

Common Random Variables:

- $X \sim \text{Binomial}(n, p)$: $0 \leq p \leq 1, n \in \{1, 2, \dots\}$

$$p_X(k) = \binom{n}{k} p^k (1-p)^{n-k} \quad k \in \{0, 1, \dots, n\}$$

$$E[X] = np$$

$$\text{Var}[X] = np(1-p)$$

$$M_X(t) = (pe^t + 1 - p)^n$$

- $X \sim \text{Geometric}(p)$: $0 \leq p \leq 1$

$$p_X(k) = p(1-p)^{k-1} \quad k \in \{1, 2, \dots\}$$

$$E[X] = \frac{1}{p}$$

$$\text{Var}[X] = \frac{1-p}{p^2}$$

$$M_X(t) = \frac{pe^t}{1 - (1-p)e^t}$$

- $X \sim \text{Poisson}(\lambda)$: $\lambda > 0$

$$p_X(k) = \frac{\lambda^k}{k!} e^{-\lambda} \quad k \in \{0, 1, 2, \dots\}$$

$$E[X] = \lambda$$

$$\text{Var}[X] = \lambda$$

$$M_X(t) = \exp(\lambda(e^t - 1))$$

- $X \sim U(a, b)$: $b > a$

$$f_X(x) = \begin{cases} \frac{1}{b-a} & a < x < b \\ 0 & \text{else} \end{cases}$$

$$E[X] = \frac{a+b}{2}$$

$$Var[X] = \frac{(b-a)^2}{12}$$

$$M_X(t) = \frac{e^{tb} - e^{ta}}{t(b-a)}$$

- $X \sim \text{Exp}(\lambda)$: $\lambda > 0$

$$f_X(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{else} \end{cases}$$

$$E[X] = \frac{1}{\lambda}$$

$$Var[X] = \frac{1}{\lambda^2}$$

$$M_X(t) = \frac{\lambda}{\lambda - t}$$

- $X \sim \mathcal{N}(\mu, \sigma)$: $\sigma > 0$

$$f_X(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

$$E[X] = \mu$$

$$Var[X] = \sigma^2$$

$$M_X(t) = \exp\left(\mu t + \frac{\sigma^2 t^2}{2}\right)$$