Measuring the Rydberg Constant with a Diffraction Grating

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***Abstract*— In this experiment, I used a spectrometer to determine the value of the Rydberg constant, by analyzing the line spectrum emitted by a hydrogen lamp. The hydrogen lamp emitted light with specific wavelengths, which were diffracted by different angles as they passed through a diffraction grating. By measuring such angles in various orders of diffraction, I calculated the wavelengths of red, cyan, and blue light beams. I deduced the energy levels involved in the emission of these wavelengths and substituted the values into the Rydberg formula to calculate the Rydberg constant as 1.0866 ± 0.01256·10^7 m^{-1}. This value falls within the range of uncertainty and has an error of 0.99%, indicating that it is in agreement with the theoretical value of the Rydberg constant, 10973731.6 m^{-1}.**

I. Introduction

Atomic spectrum refers to the specific wavelengths of electromagnetic radiation that are emitted or absorbed by an electron when it undergoes a transition between different energy levels within an atom [1]. The Rydberg formula describes the relationship between the wavelengths of such radiation and their associated energy levels for hydrogen atoms, the constant involved in the formula was later given the symbol R_∞ in honour of Rydberg [2].

In 1885, Johann J. Balmer proposed a mathematical formula to represent the wavelengths of the hydrogen spectrum [3], and Johannes Rydberg extended it to include all spectral series of lines for the hydrogen atom [4]. However, it was Bohr's theory that revealed the significance of the Rydberg constant as a combination of fundamental constants, including the electron's mass, charge, Planck's constant, and the speed of light [5]. Thus, the Rydberg constant acquired a theoretical basis and profound significance, making it one of the most important constants in physics today.

The goal of this experiment was to use a spectrometer to measure the Rydberg constant, R_∞, to a reasonable degree of accuracy. By diffracting the light beams of particular wavelengths emitted by a hydrogen lamp and measuring their various angles of diffraction, a spectral "fingerprint" of the hydrogen atom was obtained [6]. It contains valuable information about the wavelengths of the light beams and can be used to determine the Rydberg constant.

II. Theory

A. Electronic transition and the Rydberg formula

Electrons in an atom orbit the nucleus in discrete energy levels, or shells, where they cannot exist in between the levels. When an electron loses energy, it can drop from a higher energy level to a lower one. The energy difference between the two levels determines the frequency and wavelength of the electromagnetic radiation emitted or absorbed by the atom.

The Balmer series corresponds to transitions between the second excited state (n = 2) and higher energy levels (n > 2) in hydrogen atoms. These transitions result in the emission of electromagnetic radiation in the visible region of the spectrum, specifically in the range of 400 to 700 nanometers, which is visible as different colours of light [3]. Figure 1 shows the generation of these light in a hydrogen atom.

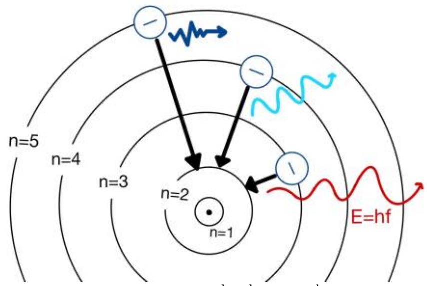

Fig 1. When electrons fall from the 3rd, 4th, and 5th energy levels to the 2nd energy level, electromagnetic radiations of red, cyan, and blue light are released. The lower the energy level the electron falls from, the smaller the energy difference between shells, and thus the lower the energy of the electromagnetic radiation and the smaller the frequency of light.

Rydberg concluded the relationship between the energy levels involved in an electronic transition and the wavelength of the emitted radiation, λ, into:

$$\frac{1}{\lambda} = R_\infty\left(\frac{1}{p^2} - \frac{1}{n^2}\right) , \tag{1}$$

where R_∞ is the Rydberg constant, and n and p is the final and initial energy level of the electron respectively. In this experiment, n is always equal to 2 since we only consider the transitions in the Balmer series, where the electromagnetic radiation is in the visible spectrum.

B. Diffraction gratings

Diffraction of light refers to the bending or spreading of light waves as they pass through an opening or around an obstacle [7]. A diffraction grating consists of a flat surface with equally spaced parallel grooves, which act as multiple narrow slits for light to pass through. As the beam of light passes through the grating, the grooves diffract it and produce multiple wavelets, since each point along the wavefront of the light that passes through the slit becomes a source of secondary wavelets, according to Huygens' construction [8]. These wavelets interfere and create a pattern of bright and dark fringes. At specific angles where the difference in the distance travelled by the waves is an integer multiple of the wavelength, constructive interference occurs, resulting in the appearance of bright fringes known as maxima. This configuration is shown in Figure 2.

