34. Properties of Expectations

Computing Probabilities by Conditioning

We can use conditioning to compute probabilities:

Let A be an event.

Let random variable $Y \in \{y_1, y_2, \ldots\}$ and $B_i = \{Y = y_i\}$.

Then $B_1, B_2, \ldots$ partition the sample space S. So by law of total probability:

$$\begin{aligned} P[A] &= P[A|B_1]P[B_1] + P[A|B_2]P[B_2] + \cdots \\ &= P[A|Y = y_1]P[Y = y_1] + P[A|Y = y_2]P[Y = y_2] + \cdots \\ &= \sum_n P[A|Y = y_n]P[Y = y_n] \end{aligned}$$

Similarly, if Y is continuous:

$$P[A] = \int_{-\infty}^{\infty} P[A \mid Y = y] f_Y(y) dy$$

Example 34.1: Say X and Y are independent random variables with densities $f_X(x)$ and $f_Y(y)$.

Find $P[X < Y]$.

Solution:

Example 34.2: Say X and Y are independent with densities $f_X(x)$ and $f_Y(y)$. Find the cdf and pdf of $X + Y$ by conditioning on Y.

Solution: