CSCI-C 241 HW #8 Assigned Thursday, October 11th, 2018
Instructor: Wennstrom Due Tuesday, October 18th, 2018

As always, when writing informal proofs (otherwise known as "proofs"), follow the guidelines I laid out in the lecture. This means that you may use any logical rules from propositional or predicate logic, and you do not need to cite them. When using the conclusion from the previous line, you don't need to repeat it (but you do need to repeat any facts you are using from earlier lines in the proof). You are allowed to skip steps involving $\wedge$-introduction, $\wedge$-elimination, or double negation. You can combine one step involving using the definition of a symbol/term with one step involving a logical rule (such as combining application with the definition of $\subseteq$).

The purpose of the subset proofs is to learn how to use direct proof for universal claims, so do *not* use rules about how intersections, unions, etc. interact with subsets. For example, while it's true that $A \cap B \subseteq A$, you may not use this fact in your proofs, and the problem will be rejected if you do so. **Always use direct proof to prove any universal claim** (such as proving that one set is a subset of another).

For similar reasons, you must **always use proof by contradiction to prove any negative claim** (such as to prove that an object is *not* a member of a set). You may be tempted to use modus tollens (if you know $A \subseteq B$ and $x \notin B$, then conclude $x \notin A$). This is logically correct, but **do not use this rule on this assignment**. You should be using proof by contradiction instead. For similar reasons, **do not use de Morgan's Laws**.

Here are the definitions about sets that you will need for these proofs:

Definition. $x \in A \cap B$ means that $x \in A$ and $x \in B$.

Definition. $x \in A \cup B$ means that $x \in A$ or $x \in B$.

Definition. $x \in A \setminus B$ means that $x \in A$ and $x \notin B$.

Definition. $A \subseteq B$ means that every member of A is also a member of B.

1. For the following claims, first decide if they are true or false. If they are true, then give an informal proof of the claim. If they are false, then give a counterexample that disproves the claim.
   (a) For all sets A and B: $A \cup B \subseteq A$.
   (b) For all sets H, I, J, and K: if $H \subseteq I$ and $J \subseteq K$, then $(H \cap J) \subseteq (I \cap K)$.
   (c) For all sets A, B, C, D, and E: if $A \cup B \subseteq C$, then $D \setminus C \subseteq D \setminus (A \cap E)$.
   (d) For all sets A, B, and C: if $A \subseteq C$, then $A \cup B \subseteq B \cup C$.
   (e) For all sets A, B, and C: if $A \subseteq C$, then $A \cup B \subseteq B \cap C$.
   (f) For all sets X, Y, and Z: if $(X \cap Y) \subseteq Z'$, then $X \subseteq (Y \cap Z)'$.

2. For this problem, you'll be creating toy models to meet the given requirements.

   This time, you'll write down a collection of letters to serve as a universe for each problem. Some of the letters will be vowels (A, a, E, e, I, i, ...) and some will be consonants (B, b, D, d, F, f, G, g, ...) Some will be in uppercase (A, B, D, E, ...), and some will be in lowercase (a, b, d, e, ...).

To avoid confusion, avoid letters where it's hard to tell the difference between upper and lowercase (e.g., don't use C or O or X), and for this assignment, we will treat Y as a vowel.

In addition, you will be drawing arrows between the letters. Arrows are allowed to point from a letter to any other letter or to itself.

Use the following interpretations for the predicate symbols. $V(x)$: "x is a vowel." $L(x)$: "x is lowercase." $P(x, y)$: "There is an arrow pointing from x to y."

There must be at least three letters for each model.

If you think that there is no such model, you must explain why not.

(a) Satisfying $\neg\forall x\big(V(x) \to L(x)\big)$ and $\exists x\big(V(x) \wedge L(x)\big)$.

(b) Satisfying $\neg\exists x\big(V(x) \wedge L(x)\big)$ and $\exists x V(x) \wedge \exists x L(x)$.

(c) Satisfying $\forall x\big(L(x) \to \neg V(x)\big)$ and $\exists x L(x) \wedge \exists x V(x)$.

(d) Satisfying $\exists x\big(L(x) \wedge \neg V(x)\big)$, $\forall x\big(V(x) \to L(x)\big)$, and $\exists x V(x)$.

(e) Satisfying $\forall x\big(\neg V(x) \to L(x)\big)$, $\forall x\big(V(x) \to L(x)\big)$, and $\exists x V(x) \wedge \exists x \neg V(x)$.

(f) Satisfying $\forall x \forall y P(x, y)$.

(g) Satisfying $\forall x \exists y P(x, y)$, but not $\exists y \forall x P(x, y)$.

(h) Satisfying $\exists y \forall x P(x, y)$, but not $\forall x \exists y P(x, y)$.

For problems 3 through 4, let the universe be the set of all websites, and use the following definitions. $B(x)$: "x is a blog." $G(x)$: "x is hosted by Google." $L(x, y)$: "x links to y." $V(x, y)$: "x has more views than y."

3. Translate the following English sentences into predicate logic.

   (a) There are no websites hosted by Google.

   (b) Not all websites are blogs.

   (c) Some websites are not hosted by Google.

   (d) No blogs are hosted by Google.

   (e) Not every blog is hosted by Google.

   (f) Every website that is not hosted by Google is a blog.

   (g) There aren't any blogs that are hosted by Google.

   (h) Every website is either a blog or is not hosted by Google.

4. Translate the following predicate logic formulas into English sentences. These should all be natural-sounding English sentences, with no variables.

   Hint: Some of these are equivalent to each other, so you can reuse the same answer for multiple problems.

   (a) $\forall x \exists y L(x, y)$

(b) $\exists x \forall y L(x, y)$

(c) $\exists x \forall y L(y, x)$

(d) $\exists y \forall x L(x, y)$