope), and another without. I will compare the two values of capacitance I obtain and examine the extent of influence that the internal capacitance of the oscilloscope can make on the total capacitance of the circuit.

II. Theory

A. *Discharging decay of a capacitor*

Capacitance is measured by the change in charge in response to a difference in potential difference [3]. The charge stored in the capacitor, Q, is given by:

$$Q = C \cdot V, \tag{1}$$

where V is the voltage across the capacitor, and C is its capacitance in farads (F). According to Ohm's law, V and the current flowing through the circuit, I, have a relationship:

$$V = I \cdot R, \tag{2}$$

where R is the resistance in the circuit. Since the current is the rate of change of charge, equation (2) can be rewritten as:

$$\frac{dQ}{dt} = -\frac{1}{RC} \cdot Q, \tag{3}$$

where the minus sign indicates there is a reduction in the amount of charge remaining on the capacitor during its discharge. By solving equation (3), a first-order ordinary differential equation, with charge Q_0 as the initial condition, the charge can be expressed as a function of time:

$$Q = Q_0 e^{\frac{-t}{RC}}. \tag{4}$$

Therefore, the amount of charge on the capacitor experiences an exponential decay during the discharge of the capacitor. Since voltage is directly proportional to the charge according to equation (1), the rate of change of voltage is always the same as that of the charge. Thus, monitoring how the voltage across the capacitor varies with time and fitting the data points with an exponential decay function allows the time constant RC, and hence the capacitance, C, to be determined.

B. *Phase shift in AC circuit*

The aforementioned claims hold true for any DC circuit consisting of a capacitor and a resistor connected in series. However, the concept of reactance comes into play if the power source in such a circuit produces an AC signal. Reactance is a form of opposition that electronic components exhibit to the change of current [4]. It is proportional to the reciprocal of total capacitance C_{total} in the circuit, given by:

$$C_{total} = \frac{1}{2\pi f X_c}, \tag{5}$$

where f is the driving frequency of the circuit, and X_c is the total reactance in the circuit, measured in ohms (Ω).

The direction and amplitude of the current in an AC circuit are determined by the difference between the driving voltage and the capacitor voltage [5]. When the driving voltage equals the capacitor voltage, the current reverses direction, and the instant current equals zero. This is when the capacitor voltage reaches its peak, as the current and voltage of a capacitor are out of phase by 90 degrees [6]. There is a phase difference between the input voltage and output voltage because the peaks of the capacitor's output signal and the input signal emerge at different times. This phase shift is related to the magnitude of the reactance and resistance in the circuit. When the capacitor dominates the circuit, where $X_c >> R$, the phase difference tends to 90 degrees [7]. In the opposite case, the phase difference tends to 0 degrees. This relationship is described by:

$$X_C = \frac{v_x \cdot R}{V_g \cdot \sin\alpha}, \tag{6}$$

where Vx and Vg are the amplitude of the capacitor's output voltage and the input voltage respectively, and α is the phase difference. Vx is always smaller than Vg because the resistor in the circuit dissipates energy. Thus, by measuring Vx, Vg, α, and R, we can calculate the value of X_c, and therefore the value of C_{total} in the circuit.

In this experiment, the capacitor under investigation and the internal capacitor of the oscilloscope were connected in parallel. Suppose they had capacitance C_1 and C_2 respectively, their combined capacitance C_{total} is given by:

$$C_{total} = C_1 + C_2 \qquad (7)$$

Therefore, subtracting the capacitance of the oscilloscope from the calculated value of C_{total} yields the capacitance of the capacitor under investigation.

III. Method

A. *Measuring the capacitance using discharge decay*

There were three main devices I used in this method of measuring capacitance: an oscilloscope, a capacitor, and a resistor. I connected both the signal generator and the capacitor to the input port of the oscilloscope; therefore, the patterns of the driving signal and the voltage across the capacitor were displayed on the oscilloscope simultaneously.