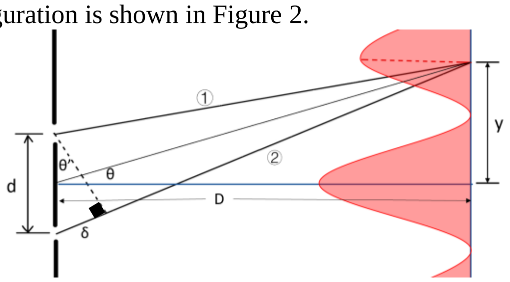

Fig 2. The red region shows the distribution of maxima and minima of the diffracted pattern, a maximum is always situated in the middle because two waves travel the same distance and thus the path difference is zero. Other

maxima appear at specific angles. Waves from two adjacent grooves separating at a distance d constructively interfere at an angle θ to the horizontal, as a result, a bright fringe will be produced at a distance y from the central maximum. The light from path ① travels a distance δ less than the light from path ②.

When the distance D from the diffraction grating to the fringes is significantly larger than the slit width d, the angle θ' can be estimated to have the same value as the angle of diffraction, θ. This is true in this experiment as d is to the order of 10^{-5}m whereas D is about 10 cm. Therefore, as shown in Figure 2, the path difference δ of the waves can be represented by:

$$\delta = d \cdot sin\theta\ . \tag{2}$$

As the maxima appear only when the path difference is an integer multiple of the wavelength λ of the waves, which means at the positions of the maxima, θ must satisfy the:

$$d \cdot sin\theta = \ m \cdot \lambda\ , \tag{3}$$

where m is the order of diffraction.

When a maximum occurs, light with varying wavelengths will exhibit different angles of diffraction, which increase with the wavelength of the light, resulting in wider fringe spreading. By measuring the angle of diffraction at different orders, the wavelength of light with a particular colour can be determined.

III. Methods

The device I used in this experiment was a spectrometer, containing a collimator, a rotational spectrometer table, and a telescope. Its top view is shown in Figure 4.

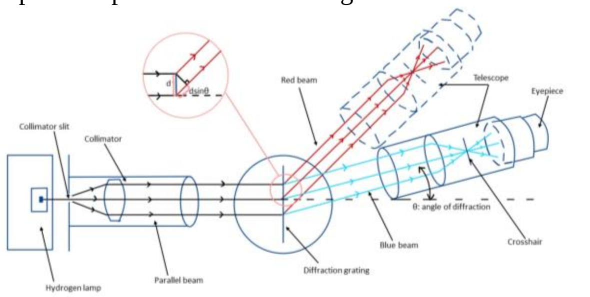

Fig 4. A narrow slit located at the focal point of the collimator lens allowed the light emitted from the hydrogen lamp to enter. The collimator produced a thin, parallel beam of light, ensuring the light struck the diffracting grating at a consistent angle of incidence [9]. The diffracting grating bent the beam composed of various colours, with each colour diffracting at a distinct angle.

A. *Instrument focusing and alignment*

The first step of the experiment is to focus the telescope. To do so, I adjusted its focus until the sharp image of a distant object can be seen on the crosshair between the eyepiece and the camera inside the telescope.

It is also important to align the apparatus to ensure that light enters the diffraction grating perpendicularly. The vernier scale on the rotational table allowed me to note down the exact angle of the grating mount and the telescope thus enabling me to rotate these components by a precise angle.

I started by aligning the collimator and telescope with the aid of a yellow sodium bulb illuminating the collimator slit. When the slit's image appeared in the middle of the crosshair, the collimator was positioned at a 180° angle to the telescope. I then precisely rotated the telescope 90° and set a mirror in the grating holder. The entry slit appeared as a reflected image in the eyepiece when I turned the grating table to a particular angle, indicating that the mirror surface is at a 45° angle to the incident beam from the collimator [6]. Finally, by rotating the mirror by 45° and replacing a diffraction grating for the mirror, the light incident from the collimator and the grating was perpendicular to each other.

B. *Measuring the angle of diffraction*

As the hydrogen lamp emits red, cyan, and blue light in the visible spectrum, its diffraction pattern can be shown in Figure 3.

