B365 Homework 1

1. Three people, A,B, and C, each flip their coins until one person has a different result from the others. The person having the different coin wins. For instance, if the three players flip HHH, TTT, HHT, C would win.

   (a) Simulate this experiment 10000 times and give the resulting estimate of P(A wins).

   (b) How mamy trials are necessary before the 95% confidence interval has width .01?

   (c) Interpret the statment "the 95% confidence interval has width .01" in terms of the true value of P(A wins) and your interval.

   (d) Argue for what you believe is the true probability of P(A wins).

2. A and B alternate drawing cards from a shuffled pack, replacing each card when done. A goes first. Play continues until a heart is drawn. Simulate this experiment to compute P(A draws first heart) to 2 decimal places ($\pm .005$)

3. Two cards are drawn from a shuffled deck.

   (a) Give the sample space for the experiment, Ω, and calculate $|\Omega|$ — the number of elements in Ω.

   (b) How many elements of Ω have both cards having the same rank? (i.e. both aces or both kings etc.)

   (c) What is the probability of drawing a pair? (both same rank)

4. An olympic archer hits the "bullseye" (the center of the target) half of the time. Suppose the archer shoots 10 arrows. Compute:

   (a) P(0 bullseyes)

   (b) P(1 bullseyes)

   (c) P(2 bullseyes)

   (d) P(3 bullseyes)

5. Suppose we are interested in P(A) for some event A. A better 95% confidence interval than the one presented in class is

$$\hat{p} \pm 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

   where $\hat{p}$ is still the proportion of times A occurs in our sample. This confidence is smaller, which is better, but covers the true value with probability only approximately .95.

   Suppose our event of interest has P(A) = 1/10.

(a) Give a code fragment to simulate 1000 trials of the experiment and print out the resulting confidence interval using the formula above.

(b) Repeat the experiment above 1000 times and compute the fraction of the time the confidence interval contains the true probability, 1/10. This should happen about 95% of the time.

6. A bag contains 10 numbers: 1.2, 1.5, 3.2, 3.3, 3.4, 5.3, 6.3, 7.2, 8.9, 9.1. One person A draws a number at random, while person B draws a number from the remaining choices. What is the exact probability that A's number is greater than B's number? Explain your reasoning in detail.

7. Suppose we want to simulate an experiment that can take outcomes $1, \ldots, n$ with probabilies $p_1, \ldots, p_n$. To be specific, suppose an R-vector

```
p=c(.1,.2,.3,.35, .02, .03)
```

giving the desired probabilities. Write R code that produces a number from 1 to 6 with the given probabilities. Avoid using "if" statements. I recommend using the R command `cumsum` to do this, though there many possible approaches.