[CS502]-Graph Theory and Applications		Fall 2022-23

Independent Study

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SUDOKU SOLVING USING PATTERNS AND GRAPH THEORY

1.INTRODUCTION

Sudoku is a puzzle that has grown in popularity since 2005. One criterion for solving a Sudoku puzzle is to combine reasoning with experimenting. Additional math is given in the background, such as the computational complexity involved in solving Sudokus and the use of combinatory to verify that Sudoku puzzle frameworks are genuine.

A 9 by 9 square framework with 81 cells makes up the classic Sudoku adaption. There are nine 3 by 3 squares divided up throughout the lattice. There are numbers from the set ”1,2,3,4,5,6,7,8,9” in some of the 81 cells. Givens are the cells that have been filled in. The goal is to use the nine digits to fill in the complete lattice so that each number appears exactly once on each line, segment, and square. On the lines, segments, and square, we refer to this necessity as the One Rule.

The below-portrayed riddle is known as a Sudoku of rank 3. A Sudoku of rank n is a $n^2 \times n^2$ square lattice, partitioned into n^2 blocks, every one of size $n \times n$. The numbers used to fill the network in are 1, 2, 3, ..., n^2 , and the One Rule actually applies.

	7	5		9				6
	2	3		8			4	
8					3			1
5			7		2			
	4		8		6		2	
			9		1			3
9			4					7
	6			7		5	8	
7				1		3	9	

1	7	5	2	9	4	8	3	6
6	2	3	1	8	7	9	4	5
8	9	4	5	6	3	2	7	1
5	1	9	7	3	2	4	6	8
3	4	7	8	5	6	1	2	9
2	8	6	9	4	1	7	5	3
9	3	8	4	2	5	6	1	7
4	6	1	3	7	9	5	8	2
7	5	2	6	1	8	3	9	4

Here’s an example of a sudoku and its solution.

Sudoku puzzles are currently becoming more and more popular among people all around the world. Numerous engineers have tried to develop significantly more complex and intriguing riddles as the game has become mainstream in a large number of countries. The game is now mentioned in almost every newspaper, book, and website.

2.PENCIL-AND-PAPER ALGORITHM

The pencil-and-paper calculation is formed dependent on human procedures. This implies that the calculation is executed dependent on human insights. In this way the name of the solver is pencil-and-paper calculation. The productivity of the suggested computation is then evaluated by comparison with the Brute power calculation and comparison with this calculation as a whole. The savage power is a general computation that can be used to solve any problem that can be imagined. Any possible configurations are created by this calculation until the right solution is identified.

NAKED SINGLES:

When the contents of various squares on a similar line, segment, and box are taken into consideration, this strategy is useful for locating a square that can only accept one single value. Additionally, at this time, the line, section, and box each contain 8 distinct numbers, leaving just one number for that square to display as.

	a	b	c	d	e	f	g	h	i
1				1		4			
2			1				9		
3		9		7		3		6	
4	8		7				1		6
5							3		
6	3		4				5		9
7		5		4		2		3	
8			8				6		
9				8		6			

	a	b	c	d	e	f	g	h	i
1				1		4			
2			1				9		
3		9		7		3		6	
4	8	2	7				1		6
5							3		
6	3		4				5		9
7		5		4		2		3	
8			8				6		
9				8		6			

A description of the naked single method. In the left figure square 4b can hold just one possible number, which is 2 as it is inserted in the right figure.

As we find in above figures, it is feasible to list every one of the applicants from 1 to 9 in each unfilled square, for example square 4b can just hold number 2 since it is the lone contender for this position. The main perspective is that when an up-and-comer is found for a specific position then it very well may be taken out from the rundown as a potential competitor in the line, section and box [7]. The explanation that it is known as the ”exposed single” technique is that this sort of square contains just a single conceivable up-and-comer

NAKED PAIRS & TRIPLETS:

The exposed single technique is quite comparable to these strategies, but in this strategy we find identical two applications in two squares. Using this information, we may identify probable connections between various squares.

	a	b	c	d	e	f	g	h	i
1	9	6			1			3	
2	3		3				9		4
3								9	6
4				3		8			
5	6		9					8	5
6				4		9			
7		2		5	8	4		6	
8	5		8				2		7
9		4		2,7	9	2,7	3		5

For instance in the above figure , squares 9d and 9f can just contain values 2 and 7. By having this information, clearly square 9d and 9f can't contain 1 or 6 so those applicants are taken out. The solitary applicants are 2 and 7 in squares 9d and 9f.

