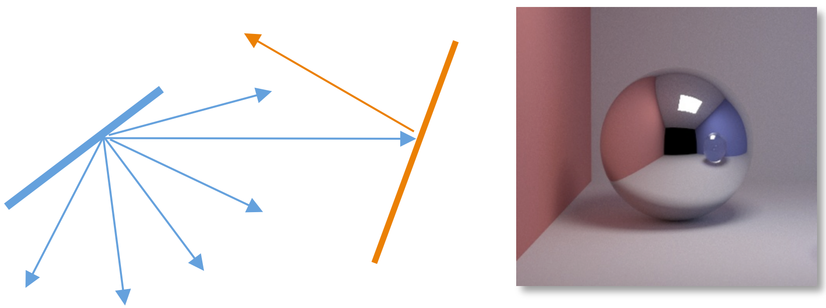
Rendering Equation

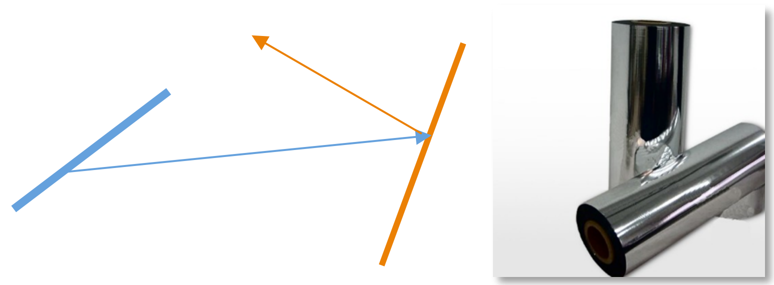
You’ve learned how to do this

Now we'll see how to do this

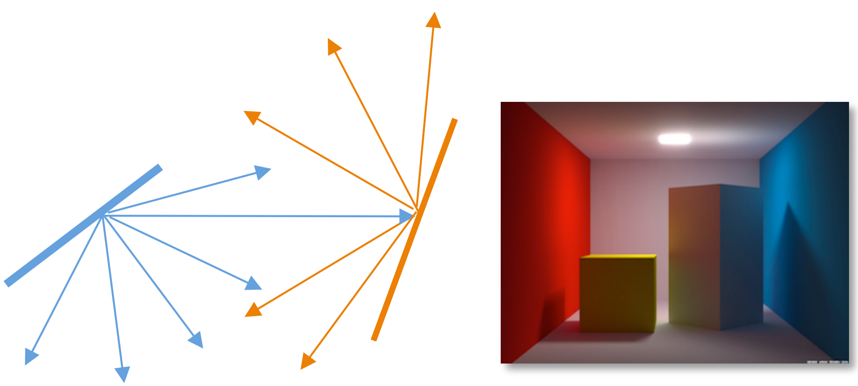
Types of Light Transport



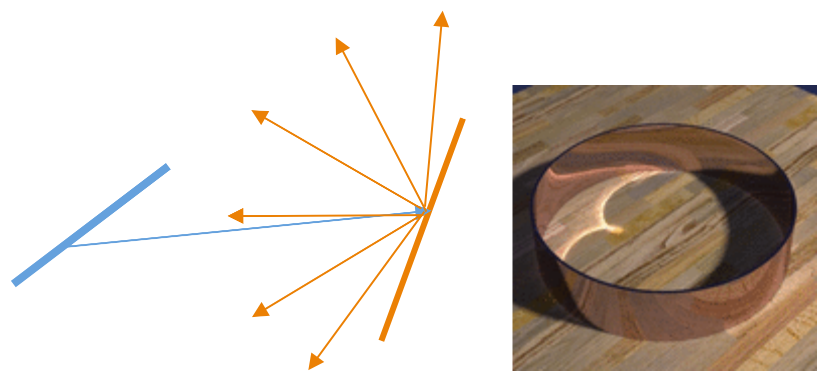
Diffuse-specular



Specular-specular

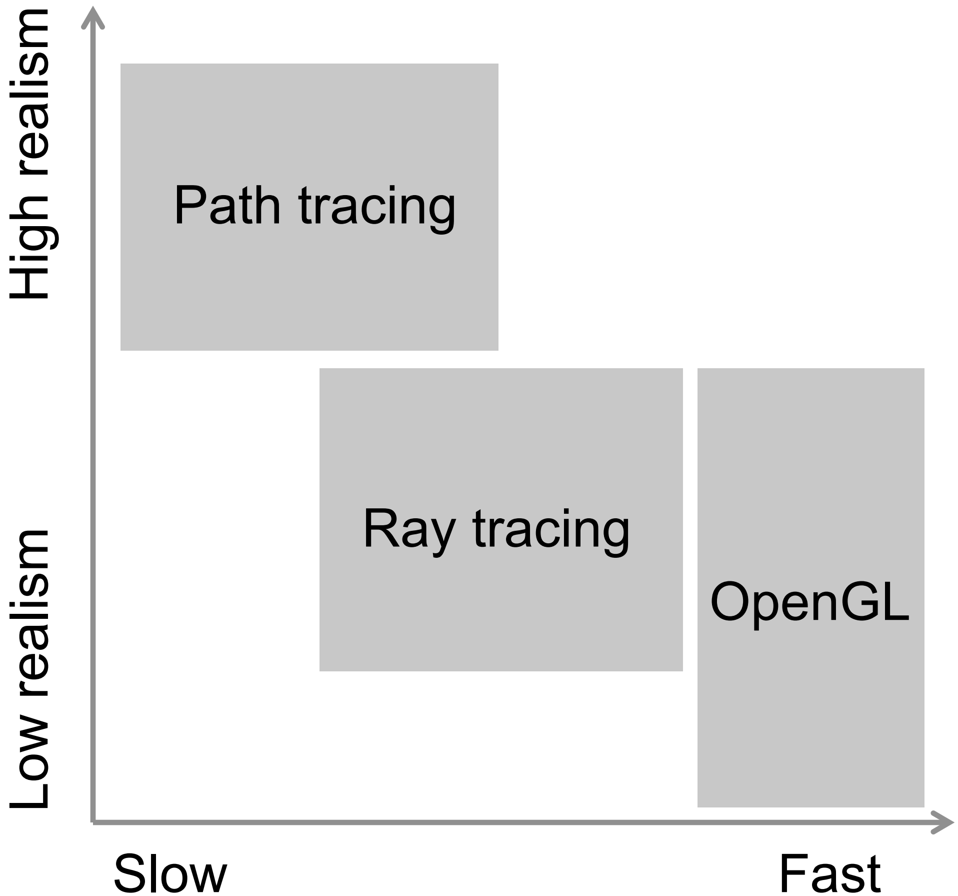


Diffuse-diffuse



Specular-diffuse

Speed/quality Domain



In the next lectures

- This lecture (1h): The rendering equation
  - Definition
  - Light
  - Reflectance
  - Time, lens and spectral domain
- Next lectures (1+2hs): Methods to solve it
  - Path tracing
  - Photon mapping

Physically-based Rendering

- Simulation of light transport
