4. Sample Spaces with Equally Likely Outcomes

[Ross S2.5]

Say $S = \{1, 2, \ldots N\}$.

Then $$1 = P[S] = P[1] + P[2] + \cdots + P[N]. \tag{4.1}$$

If each outcome is equally likely:

$$P[1] = P[2] = \cdots = P[N] \tag{4.2}$$

Combining (4.1) and (4.2):

$$P[1] = P[2] = \cdots = P[N] = 1/N \tag{4.3}$$

Then, for any subset $E \subset S$:

$$P[E] = P\left[\bigcup_{i \in E} \{i\}\right] = \sum_{i \in E} P[i] = \sum_{i \in E} 1/N = |E|/N = |E|/|S|.$$

Example 4.1: If 2 dice are rolled, what is the probability that the sum is 9? Assume equally likely outcomes.

Solution:

Example 4.2: An urn has 7 white balls and 5 black balls.

If we draw 3 balls at random, what is the probability that 1 is white and 2 are black?

Solution:

Order doesn't matter:

Order matters:

These problems all boil down to counting combinations. I'll assume you learned counting in ECE108 and skip the topic, except for the next problem which is a nice application of the inclusion/exclusion principle.

Example 4.3: [Cover if time] Matching Problem

Each of n persons throws their hat into the center of a room and picks a hat at random.

What is the probability that no person selects their own hat? [Hard]

Solution: