CSCI-C 241 HW #4 Assigned Thursday, September 13th, 2018
Instructor: Wennstrom Due Tuesday, September 18th, 2018

1. Provide "semi-formal" natural deduction proofs of the following claims. You may not use any equivalence rules, only the eight natural deduction inference rules.
   (a) $P \vee (Q \wedge \neg\neg R), P \rightarrow R \vdash R$
   (b) $(F \wedge G) \vee (H \rightarrow I), H \vdash G \vee I$
   (c) $J \rightarrow K \vdash (J \vee \neg\neg K) \rightarrow K$
   (d) $(A \wedge C) \rightarrow D, A \wedge \neg B \vdash \neg(A \rightarrow B) \wedge (C \rightarrow D)$
2. The formulas $A \rightarrow (B \wedge C)$ and $(A \rightarrow B) \wedge (A \rightarrow C)$ are logically equivalent. For this problem, you are going to prove that they are equivalent in two different ways.
   (a) Prove that $A \rightarrow (B \wedge C) \equiv (A \rightarrow B) \wedge (A \rightarrow C)$ by writing two semi-formal proofs: one proving $A \rightarrow (B \wedge C) \vdash (A \rightarrow B) \wedge (A \rightarrow C)$ and another proving $(A \rightarrow B) \wedge (A \rightarrow C) \vdash A \rightarrow (B \wedge C)$ (using only the eight natural deduction inference rules and no equivalence rules).
   (b) Prove that $A \rightarrow (B \wedge C) \equiv (A \rightarrow B) \wedge (A \rightarrow C)$ by giving a "semi-formal" equivalence proof (using only equivalence rules and no inference rules).
3. For each of the following pairs of formulas, determine whether the two formulas are logically equivalent or not. If they are not equivalent, say so and give a truth assignment that proves this. If they are equivalent, say so and write an equivalence proof to prove this. Please follow the guidelines for equivalence proofs as laid out in the C241 lecture notes.
   (a) $A \rightarrow (B \wedge C)$ and $(A \rightarrow B) \wedge (A \rightarrow C)$
   (b) $(A \wedge B) \rightarrow C$ and $(A \rightarrow C) \wedge (B \rightarrow C)$
   (c) $\neg\big((W \wedge \neg X) \rightarrow (\neg Y \vee Z)\big)$ and $(\neg W \vee X) \wedge (Y \wedge \neg Z)$
   (d) $\neg\big((W \wedge \neg X) \rightarrow (\neg Y \vee Z)\big)$ and $(Y \wedge \neg Z) \wedge (W \wedge \neg X)$
   (e) $P \leftrightarrow Q$ and $\neg(P \oplus Q)$
4. **Bonus:** Give a semi-formal natural deduction proof of the claim $\vdash P \vee \neg P$. You may not use any equivalence rules, only the eight natural deduction inference rules. (Hint: This one's a lot harder than it looks. You might want to start by using $\neg$-Introduction to try and prove $\neg\neg(P \vee \neg P)$.)