- Light
  - The nature of light, how it travels in the environment
- Material
  - Anything that interacts with light, how it reflects, refracts or scatters light
  - Bidirectional Reflectance Distribution Function
- Geometry

Light

Visible light is electromagnetic radiation with wavelengths approximately in the range from 400 nm to 700 nm

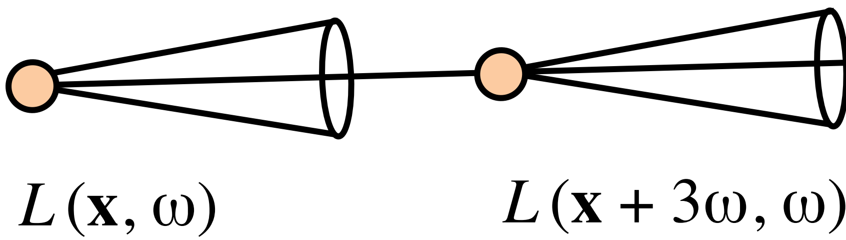
What is light?

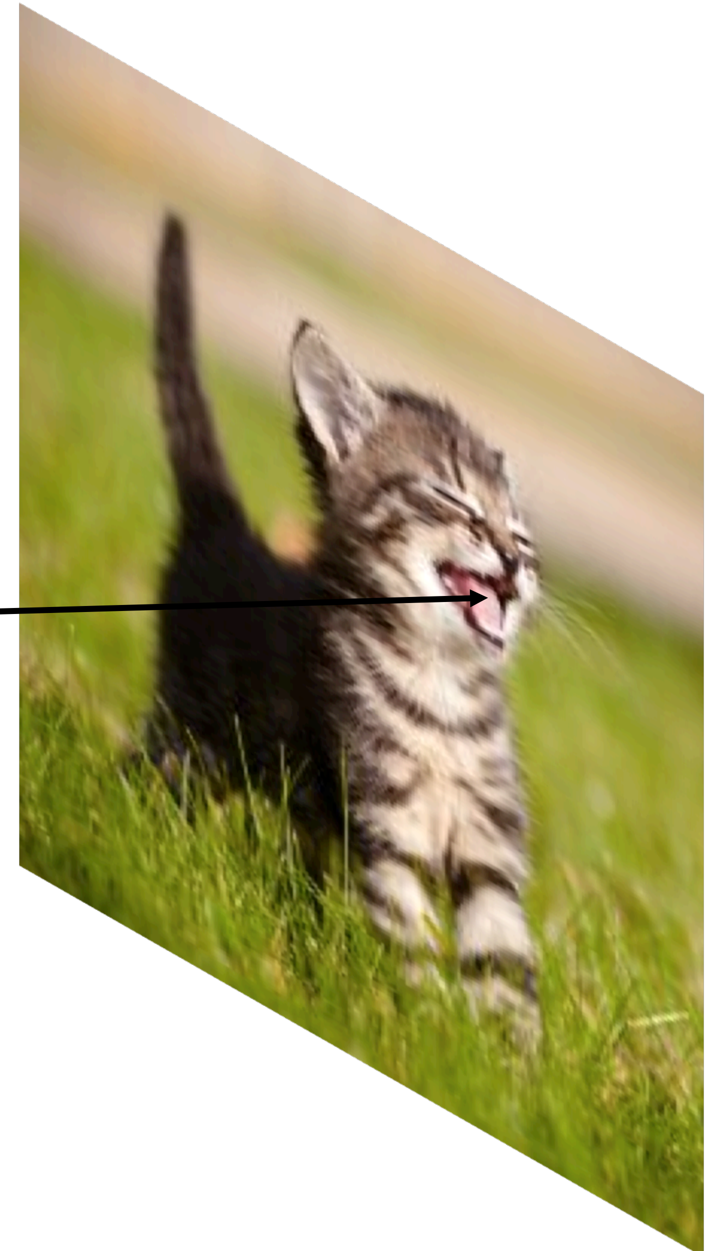
- Light can be viewed as
  - Wave or
  - Particle phenomenon
- Particles are photons
  - Packets of energy which travel in a straight line in vacuum with velocity c (~300,000 km/s)
- For us here:
  Continuous quantity at infinite speed

