

Remember that when writing semi-formal proofs for this class, you need to use the style I introduced in the lectures. In particular, you must follow the following guidelines:

- Always write the claim before the proof. Clearly label where the proof starts and where it ends.
- Clearly indicate all assumptions.
- Only use one rule per step, cite the name of the rule, and indicate which formulas and/or subproofs you used.
- Indent all subproofs, and indicate what kind of subproofs they are.
- Only use the eight natural deduction rules:  $\wedge$ -Elimination,  $\wedge$ -Introduction, Application ( $\rightarrow$ -Elim.), Direct Proof ( $\rightarrow$ -Intro.), Double Negation ( $\neg$ -Elim.), Proof by Contradiction ( $\neg$ -Intro.), Weakening ( $\vee$ -Intro.), and Proof by Cases ( $\vee$ -Elim.).

For this assignment, you won't need proof by cases.

1. Provide semi-formal natural deduction proofs of the following claims. You may only use the eight natural deduction inference rules.
  - (a)  $(P \wedge Q) \rightarrow R, P \wedge S, \neg \neg Q \vdash R$
  - (b)  $X \wedge (X \rightarrow (Z \wedge Y)) \vdash X \wedge Y$
  - (c)  $A \wedge \neg \neg B \vdash B \vee (A \rightarrow \neg C)$
  - (d)  $(K \vee L) \rightarrow N, K \wedge M \vdash N \wedge M$
  - (e)  $(A \wedge B) \rightarrow C, B, A \wedge \neg D \vdash C \wedge \neg D$
  - (f)  $(A \wedge B) \rightarrow C, B \vdash (A \wedge \neg D) \rightarrow (C \wedge \neg D)$
  - (g)  $Z \rightarrow \neg X, Z \wedge \neg \neg Y \vdash \neg X \vee Y$
  - (h)  $\vdash ((Z \rightarrow \neg X) \wedge (Z \wedge \neg \neg Y)) \rightarrow (\neg X \vee Y)$
  - (i)  $(W \wedge X) \rightarrow \neg Y, X \vdash \neg(W \wedge Y)$
  - (j)  $\vdash (M \rightarrow \neg N) \rightarrow \neg(M \wedge N)$
  - (k)  $Z \rightarrow A, B \rightarrow A, \neg A \vdash \neg Z \wedge \neg B$