C241 HW12

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1. (a) $f_a = \{(1,1),(2,2),(3,3)\}$

   (b) No such function can exist. If a function on A is one-to-one, then each input to the function would give a unique output value that is in A, and thus every member of A is accounted for as an output, so the function would be onto as well.

   (c) No such function can exist. If a function on A is onto, then each possible output of the function in A is accounted for, and due to the well-defined aspect of functions, no one input can give two different outputs, so since both the domain and the codomain of the function is A, any onto function on A is also one-to-one.

   (d) $f_d = \{(1,1),(2,1),(3,1)\}$

   (e) $f_e = \{(1,a),(2,b),(3,c)\}$

   (f) No such function can exist because since it'd be a function, it must be well-defined, and so each input can map to only one output. There are three possible input values and four possible output values, so at least one of the output values must lack an input value, so no function from A to B can be onto.

   (g) $f_g = \{(1,a),(2,a),(3,b)\}$

   (h) No such function can exist because since it'd be a function, it must be total, and so each input in the domain must map to an output that exists in the codomain. The domain B has 4 elements while the codomain A has only 3 elements, so at least two inputs will have to have the same output, so the function would not be one-to-one.

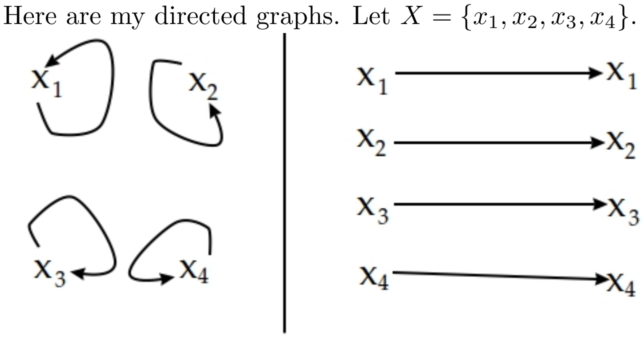
   (i) $f_i = \{(a,1),(b,2),(c,3),(d,3)\}$

   (j) $f_j = \{(a,1),(b,2),(c,2),(d,1)\}$

   (k) $f_k(n) = |n|$

   (l) $f_l(n) = \begin{cases} 2n & n \geq 0 \\ -2n+1 & n < 0 \end{cases}$

2. (1) Here are my directed graphs. Let $X = \{x_1, x_2, x_3, x_4\}$.



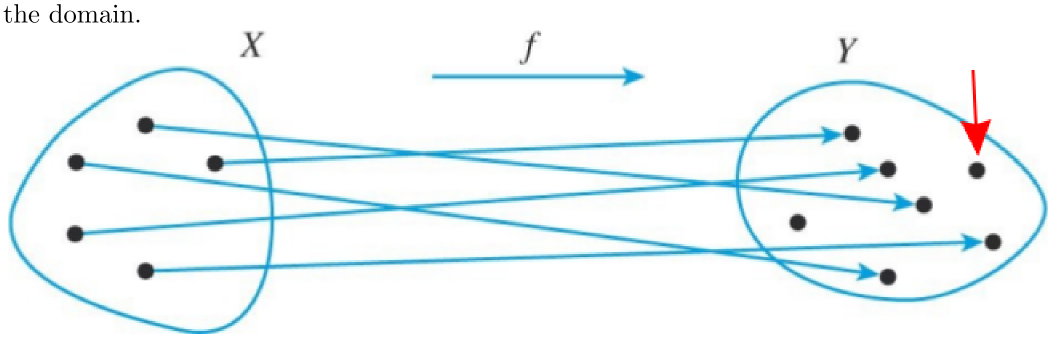
(2) A function $f : A \to B$ is one-to-one if $\forall a \forall b(((a \in A) \wedge (b \in B) \wedge (f(a) = f(b))) \to (a = b))$.
A function $f : A \to B$ is onto if $\forall b((b \in B) \to \exists a((a \in A) \wedge (f(a) = b)))$

(3) s is not one-to-one because $s(\{1, 2, 3\}) = s(\{6\})$ and $\{1, 2, 3\} \neq \{6\}$.