3. PENCIL-AND-PAPER SOLVER

Human players of Sudoku have a limited number of strategies they use. However, it could be challenging to put each of these strategies into practice. Since a human player has a better understanding of the complete Sudoku board than the PC programming does, it is discovered that the hidden single strategy or pair technique is difficult to apply in PC programming. This is due to the way a human player can filter two lines or two sections to determine whether a certain digit is allowed to be in an empty square that needs to be filled in with the correct number. The aforementioned PC programming activity requires significant time to complete.

The strategies that are utilized in this calculation are the accompanying:

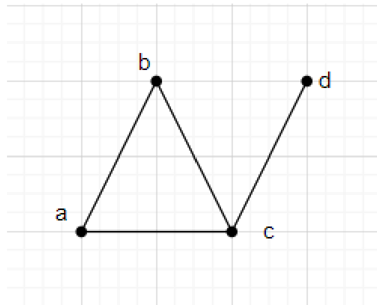
1. Unique missing candidate
2. Naked single method
3. Backtracking

4.THE RELATIONSHIP BETWEEN SUDOKU AND GRAPH

Let's go over some fundamental definitions because we'll be using graph theory to solve sudoku.

GRAPH

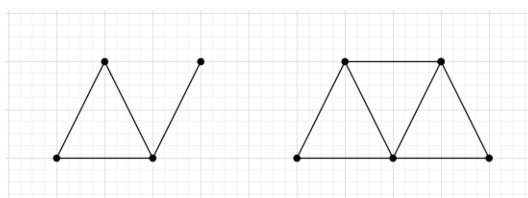
A graph G=(V, E) consists of a set of objects called vertices, and another set whose elements E = $\{ e_1, e_2, ... \}$ are called edges, such that each edge e_k is identified with an unordered pair (v_i, v_j) of vertices. The vertices v_i, v_j associated with edge e_k are called the end vertices of e_k .



For example, above figure represents the graph G whose vertex set V (G) is {a, b, c, d}, and whose edge set E(G) consists of the edges ab, ac, bc and cd. The numbers of elements in V (G) and E(G) are denoted by $|V(G)|$ and $|E(G)|$ respectively.

SUB GRAPH

A graph g is said to be a subgraph of graph G if all the vertices and all the edges of g are in G, and each edge of g has the same end vertices in g as in G.

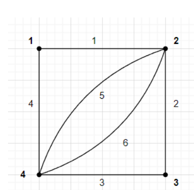


ADJACENT GRAPH

In a graph, two vertices are said to be adjacent, if there is an edge between the two vertices. Let V= /((V, E) /) be a graph with V = { $v_1, v_2, v_3, ...v_n$}, E = { $e_1, e_2, e_3, ...e_n$} and without parallel edges. The adjacency matrix of G is an n x n Symmetric binary matrix X = [x_{ij}] defined over the ring of integers such that

$$x_{\text{ij}} = \begin{cases} 1; & \text{if } v_i v_j \in \text{E.} \\ 0; & \text{otherwise.} \end{cases}$$

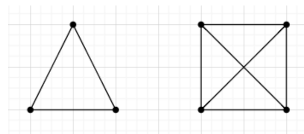
Figures below shows a labeled graph G with its adjacency matrix A and incidence matrix respectively



$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 2 \\ 0 & 1 & 0 & 1 \\ 1 & 2 & 1 & 0 \end{pmatrix} \quad \mathbf{M} = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 1 \end{pmatrix}$$

COMPLETE GRAPH

A simple graph G is said to be complete if every vertex in G is connected with every other vertex. i.e., if G contains exactly one edge between each pair of distinct vertices. A complete graph is usually denoted by K_n.