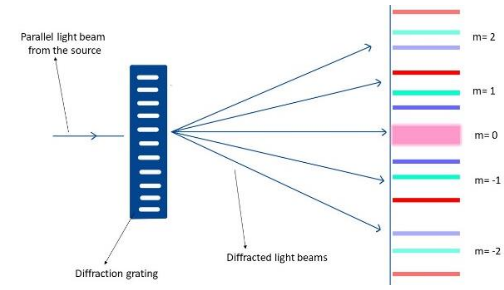

Fig 3. The red light with the longest wavelength spreads out the greatest and has the biggest angle of diffraction in each order where maxima appear, followed by the cyan light and the blue light. The three lights' central maxima are all present in the same spot, where they combine to create a pink fringe that can be seen in the center.

To measure the angle of diffraction, I first adjusted the slit width of the collimator slit to generate a clear and separated pattern of fringes. Then, the undeflected beam's vernier reading, θ_0, was first since all angles of diffraction were calculated with respect to that initial reading [6], as shown in Figure 5.

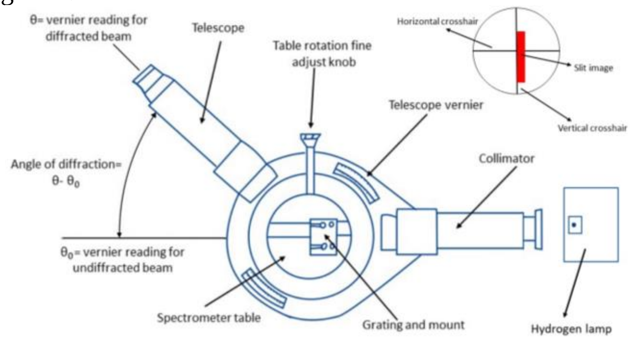

Fig 5. The telescope vernier was used to measure the angles of diffraction when examining a light coming from the hydrogen lamp. The position of the central maximum for all lights was where the pink fringe occurred, and the reading on the vernier scale when the vertical crosshair was aligned with one edge of this fringe was θ. The diffracted light can be captured at highly accurate and measured angles by spinning the telescope.

For each fringe with the same colour, I recorded their readings on the vernier scale as θ. When taking measurements, I made sure every fringe's left edge was aligned to the vertical crosshair, to reduce the random error in my measurements. Thus, the angle of diffraction, θ', can be calculated by:

$$\theta' = \ \theta - \theta_0 . \tag{4}$$

Based on equation (3), the diffracted angle for a specific wavelength increases as the slit width, d, of a diffraction grating increases. This would help to reduce the percentage uncertainty in each measurement of angle. As a result, I opted to use a diffraction grating with a wider slit width, which had 78.8 lines per millimeter, giving a slit width of $1.27 \cdot 10^{-5}$ m.

I observed the angles of diffraction of each light beam in each order. According to equation (3), the gradient of the graph of $d \cdot sin\theta$ against the order number, m, gives the wavelength of the light under investigation. After determining the wavelength of the red, cyan, and blue lights and recognizing their energy levels associated with them, the graph of $\frac{1}{\lambda}$ against $\frac{1}{p^2} - \frac{1}{n^2}$ can be plotted, where its gradient represents the Rydberg constant, according to equation (1).

IV. Data Analysis, Discussions, and Uncertainties

A. *Calculation of the wavelengths*

Using the Polyfit function in Python, I managed to plot the graph of $d \cdot sin\theta$ against the order number, m, for the light beams with three different colours. For the data points that correspond to each colour, I fitted them with a linear function and obtained the equations of the fitted lines. The plots are shown in Figure 6.

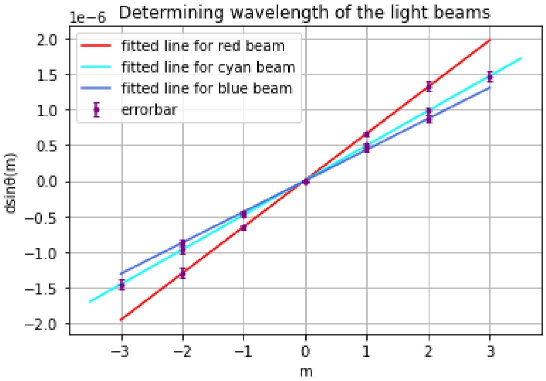

Fig 6. Datapoints representing dsinθ and m of the red, cyan, and blue beam, with their best-fitted lines and the error bars in the y-axis. The red light with the largest wavelength has the largest gradient, followed by the cyan light and the blue light.

The wavelengths of the light beams were determined by computing the gradients of the fitted lines and their uncertainties, as given in Table I. I also deduced the final energy level, n, and the initial energy level, p, associated with each radiation using a standard colour spectrum chart. Because the light with the longest wavelength had the least amount of energy, it fell from the lowest energy level. This is consistent with what I mentioned in the theory session.