Units: Radiance

- There is a large number of radiometric units
- We will simulate in units of **radiance**
- Radiance $L(\mathbf{x}, \omega)$ is the quantity that is high if you look at a bright point $\mathbf{x}$ from angle ω
- How many photons at a wavelength per unit time, unit area and unit solid angle
- Does not change when moving along ω in free space

Radiance does not change when moving along ω in free space





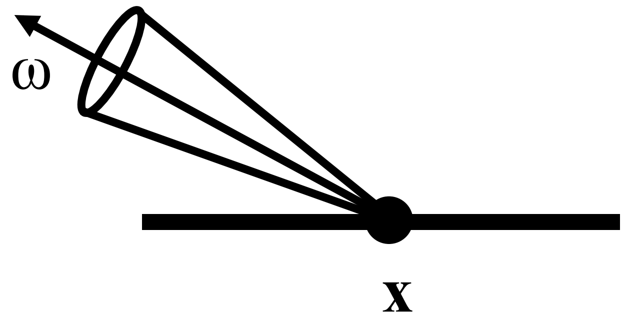
Simplifying Assumptions

1. Wavelength-independence
   – No interaction between wavelengths (no fluorescence)
2. Time-invariance
   – Solution valid over time unless scene changes (no phosphorescence)
3. Vacuum
   – Interaction only occurs at the surfaces of objects (non-participating medium)

The Rendering Equation

- Rendering Equation [Kajiya 1986]
  - Integral equation
  - Solution is a radiance distribution over space and angle
- A solution of this equation =
  A solution to the whole rendering problem
- Each approach to rendering is a different type of solution to this equation
- Popular approaches:
  Finite Element, Monte Carlo, Density Estimation

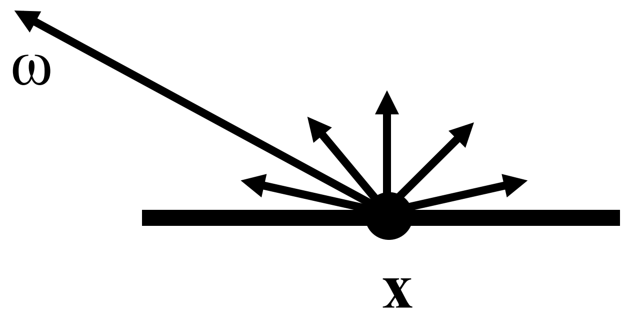
The Rendering Equation



$$L(\mathbf{x}, \omega) = L_e(\mathbf{x}, \omega) + \int f_r(\mathbf{x}, \omega_i, \omega)\, L(\mathbf{y}, -\omega_i) \cos\theta_i \, d\omega_i$$

$L(\mathbf{x}, \omega)$ is the radiance from a point on a surface in a given direction ω

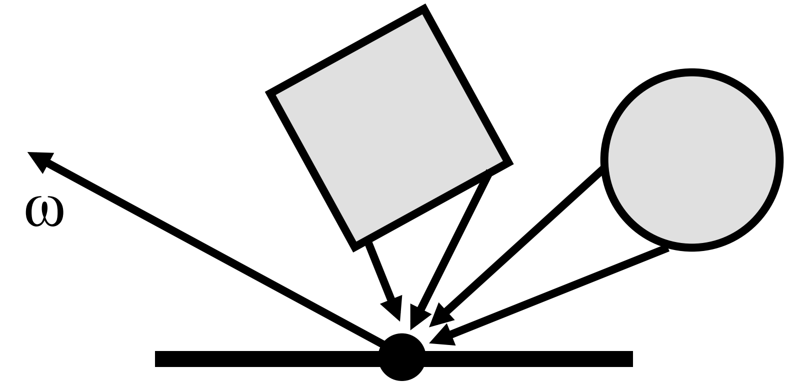
The Rendering Equation



$$L(\mathbf{x}, \omega) = L_e(\mathbf{x}, \omega) + \int f_r(\mathbf{x}, \omega_i, \omega)\, L(\mathbf{y}, -\omega_i) \cos\theta_i \, d\omega_i$$

$L_e(\mathbf{x},\omega)$ is the emitted radiance from a point: L_e is non-zero only if $\mathbf{x}$ is emissive, i.e., a light source.

