**CSCI-C 241 HW #7** Assigned Thursday, October 4th, 2018
**Instructor: Wennstrom** Due Tuesday, October 9th, 2018

1. For this problem, we will be making little toy models fitting certain requirements. For each part, you will draw a few small shapes to serve as a universe. Some of the shapes will be triangles (like △ or ▲), and some of them will be other shapes (like ○ or ■). Some of the shapes will be outlined (like △, □, or ○), and others will be solid (like ▲, ■, or ●). For each part, you should draw a separate picture, meeting the requirements for that part.

   To rule out trivial examples, you must include at least three shapes in your picture. You can use the same shape more than once in each problem if you want. I may add other requirements as well. You can use the same shape to fit more than one of these requirements. For example a solid square fills both a requirement for a solid shape and a requirement for a non-triangle.

   Use the following definitions. $T(x)$: "$x$ is a triangle." $S(x)$: "$x$ is solid." Remember that a model satisfies a formula if the formula is true under that model.

   If you think that the request is impossible, you must explain why.

   (a) Draw a toy model satisfying $\forall x\big(S(x) \to T(x)\big)$. Just to keep things from being too easy, there must be at least two triangles, at least two non-triangles, at least two outlined shapes, and at least two solid shapes.

   (b) Draw a toy model satisfying $\exists x\big(S(x) \land T(x)\big)$, but not $\forall x\big(S(x) \to T(x)\big)$.

   (c) Draw a toy model satisfying $\forall x\big(T(x) \land S(x)\big)$.

   (d) Draw a toy model satisfying $\forall x \neg T(x)$ and $\neg \exists x S(x)$

   (e) Draw a toy model satisfying $\forall x\big(T(x) \to S(x)\big)$, $\neg \forall x T(x)$, and $\exists x \neg S(x)$. There must be at least **three** triangles.

   (f) Draw a toy model that satisfies $\exists x T(x) \land \exists x S(x)$ and $\neg \exists x\big(T(x) \land S(x)\big)$ .

   (g) Draw a toy model that satisfies $\forall x\big(S(x) \to \neg T(x)\big)$. In addition, there must be at least two triangles, at least two outlined shapes, and at least two solid shapes.

For problems 2 through 3, let the universe be $\mathbb{Z}$, and use the following definitions. $P(x)$: "$x$ is prime." $N(x)$: "$x$ is negative." $F(x)$: "$x$ is greater than five." $E(x)$: $x$ is even."

2. Translate the following English sentences into first-order logic.

   Because these are pure first-order logic formulas, the only symbols that are allowed are the propositional connectives ($\land$, $\lor$, $\to$, $\neg$, ...), the quantifiers ($\forall$ and $\exists$), the atomic predicates (in this case $P$, $N$, $F$, and $E$), variables ($x$, $y$, etc.), and parentheses. Specific numbers or the symbol $\mathbb{Z}$ are not allowed. (Since the universe is already declared to be the set of all integers, there's no need to specifically mention that $x$ is an integer.)

   (a) Every integer is even.

   (b) All integers are prime.

   (c) Some integers are negative.

   (d) There exists an even integer.

(e) At least one integer is greater than five.

(f) Every prime number is greater than five.

(g) Some even integer is greater than five.

(h) There is at least one negative even integer.

(i) All numbers greater than five are even.

(j) At least one even number is prime.

(k) All integers are even and prime.

(l) There are no even integers.

(m) No integer is negative.

(n) Some integers are not greater than five.

(o) Not all integers are prime.

(p) None of the negative integers are greater than five.

(q) Not every even integer is prime.

(r) There aren't any even integers that are negative.

(s) No integer that is greater than five is even.

3. Translate the following first-order logic formulas into English sentences. You may not use variables in your translation.

   (a) $\forall x P(x)$

   (b) $\exists x N(x)$

   (c) $\forall x \big(P(x) \wedge E(x)\big)$

   (d) $\forall x \big(P(x) \vee N(x)\big)$

   (e) $\forall x \big(F(x) \rightarrow P(x)\big)$

   (f) $\exists x \big(P(x) \wedge E(x)\big)$

   (g) $\exists x \big(N(x) \rightarrow E(x)\big)$ **Warning:** This is a really weird formula, and if you try to translate it directly in English, the result will be awkward and forced-sounding. Try rewriting the formula using the material implication rule first to get a better result.

   (h) $\forall x \big(P(x) \rightarrow \neg N(x)\big)$

   (i) $\exists x \big(N(x) \wedge \neg P(x)\big)$

   (j) $\neg \exists x \big(N(x) \wedge E(x)\big)$