5.1 $(5E)^{-1}$

$$, \quad E = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$5E = \begin{pmatrix} 5 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 5 \end{pmatrix} \equiv A$$

$$A^{-1} = A_*^T / \det A$$

$$\det A = 5 \begin{vmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 5 \end{vmatrix} = 5 \cdot 5 \begin{vmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{vmatrix} = 5 \cdot 5 \cdot 5 \begin{vmatrix} 5 & 0 \\ 0 & 5 \end{vmatrix} = 5^5 = 3125$$

$$A_* = \begin{pmatrix} A_{00} & A_{01} & A_{02} & A_{03} & A_{04} \\ A_{10} & \cdots & & & \\ \vdots & & & & \\ A_{40} & \cdots & & & \end{pmatrix} \qquad A_{ik} = (-1)^{k+i} \cdot M_k^i$$

$$A_{00} = 1 \cdot \begin{vmatrix} 5 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 5 \end{vmatrix} = 5^4 = 625 = A_{11} = A_{22} = A_{33} = A_{44}$$

$$A_{01} = -1 \begin{vmatrix} 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 5 \end{vmatrix} = 0 = A_{02,03,04} = A_{10,13,14,12} = A_{20,21,23,24} = A_{30,31,32,34}$$

$$(5E)^{-1} = \frac{1}{3125} \begin{pmatrix} 625 & 0 & 0 & 0 & 0 \\ 0 & 625 & 0 & 0 & 0 \\ 0 & 0 & 625 & 0 & 0 \\ 0 & 0 & 0 & 625 & 0 \\ 0 & 0 & 0 & 0 & 625 \end{pmatrix} = \begin{pmatrix} \frac{1}{5} & 0 & 0 & 0 & 0 \\ 0 & 1/5 & 0 & 0 & 0 \\ 0 & 0 & 1/5 & 0 & 0 \\ 0 & 0 & 0 & 1/5 & 0 \\ 0 & 0 & 0 & 0 & 1/5 \end{pmatrix}$$

5.2 $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 0 & 6 \\ 7 & 8 & 9 \end{pmatrix}$, $\det A$?

$$\det A = 1 \cdot \begin{vmatrix} 0 & 6 \\ 8 & 9 \end{vmatrix} - 2 \begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} + 3 \begin{vmatrix} 4 & 0 \\ 7 & 8 \end{vmatrix} = -48 - 72 + 84 + 96 = 60$$

Ответ: $\det A = 60$

5.3 (1) $A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 0 & 6 \\ 7 & 8 & 9 \end{pmatrix}$, A^{-1} – ?

$A^{-1} = A_*^T / \det A$, $\det A = 60$, $A_* = \begin{pmatrix} A_{00} & A_{01} & A_{02} \\ A_{10} & A_{11} & A_{12} \\ A_{20} & A_{21} & A_{22} \end{pmatrix}$

$A_{00} = (-1)^0 \cdot \begin{vmatrix} 0 & 6 \\ 8 & 9 \end{vmatrix} = -48$

$A_{01} = (-1)^1 \begin{vmatrix} 4 & 6 \\ 7 & 9 \end{vmatrix} = -36 + 42 = 6$

$A_{02} = (-1)^2 \begin{vmatrix} 4 & 0 \\ 7 & 8 \end{vmatrix} = 32$

$A_{10} = (-1)^1 \begin{vmatrix} 2 & 3 \\ 8 & 9 \end{vmatrix} = -18 + 24 = 6$

$A_{11} = (-1)^2 \begin{vmatrix} 1 & 3 \\ 7 & 9 \end{vmatrix} = 9 - 21 = -12$

$A_{12} = (-1)^3 \begin{vmatrix} 1 & 2 \\ 7 & 8 \end{vmatrix} = -8 + 14 = 6$

$A_{20} = (-1)^2 \begin{vmatrix} 2 & 3 \\ 0 & 6 \end{vmatrix} = +12$

$A_{21} = (-1)^3 \begin{vmatrix} 1 & 3 \\ 4 & 6 \end{vmatrix} = -6 + 12 = +6$

$A_{22} = (-1)^4 \begin{vmatrix} 1 & 2 \\ 4 & 0 \end{vmatrix} = -8.$

$$A_* = \begin{pmatrix} -48 & 6 & 32 \\ 6 & -12 & 6 \\ 12 & 6 & -8 \end{pmatrix} \Rightarrow A_*^T = \begin{pmatrix} -48 & 6 & 12 \\ 6 & -12 & 6 \\ 32 & 6 & -8 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} -4/5 & 1/10 & 1/5 \\ 1/10 & -1/5 & 1/10 \\ 8/15 & 1/10 & -2/15 \end{pmatrix}$$

(2) Матрица 4x4 ранга 1:

$$\begin{pmatrix} 3 & 0 & 0 & 3 \\ 3 & 0 & 0 & 3 \\ 3 & 0 & 0 & 3 \\ 3 & 0 & 0 & 3 \end{pmatrix} \text{ или } \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix} \text{ или } \begin{pmatrix} 5 & 5 & 5 & 5 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 5 & 5 & 5 & 5 \end{pmatrix},$$

или все элементы матрицы равны, и т.д.

5.4. $\vec{a} = (1,5)$

$\vec{b} = (2,8)$. Найти $c = \vec{a} \cdot \vec{b}$.

$c = a \cdot b \cdot \cos\alpha = a_x \cdot b_x + a_y \cdot b_y = 1 \cdot 2 + 5 \cdot 8 = 42$

5.5 $\vec{a} = (1, 5, 0)$

$\vec{b} = (2, 8, 7)$

$\vec{c} = (7, 1.5, 3)$ Найти $(\vec{a} \times \vec{b}) \cdot \vec{c}$

$$\vec{d} = \vec{a} \times \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 5 & 0 \\ 2 & 8 & 7 \end{vmatrix} = \vec{i} \begin{vmatrix} 5 & 0 \\ 8 & 7 \end{vmatrix} - \vec{j} \begin{vmatrix} 1 & 0 \\ 2 & 7 \end{vmatrix} + \vec{k} \begin{vmatrix} 1 & 5 \\ 2 & 8 \end{vmatrix} =$$

$= 35\vec{i} - 7\vec{j} - 2\vec{k}$, т.е.

$\vec{d} = (35, -7, -2)$, тогда

$\vec{d} \cdot \vec{c} = d_x c_x + d_y c_y + d_z c_z = 35 \cdot 7 - 7 \cdot 1{,}5 - 2 \cdot 3 =$

$= 245 - 10{,}5 - 6 = 228{,}5$