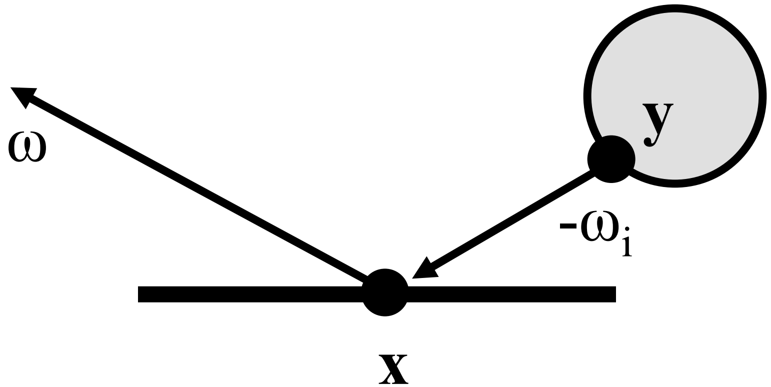
The Rendering Equation



$$L(\mathbf{x}, \omega) = L_e(\mathbf{x}, \omega) + \underbrace{\int f_r(\mathbf{x}, \omega_i\ , \omega)\ L(\mathbf{y}, -\omega_i) \cos\theta_i \ d\omega_i}$$

Reflected light. Summed contribution from all other surfaces in the scene

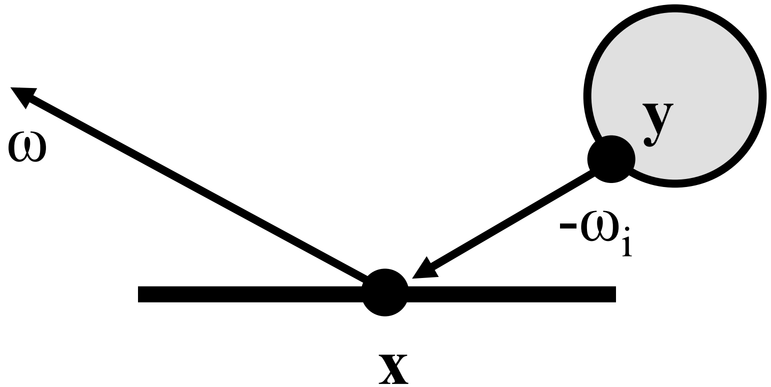
The Rendering Equation



$$L(\mathbf{x}, \omega) = L_e(\mathbf{x}, \omega) + \int f_r(\mathbf{x}, \omega_i, \omega)\, L(\mathbf{y}, -\omega_i) \cos\theta_i \, d\omega_i$$

For each ω_i, compute $L(\mathbf{y}, -\omega_i)$: the radiance at point $\mathbf{y}$ in the direction $-\omega_i$ (i.e., radiance arriving at $\mathbf{x}$)

