1. Introduction

What is probability theory?

How is it related to statistics?

Probability Theory $\neq$ Statistics

This is similar to how Math $\neq$ Physics.

Physics: Physics uses math to create models of nature [i.e., hypotheses]. Physics tests the models based on measured data.

Statistics: Statistics is the science of collecting, analyzing, presenting, and interpreting data.[1]

Probability Theory: The branch of mathematics concerned with the analysis of random phenomena.[2]

Statistics uses probability theory to create models/hypotheses for how data is generated. Statistics uses these models and data to infer conclusions or make predictions.

Example 1.1: We all throw our hat into the center of the room, and pick one back at random. What is the probability that at least one of us gets our hat back?

Is this probability theory or statistics?

Answer: [Probability Theory]

[1] source: https://www.stat.uci.edu/what-is-statistics/
[2] source: Encyclopedia Britannica

Example 1.2: In a radar system there are two possibilities (or hypotheses): there is a plane, or there is no plane. For each hypothesis, you create a model for how the data you collect should behave.

You now collect data, and from this, infer which hypothesis you believe to be true.

Probability theory or statistics?

Ans: Statistics

Second Ans: Probability theory is used to model how data is generated in each case.

Example 1.3: In an election, $x\%$ of voters will vote for candidate A, and $(100 - x)\%$ will vote for B.

A polling company asks 1000 people their voting intention and based on this, makes an estimate $\hat{x}$ of x.

Is this prob. theory or statistics?

This is a model parameterized by x, and the company is estimating the model parameter x from the measured data $\rightarrow$ statistics.

In this example, we can also use probability theory to compute the prob. that the estimate is within a tolerance of the true value x, i.e., prob. that $|x - \hat{x}| < 3\%$

Example 1.4: In a bank, there is one common line for all tellers, whereas in a grocery store, each checkout counter has its own line.

Which is more efficient and why?

Is this prob. theory or statistics?

Answer: Probability Theory

Example 1.5: An online retailor has data on past purchases of all customers, including your past purchase history.

From this data, they try to infer what else you might be interested in buying.

Is this prob. theory or statistics?

Answer: Statistics

Example 1.6: In a room of 100 people, the odds that 2 people share the same birthday is not $100/365$ but $\approx 99.99997\%$.

Is this prob. theory or statistics?

Answer: Probability Theory

Example 1.7: A radar system measures the position of a car at various times, and makes a prediction of where the car will be over the next 2 seconds.

Is this prob. theory or statistics?

Answer: Statistics

Example 1.8: A radar system measures the position of a rocket car at various times, and based on this, the car is steered by computer.

What area is this?

Answer: Stochastic control.

Example 1.9: In a digital communication system, you wish to infer if the transmitted bit is a 1 or a 0 from a noisy observation of this bit.

What area is this?

Answer: Digital communications

In ECE 203, you will learn mostly probability theory.

In ECE 307, you will learn mostly statistics.

Example applications that use probability theory:

- Quantum mechanics (e.g., quantum computing)
- Machine learning (e.g., reinforcement learning)
- Stochastic control (e.g., autonomous vehicles, Kalman filter)
- Digital/wireless communications
- Statistical signal processing
- Statistical inference / Detection and estimation theory
- Randomized algorithms
- Stochastic processes (e.g., queueing theory)
- Finance (e.g., risk management, derivative pricing)