**CSCI-C 241 Mini-HW #4** Assigned Tuesday, September 11th, 2018
**Instructor: Wennstrom** Due Thursday, September 13th, 2018

1. Here are some attempts at proofs given by students in previous semesters. For each proof, I would like you to explain which steps (if any) are logically incorrect and why. Note that I am not asking you to to explain how they "should" have written the proof. In particular, keep in mind that an assumption is *never* logically incorrect.

   I've included several correct proofs that happen to include very roundabout reasoning, false starts, or other weird things that aren't actually *wrong*. These proofs are not *good* proofs, and there are certainly better ways of proving the same claims, but the proofs *are logically correct*. As long as every step in the proof is logically valid deduction, then you should state that the proof is correct.

   Since I want you to focus on the individual steps in the proofs and not the overall strategies, I've omitted the statements of the claims.

   I've included line numbers just to make it easier to talk about the different steps.

   To give you an idea of what I'm expecting, here's an example of a proof and the kind of answer I want from you:

   *Proof.*

   | | | |
   |---|---|---|
   | 1 | Assume $(F \to G) \vee \neg\neg G$ and $F$. | |
   | 2 |     Assume $F \to G$. | |
   | 3 |     Since $F$ and $F \to G$ are true, so is $G$. | (Appl.) |
   | 4 | Because we have $F$ and $G$, we can conclude $F \wedge G$. | ($\wedge$-Intro.) $\square$ |

   **Answer:** Line 4 is wrong because you can't use formulas from a subproof after the subproof has ended.

   Finally, note that there are no mistakes in format, phrasing, citations, or other aspects of presentation, so don't think about that. Just pay attention to what rule they are using, and determine if they are using it correctly or not.

   (a) *Proof.*

   | | | |
   |---|---|---|
   | 1 | Assume $(F \to G) \vee \neg\neg G$ and $F$. | |
   | 2 | Since $(F \to G) \vee \neg\neg G$ is true, so is $\neg\neg G$. | ($\vee$-Elim.) |
   | 3 | From $\neg\neg G$, we can derive $G$. | (Dbl. Neg.) |
   | 4 | Because we have $F$ and $G$, we can conclude $F \wedge G$. | ($\wedge$-Intro.) $\square$ |

   (b) *Proof.*

   | | | |
   |---|---|---|
   | 1 | Assume $P \to Q$ and $R$. | |
   | 2 |     Suppose $P$. | |
   | 3 |     Since $P$ and $P \to Q$, $Q$. | (Appl.) |
   | 4 |     From $P$, we get $P \vee \neg\neg Q$. | (Weak.) |
   | 5 |     Because $Q$ and $R$, $Q \wedge R$ | ($\wedge$-Intro.) |
   | 6 | Since we proved $P \vee \neg\neg Q$ and $Q \wedge R$, we have $(P \vee \neg\neg Q) \to (Q \wedge R)$. | (Dir. Pf) $\square$ |

(c) *Proof.*

| | | |
|---|---|---|
| 1 | Assume $Y \wedge Z$ and $X \rightarrow \neg Y$. | |
| 2 | From $Y \wedge Z$, we can derive $Y$ and $Z$ | ($\wedge$-Elim.) |
| 3 | Suppose towards a contradiction that $X$ is true. | |
| 4 | From $X$ and $X \rightarrow \neg Y$, we get $\neg Y$. | (Appl.) |
| 5 | Under the assumption $X$, we proved $\neg Y$, which contradicts our earlier statement $Y$. Therefore $\neg X$. | (contrad.) |
| 6 | Suppose towards a contradiction that $X \wedge Z$ is true. | |
| 7 | Since $X \wedge Z$, we get $X$. | ($\wedge$-Elim.) |
| 8 | We assumed $X \wedge Z$ and derived $X$ and $\neg X$, which can't both be true, and so $\neg(X \wedge Z)$. | (contrad.) $\square$ |

(d) *Proof.*

| | | |
|---|---|---|
| 1 | Assume $Y \wedge Z$ and $X \rightarrow \neg Y$. | |
| 2 | Suppose towards a contradiction that $X$ is true. | |
| 3 | Because $X$ and $X \rightarrow \neg Y$ hold, so does $\neg Y$. | (Appl.) |
| 4 | Since $Y \wedge Z$, we have $Y$ and $Z$ | ($\wedge$-Elim.) |
| 5 | From $X$ and $Z$, we can conclude $X \wedge Z$ | ($\wedge$-Intro.) |
| 6 | From $X$, we showed a contradiction ($\neg Y$ and $Y$), and therefore $\neg(X \wedge Z)$. | (contrad.) $\square$ |

(e) *Proof.*

| | | |
|---|---|---|
| 1 | Assume $(A \vee \neg B) \rightarrow C$ and $A$. | |
| 2 | Case 1: Assume $A$. | |
| 3 | From $A$, we can derive $A \vee \neg B$. | (Weak.) |
| 4 | From $A \vee \neg B$ and $(A \vee \neg B) \rightarrow C$, we get $C$. | (Appl.) |
| 5 | Case 2: Assume $\neg B$. | |
| 6 | From $\neg B$, we can derive $A \vee \neg B$. | (Weak.) |
| 7 | From $A \vee \neg B$ and $(A \vee \neg B) \rightarrow C$, we get $C$. | (Appl.) |
| 10 | We have $(A \vee \neg B) \rightarrow C$, and so in either case, we get $C$. | (Cases) |
| 11 | From $C$ and $A$, we can conclude $A \wedge C$. | ($\wedge$-Intro.) $\square$ |

(f) *Proof.*

| | | |
|---|---|---|
| 1 | Suppose $(K \to J) \vee \neg\neg J$ and $K$. | |
| 2 | Case 1: Assume $K \to J$ | |
| 3 | Suppose $K$. | |
| 4 | Since $K$ and $K \to J$, we have $J$. | (Appl.) |
| 5 | I assumed $K$ and proved $J$; therefore $K \to J$. | (Dir. Pf) |
| 6 | Case 2: Assume $\neg\neg J$. | |
| 7 | Suppose $K$. | |
| 8 | Since $\neg\neg J$, we have $J$. | (Dbl. Neg.) |
| 9 | I assumed $K$ and proved $J$; therefore $K \to J$. | (Dir. Pf) |
| 10 | Because $(K \to J) \vee \neg\neg J$, these are the only possible cases, and hence $K \to J$. | (Cases) |
| 11 | Since $K$ and $K \to J$, we have $J$. | (Appl.) |
| 12 | From $K$ and $J$, we can conclude $J \wedge K$. | ($\wedge$-Intro.) $\square$ |