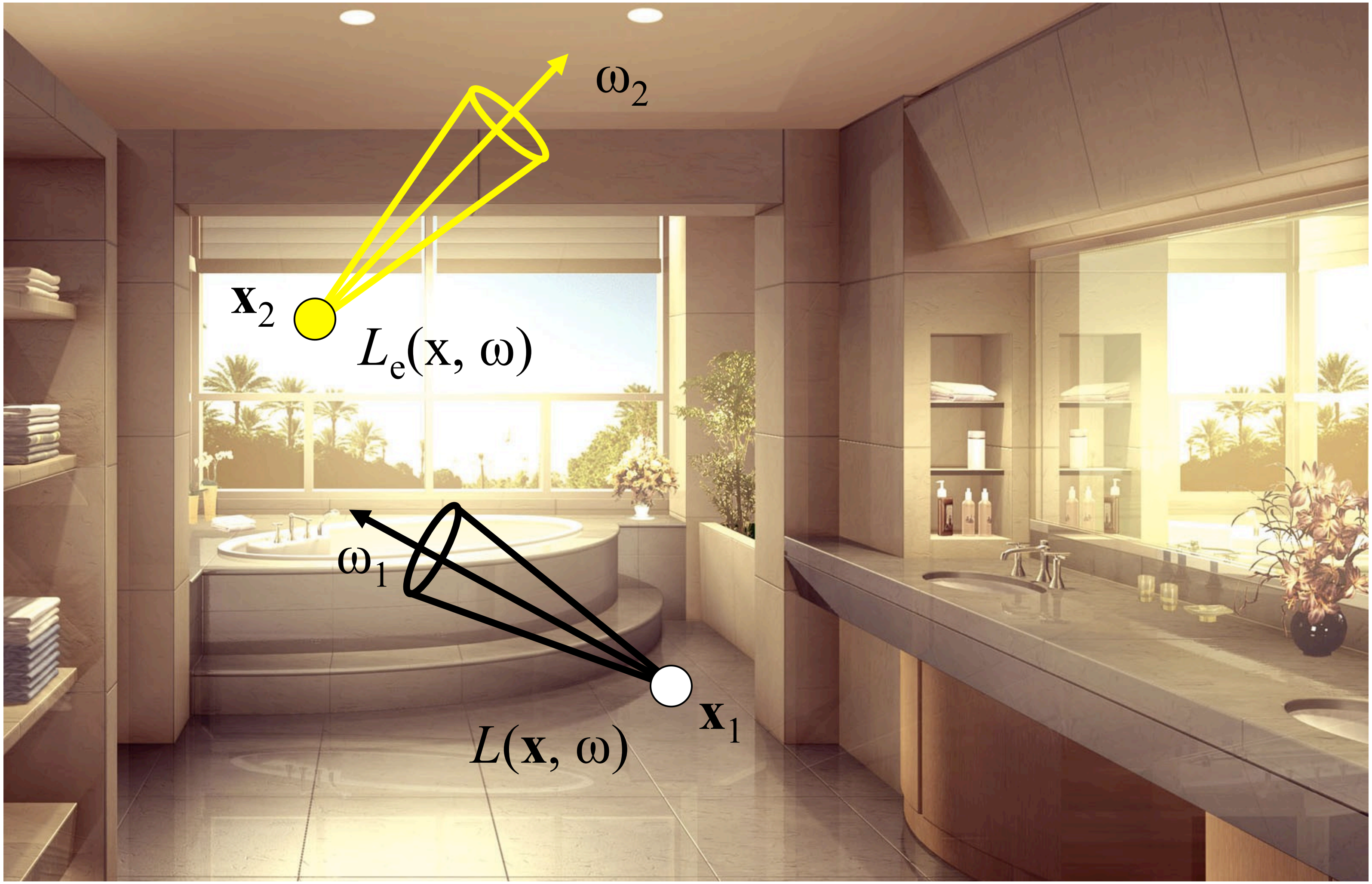
The Rendering Equation



$$L(\mathbf{x}, \omega) = L_e(\mathbf{x}, \omega) + \int f_r(\mathbf{x}, \omega_i, \omega)\, L(\mathbf{y}, -\omega_i) \cos \theta_i \, d\omega_i$$

Scale the contribution by $f_r(\mathbf{x}, \omega_i, \omega)$, the reflectivity (BRDF) of the surface at **x**,

There is no image



Recap

- What are the players?
  1. Emission, i.e., light sources
  2. Spherical integration
  3. Visibility, i.e., finding $\mathbf{y}$
  4. Reflectivity, i.e., BRDF aka. material
- Will see all of them in detail next

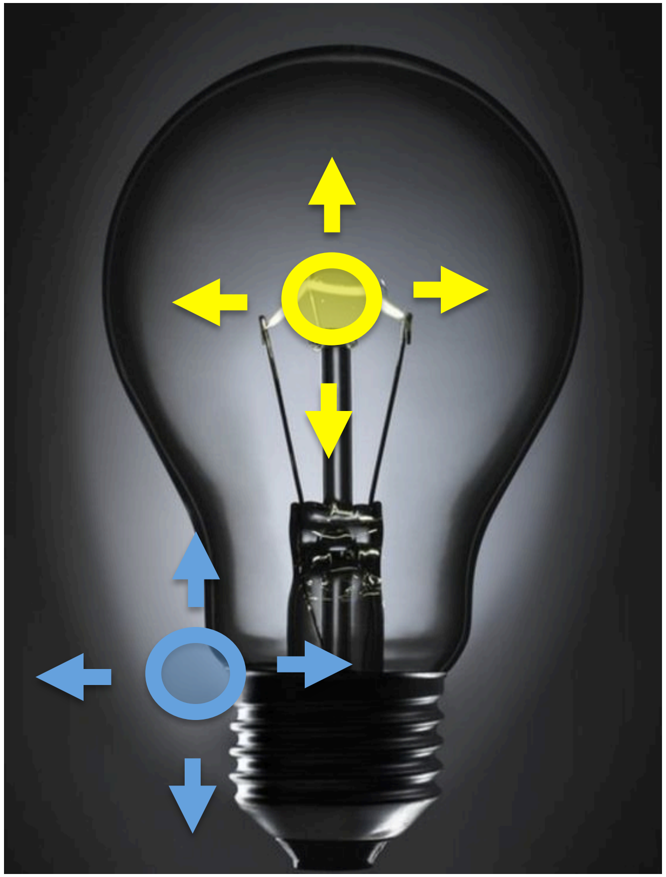
Light sources

Light sources

- Forget about points lights
- From now on, every location $\mathbf{x}$ can send light into every direction ω
- Emission function $L_{\mathrm{e}}(\mathbf{x}, \omega)$

