Name: ______________________ Score: ______/100

EXAM 1 **M301** **Prof. Lyons** **Spring 1995**

There are 13 problems in all worth a total of 100 points. To get full credit, you must explain your answers to every single question: give your reasoning or show the work in a clear manner. This will not be repeated in the questions; that's right — this will not be repeated. The problems appear in random order. Decide for yourself which ones to do first; I recommend doing the ones you find easy first. You may write on the reverse sides of the paper; indicate when you do so.

1. (9 points) Choose h and k so that the system below has

$$x_1 + hx_2 = 1$$
$$2x_1 + 3x_2 = k$$

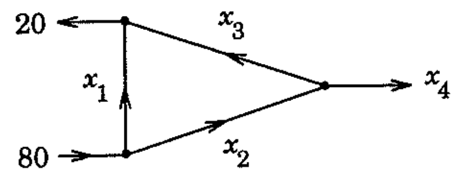
(a) no solution

(b) a unique solution

(c) many solutions.

2. (8 points) If a linear transformation $T : \mathbf{R}^n \to \mathbf{R}^m$ is one-to-one, what can you say about m and n?

3. (10 points) Find the general flow pattern of the network below. Assuming that the flows are all nonnegative, what is the largest possible value for x_3?



4. (7 points) Find the standard matrix for the linear transformation that reflects points in the plane through the line $x_1 = -x_2$.

5. (7 points) Suppose that the solution set of a certain system of equations can be described as $x_1 = 7 + x_4$, $x_2 = -5 - 2x_4$, $x_3 = 1 - 3x_4$, with x_4 free. Use vectors to describe this set as a line in $\mathbf{R}^4$.

6. (8 points) Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -1 \\ 3 \\ 7 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 3 \\ -2 \\ -2 \end{bmatrix}.$$

Does $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ span $\mathbf{R}^3$?

7. (8 points) Let $A = \begin{bmatrix} -3 & 1 \\ 6 & -2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$. Describe the set of all $\mathbf{b}$ for which $A\mathbf{x} = \mathbf{b}$ is consistent.

8. (10 points) Let

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -2 \\ 1 \\ 7 \end{bmatrix}, \quad \mathbf{y} = \begin{bmatrix} h \\ -3 \\ -5 \end{bmatrix}.$$

For what value(s) of h will $\mathbf{y}$ be in the plane spanned by $\mathbf{v}_1$ and $\mathbf{v}_2$?

9. (8 points) Fill in the blank: "If A is an $m \times n$ matrix, then the columns of A are linearly independent iff A has ________ pivot columns."

10. (6 points) In a certain region, about 4% of the city's population moves to the surrounding suburbs each year and about 3% of the suburban population moves into the city. In 1990, there were 600,000 residents in the city and 400,000 in the suburbs. Set up a difference equation that describes this situation, where $\mathbf{x}_0$ is the initial population in 1990. (Since you don't have Matlab available, I won't ask you to actually calculate the population for other years.)

11. (6 points) Which of the following sets of vectors are linearly independent?

(a) $\begin{bmatrix} 5 \\ 5 \end{bmatrix}, \begin{bmatrix} 6 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 4 \end{bmatrix}, \begin{bmatrix} 3 \\ 6 \end{bmatrix}$

(b) $\begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}, \begin{bmatrix} -6 \\ 5 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(c) $\begin{bmatrix} 6 \\ 2 \\ -8 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -2 \end{bmatrix}$

12. (7 points) Describe all solutions of $A\mathbf{x} = \mathbf{0}$ in parametric vector form if $A = \begin{bmatrix} 1 & -2 & -5 & 5 \\ 0 & 0 & -3 & 2 \end{bmatrix}$.
Could the answer be different if A is not equal to this matrix but is only row equivalent to it?

13. (6 points) Write the following equation as a matrix equation:

$$y_1 \begin{bmatrix} 3 \\ 1 \end{bmatrix} + y_2 \begin{bmatrix} -5 \\ 2 \end{bmatrix} + y_3 \begin{bmatrix} 0 \\ -1 \end{bmatrix} = \begin{bmatrix} -4 \\ 7 \end{bmatrix}.$$