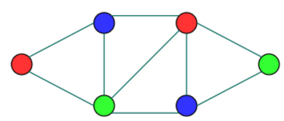
It should be noted that Kn has exactly $\frac{n(n-1)}{2}$ edges. K3 and K4 are shown in the above figures.

Filling the table with the numbers must follow these rules:

1. Numbers in rows are not repeated.
2. Numbers in columns are not repeated.
3. Numbers in 3 ×3 blocks are not repeated.
4. Order of the numbers when filling is not important.

GRAPH COLOURING:

Graph Coloring is the assignment of colors to vertices of a graph such that no two adjacent vertices have the same color.



5.CONVERTING SUDOKU TO GRAPH COLORING

1. The graph will have 81 vertices with each vertex corresponding to a cell in the grid.

2. Two distinct vertices will be adjacent if and only if the corresponding cells in the grid are either in the same row, or same column, or the same sub-grid.

3. Each completed Sudoku square then corresponds to a k-coloring of the graph.

Consider an n_2xn_2 grid, To each cell in the grid, we associate a vertex labeled (i, j) with $1 \leq i, j \leq n_2$.
We will say that (i, j) and (i', j') are adjacent if i= i'or j = j' or [i/n]= [i'/n]and [j/n]= [j'/n].

Graph is called regular if the degree of every vertex is the same.

Each vertex has degree 20, thus the number of edges is: $|H| = 20 * 81 / 2 = 810$

[1,1]	[1,2]	[1,3]	[1,4]
[2,1]	[2,2] **3**	[2,3]	[2,4]
[3,1] **1**	[3,2]	[3,3] **2**	[3,4]
[4,1]	[4,2]	[4,3]	[4,4] **4**

The following theorems are some theorems we are going to use in the later part.

Theorem: An n x n Sudoku must have at least n - 1 starting colors.

Theorem: Let G be a graph with its chromatic number X(G) and let C be a partial coloring of G using X(G) - 2 colors. If the partial coloring can be finished to a proper coloring of G, then there must be at least two different ways of completing the coloring.

GRAPH COLORING TECHNIQUE

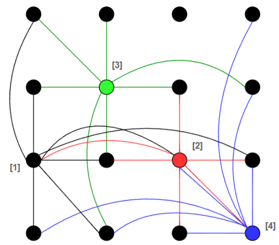
The whole algorithm can be divided into the following steps:

1. The vertex that is as of now hued is chosen and connected by edges of same tone with any remaining vertices of sets in which the vertex is found. These vertices can at this point don't be hued with a similar tone. This is rehashed for all the vertices for which clues are given.

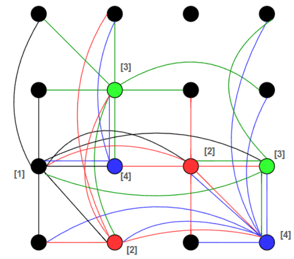
2. The vertices where the biggest number of shaded edges merge are discovered (all things considered, there will be just a single competitor).

3. If there are vertices among them that can be shaded exclusively by one tone, at that point they are hued with it and the strategy proceeds from the initial step (there is no compelling reason to bring those edges into the diagram that lead to a vertex where there is now another edge of a similar tone). In the event that there are no such vertices, the technique proceeds with the fourth step.

4. From the arrangement of those chosen vertices, the one that is nearby the biggest number of uncolored vertices is picked and hued to the tone with the most minimal worth that isn't utilized for its neighbors. In the event that there are all the more such vertices one of them is chosen randomly. In the subsequent stage the technique proceeds from the initial step.



Sudoku graph after step one



Sudoku graph after step two

6. CONCLUSION

The results of this investigation demonstrate that the pencil-and-paper algorithm may be used to solve any Sudoku puzzle. The algorithm is a good way to locate answers more quickly and effectively, and graph colouring is a good way to identify strategies and solve Sudoku puzzles quickly.

7. REFERENCES

[1] J.F. Crook, A pencil and paper algorithm for solving Sudoku Puzzles, [Cited 2013 February , Winthrop University, Webpage: http://www.ams.org/notices/200904/tx090400460p.pdf

[2] Drawing tool: https://www.diagrams.net/