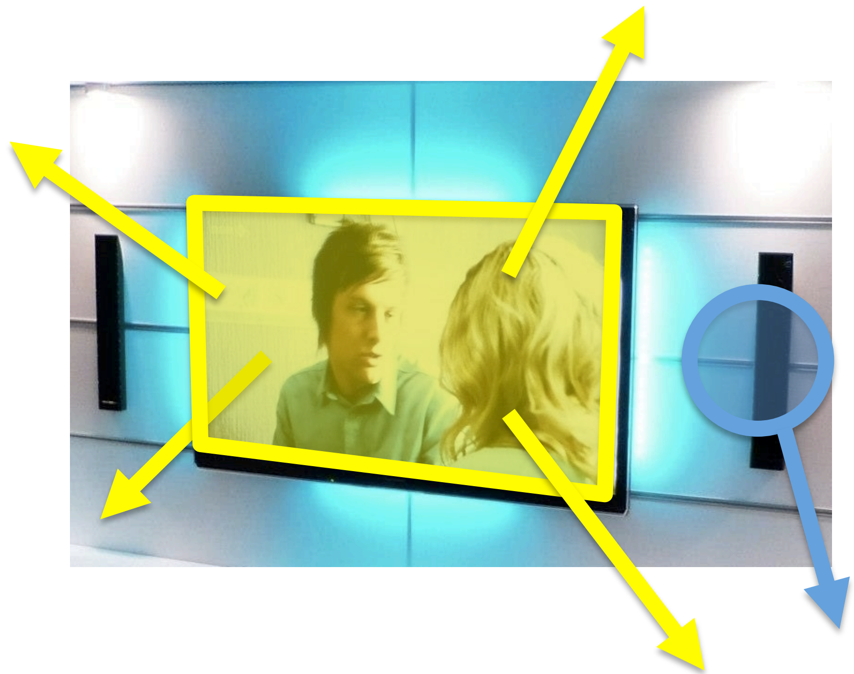
Example light

Emission is zero, except at the center, where it is L_e for all directions



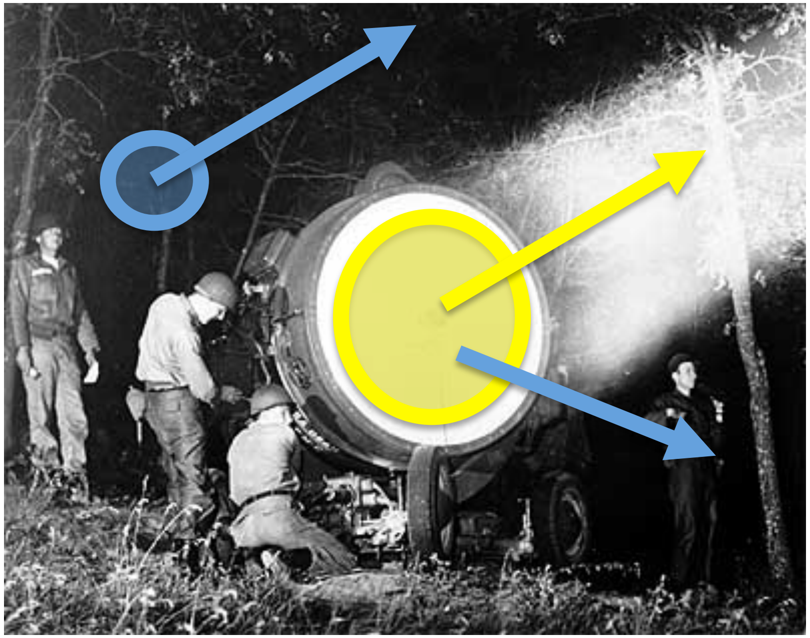
Example light

Emission is zero except at all points on the TV in all direction, where it is L_e

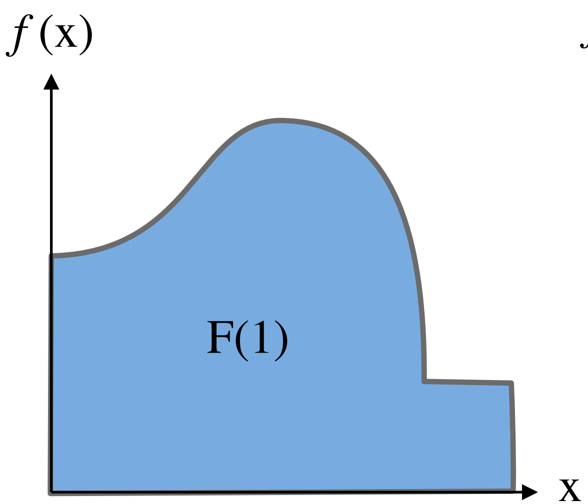


Example light

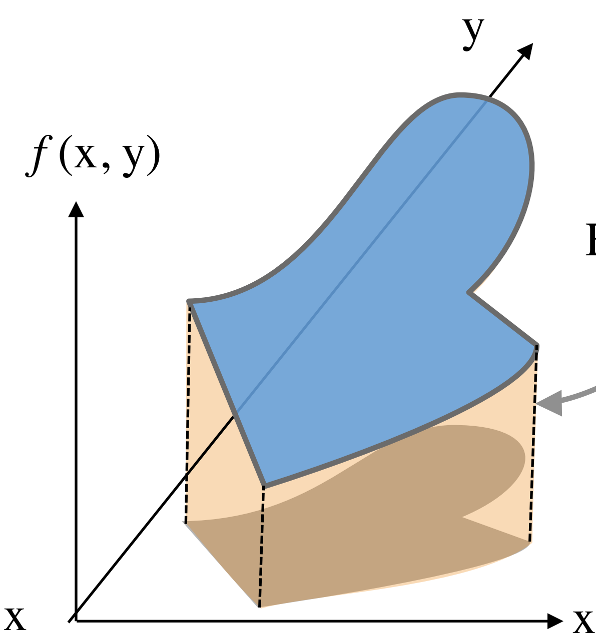
Emission is zero except at all points on the surface in direction of the search light, where it is L_e



