

## 15. Continuous Random Variables

[Ross S5.1]

We saw random variables where the set of possible outcomes was discrete. In some cases, a random variable can take a continuum of values:

$X$  = time at which a train arrives

$Y$  = voltage across a resistor

$Z$  = rainfall measured in mm

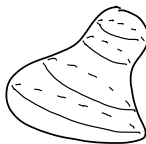
**Definition 15.1:** We say  $X$  is a continuous random variable if there is a non-negative function  $f_X(x)$  such that

$$P[X \in B] = \int_B f_X(x) dx = \int_B f_X(u) du$$

$f_X(x)$  is called **probability density function** (pdf).

[ Textbook omits subscript  $X$  on  $f_X(x)$ ...]

This is similar to mass density: if I know  $\rho(x)$ , the **density of mass** in  $\text{kg/m}^3$  at every point  $x \in \mathbb{R}^3$ , then the mass inside any volume  $V$  is:

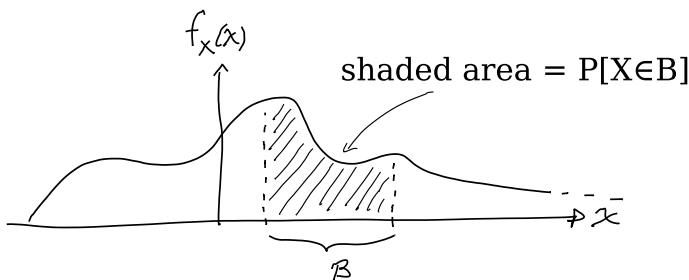


3D volume  $V \subset \mathbb{R}^3$   
density  $\rho(\underline{x})$  at  $\underline{x} \in \mathbb{R}^3$

$$m(V) = \iiint_V \rho(\underline{x}) \, d\underline{x}$$

$f_X(x)$  is similar, except it measures the *density of probability*, not mass:

$$P[X \in B] = \int_B f_X(x) \, dx$$



Since  $X$  must take some value:

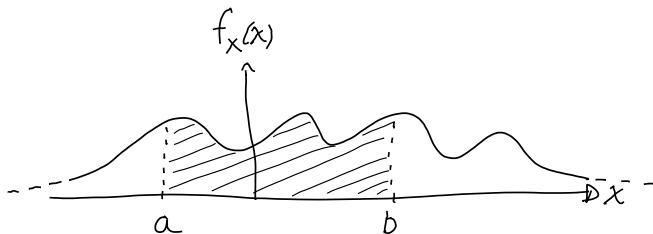
$$1 = P[X \in (-\infty, \infty)] = \int_{-\infty}^{\infty} f_X(x) \, dx. \quad (15.1)$$

*Note:* Say  $X$  has units of kg. Since  $dx$  has units of kg,  $f_X(x)$  has units of  $\text{kg}^{-1}$ .

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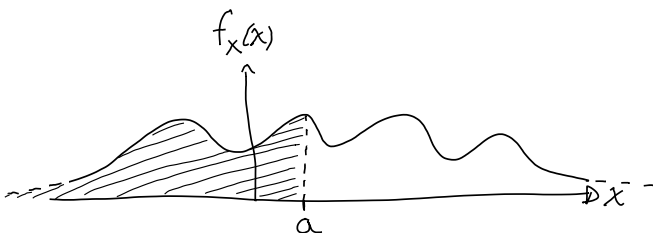
Once we know  $f_X(x)$ , all probability statements about  $X$  can be answered:

$$1) P[X \in [a, b]] = \int_a^b f_X(x) dx$$



$$2) P[X = a] = P[X \in [a, a]] = \int_a^a f_X(x) dx = 0$$

$$3) F_X(a) = P[X \leq a] = P[X \in (-\infty, a]] = \int_{-\infty}^a f_X(x) dx$$



$$4) f_X(a) = \frac{d}{da} F_X(a)$$


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**Example 15.1:** The lifetime of a motor in months is a random variable with pdf

$$f_X(x) = \begin{cases} \lambda e^{-x/100} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

for some unknown constant  $\lambda$ . What is the probability that it functions for

a) between 50 and 150 months?

b) fewer than 100 months?

*Solution:*

**Example 15.2:** Let  $X$  have pdf  $f_X(x)$ , and  $Y = 2X$ . Find  $f_Y(y)$ .

*Solution:*