UNIVERSITY OF WATERLOO

MTE202 - ORDINARY DIFFERENTIAL EQUATIONS

Mathematical Modelling (Lecture 9)

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Jean-Pierre Hickey

Mechanical and Mechatronics Engineering

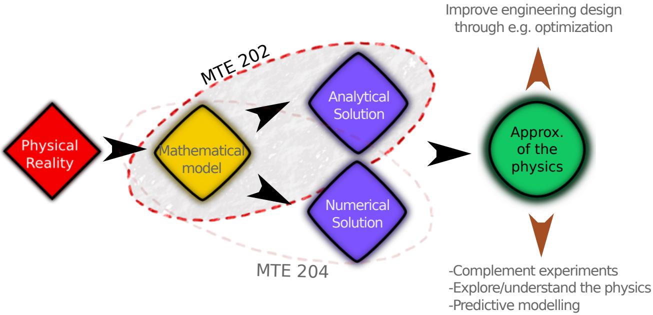
TODAY'S LECTURE

- We take a step outside of MTE202.
- Discuss mathematical modelling within a broader engineering context
- Links together engineering/design, mathematics, and physics

Lecture plan:

- Contextualize mathematical modelling
- Key Principles of mathematical modelling
- Sample problem: Injector plate in LRE for Vulcain 2
- Exercise: Wood pyrolysis

MATHEMATICAL MODELLING



- The ability to develop mathematical models is (IMHO) one of the most important skills for an engineering undergrad
- The solution strategies to ODEs are tools, modelling is the objective.
- Modelling is used to study heat transfer, fluid dynamics, material science etc. (..and banking and finance, quants?)

BACKGROUND

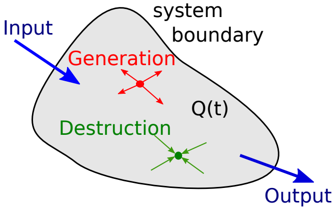
- Generally, we need to create mathematical models from a physical problem
- The problem is rarely defined in mathematical terms
- Often contains incomplete information, ambiguities, too much/too little information
- Need to use engineering intuition to focus on what is relevant and make judicious assumptions
- Skills are transferable/generalizable (to other classes and/or other fields). The general approach towards mathematical modelling remains the same!
- Problems seen in MTE202 will be academic/simple. The approach to a real problem is identical.

QUESTIONS TO ANSWER

- Why? What do we want to know?
- What is given?
- What are the governing physical principles?
- What are my assumptions?
- What will the model predict?
- How can I validate and verify the model?

KEY PRINCIPLES OF MATHEMATICAL MODELLING

- Dimensional Homogeneity and Consistency
  - The dimensions of a mathematical model must be consistent. If we model an equation for the mass balance of a system, all the terms must have dimensions of mass. Units must also be consistent.
- Abstraction and Scaling
  - The model should have the correct level of detail and abstraction for the desired output. If we model a spring-damper system, we can, in most cases, consider the spring as a linear system (force $\propto$ displacement). We abstract the complex material properties into a single spring constant.
- Conservation and Balance Principles



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LIQUID ROCKET ENGINE INJECTOR HEAD

Context: A company wanted to know how well we can predict the heat load in a new injector design for liquid rocket engine combustion chamber.

- Why? What do we want to know?
- What is given?
- What are the governing physical principles?
- What are my assumptions?
- What will the model predict?
- How can I validate and verify the model?

WOOD PYROLYSIS

Pyrolysis is a thermochemical decomposition of organic material at elevated temperatures in the absence of oxygen.

Video: https://youtu.be/7lfpLStXONo

- What do we want to know?
  - How can we predict the physical behaviour of wood pyrolysis? Fire modellers would like to easily predict how wood pyrolyses.
- What is given?
  - Typically, we know type of wood, moisture content of wood etc.
- What are the governing physical principles?
  - This is where your job starts...

Exercise:

- Take about 10 minutes to analyze this problem in groups of 5-10
- You may not have all the background needed (for heat transfer etc), but you are some of the smartest in the country!
- The goal is not to get an exact answer but to develop the mindset needed for mathematical modelling

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