A resistor was connected in series with the capacitor. I used a multimeter to test the resistor's resistance; the resistance rises with time owing to the effect of heating. It fluctuated by 0.08 kΩ over a 20-second interval (roughly the duration of the experiment), which is a reasonable estimation of the resistance's uncertainty. Thus, the resistance was measured to be 99.06 +/- 0.08 kΩ. A detailed demonstration of the setup of the experiment is shown in Figure 1:

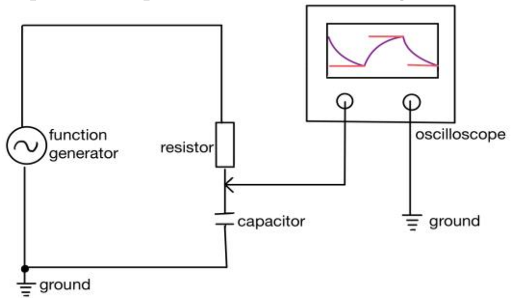

Fig.1. The circuit arrangement of measuring how the voltage across the capacitor varies with time. The oscilloscope measured the voltage across the capacitor and displayed its characteristics. The output signal generated by the oscilloscope was also displayed. One probe of the oscilloscope was connected to the ground, enabling the oscilloscope's correct functioning.

A square wave with a 2v amplitude was generated across the circuit. I adjusted the offset of this voltage signal to be 1v, therefore the voltage was oscillating between 0v and 2v. The capacitor was charged when the voltage is 2v and was discharged when the voltage is 0v. I adjusted the frequency of the driving signal until the end of the charging and discharging cur was barely horizontal, indicating the capacitor was fully charged, i.e., the amount of charge on the capacitor reached its maximum. The time scale of the oscilloscope was altered to allow at least one full cycle of charging and discharging pattern was shown. The patterns presented on the oscilloscope are demonstrated in Figure 2.

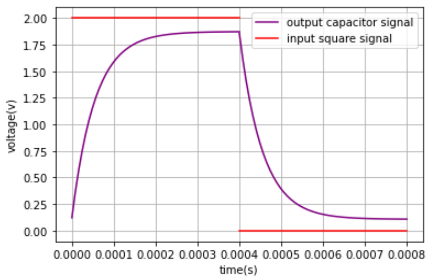

Fig.2. An illustration of patterns shown on the oscilloscope. The capacitor was charged during the time interval 0~400 μs, where the input voltage is 2v; and was discharged during 400 ~ 800 μs, where the input voltage was 0v. The exponential charging and discharging curves were steady at the ends, indicating there would be almost no further changes in the voltage across the capacitor and thus the capacitor was fully charged or discharged.

I imported the voltage-time data into Python and selected the data points corresponding to the discharging of the capacitor. An exponential decay function was fitted, and the value of the capacitance was calculated.

B. *Measuring the capacitance using phase shift*

In this method, the circuit I used was the same as the first method, apart from replacing the resistor with one with a smaller resistance, 6833 +/- 5 Ω. The capacitance of the oscilloscopes was taken into account in this method; thus, the circuit diagram was modified as shown in Figure 3.

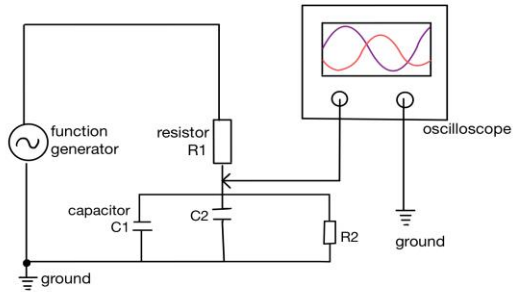

Fig.3. The circuit arrangement when the oscilloscope's resistance R_2 (1MΩ) and capacitance C_2 (20pF) were factored in. The voltage across C_1, C_2, and R_2 was measured by the oscilloscope.

A sine wave with a frequency of 50kHz was generated across the circuit. The input peak voltage Vg and the output peak voltage Vx were recorded respectively. The phase difference between the two waves was also measured by the oscilloscope. A graph of wavefronts demonstrated on the oscilloscope is shown in Figure 4.

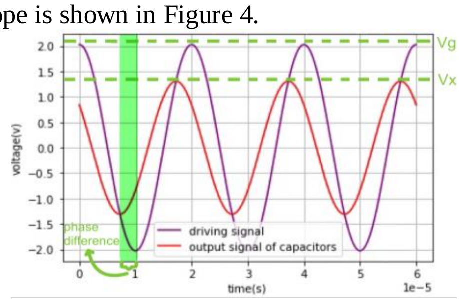

Fig.4. An illustration of wavefronts shown on the oscilloscope. Vg and Vx are the peak voltages of input and output signals. As predicted, the capacitor's voltage reached its peak when it was equal to the driving voltage, and Vx was smaller than Vg. The ratio of the time difference between the two peaks to the period of the waves was the same as that of the phase difference to 2π.