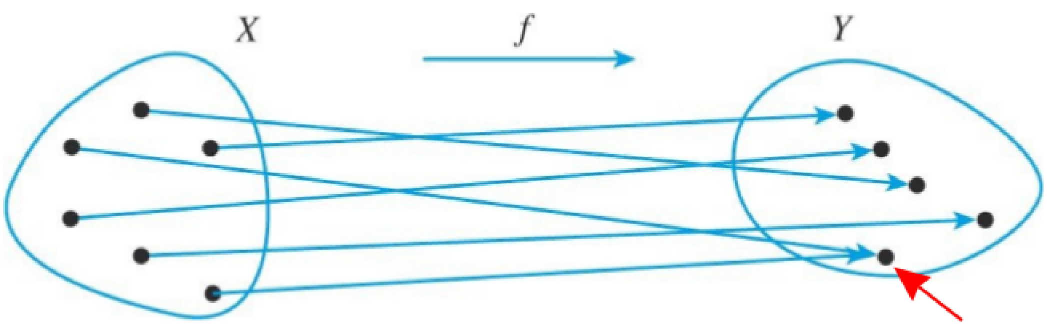
(6) For f to be a well-defined function, every person in P must have exactly one job/occupation.

(7) m is not onto. I am a member of the set P, and I am not a birth mother of anyone, so there is at least one member of the codomain that does not have a corresponding input. m is also not one-to-one; my sister and I are each members of P, and we have the same birth mother. $m(\text{me}) = m(\text{my sister})$, but I am not the same person as my sister.

(9) No, the function is not onto. There is a member of the codomain that has no corresponding input from the domain.



(10) No, the function is not one-to-one. There is one value in the codomain that is mapped to by two different values in the domain.



(14) $f(x) = 3x - 5$

(a) **Claim**: f is one-to-one.

Proof. Choose some $x_1, x_2 \in \mathbb{R}$ with $f(x_1) = f(x_2)$.
Thus $3x_1 - 5 = 3x_2 - 5$.
So $3x_1 = 3x_2$.
So $x_1 = x_2$. □

(b) **Claim**: f is onto.

Proof. Choose some $y \in \mathbb{R}$.
Let $x_t = \frac{y+5}{3}$.
Since $3, 5, y \in \mathbb{R},\ x_t \in \mathbb{R}$.
Since $x_t = \frac{y+5}{3},\ y + 5 = 3x_t$.
So $y = 3x_t - 5$.
Since $x_t, y \in \mathbb{R}$ and $y = 3x_t - 5,\ y = f(x_t)$. □

(26) $g : \mathbb{Z} \to \mathbb{N}$; $g(z) = z^2 + 1$.

(a) $g(2) = g(-2) = 5$ and $2 \neq -2$, so g is not one-to-one.

(b) $6 \in \mathbb{N}$. Looking for a $z_t \in \mathbb{Z}$ for which $g(z_t) = 6$ leads to this equation:

$$6 = z_t^2 + 1. \tag{1}$$

Solving (1) for z_t, we get

$$z_t = \sqrt{5}. \tag{2}$$

$\sqrt{5} \approx 2.2360679775$, so $\sqrt{5} \notin \mathbb{Z}$, so there is no input from $\mathbb{Z}$ for which 6 is an output of g, so g is not onto.

3. (a) a is not onto. There is no input set X_t for which $a(X_t) = \{1\}$. This is because the input set would have to be $\{\frac{1}{2}\}$, and $\frac{1}{2} \notin \mathbb{Z}$.

(b) a is one-to-one.

Proof. Choose some $X_1 \subseteq \mathbb{Z},\ X_2 \subseteq \mathbb{Z}$ with $a(X_1) = a(X_2)$.
So $\{2n \mid n \in X_1\} = \{2n \mid n \in X_2\}$.
Thus for every $x_1 \in \{2n \mid n \in X_1\},\ x_1 \in \{2n \mid n \in X_2\}$, and for every $x_2 \in \{2n \mid n \in X_2\}$, $x_2 \in \{2n \mid n \in X_1\}$.
Define $d(n) = 2n$.

> Choose some $n_1, n_2 \in \mathbb{Z}$ with $d(n_1) = d(n_2)$.
> So $2n_1 = 2n_2$.
> So $n_1 = n_2$.

So d is one-to-one.
HMMMMMM how do I move forward? □