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B365

HW1

1. Problem 1:
   1. Run prob1.r to see an output of an estimate of P(A wins). One estimate that I received by running that file was 0.3377 +- 0.0098
   2. By solving the equation:

for M, we are led to the equation:

So, if I want my error to be .005 (+- .005 indicates that the width of the confidence interval is 2 \* .005, or .01), then I should conduct M = 38,416 trials.

* 1. This statement means that if one were to conduct 38,416 trials in the same way, then 95% of the time, the interval defined by the estimated value of P(A wins) +- the radius of the confidence interval (P\_hat(A wins) +- .005, in other words) will contain the true value of P(A wins). So, one can say that the true value of P(A wins) is different from P\_hat(A wins) by 0.5% or less with 95% confidence (i.e., that statement will be true 95% of the time).
  2. I believe that the true probability of P(A wins) is 1/3. This is because there are 2 cases out of the possible 6 final possibilities for the outcomes of each round in which player A wins. I have included a table that helps illustrate this point.

|  |  |  |  |
| --- | --- | --- | --- |
| A | B | C | Winner |
| H | H | H | None. Replay |
| H | H | T | C |
| H | T | H | B |
| H | T | T | A |
| T | H | H | A |
| T | H | T | B |
| T | T | H | C |
| T | T | T | None. Replay |

1. Each time A or B draws a card, there is a ¼ chance that that card is a heart. I simulated the experiment a total of 38,416 times, to get an error value of .005. My comments in prob2.r help explain my usage of some particular built-in R functions.
2. Problem 3:
   1. The sample space Ω for the experiment is all possible pairings of cards that are drawn from the same deck.

|Ω|, the number of elements in Ω, is choose(52,2), or 1326.

* 1. Each rank is used by each of the four suits. There are 13 ranks. There should therefore be 13 \* choose(4,2) elements in Ω whose cards have the same rank. This number comes out to be 78.
  2. The probability of drawing a pair is the proportion of pairs in Ω to |Ω| (the number of elements in the sample space). This is 78/1326, or roughly 0.058823.

1. Problem 4:
   1. P(0 bullseyes) is choose(10,0)/(2^10), or 1/1024, or roughly 0.00097656.
   2. P(1 bullseyes) is choose(10,1)/(2^10), or 10/1024, or roughly 0.0097656.
   3. P(2 bullseyes) is choose(10,2)/(2^10), or 45/1024, or roughly 0.043945.
   4. P(3 bullseyes) is choose(10,3)/(2^10), or 120/1024, or roughly 0.11719.
2. Problem 5:
   1. See prob5.r
   2. I reused some of the code that I used for part (a) in a loop for part (b). I got roughly 0.95.
3. First off, there are 10!/(10-2)! = 10\*9 = 90 possible orderings of numbers that A and B can grab. This comes from the permutations function, p(n,k) = n!/(n-k)!.  
     
   P(A’s number > B’s number) = 45/90 = 0.5. This is because if A draws the smallest number, B is guaranteed to draw a larger number. So from the scenario in which A draws the smallest number, there are 0 outcomes in which that number is greater than the number that B draws.  
     
   If A draws the second smallest number, there is only one possible number of the remaining 9 that B can choose from that is smaller than A’s number.  
     
   If A draws the third smallest number, there are two possible numbers of the remaining 9 that B can choose from that are smaller than A’s number.  
     
   If A draws the fourth smallest number, there are three possible numbers of the remaining 9 that B can choose from that are smaller than A’s number.  
     
   These possibilities continue as so until A draws the greatest number in the bag. Of course, then, all 9 numbers that B can possibly draw are smaller than A’s number. So the total possible outcomes in which A’s number > B’s number is the 9th triangle number, or 9+8+7+6+5+4+3+2+1, or 45. The proportion of this number to the total number of equally likely outcomes is then the probability that A chooses a number greater than what B chooses. This number is, again, 0.5.
4. See prob7.r to view my approach to producing integers 1-6 with the desired probabilities.