

As always, when writing informal proofs (otherwise known as “proofs”), follow the guidelines I laid out in the lecture. This means that you may use any logical rules from propositional or predicate logic, and you do not need to cite them. When using the conclusion from the previous line, you don’t need to repeat it (but you do need to repeat any facts you are using from earlier lines in the proof). You are allowed to skip steps involving \wedge -introduction, \wedge -elimination, or double negation.

Always use direct proof to prove any universal claim (such as proving that one set is a subset of another).

The purpose of the subset proofs is to learn how to use direct proof for universal claims, so do *not* use rules about how intersections, unions, etc. interact with subsets. For example, while it’s true that $A \cap B \subseteq A$, you may not use this fact in your proofs, and the problem will be rejected if you do so. Make sure you are using direct proof to prove that one set is a subset of another.

Here are the definitions about sets that you will need for these proofs:

Definition. $x \in A \cap B$ means that $x \in A$ and $x \in B$.

Definition. $x \in A \cup B$ means that $x \in A$ or $x \in B$.

Definition. $x \in A \setminus B$ means that $x \in A$ and $x \notin B$.

Definition. $A \subseteq B$ means that every member of A is also a member of B .

1. Prove the following claim:

Claim. For all sets A , B , C , and D , if $A \cup C \subseteq D$, then $A \setminus B \subseteq D \setminus B$.