TABLE I

Colour	Wavelength (nm)	n	p
Red	654.11± 1.15	3	2
Cyan	486.95± 0.38	4	2
Blue	434.56± 1.26	5	2

B. *Calculation of the Rydberg constant*

I plotted the graph of $\frac{1}{\lambda}$ against $\frac{1}{p^2} - \frac{1}{n^2}$, and fitted the points with a linear function, as shown in Figure 7.

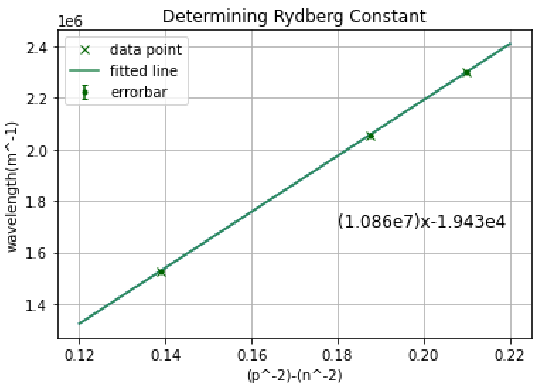

Fig 7. The data points, the fitted straight line and the error bars in the y-axis. The error bars are almost unseen because the percentage error in $\frac{1}{\lambda}$ was tiny (~ 0.1%). The gradient of this graph represents the Rydberg constant.

Therefore, the Rydberg constant was calculated to be $1.0866 \pm 0.01256 \cdot 10^7$ m^{-1}. Compared to its theoretical value of 10973731.6 m^{-1}, there is a percentage difference of 0.99%.

C. *Uncertainties and discussions*

The sole measurement included in this experiment was the angle of diffraction. As a result, all uncertainties were derived from systematic or random errors in measuring this angle. The measurement apparatus, a vernier scale, had a division of 1/60 degrees (1 minute), although its uncertainty in practice was undoubtedly larger. It was difficult to discern whether two lines on the top and lower scales aligned the most closely while reading the vernier scale. Since there were often 3~5 pairs of lines aligned, I recorded the angle corresponding to the central lines as the measured value and the range of these lines as the uncertainty. Angles could not be expressed with limited decimal places due to the involvement of 1/60 degrees in the measurements. Since the spectrometer could detect the diffraction relatively precisely, I decided to confine my data and all subsequent calculations to 5 significant figures.

Throughout the experiment, I measured the angle from the left edge of each fringe, which introduced fewer random errors than measuring the angle from the center of the fringes. My choice of the 78.8 lines per mm grating over the 300 lines per mm grating ensured a wider separation of fringes and lowered the percentage uncertainty in the measurement of the diffracted angle. However, this limited the number of maxima visible via the eyepiece, and I was only able to collect data up to the 2nd order maxima for the red and blue beams and the 3rd order maxima for the cyan beam. Taking fewer data points means the random errors cannot be averaged out sufficiently. Therefore, it is uncertain if this decision was worthwhile.

Since $d \cdot sin\theta$ is a function of the angle of diffraction, θ, knowing the uncertainty in each θ allows the calculation of the uncertainty of $d \cdot sin\theta$, by the virtue of:

$$\sigma_z = \sqrt{(\frac{df}{dx})^2 \sigma^2 x} , \quad (5)$$

where z and x correspond to $d \cdot sin\theta$ and θ respectively.

The covariant matrix generated by Python during fitting $d \cdot sin\theta$ and m provided the uncertainty in the gradient of each curve, which considered the uncertainty of $d \cdot sin\theta$. These values were below 0.5% of the wavelength so I would say my measurements were relatively precise in terms of getting a small uncertainty in the wavelengths of the light beams. Using equation (5) to compute the uncertainty in $\frac{1}{\lambda}$ and following the same process to obtain the uncertainty of the Rydberg constant allowed me to calculate the percentage uncertainty in my final result of R_∞ as 1.12%. The theoretical value of R_∞ falls in this range, indicating my result was relatively accurate.

V. Conclusion

In this experiment, I measured the Rydberg constant to be $1.0866 \pm 0.01256 \cdot 10^7$ m^{-1} and I showed this value was consistent with the theoretical value of R_∞. However, several evaluations can be made on my measuring techniques. I could take a picture of the vernier scale horizontally and zoom in to obtain a clearer reading. Additionally, I may conduct the experiment using both the 78.8 and 300 lines per millimeter gratings to obtain a larger database. By doing this, the random errors in my measurements would be further averaged out. This experiment can be applied to analyze the spectrum of other atoms, so their electronic structures can be explored.

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