C241 HW8

Zac Monroe

October 2018

1. (a) **Claim**: For all sets A and B, $A \cup B \subseteq A$.
   This claim is false. Choose sets $A = \{1, 2, 3\}$ and $B = \{4, 5\}$. $A \cup B = \{1, 2, 3, 4, 5\} \not\subseteq A$ because $4 \in A \cup B$ but $4 \notin A$.

   (b) **Claim**: For all sets H, I, J, and K, if $H \subseteq I$ and $J \subseteq K$, then $(H \cap J) \subseteq (I \cap K)$.

   Proof. Choose sets H, I, J, K and assume $H \subseteq I$ and $J \subseteq K$.
   Choose some $x \in H \cap J$.
   Since $x \in H \cap J$, $x \in H$ and $x \in J$.
   Since $x \in H$ and $H \subseteq I$, $x \in I$.
   Since $x \in J$ and $J \subseteq K$, $x \in K$.
   Since $x \in I$ and $x \in K$, $x \in I \cap K$.
   Therefore $(H \cap J) \subseteq (I \cap K)$. $\square$

   (c) **Claim**: For all sets A, B, C, D, and E, if $A \cup B \subseteq C$, then $D \setminus C \subseteq D \setminus (A \cap E)$.

   Proof. Choose sets A, B, C, D, E and assume $A \cup B \subseteq C$.
   Choose some $x \in D \setminus C$.
   Since $x \in D \setminus C$, $x \in D$ and $x \notin C$.
   Suppose towards a contradiction that $x \in A$.
   Thus $x \in A \cup B$.
   Since $x \in A \cup B$ and $A \cup B \subseteq C$, $x \in C$, which contradicts $x \notin C$.
   Therefore $x \notin A$.
   Thus $x \notin A \cap E$.
   Since $x \in D$ and $x \notin A \cap E$, $x \in D \setminus (A \cap E)$.
   Therefore $D \setminus C \subseteq D \setminus (A \cap E)$. $\square$

   (d) **Claim**: For all sets A, B, and C, if $A \subseteq C$, then $A \cup B \subseteq B \cup C$.

   Proof. Choose sets A, B, C assume $A \subseteq C$.
   Choose some $x \in A \cup B$.
   Thus $x \in A$ or $x \in B$.
   Case 1: $x \in A$
   Since $x \in A$ and $A \subseteq C$, $x \in C$.
   Thus since $x \in C$, $x \in B \cup C$.

Case 2: $x \in B$

Since $x \in B, x \in B \cup C$.

In either case of $x \in A$ or $x \in B$, $x \in B \cup C$, so $x \in B \cup C$ in general.

Therefore $A \cup B \subseteq B \cup C$. $\square$

(e) **Claim**: For all sets A, B, and C, if $A \subseteq C$, then $A \cup B \subseteq B \cap C$.

This claim is false. Choose sets $A = \{1, 2\}, B = \{3, 4\}, C = \{1, 2, 3\}$. $A \cup B = \{1, 2, 3, 4\}$ and $B \cap C = \{3\}$, so $A \cup B \nsubseteq B \cap C$ because $1 \in A \cup B$ but $1 \notin B \cap C$.

(f) **Claim**: For all sets X, Y, and Z, if $(X \cap Y) \subseteq Z'$, then $X \subseteq (Y \cap Z)'$.

Proof. Choose sets X, Y, Z and assume $(X \cap Y) \subseteq Z'$.

Choose some $x \in X$.

Suppose $x \in Y \cap Z$. (We will show a contradiction)

Since $x \in Y \cap Z$, $x \in Y$ and $x \in Z$.

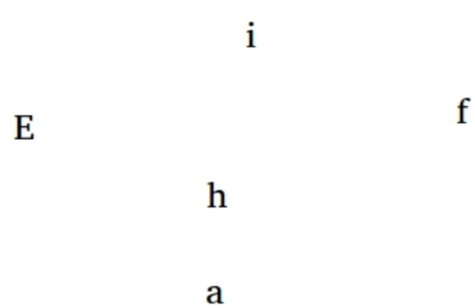
Since $x \in X$ and $x \in Y$, $x \in X \cap Y$.

Since $x \in X \cap Y$ and $X \cap Y \subseteq Z'$, $x \in Z'$, which contradicts $x \in Z$.

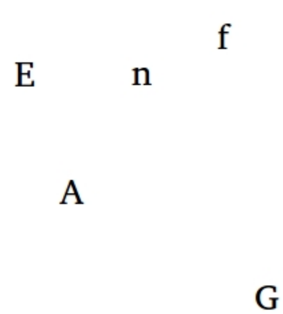
Therefore $x \notin (Y \cap Z)$, or $x \in (Y \cap Z)'$.

Therefore $X \subseteq (Y \cap Z)'$. $\square$

2. (a) Here is my toy model for part (a).



(b) Here is my toy model for part (b).



(c) Here is my toy model for part (c).

n

f

g

I

E

(d) Here is my toy model for part (d).

g

L

e

i

(e) Here is my toy model for part (e).

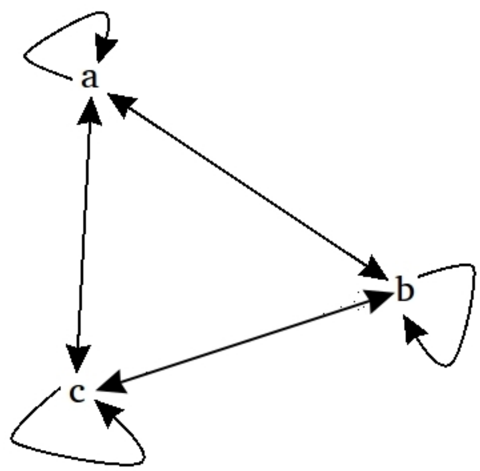
e

g

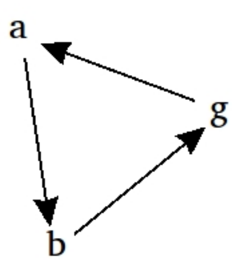
p

n a

(f) Here is my toy model for part (f).



(g) Here is my toy model for part (g).



(h) No such model can exist. $\exists y \forall x P(x, y)$ requires that all x in the universe be pointing at a letter (at least at a common y), so all letters therefore *must* have another letter that they point to, or in other words, under $\exists y \forall x P(x, y)$ it is guaranteed that for all x there exists a y such that x points to y, or $\forall x \exists y P(x, y)$.

3. (a) $\neg \exists x G(x)$

   (b) $\neg \forall x B(x)$

   (c) $\exists x \neg G(x)$

   (d) $\forall x (B(x) \to \neg G(x))$

   (e) $\neg \forall x (B(x) \to G(x))$

   (f) $\forall x (\neg G(x) \to B(x))$

   (g) $\neg \exists x (B(x) \wedge G(x))$

   (h) $\forall x (B(x) \vee \neg G(x))$

4. (a) All websites link to some site.

   (b) There is some site that links to all sites.

   (c) There is some site that all websites link to.

   (d) There is some site that all websites link to.