

## B365 Homework 2

1. 100,020 Massachusetts adults were randomly sampled with two factors recorded: whether or not the individual had diabetes, and whether or not the person ate Kale. The following gives a table of the results.

	Diabetes	No Diabetes
Kale	801	9192
No Kale	9905	80122

- (a) We write  $p(\text{Diabetes}|\text{Kale})$  for the probability that a Massachusetts adult who eats kale has diabetes. Either give a value for  $p(\text{Diabetes}|\text{Kale})$  or explain why it cannot be computed.
  - (b) We write  $\hat{p}(\text{Diabetes}|\text{Kale})$  for the proportion of Kale-eating members of our sample above that had diabetes. Either give a value for  $\hat{p}(\text{Diabetes}|\text{Kale})$  or explain why it cannot be computed.
  - (c) Give 95% confidence intervals for the probability of having diabetes for both the kale-eating and non-kale-eating members of our Massachusetts adults.
  - (d) Can you conclude that the kale-eaters are less likely to have diabetes? Explain your reasoning.
  - (e) Can you conclude that kale consumption causes a lower diabetes rate in this population? Explain your reasoning.
  - (f) Come up with a possible theory that explains why kale-eaters have a lower rate of diabetes, but does not assume that kale *causes* the lower rate.
2. Consider the same numerical data, but imagine that the people were assigned to eat kale for 10 years, according to the following mechanisms. In each case say if you believe there is evidence that Kale causes a lower rate of diabetes and explain why.
- (a) The kale-eaters were chosen to be those whose zip-code ended in an odd digit. The rest chosen to not eat kale.
  - (b) People were asked if they considered themselves to be health-conscious. The health-conscious people were not allowed to eat kale while the non-health-conscious people were forced to eat kale.
  - (c) Each person used the R program and was assigned to the kale-eating group if and only
 

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runif(1) < .2
```

 in their program.
3. Consider the following computer experiment. Generate two “uniformly distributed” numbers  $x, y$  in the interval  $[0, 1]$  (this is what `runif` does). Let  $A$  be the event that  $x + y < 1$  and  $B$  be the event that  $x - y < 0$ .
- (a) Generate 1000  $(x, y)$  values and plot these in R. Create the 1000-length boolean vectors  $a$  and  $b$  according to whether or not the above events  $A$  and  $B$  are satisfied. There are 4 possible truth assignments of  $A$  and  $B$ :  $\{TT, TF, FT, FF\}$ . Use a different plot character for each possible truth assignment. Thus, for example, all of points where both  $A$  and  $B$  occur would have the same plot symbol, and similarly for the other possible truth assignments. From this picture argue that  $A$  and  $B$  either *are* or *are not* independent.
  - (b) Using your samples from the first part, compute 95% confidence intervals for  $P(A)$  and  $P(A|B)$ . Are these confidence intervals consistent with  $A$  and  $B$  being independent?
4. Consider the following experiment for generating two boolean variables corresponding to the events  $A, B$ . Here  $x\%y$  is the remainder when  $x$  is divided by  $y$ , so  $x\%1$  is the “decimal part” of  $x$ .

```
x = runif(1);
A = (x < .5)
B = ((2*x) %% 1) < .5
```

- (a) Simulate this experiment 1,000,000 times and generate confidence intervals for  $P(A)$ ,  $P(B)$ , and  $P(A, B)$  — the prob of  $A$  and  $B$  both occurring.
  - (b) Do  $A$  and  $B$  appear to be independent events?
5. Say if the following pairs of events should be modeled as independent or dependent. Explain your reasoning.
- (a) We choose a voter at random (all voters equally likely) from Bloomington and let  $A$  be the event that the voter favors mayor and  $B$  be the event that the voter favors the police chief.
  - (b) Two people are selected at random from Bloomington and let  $A$  be the event that the first person favors the mayor, while  $B$  is the event that the 2nd person favors the mayor.
  - (c) Flip a coin and let  $A$  be the event that the coin is heads and  $B$  be the event that the coin is tails.
  - (d) A person is selected at random from Bloomington.  $A$  is the event that the person likes the movie “The Incredibles” while  $B$  is the event that the person likes “The Incredibles 2.”