Since R_2 was significantly larger than the reactance of the capacitors, the current flowing through R_2 was negligible. We can therefore treat this circuit as a simple RC circuit, where C_1 and C_2 formed a combined capacitor with capacitance C_{total}, connected in series with R_1. Thus, equation (5) and equation (6) can be used to compute the total reactance and total capacitance in the circuit. Knowing the capacitance of the oscilloscope, the capacitance of the capacitor was calculated by equation (7).

IV. Results, Uncertainties, and Discussions

A. *Analyzing discharging data of the capacitor*

The oscilloscope recorded over 120,000 voltage-time data points during the discharge of the capacitor. Where the uncertainty of a single measurement is hard to be determined, I decided to select 50 representative data points to carry out the subsequent analysis. I separated the voltage and time data equally into 50 sub-lists respectively and calculated their mean values in each sub-list. This lessened random errors in the measurements, as there were up-and-down fluctuations of voltage in each time interval. I estimated the uncertainty of voltage to be the standard deviation of voltage measurements in each sub-list. The mean values of data and their best-fit exponential decay curve were plotted, shown in Figure 5.

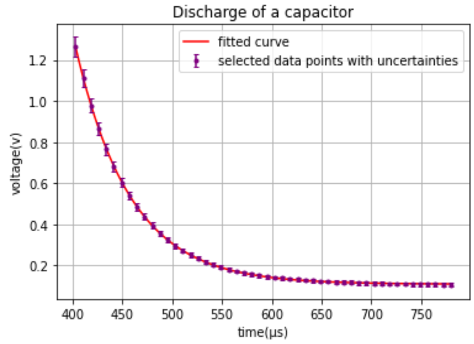

Fig.5. A plot of voltage across the capacitor against time generated by Python, where data points were fitted by the equation A· exp (b· t) + c with a weight of 1/uncertainty. This was done by applying the curve_fit function in Python. Error bars represent the uncertainty in each mean of the voltage.

The decay constant -b of the fitted function was 18163.41 +/- 1791.09 s-1. According to equation (4), this value should be equal to $\frac{1}{RC}$. Thus, the capacitance, C, can be expressed as a function in terms of R and -b. Its uncertainty can be computed by the formula:

$$z = f(x, y)$$

$$\sigma_z = \sqrt{(\frac{\partial f}{\partial x})^2 \sigma^2 x + (\frac{\partial f}{\partial y})^2 \sigma^2 y}\,, \tag{8}$$

where σ represents the uncertainty. C, R, and -b correspond to z, x, y respectively.

By substituting the values of R and -b, the capacitance of the capacitor was calculated to be 555.73 +/- 27.46 pF.

B. Analyzing phase shift data

The oscilloscope measured the mean values of the peak voltages of the input and output signals, as well as their phase differences. These mean values served as my record for Vg, Vx, and α. The standard deviations were used to determine their uncertainties. Values and uncertainties of Vg, Vx, R, and α are presented in Table I.

TABLE I

Vg(v)	Vx(v)	R(Ω)	α(rad)
2.0335±0.0036	1.2989±0.0317	6833±5	0.861±0.011

Using equation (5) (6) (7), the values of total reactance X_c, total capacitance C_{total}, and the capacitance of the capacitor C1 can be calculated. Since X_c is a function of Vg, Vx, R and α, its uncertainty is given by:

$$z = f(x, y, p, q)$$

$$\sigma_z = \sqrt{(\frac{\partial f}{\partial x})^2 \sigma^2 x + (\frac{\partial f}{\partial y})^2 \sigma^2 y + (\frac{\partial f}{\partial p})^2 \sigma^2 p + (\frac{\partial f}{\partial q})^2 \sigma^2 q}, \tag{9}$$

where X_c, Vx Vg, Vx, R and α correspond to z, x, y, p, q respectively.

As C_{total} is a function of X_c, and C_1 is a function of C_{total}, their uncertainties can be computed by the formula:

$$z = f(x)$$

$$\sigma_z = \sqrt{(\frac{df}{dx})^2 \sigma^2 x}, \tag{10}$$

where C_1 and C_{total} correspond to z and x respectively.

