332:345 – Linear Systems & Signals – Fall 2016 – S. J. Orfanidis

$f(t)$	$F(s)$	$z = e^{sT}, \quad F(z) = \sum_{n=0}^{\infty} f(nT)\,e^{-snT} = \sum_{n=0}^{\infty} f(nT)\,z^{-n}$
$u(t)$	$\dfrac{1}{s}$	$\dfrac{1}{1-z^{-1}}$
$tu(t)$	$\dfrac{1}{s^2}$	$\dfrac{Tz^{-1}}{(1-z^{-1})^2}$
$t^2u(t)$	$\dfrac{2}{s^3}$	$\dfrac{T^2z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$
$e^{-at}u(t)$	$\dfrac{1}{s+a}$	$\dfrac{1}{1-z^{-1}e^{-aT}}$
$te^{-at}u(t)$	$\dfrac{1}{(s+a)^2}$	$\dfrac{Te^{-aT}z^{-1}}{(1-z^{-1}e^{-aT})^2}$
$t^2e^{-at}u(t)$	$\dfrac{2}{(s+a)^3}$	$\dfrac{T^2e^{-aT}z^{-1}(1+z^{-1}e^{-aT})}{(1-z^{-1}e^{-aT})^2}$
$\dfrac{1}{a}(1-e^{-at})u(t)$	$\dfrac{1}{s(s+a)}$	$\dfrac{(1-e^{-aT})z^{-1}}{a(1-z^{-1})(1-z^{-1}e^{-aT})}$
$tu(t)-\dfrac{1}{a}(1-e^{-at})u(t)$	$\dfrac{a}{s^2(s+a)}$	$\dfrac{z^{-1}(aT+e^{-aT}-1)+z^{-2}(1-e^{-aT}-aTe^{-aT})}{a(1-z^{-1})^2(1-z^{-1}e^{-aT})}$
$e^{-at}u(t)-e^{-bt}u(t)$	$\dfrac{b-a}{(s+a)(s+b)}$	$\dfrac{(e^{-aT}-e^{-bT})z^{-1}}{(1-z^{-1}e^{-aT})(1-z^{-1}e^{-bT})}$
$e^{-at}e^{j\omega_0 t}u(t)$	$\dfrac{1}{s+a-j\omega_0}$	$\dfrac{1}{1-z^{-1}e^{-aT}e^{j\omega_0 T}}$
$e^{-at}\sin(\omega_0 t)u(t)$	$\dfrac{\omega_0}{(s+a)^2+\omega_0^2}$	$\dfrac{z^{-1}e^{-aT}\sin(\omega_0 T)}{1-2z^{-1}e^{-aT}\cos(\omega_0 T)+z^{-2}e^{-2aT}}$
$e^{-at}\cos(\omega_0 t)u(t)$	$\dfrac{s+a}{(s+a)^2+\omega_0^2}$	$\dfrac{1-z^{-1}e^{-aT}\cos(\omega_0 T)}{1-2z^{-1}e^{-aT}\cos(\omega_0 T)+z^{-2}e^{-2aT}}$

$f(n)$	$F(z)$
$\delta(n-D)$	z^{-D}
$u(n)$	$\dfrac{1}{1-z^{-1}}$
$nu(n)$	$\dfrac{z^{-1}}{(1-z^{-1})^2}$
$n^2u(n)$	$\dfrac{z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$
$a^nu(n)$	$\dfrac{1}{1-az^{-1}}$
$na^nu(n)$	$\dfrac{az^{-1}}{(1-az^{-1})^2}$

Laplace transform properties

$f(t-t_0)$	$e^{-st_0}F(s)$	delay
$e^{-at}f(t)$	$F(s+a)$	modulation
$\dot{f}(t)$	$sF(s)-f(0^-)$	t-differentiation
$\ddot{f}(t)$	$s^2F(s)-sf(0^-)-\dot{f}(0^-)$	t-differentiation
$tf(t)$	$-\dfrac{dF(s)}{ds}$	s-differentiation
$f(0^+)$	$\lim_{s\to\infty} sF(s)\,,\quad \lim_{z\to\infty} F(z)$	initial value
$f(\infty)$	$\lim_{s\to 0} sF(s)\,,\quad \lim_{z\to 1}(1-z^{-1})F(z)$	final value

Z-transform properties

$f(n-D)$	$z^{-D}F(z)$	delay
$a^nf(n)$	$F(z/a)$	modulation
$nf(n)$	$-z\dfrac{dF(z)}{dz}$	z-differentiation
$f(n)*g(n)$	$F(z)G(z)$	convolution

Padé approximations of a delay:

$$e^{-\tau s} \approx \frac{1 - \tau s/2}{1 + \tau s/2}$$

$$e^{-\tau s} \approx \frac{1 - \tau s/2 + \tau^2 s^2/12}{1 + \tau s/2 + \tau^2 s^2/12}$$

$$e^{-\tau s} \approx \frac{1 - \tau s/2 + \tau^2 s^2/10 - \tau^3 s^3/120}{1 + \tau s/2 + \tau^2 s^2/10 + \tau^3 s^3/120}$$

Hermite interpolation formula:

$$P(t_1) = a_1\,, \quad \dot{P}(t_1) = b_1$$

$$P(t_2) = a_2\,, \quad \dot{P}(t_2) = b_2$$

$$T = t_2 - t_1$$

$$P(t) = \left(\frac{t - t_2}{T}\right)^2 \left[a_1 + (Tb_1 + 2a_1)\left(\frac{t - t_1}{T}\right)\right] + \left(\frac{t - t_1}{T}\right)^2 \left[a_2 + (Tb_2 - 2a_2)\left(\frac{t - t_2}{T}\right)\right]$$