

1. Let  $A = \{1, 2, 3\}$  and  $B = \{a, b, c, d\}$ . Give an example of the following. If you think no such example exists, you must explain why not. You may not use any examples from elsewhere in this assignment. (Hint: Only four of these are impossible.)
  - (a) A function on  $A$  that is one-to-one and onto
  - (b) A function on  $A$  that is one-to-one, but not onto
  - (c) A function on  $A$  that is onto, but not one-to-one
  - (d) A function on  $A$  that is neither one-to-one nor onto
  - (e) A function from  $A$  to  $B$  that is one-to-one
  - (f) A function from  $A$  to  $B$  that is onto
  - (g) A function from  $A$  to  $B$  that is not one-to-one
  - (h) A function from  $B$  to  $A$  that is one-to-one
  - (i) A function from  $B$  to  $A$  that is onto
  - (j) A function from  $B$  to  $A$  that is not onto
  - (k) A function from  $\mathbb{Z}$  to  $\mathbb{N}$  that is onto.
  - (l) **Bonus:** A function from  $\mathbb{Z}$  to  $\mathbb{N}$  that is one-to-one.
2. Do problems 1-3, 6-7, 9-10, 14, and 26 from section 2.3 (pages 103-107) in the textbook.
3. **Bonus:** Consider the function  $a$  on the set of all sets of integers defined by  $a(X) = \{2n \mid n \in X\}$ . Justify your answers with a proof and/or counterexample as appropriate.
  - (a) Is  $a$  onto?
  - (b) Is  $a$  one-to-one?