The values and uncertainties of X_c, C_{total} and C_1 are presented in Table II.

TABLE II

X_c(Ω)	C_{total}(pF)	C_1(pF)
5754.21±151.51	553.17±14.56	533.17±14.56

Therefore, the capacitance of the capacitor was calculated to be 533.17 +/- 14.56 pF.

C. Comparison of the two methods and discussion

The internal capacitance of the oscilloscope was not taken into consideration in the first method. As a result, the oscilloscope was measuring the overall capacitance of the two capacitors connected in parallel. Consequently, the measured capacitance would be greater than the capacitance of the capacitor under investigation.

In the second method, the capacitance C_{total} of the combined capacitor was calculated, and the capacitance of the oscilloscope was subtracted from it. Therefore, C_{total} should equal the capacitance C I determined in the first method. This was verified in my results, which showed that C_{total} was 553.17 +/- 14.56 pF and C was 555.73 +/- 27.46 pF. The percentage difference between these two numbers is less than 0.5%. Thus, the outcomes of this experiment were consistent with the expectation.

The capacitance of the oscilloscope, 20pF, affected the value of capacitance determined to vary by about 3.75%. This is because it accounted for 3.75% of the total capacitance in the circuit. Thus, when the capacitance of the measuring device is significantly less than the capacitance of other capacitors in the circuit, it is appropriate to disregard it. However, when it is comparable to that of other capacitors, its influence cannot be overlooked.

V. CONCLUSION

This experiment aims to find out the value of the capacitance of a small capacitor, by which two approaches were used: determining the decay constant of the capacitor's discharge and measuring the phase shift of the capacitor's voltage output to the signal input. The first approach ignored the capacitance of the measuring device while the second method did not. The capacitance was determined to be 555.73 +/- 27.46 pF and 533.17 +/- 14.56 pF respectively. The experiment was generally successful, as the difference between the two results is consistent with the capacitance of the oscilloscope, which is what was intended.

However, a number of evaluations can be made on my methods of conducting the experiment. I only performed one set of measurements to determine the capacitance in the second method. Measuring the phase shift data with multiple input voltages and analyzing the results graphically may be a better technique to minimize systematic errors in the measurements. Furthermore, several uncertainties had not been considered in the experiment, such as the values of the oscilloscope's capacitance and resistance, as they were given by the lab manual rather than being tested by myself. Knowing their uncertainties would make my calculations more rigorous. Finally, the theoretical capacitance of the capacitor under investigation was unknown. Thus, I was unable to determine the error in my results and make fair remarks on the accuracy of the experiment's outcomes.

REFERENCES

[1] Harrington, Roger F. (2003). Introduction to Electromagnetic Engineering (1st ed.). Dover Publications. p. 43. ISBN 0-486-43241-6.

[2] Floyd, Thomas (2005) [1984]. Electronic Devices (7th ed.). Upper Saddle River, New Jersey, USA: Pearson Education. p. 10. ISBN 0-13-127827-4.

[3] Y. Uchida (Ed.) Year 1 Laboratory Manual: Measuring Capacitance', Imperial College London, 2022

[4] Veley, Victor F. C. (1987). The Benchtop Electronics Reference Manual (1st ed.). New York: Tab Books. pp. 229, 232.

[5] Mechkov, Cyril. (2013). Re: Why is there a phase shift in RC circuits? How do we make it exactly 90 degrees?. Retrieved from: https://www.researchgate.net/post/Why-is-there-a-phase-shift-in-RC-circuits-How-do-we-make-it-exactly-90-degrees/511ebdc3e5438f8c1900003b/citation/download.

[6] "Alternating Current | Class 12 Physics (India) | Science | Khan Academy." Khan Academy. Accessed December 8, 2022. https://www.khanacademy.org/science/in-in-class-12th-physics-india/x51bd77206da864f3:alternating-current.

[7] Zekry, Abdelhalim. (2013). Re: Why is there a phase shift in RC circuits? How do we make it exactly 90 degrees?. Retrieved from: https://www.researchgate.net/post/Why-is-there-a-phase-shift-in-RC-circuits-How-do-we-make-it-exactly-90-degrees/511cd2a0e39d5ed717000031/citation/download