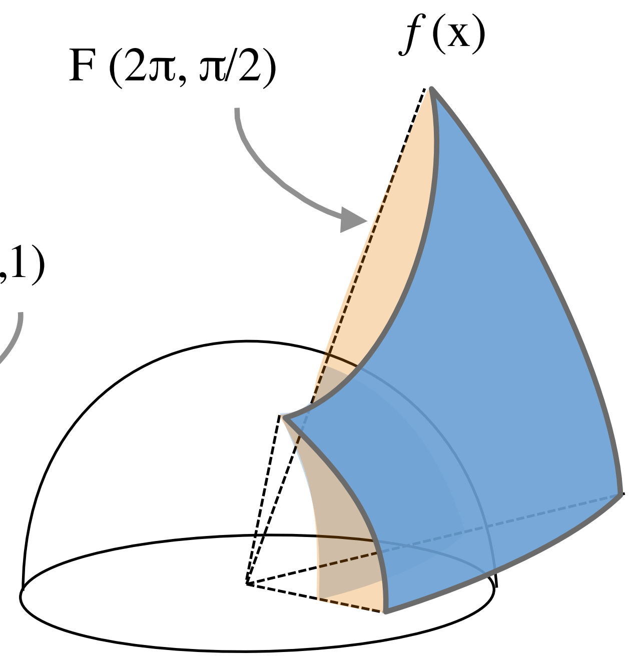
(Hemi)-spherical integration



1D Integration

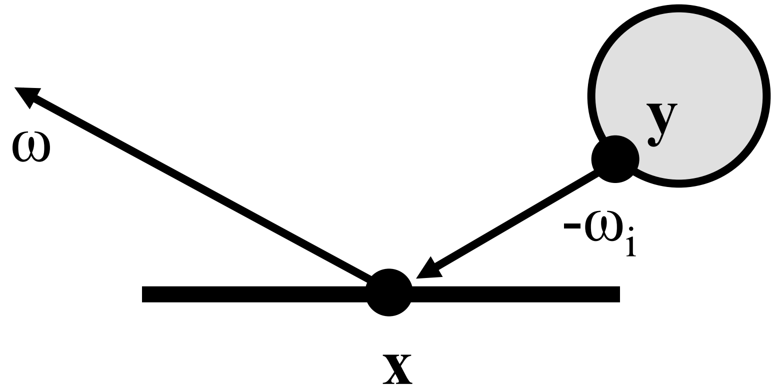


2D Integration



Spherical Integration

What is $\mathbf{y}$?

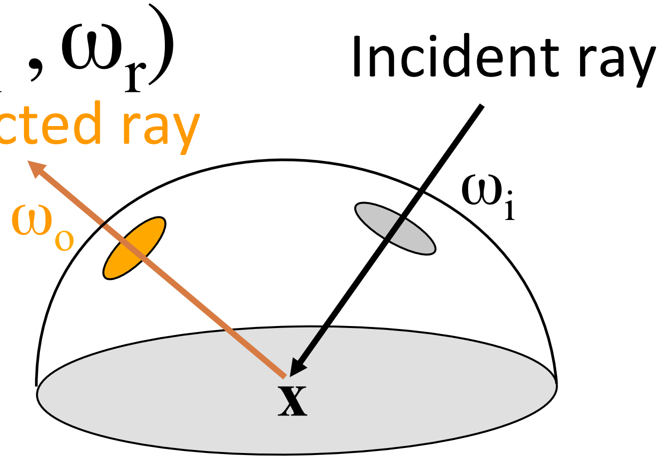


- $\mathbf{y}$ is the first point along ω
- Easy to say but hard to compute: Ray-tracing
- The source of infinite frequencies

BRDF

Bi-directional **R**eflectance **D**istribution **F**unction

- Radiance reflected at direction ω_o from irradiance at direction ω_i
- Symbol $f_r(\omega_i, \omega_r)$



