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B365

HW1

1. Problem 1:
    (a) Run prob1.r to see an output of an estimate of P(A wins). One estimate that I received by running that file was 0.3377 +- 0.0098
    (b) By solving the equation:

    $$error = 1.96/\sqrt{(4*M)}$$

    for M, we are led to the equation:

    $$M = (.98/error)^2$$

    So, if I want my error to be .005 (+- .005 indicates that the width of the confidence interval is 2 * .005, or .01), then I should conduct M = 38,416 trials.
    (c) This statement means that if one were to conduct 38,416 trials in the same way, then 95% of the time, the interval defined by the estimated value of P(A wins) +- the radius of the confidence interval (P_hat(A wins) +- .005, in other words) will contain the true value of P(A wins). So, one can say that the true value of P(A wins) is different from P_hat(A wins) by 0.5% or less with 95% confidence (i.e., that statement will be true 95% of the time).
    (d) I believe that the true probability of P(A wins) is 1/3. This is because there are 2 cases out of the possible 6 final possibilities for the outcomes of each round in which player A wins. I have included a table that helps illustrate this point.

A	B	C	Winner
H	H	H	None. Replay
H	H	T	C
H	T	H	B
H	T	T	A
T	H	H	A
T	H	T	B
T	T	H	C
T	T	T	None. Replay

2. Each time A or B draws a card, there is a ¼ chance that that card is a heart. I simulated the experiment a total of 38,416 times, to get an error value of .005. My comments in prob2.r help explain my usage of some particular built-in R functions.
3. Problem 3:
   (a) The sample space Ω for the experiment is all possible pairings of cards that are drawn from the same deck.

   |Ω|, the number of elements in Ω, is choose(52,2), or 1326.

   (b) Each rank is used by each of the four suits. There are 13 ranks. There should therefore be 13 * choose(4,2) elements in Ω whose cards have the same rank. This number comes out to be 78.

   (c) The probability of drawing a pair is the proportion of pairs in Ω to |Ω| (the number of elements in the sample space). This is 78/1326, or roughly 0.058823.
4. Problem 4:
   (a) P(0 bullseyes) is choose(10,0)/(2^10), or 1/1024, or roughly 0.00097656.

   (b) P(1 bullseyes) is choose(10,1)/(2^10), or 10/1024, or roughly 0.0097656.

   (c) P(2 bullseyes) is choose(10,2)/(2^10), or 45/1024, or roughly 0.043945.

   (d) P(3 bullseyes) is choose(10,3)/(2^10), or 120/1024, or roughly 0.11719.

5. Problem 5:
   (a) See prob5.r
   (b) I reused some of the code that I used for part (a) in a loop for part (b). I got roughly 0.95.
6. First off, there are 10!/(10-2)! = 10*9 = 90 possible orderings of numbers that A and B can grab. This comes from the permutations function, p(n,k) = n!/(n-k)!.

   P(A's number > B's number) = 45/90 = 0.5. This is because if A draws the smallest number, B is guaranteed to draw a larger number. So from the scenario in which A draws the smallest number, there are 0 outcomes in which that number is greater than the number that B draws.

   If A draws the second smallest number, there is only one possible number of the remaining 9 that B can choose from that is smaller than A's number.

   If A draws the third smallest number, there are two possible numbers of the remaining 9 that B can choose from that are smaller than A's number.

   If A draws the fourth smallest number, there are three possible numbers of the remaining 9 that B can choose from that are smaller than A's number.

   These possibilities continue as so until A draws the greatest number in the bag. Of course, then, all 9 numbers that B can possibly draw are smaller than A's number. So the total possible outcomes in which A's number > B's number is the 9^{th} triangle number, or

9+8+7+6+5+4+3+2+1, or 45. The proportion of this number to the total number of equally likely outcomes is then the probability that A chooses a number greater than what B chooses. This number is, again, 0.5.

7. See prob7.r to view my approach to producing integers 1-6 with the desired probabilities.