CSCI-C 241 HW #5 Assigned Thursday, September 20th, 2018
Instructor: Wennstrom Due Tuesday, September 25th, 2018

Because I know some of you don't have the textbook yet, I will include the problems from the textbook with this assignment. I will *not* do this for future assignments, so make sure you **get a copy of the book soon!**

Book Problems: Do problems 5-9 from section 2.2 (pages 88-92) in the textbook.

5. Consider the following sets. The universal set for this problem is $\mathbb{N}$.

$$A = \text{The set of all even numbers.}$$
$$B = \text{The set of all prime numbers.}$$
$$C = \text{The set of all perfect squares.}$$
$$D = \text{The set of all multiples of 10.}$$

Using **only** the symbols 3, A, B, C, D, $\mathbb{N}$, $\in$, $\subseteq$, $=$, $\neq$, $\cap$, $\cup$, $\times$, $'$, $\varnothing$, (, and), write the following statements in set notation.

(a) None of the perfect squares are prime numbers.
(b) All multiples of 10 are even numbers.
(c) The number 3 is a prime number that is not even.
(d) If you take all the prime numbers, all the even numbers, all the perfect squares, and all the multiples of 10, you still won't have all the natural numbers.

6. Consider the following sets. The universal set for this problem is the set of all residents of India.

$$A = \text{The set of all English speakers.}$$
$$B = \text{The set of all Hindi speakers.}$$
$$C = \text{The set of all Urdu speakers.}$$

Express the following in the symbols of set theory.

(a) Residents of India who speak English, Hindi, and Urdu.
(b) Residents of India who do not speak English, Hindi, or Urdu.
(c) Residents of India who speak English, but not Hindi or Urdu.

7. Consider the following sets. The universal set for this problem is the set of all quadrilaterals..

$$A = \text{The set of all parallelograms.}$$
$$B = \text{The set of all rhombuses.}$$
$$C = \text{The set of all rectangles.}$$
$$D = \text{The set of all trapezoids.}$$

Using **only** the symbols x, A, B, C, D, $\mathbb{N}$, $\in$, $\subseteq$, $=$, $\neq$, $\cap$, $\cup$, $\times$, $'$, $\varnothing$, (, and), write the following statements in set notation.

(a) The polygon x is a parallelogram, but it isn't a rhombus

(b) There are other quadrilaterals besides parallelograms and trapezoids.

(c) Both rectangles and rhombuses are types of parallelograms.

8. Let the following sets be given. The universal set for this problem is the set of all students at some university.

$$F = \text{The set of all freshmen.}$$

$$S = \text{The set of all seniors.}$$

$$M = \text{The set of all math majors.}$$

$$C = \text{The set of all CS majors.}$$

(a) Using only the symbols F, S, M, C, $||$, $\cap$, $\cup$, $'$, and $>$, translate the following statement into the language of set theory.

There are more freshmen who aren't math majors then there are senior CS majors.

(b) Translate the following statement in set theory into everyday English.

$$(F \cap M) \subseteq C$$

9. Let E be the set of even numbers, and let P be the set of prime numbers. Use set notation to express the following statement: "2 is the only even prime number."

Non-Book Problems:

For the rest of this assignment, use the following definitions:

$A = \{1, 2, 3, 4, 5\}$

$B = \{0, 2, 4, 6, 8\}$

$C = \{1, 3, 5\}$

$S = \big\{\{1\}, \{1, 2\}, \{1, 3\}, \{1, 2, 3\}\big\}$

$X = \{x \mid x \in \mathbb{N} \wedge x \leq 5\}$

$Y = \{x \mid x \in \mathbb{N} \wedge x + 2 \leq 5\}$

$W = \{x + 2 \mid x \in \mathbb{N} \wedge x \leq 5\}$

$Q = \{x^3 \mid x \in \mathbb{N}\}$

$P = \{a + b \mid a \in A \wedge b \in B\}$

$H = \left\{\frac{n}{2} \;\middle|\; n \in \mathbb{N}\right\}$

Remember that $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$ is the set of natural numbers, $\mathbb{Z}$ is the set of integers $\{\ldots, -2, -1, 0, 1, 2, \ldots\}$, $\mathbb{Q}$ is the set of rational numbers $\left\{\frac{p}{q} \;\middle|\; p \in \mathbb{Z} \wedge q \in \mathbb{Z} \wedge q \neq 0\right\}$, and $\mathbb{R}$ is the set of real numbers (anything on the number line).

1. Give an example of each of the following. If you think no such example exists, you must explain why.

(a) a member of $A \cap B$

(b) a member of $A \setminus C$

(c) a member of H.

(d) a member of $H \setminus \mathbb{Z}$.

(e) a member of $\mathbb{Z} \setminus H$.

(f) three different members of Y

(g) a member of $W \cap Y$

(h) three different members of Q

(i) a member of S

(j) a member of $\varnothing$

(k) a proper subset of C that is not empty

(l) a subset of Q with at least three members

(m) a subset of S with at least two members

(n) a superset of C

2. Decide whether the following statements are true or false. No justification is needed.

(a) $2 = \{2\}$

(b) $\{2\} \in A$

(c) $\varnothing = \{\}$

(d) $\varnothing \in A$

(e) $\{2\} \in A$

(f) $\{1\} \in S$

(g) $3 \in S$

(h) $2 \in Y$

(i) $4 \in Y$

(j) $6 \in Y$

(k) $8 \in Q$

(l) $-8 \in Q$

(m) $C \subseteq A$

(n) $\{3, 1\} \subseteq C$

(o) $2 \subseteq A$

(p) $\{2\} \subseteq A$

3. Decide whether the following statements are true or false. Give a brief justification (for most of these, one sentence should be enough) for your answer.

(a) $11 \in P$

(b) $14 \in P$

Remember that if you think two sets are *not* equal, you need to give an example of something that is a member of one set, but not the other. If you think they *are* equal, then you need to explain why.

(c) $\{1,2,3\} = S$

(d) $\{3,5,1\} = C$

(e) $\{1,5,1,3,1,5,5,1,3\} = C$

(f) $\varnothing = \{\varnothing\}$

Remember that if you think the left set is *not* a subset of the right set, you need to give an example of something that is a member of the left set, but not a member of the right set. If you think it *is* a subset, you need to explain why *every* member of the left set is a member of the right set.

(g) $B \subseteq A$

(h) $C \subseteq X$

(i) $C \subseteq Y$

(j) $\varnothing \subseteq B$

4. For this problem, let the universe be $\mathcal{U} = \{0,1,2,3,4,5,6,7,8,9,10\}$. Write the following sets in set-list notation:

   (a) B'

   (b) $(C \cup \{1,2,3\})' \cap \{2,3,4\}$

   (c) $\varnothing'$

   (d) $\{0,1,2,3,4,5,6,7,8,9,10\}'$

   (e) $(A \setminus B) \cap \{1,2,3\}$

   (f) $A \cap C$

   (g) $A \cup C$

   (h) $A \setminus C$

   (i) $C \setminus A$

   (j) $B \cap C$

   (k) $B \setminus C$

5. Give a definition of the set B using set-builder notation. There are many possible correct answers here.

6. **Bonus**: Give a second, different definition of B using set-builder notation.

7. Calculate the following. If the cardinality is infinite, just say "infinite".

   (a) $|B|$

   (b) $|S|$

   (c) $|\{x \mid x \in \mathbb{N} \wedge x \leq 4\}|$

   (d) $|\{x \mid x \in \mathbb{N} \wedge x \leq 1000\}|$

   (e) $|\varnothing|$

   (f) $|Q|$