23. Jointly Distributed Random Variables

Examples [Ross S6.1]

Example 23.1: The joint pdf of X and Y is given by

$$f_{XY}(x,y) = \begin{cases} 2e^{-x}e^{-2y} & x > 0 \text{ and } y > 0 \\ 0 & \text{else} \end{cases}$$

Compute

a) $P[X > 1, Y < 1]$

b) $P[X < Y]$

c) $P[X < a]$ (assume $a > 0$)

Solution:

Example 23.2: Given $R > 0$, consider the joint pdf

$$f_{XY}(x,y) = \begin{cases} c & \text{if } x^2 + y^2 \leq R^2 \\ 0 & \text{else} \end{cases}$$

for some $c > 0$.

a) Find c.

b) Find the marginal pdf of X.

c) Let $D = \sqrt{X^2 + Y^2}$ be the distance of the pair (X, Y) from the origin. Find $P[D \leq a]$.

d) Find $E[D]$.

Note: This is the uniform distribution on a disk of radius R.

Solution:

Example 23.3: [Cover if time] The joint pdf of X and Y is

$$f_{XY}(x,y) = \begin{cases} e^{-(x+y)} & x > 0 \text{ and } y > 0 \\ 0 & \text{else} \end{cases}$$

Find the pdf of $Z = X/Y$.

Solution: