30. Properties of Expectations

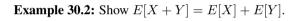**Expectation of Sums of Random Variables** [Ross S7.2]

Recall that the mean value of X is

$$E[X] = \begin{cases} \sum_x x p_X(x) & X \text{ is discrete} \\ \\ \int_{-\infty}^{\infty} x f_X(x) dx & X \text{ is continuous} \end{cases}$$

Proposition 30.1 *Let X and Y be two random variables. Let $g(x, y)$ be a function. Then*

$$E[g(X, Y)] = \begin{cases} \sum_y \sum_x g(x, y) p_{XY}(x, y) & X,\ Y \text{ are discrete} \\ \\ \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x, y) f_{XY}(x, y) dx dy & X,\ Y \text{ are continuous} \end{cases}$$

Example 30.1: The positions $X \sim U(0, L)$ and $Y \sim U(0, L)$ of two persons on a road are independent. What is the mean distance between them?

Solution:

Example 30.2: Show $E[X + Y] = E[X] + E[Y]$.

Solution: [Continuous case only, discrete is similar]

Note: by induction, $E[X_1 + \cdots + X_n] = E[X_1] + \cdots + E[X_n]$.

Example 30.3: Let $X_1, X_2, \ldots, X_n$ be iid with (common) mean μ. The

quantity

$$\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$$

is called the **sample mean**. What is $E[\bar{X}]$?

Solution:

Note: Since $E[\bar{X}] = $ true mean μ, we say $\bar{X}$ is an **unbiased estimate** of μ.

Example 30.4: 12 friends play a game. Each has a ball and picks one of 10 targets randomly, independently of the others. Each has probability p of hitting their chosen target.

Find the expected number of targets not hit.

Solution: