CSCI-C 241 HW #2 Assigned Thursday, August 30th, 2018
Instructor: Wennstrom Due Tuesday, September 4th, 2018

1. For each of the following formulas, determine whether the formula is satisfiable or not. Remember: You must justify your answer with truth assignment (if satisfiable) or a complete table (if not).
   (a) $A \to \neg B$
   (b) $A \to \neg A$
   (c) $(A \wedge \neg A) \vee (B \wedge \neg B)$
   (d) $\neg(A \wedge B) \leftrightarrow (\neg A \vee \neg B)$
   (e) $\neg\big((B \wedge \neg A) \to (A \vee \neg C)\big)$
   (f) $\neg A \vee \Big((D \vee \neg D) \to \big((B \wedge \neg B) \leftrightarrow (C \to C)\big)\Big)$

2. Answer the following questions. Justify your answers with an truth assignment or an entire table if appropriate. For example, if you claim that a formula is satisfiable, you have to give an example of a satisfying truth assignment. (Don't just give me the table; I want to see that you know how extract a truth assignment from the table.) If you claim a formula is *not* satisfiable, then you need to show that *every* truth assignment does *not* satisfy the formula, meaning that you need the entire truth table. Similarly, if you claim that a formula is a tautology, you need to give the whole table, and if you claim that a formula is *not* a tautology, you need to give an assignment that doesn't satisfy the formula.
   (a) Is $\neg A \to \neg(A \vee B)$ a tautology?
   (b) Is $\neg A \to \neg(A \vee B)$ a contingency?
   (c) Is $\big((A \to B) \wedge (C \vee \neg B)\big) \to \big(A \to C\big)$ a tautology?
   (d) Is $\big((A \to B) \wedge (C \vee \neg B)\big) \to \big(A \to C\big)$ satisfiable?
   (e) Is $\neg\big(A \wedge B\big) \leftrightarrow (\neg A \vee \neg B)$ a contradiction?
   (f) Is $\neg\big(A \wedge B\big) \leftrightarrow (\neg A \vee \neg B)$ a contingency?
   (g) Is $(A \to B) \to (\neg A \to \neg B)$ a contradiction?
   (h) Is $(A \to B) \to (\neg A \to \neg B)$ a tautology?

3. Check whether each set of formulas is consistent or not. If it is consistent, you must justify this with an example of an assignment that satisfies all of the formulas. If it is inconsistent, you must prove this by giving all rows of the table.
   (a) $\{A \wedge \neg B, A \vee B\}$
   (b) $\{P \to Q, P, \neg Q\}$
   (c) $\{F \to E, \neg G \leftrightarrow F, G\}$

4. For each of the following pairs of formulas, decide if they are logically equivalent. If they are *not* logically equivalent, you must give a truth assignment to disprove this. If they *are* logically equivalent, prove this by giving all rows of the table.

(a) $P \leftrightarrow \neg Q$ and $(P \wedge \neg Q) \vee (\neg P \ \wedge Q)$

(b) $A \rightarrow B$ and $\neg B \rightarrow A$

5. The **converse** of an implication ($A \rightarrow B$ for example) is the implication you get by switching the positions of the premise (A in our example) and conclusion (B). So the converse of $A \rightarrow B$ is $B \rightarrow A$. These two formulas are *not* logically equivalent. The **contrapositive** of an implication is the what you get by both switching the premise and conclusion *and* replacing them with their negations. So the contrapositive of $A \rightarrow B$ is $\neg B \rightarrow \neg A$. It turns out that a formula *is* equivalent to its contrapositive.

   (a) Prove that an implication is *not* logically equivalent to its converse. In other words, find a counterexample that proves $A \rightarrow B$ is not logically equivalent to $B \rightarrow A$.

   (b) Prove that an implication *is* logically equivalent to its contrapositive. In other words, show that every truth assignment gives the same outcome for $A \rightarrow B$ as it does for $\neg B \rightarrow \neg A$.

6. Check whether each argument is valid or not. Remember: If the argument is invalid, you must prove this by giving a counterexample (an assignment that makes the premises true and the conclusion false) that proves this. If it is valid, you must give all rows of the table.

   (a) $$\begin{array}{l} \neg(A \wedge B) \\ \neg A \\ \hline \neg(B \rightarrow A) \end{array}$$

   (b) $$\begin{array}{l} Y \rightarrow X \\ X \rightarrow Y \\ \hline \neg Y \vee X \end{array}$$

   (c) $$\begin{array}{l} P \leftrightarrow Q \\ \neg Q \wedge P \\ \hline Q \wedge \neg P \end{array}$$

7. Give an example of each of the following. If you think no such example exists, explain why not.

   (a) A formula that is logically equivalent to $A \leftrightarrow B$.

   (b) A contingency and a tautology that are logically equivalent.

   (c) A consistent set formulas that includes the formula $P \wedge \neg Q$. (There have to be at least two formulas in your set.)

   (d) An inconsistent set of formulas where every formula in the set is satisfiable.

   (e) A consistent set of formulas where one of the formulas is a contradiction.

   (f) A consistent set of formulas that contains both a contingency and a tautology.

   (g) An invalid argument with two premises where the premises and the conclusion are all contingencies. (The contingency requirement is just there to rule out some very silly answers.)

(h) Two formulas (call them "p" and "q") such that the argument $\dfrac{\begin{array}{l} p \\ q \end{array}}{X \wedge Y}$ is valid.

Write your answer in the form: "$p =$ ______ and $q =$ ______".

(i) Two formulas (call them "p" and "q") where the set $\{p, q\}$ is consistent, but the argument $\dfrac{p}{q}$ is invalid.

Write your answer in the form: "$p =$ ______ and $q =$ ______".

(j) A valid argument where the conclusion is a contradiction.

8. Answer each of the following questions. If the answer is yes, you must explain why. If the answer is no, you must give a counterexample.

   (a) Is every tautology also satisfiable?

   (b) Is every satisfiable formula also a tautology?

   (c) Is every contingency required to be satisfiable?

9. Consider the following argument. Without building a truth table, decide whether or not you think it is valid. Write down your answer, and explain in your own words why you think the argument is valid or invalid. **Remember, do not build a truth table!**

$$\begin{array}{l} A \wedge B \\ A \to C \\ C \to (D \wedge E) \\ \hline D \wedge B \end{array}$$

10. **Bonus:** Look at the two formulas below. Do *not* actually build their truth tables (I'm not that mean). Which one would have a bigger truth table? Why?

    (a) $(A \wedge B) \vee \big(C \to (D \wedge E)\big)$

    (b) $\big((A \wedge \neg A) \leftrightarrow (B \wedge \neg B)\big) \to \big((A \to A) \vee \neg\neg (A \vee B)\big)$