

8. Random Variables (rv)

Random Variables [Ross S4.1]

After an experiment is done, we are often interested in a function of the outcome:

- e.g., sum of two dice rolls
- e.g., number of heads after flipping 10 coins

A function that maps each outcome $s \in S$ to a real number is called a **random variable** [abbreviated as rv].

Example 8.1: Let $S = \{(1, 1), (1, 2), \dots, (6, 6)\}$ be outcomes of two dice rolls.

For $s = (a, b) \in S$ we have $X(s) = a + b$ is a random variable.

We often write X instead of $X(s)$ since s and S are clear from context, or don't matter.

Example 8.2: Toss 3 coins. Let $X = \#$ of heads. Then X is a rv that can only take values 0, 1, 2 or 3.

$$\{X = 0\} = \{ttt\}$$

$$\{X = 1\} = \{tth, tht, htt\}$$

$$\{X = 2\} = \{hht, hth, thh\}$$

$$\{X = 3\} = \{hhh\}$$

and

$$P[\{X = 0\}] = P[X = 0] = 1/8$$

$$P[X = 1] = 3/8$$

$$P[X = 2] = 3/8$$

$$P[X = 3] = 1/8$$

Note: Since $\{X = 0\}, \{X = 1\}, \{X = 2\}, \{X = 3\}$ are disjoint and cover all possible outcomes for X :

$$\sum_{i=0}^3 P[X = i] = 1.$$

Example 8.3: [Cover if time] Let E and F be independent events with:

$$P[E] = 0.1, \quad P[F] = 0.2$$

Let $Y = \#$ events that have occurred. Then

$$P[Y = 0] = P[E^c F^c] = P[E^c]P[F^c] = 0.9 \times 0.8$$

$$P[Y = 1] = P[EF^c \cup E^c F] = P[EF^c] + P[E^c F] = 0.1 \times 0.8 + 0.9 \times 0.2$$

$$P[Y = 2] = P[EF] = P[E]P[F] = 0.1 \times 0.2$$

Example 8.4: A flipped coin has probability p of being heads. We flip the coin until a head occurs, up to a max of n flips. Let $Z = \#$ of flips. Then

$$P[Z = 1] = P[h] = p$$

$$P[Z = 2] = P[th] = (1 - p) \times p$$

$$P[Z = 3] = P[tth] = (1 - p)^2 \times p$$

$$P[Z = n - 1] = P[n - 2 \text{ tails followed by heads}] = (1 - p)^{n-2} \times p$$

$$P[Z = n] = P[n - 1 \text{ tails followed by anything}] = (1 - p)^{n-1}$$