C241 Exam 1 Note Sheet

Zac Monroe

September 2018

1 Propositions

Proposition A statement that can be argued to be either true or false.
Ex: It is raining outside.

Tautology A proposition that is always true for any truth assignment.
Ex: $A \vee \neg A$; I am male or I am not male.

Contingency A proposition that is true for at least one truth assignment and false for at least one truth assignment.
Ex: A; I own a dog.

Satisfiable A proposition that is true for at least one truth assignment.
Ex: $A \wedge B$; You deserved to win.

Contradiction A proposition that is always false for any truth assignment.
Ex: $A \wedge \neg A$; I like cats and I don't like cats.

Truth tables These are good. Use them.

2 Arguments, Sets of Formulas

Argument A set of premise propositions, followed by a conclusion proposition.

Ex: $\dfrac{\begin{array}{c} A \\ A \rightarrow B \end{array}}{B}$;

If a person likes ice cream then they are happy., I like ice cream. $\vdash$ I am happy.

Validity of arguments An argument is valid iff, for every truth assignment that satisfies all its premises, its conclusion is also true. So an invalid argument is one where all the premises are true but the conclusion is false. For examples, the above arguments are valid.

Consistent set of formulas A set S of formulas is consistent iff there exists at least one truth assignment that satisfies every formula $s \in S$.

3 Natural Deduction Rules

$\wedge$-Elim. If $p \wedge q$, then p or q or both

$\wedge$-Intro. If p and q, then $p \wedge q$

$\rightarrow$-Elim. If p and $p \rightarrow q$, then q. Also called Application

$\rightarrow$-Intro. If you assume p and prove q, then $p \rightarrow q$. Also called Direct Proof

¬-Elim. If $\neg\neg p$, then p. Also called Double Negation

¬-Intro. If you assume p and prove any sort of contradiction to what you already knew, then $\neg p$. Also called Proof by Contradiction

∨-Intro. If p, then $p \vee q$. Also called Weakening

∨-Elim. If $p \vee q$, and you prove $p \vdash r$ and $q \vdash r$, then r holds in general. Also called Proof by Cases

4 Equivalence Rules

Double-Negation $p \equiv \neg\neg p$

Commutativity $p \wedge q \equiv q \wedge p$; $p \vee q \equiv q \vee p$

Associativity $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge r$; $p \vee (q \vee r) \equiv (p \vee q) \vee r$

Distributivity $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$; $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$

Impotency/Tautology $p \vee p \equiv p \wedge p \equiv p$

Contrapositive $p \rightarrow q \equiv \neg q \rightarrow \neg p$

De Morgan's Laws $\neg(p \wedge q) \equiv \neg p \vee \neg q$; $\neg(p \vee q) \equiv \neg p \wedge \neg q$

Implication $p \rightarrow q \equiv \neg p \vee q$; $\neg(p \rightarrow q) \equiv p \wedge \neg q$

Bi-Implication $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

Exclusive Disjunction $p \oplus q \equiv (p \wedge \neg q) \vee (q \wedge \neg p)$

5 Sets

Membership Sets are defined by their members. Any two sets with the same members are the same set. We say a is a member of A as $a \in A$

Cardinality The cardinality $|S|$ of a set S is the number of members that S has. Can be finite or infinite

Universe The conceivable domain/universe $\mathcal{U}$ that contains every possible thing that can or cannot be in a set

Complement The complement S' of a set S is the set of everything that is not a member of S but is a member of $\mathcal{U}$

Empty set The empty set $\emptyset = \{\}$ is the set with no members

Subset, superset A set A is a subset of set B if for every member $a \in A$, $a \in B$ also. B can still have other members $b \notin A$. B is then called a superset of A. $A \subseteq B; B \supseteq A$

Union The union $C \cup D$ is the set containing all members of C as well as all members of D, and nothing more

Intersection The intersection $V \cap W$ is the set $\{x : x \in V \wedge x \in W\}$

Set-list notation You know what this is. $\{1, 2, 3, \ldots\}$

Set-builder notation You know what this is as well. $\{p \mid q(p)\}$ is the set of all p for which $q(p)$ is true