B455 HW1

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1. Let $R =$ you vaguely recognize the person, $S =$ the person went to school with you.
$P(S|R) = \dfrac{P(R,S)}{P(R)} = \dfrac{P(R|S)P(S)}{P(R|S)P(S) + P(R|\neg S)P(\neg S)} = \dfrac{\frac{1}{2}\frac{1}{10}}{\frac{1}{2}\frac{1}{10} + \frac{1}{5}(1 - \frac{1}{10})} = \dfrac{.05}{.05 + .18} \approx 0.217391$

2. Let R_i ($i \in \{\text{H, C, LC}\}$) denote the risk of classifying a patient as class i.

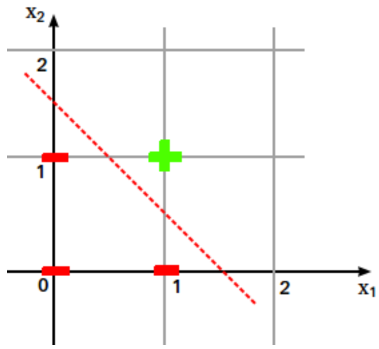
   $R_H = 0 * 0.5 + 5 * 0.2 + 20 * 0.3 = 7$
   $R_C = 1 * 0.5 + 0 * 0.2 + 10 * 0.3 = 3.5$
   $R_{LC} = 5 * 0.5 + 3 * 0.2 + 0 * 0.3 = 4.1$

   The output of the prediction with minimum risk is C, cirrhosis. I multiplied the probability that a patient is of class i, $P(i)$, by the risk of classifying as class j for all $j \neq i$ and summed them together to get weighted risk values. The class with the lowest risk value is C, cirrhosis.

3. Let S be the event that the car is stolen.
$P(color = red|S) = \frac{3}{5}, P(color = red|\neg S) = \frac{2}{5}$
$P(color = yellow|S) = \frac{2}{5}, P(color = yellow|\neg S) = \frac{3}{5}$
$P(type = sports|S) = \frac{4}{5}, P(type = sports|\neg S) = \frac{2}{5}$
$P(type = family|S) = \frac{1}{5}, P(type = family|\neg S) = \frac{3}{5}$
$P(origin = domestic|S) = \frac{2}{5}, P(origin = domestic|\neg S) = \frac{3}{5}$
$P(origin = imported|S) = \frac{3}{5}, P(origin = imported|\neg S) = \frac{2}{5}$
For a new given car with attributes $A = \{color, type, origin\}$, $S = argmax([\prod_{a \in A} P(a|S), \prod_{a \in A} P(a|\neg S)])$.
For a red family domestic car, $P(S) = \frac{3}{5}\frac{1}{5}\frac{2}{5} = \frac{6}{125}, P(\neg S) = \frac{2}{5}\frac{3}{5}\frac{3}{5} = \frac{18}{125} > \frac{6}{125}$, so the Naive Bayes' classifier would predict that such a car would not be stolen.

4. See em.ipynb (inside the zipped folder or at https://colab.research.google.com/drive/1uFmCv6GHbWuPKb1F5yJ1XLL7wm

5. The output of this model with input (0,0) is $f(-1 * 1.5 + 0 * 1 + 0 * 1) = f(-1.5) = 0$. The discriminant function has equation $1 * x_1 + 1 * x_2 - 1.5 = 0$, and is sketched on the graph below (in which the red dotted line represents the discriminator function, the green + symbol represents the location of input for which the model returns 1, and the red - symbols represent the locations of input for which the model returns 0). This function seems to correspond to the AND ($\wedge$) logic gate.



6. A single perceptron cannot learn this problem because the answers are not linearly separable.

   Proof. We want this truth table:

x1	x2	x3	class
1	1	1	0
1	1	0	1
1	0	1	1
1	0	0	0
0	1	1	1
0	1	0	0
0	0	1	0
0	0	0	1

   So we want these equations, assuming that the problem is linearly separable:

   $1 * w_1 + 1 * w_2 + 1 * w_3 - w_0 < 0 \iff w_0 > w_1 + w_2 + w_2$

   $1 * w_1 + 1 * w_2 + 0 * w_3 - w_0 \geq 0 \iff w_0 \leq w_1 + w_2$

   $1 * w_1 + 0 * w_2 + 1 * w_3 - w_0 \geq 0 \iff w_0 \leq w_1 + w_3$

   $1 * w_1 + 0 * w_2 + 0 * w_3 - w_0 < 0 \iff w_0 > w_1$

   $0 * w_1 + 1 * w_2 + 1 * w_3 - w_0 \geq 0 \iff w_0 \leq w_2 + w_3$

   $0 * w_1 + 1 * w_2 + 0 * w_3 - w_0 < 0 \iff w_0 > w_2$

   $0 * w_1 + 0 * w_2 + 1 * w_3 - w_0 < 0 \iff w_0 > w_3$

   $0 * w_1 + 0 * w_2 + 0 * w_3 - w_0 \geq 0 \iff w_0 \leq 0$

   By $w_0 \leq 0$, w_0 is not positive. So, by $w_0 > w_j$ $(j \in \{1, 2, 3\})$, w_1, w_2, and $w_3 < 0$.
   However, $w_0 \leq w_1 + w_2$ means that w_0 is less than or equal to the sum of two negative numbers that are indeed smaller than w_0, which is a contradiction. So this problem is not linearly separable. $\square$

7. $y(x) = \mathbf{w}^T x + b$. The closest point x^* to x' on the hyperplane is given by $x^* = k\mathbf{w}^T + x', k \in \mathbb{R}$.
   A unit normal vector to the hyperplane is given by $\mathbf{n} = \frac{\mathbf{w}^T}{\|\mathbf{w}^T\|}$. The shortest distance from x' to the hyperplane is the projection of $x' - x_0$ along $\mathbf{n}$ (where x_0 is the point on the hyperplane closest to x'), or
   $|(x' - x_0) \cdot \mathbf{n}| = |x' \cdot \mathbf{n} - x_0 \cdot \mathbf{n}| = \frac{|\mathbf{w}^T \cdot x' - \mathbf{w}^T \cdot x_0|}{\|\mathbf{w}^T\|} = \frac{|y(x') - b - \mathbf{w}^T \cdot x_0|}{\|\mathbf{w}^T\|} = \frac{|y(x')|}{\|\mathbf{w}\|}$
   (The last step is just justified because $y(x_0) = \mathbf{w}^T \cdot x_0 + b = 0$ because x_0 is on the hyperplane itself.)