Lab07: k-Nearest Neighbour classifier

Task 1.

In this question you will analyse the performance of the k-nearest neighbour classifier for the Iris dataset.

A. Load the Iris dataset from the dataset repository of SciKit learn.
B. Create a k-fold cross validation procedure for the dataset using as many splits as data points to perform a leave-one-out analysis.
C. Calculate the performance of the k-nearest neighbour classifier for different parameters $k = 1, \dots, 50$ using the cross-validation procedure.
D. Plot the performance against the parameter k. What is the optimal choice for k?

Task 2. (Matrix operations)

Transposing the $m \times n$ matrix

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$$

results in the $n \times m$ matrix where the roles of rows and columns are exchanged.

$$A^T = \begin{pmatrix} a_{11} & \cdots & a_{m1} \\ \vdots & \ddots & \vdots \\ a_{1n} & \cdots & a_{mn} \end{pmatrix}$$

A. Create a 3×2 matrix containing some non-zero numbers and calculate the transpose. Print both matrices.

The scalar product of two equal sized vectors

$$a = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix}$$

and

$$b = \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix}$$

is obtained by summing up the products of corresponding elements, i.e.

$$a^T b = a_1 b_1 + \cdots + a_n b_n$$

The result is a scalar (i.e. a number, not a vector).

The outer product of the two vectors is the $n \times n$ matrix collecting all products of pairs of elements of the two vectors

$$ab^T = \begin{pmatrix} a_1 b_1 & \cdots & a_1 b_n \\ \vdots & \ddots & \vdots \\ a_n b_1 & \cdots & a_n b_n \end{pmatrix}$$

B. Create two 3-vectors containing some non-zero numbers and calculate the scalar and the outer product. Print both vectors, and both results.

To multiply the $m \times n$ matrix

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$$

with the n-vector

$$b = \begin{pmatrix} b_1 \\ \vdots \\ b_n \end{pmatrix}$$

we go row-by-row through the matrix and calculate the scalar products of the matrix row with the vector, resulting in the m-vector

$$Ab = \begin{pmatrix} a_{11} b_1 + \cdots + a_{1n} b_n \\ \vdots \\ a_{m1} b_1 + \cdots + a_{mn} b_n \end{pmatrix}$$

Observe, how the sizes of matrix and vector must fit, and how the result size is determined by the matrix size.

C. Create a non-zero 3×2 matrix and a non-zero 2-vector and multiply the two. Print the matrix, the vector, and the resulting product vector.

To multiply the $m \times n$ matrix

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{m1} & \cdots & a_{mn} \end{pmatrix}$$

with the $n \times k$ matrix

$$B = \begin{pmatrix} b_{11} & \cdots & b_{1k} \\ \vdots & \ddots & \vdots \\ b_{n1} & \cdots & b_{nk} \end{pmatrix}$$

we go row-by-row through the A matrix and column-by-column through the B matrix collecting the scalar products of each row/column-pair in the $m \times k$ matrix

$$AB = \begin{pmatrix} a_{11}b_{11} + \cdots + a_{1n}b_{n1} & \cdots & a_{11}b_{1k} + \cdots + a_{1n}b_{nk} \\ \vdots & \ddots & \vdots \\ a_{m1}b_{11} + \cdots + a_{mn}b_{n1} & \cdots & a_{m1}b_{1k} + \cdots + a_{mn}b_{nk} \end{pmatrix}$$

Observe, how the sizes of the matrices must fit, and how they determine the size of the resulting matrix. Also observe, that unlike the scalar case matrix multiplication is NOT commutative, therefore even if AB is possible to compute, the dimensions of BA might not fit.

D. Create a non-zero 3×2 and a non-zero 2×4 matrix and multiply the two. Print both matrices and the result.

If we want to commute a matrix multiplication we need to transpose, because the transpose of the product of two matrices is the product of the transposed matrices in opposite order, i.e.

$$(AB)^T = B^T A^T$$

E. Use the two matrices from the previous sub-task and multiply their transposed in reverse order. Print the result and compare it to the result of the previous sub-task.