332:345 – Linear Systems & Signals – Fall 2016 – S. J. Orfanidis

$$\boxed{F(\omega) = \int_{-\infty}^{\infty} f(t)\,e^{-j\omega t}dt} \quad \text{(FT)} \quad \Leftrightarrow \quad \boxed{f(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)\,e^{j\omega t}d\omega} \quad \text{(IFT)}$$

$f(t)$	$F(\omega)$
$\delta(t)$	1
$\delta(t-t_0)$	$e^{-j\omega t_0}$
1	$2\pi\delta(\omega)$
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$
$\cos(\omega_0 t) = \frac{1}{2}(e^{j\omega_0 t} + e^{-j\omega_0 t})$	$\pi\delta(\omega-\omega_0)+\pi\delta(\omega+\omega_0)$
$\sin(\omega_0 t) = \frac{1}{2j}(e^{j\omega_0 t} - e^{-j\omega_0 t})$	$-\pi j\delta(\omega-\omega_0)+\pi j\delta(\omega+\omega_0)$
$u_h(t) =$ Heaviside unit-step	$\frac{1}{j\omega} + \pi\delta(\omega)$
$e^{-at}u_h(t)$	$\frac{1}{a+j\omega}$
$\text{sign}(t)$	$\frac{2}{j\omega}$
$\frac{1}{\pi t}$	$-j\,\text{sign}(\omega)$ (Hilbert transformer filter)
$\text{rect}_\tau(t) = \begin{cases}1, & \lvert t\rvert < \tau/2\\ 0, & \lvert t\rvert > \tau/2\end{cases}$	$2\,\frac{\sin(\omega\tau/2)}{\omega} = \tau\,\frac{\sin(\omega\tau/2)}{\omega\tau/2}$
$\frac{\sin(\omega_c t)}{\pi t}$ (lowpass filter)	$\text{rect}_{2\omega_c}(\omega) = \begin{cases}1, & \lvert\omega\rvert < \omega_c\\ 0, & \lvert\omega\rvert > \omega_c\end{cases}$
$e^{-a\lvert t\rvert}, \quad a > 0$	$\frac{2a}{a^2+\omega^2}$
$\exp\left(-\frac{t^2}{2(a+jb)}\right), \quad a,b \text{ real}, \quad a \ge 0$	$\sqrt{2\pi(a+jb)}\,\exp\left(-\frac{(a+jb)\omega^2}{2}\right)$
$e^{j\omega_0 t-(a+jb)t^2/2}, \quad a,b \text{ real}, \quad a \ge 0$ (chirped gaussian pulse)	$\sqrt{\frac{2\pi}{a+jb}}\,\exp\left(-\frac{(\omega-\omega_0)^2}{2(a+jb)}\right)$
$\sqrt{\frac{jb}{2\pi}}\,e^{j\omega_0 t - jbt^2/2}, \quad b \text{ real}$	$e^{j(\omega-\omega_0)^2/2b}$ (quadrature phase filter)
$\frac{1}{\sqrt{2\pi\sigma^2}}\,\exp\left(-\frac{(t-t_0)^2}{2\sigma^2}\right), \quad \sigma > 0$	$e^{-j\omega t_0}\,\exp\left(-\frac{\sigma^2\omega^2}{2}\right)$

†Note: $\text{rect}_a(x) \equiv u_h(x+a/2) - u_h(x-a/2)$

Properties

Given the pairs $f(t) \leftrightarrow F(\omega)$, and $g(t) \leftrightarrow G(\omega)$, then,

$f(-t) \longleftrightarrow F(-\omega)$, (reflection)

$F(-t) \longleftrightarrow 2\pi f(\omega)$, (duality/symmetry)

$F(t) \longleftrightarrow 2\pi f(-\omega)$

$f^*(t) \longleftrightarrow F^*(-\omega)$, (conjugation)

$f(t) = \text{real} \longleftrightarrow F(\omega) = F^*(-\omega)$, (hermitian)

$f(t-t_0) \longleftrightarrow e^{-j\omega t_0}F(\omega)$, (delay)

$e^{j\omega_0 t}f(t) \longleftrightarrow F(\omega-\omega_0)$, (modulation)

$e^{j\omega_0 t}f(t-t_0) \longleftrightarrow e^{-j(\omega-\omega_0)t_0}F(\omega-\omega_0)$

$\dot{f}(t) \longleftrightarrow j\omega F(\omega)$, (differentiation)

$\ddot{f}(t) \longleftrightarrow (j\omega)^2 F(\omega)$

$tf(t) \longleftrightarrow j\,\frac{dF(\omega)}{d\omega}$

$f(t/a) \longleftrightarrow \lvert a\rvert F(a\omega)$, real a

convolution/correlation properties:

$f(t)*g(t) \longleftrightarrow F(\omega)G(\omega)$

$f(t)g(t) \longleftrightarrow \frac{1}{2\pi}F(\omega)*G(\omega)$

correlation:

$r(t) = f(t)*g^*(-t) = \int_{-\infty}^{\infty} f(\tau+t)g^*(\tau)d\tau$

$r(t) \longleftrightarrow R(\omega) = F(\omega)G^*(\omega)$

Parseval identity:

$$\int_{-\infty}^{\infty} f(t)g^*(t)dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)G^*(\omega)d\omega$$

Hilbert transform and its inverse:

$\hat{f}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{f(t')}{t-t'}dt' \longleftrightarrow -j\,\text{sign}(\omega)F(\omega)$

$f(t) = -\frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\hat{f}(t')}{t-t'}dt' \longleftrightarrow j\,\text{sign}(\omega)\hat{F}(\omega)$

analytic signal:

$f(t) + j\hat{f}(t) \longleftrightarrow 2F(\omega)u_h(\omega)$