Properties of BRDFs

1. Non-negativity

$$f_r(\omega_i, \omega_r) \geq 0$$

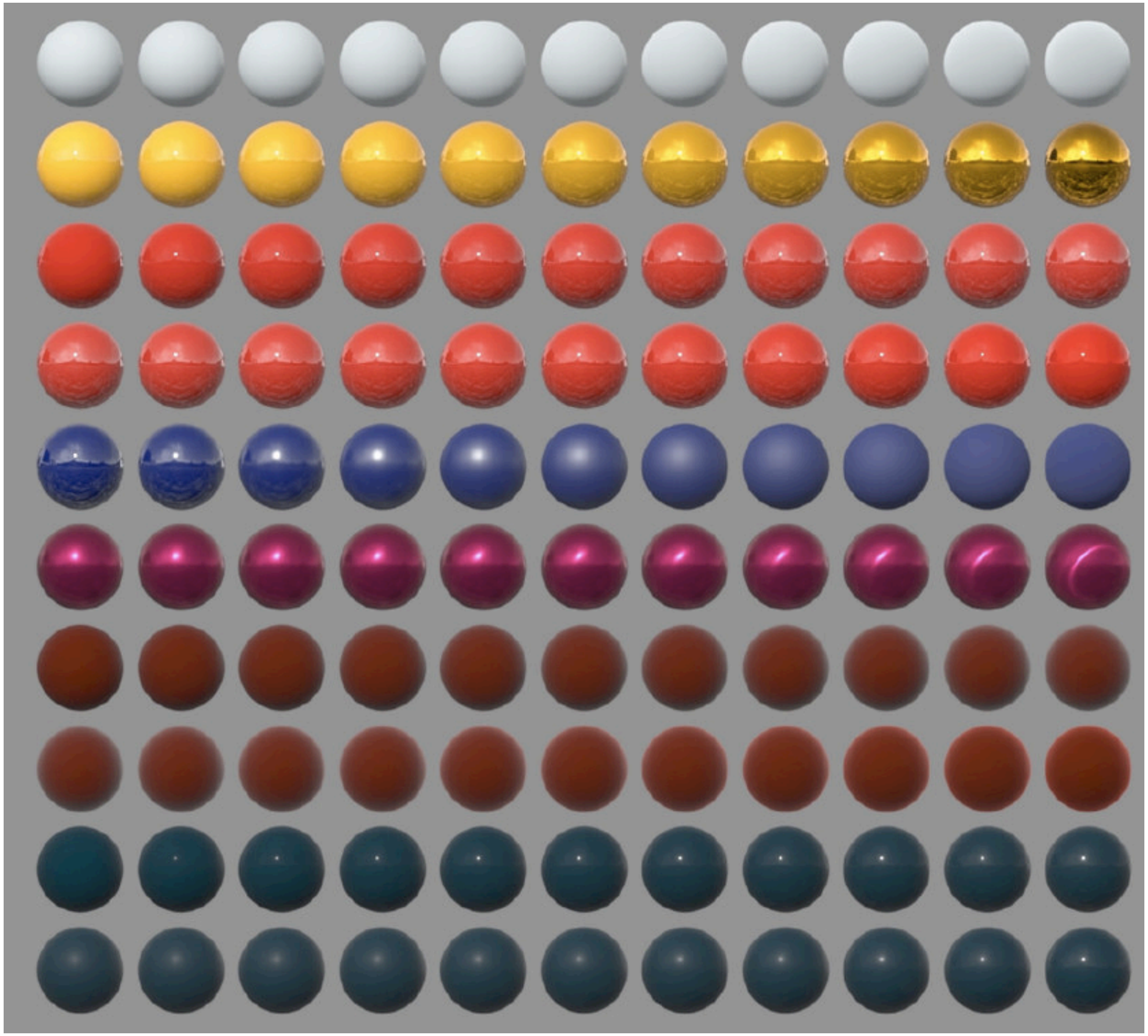
2. Energy conservation

$$\int_{\Omega} f_{\mathrm{r}}(\omega_{\mathrm{i}}, \omega_{\mathrm{r}}) \mathrm{d}\mu(\omega_{\mathrm{r}}) \leq 1 \quad \text{for all } \omega_{\mathrm{i}}$$

3. Reciprocity

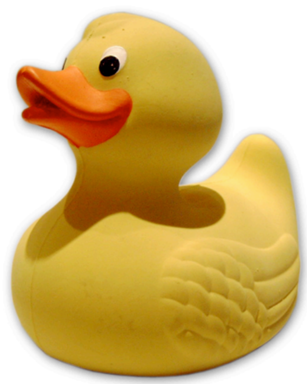
$$f_r(\omega_i, \omega_r) = f_r(\omega_r, \omega_i)$$

BRDF examples



Different types of materials

- Matte materials
  - Flour
  - Rubber
  - Matte wall paint

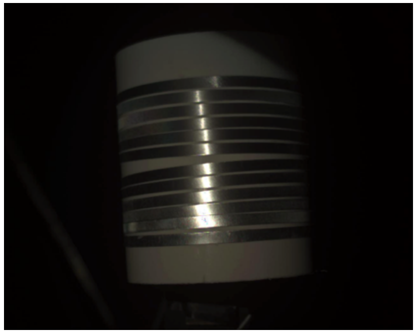


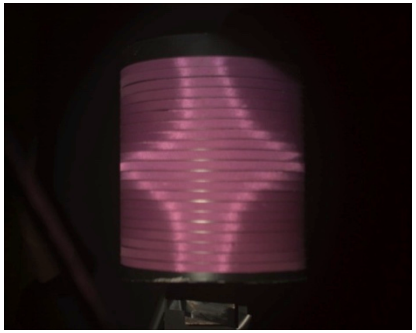
Different types of materials

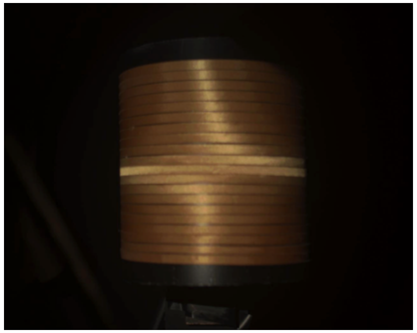
- Specular materials
  - Metals
  - Plastic
  - Glass

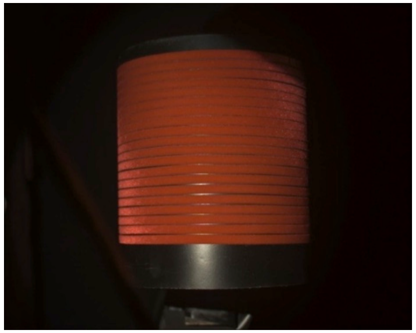
Different types of materials

- Anisotropic Materials
  - Velvet, Brushed metals









Different types of materials

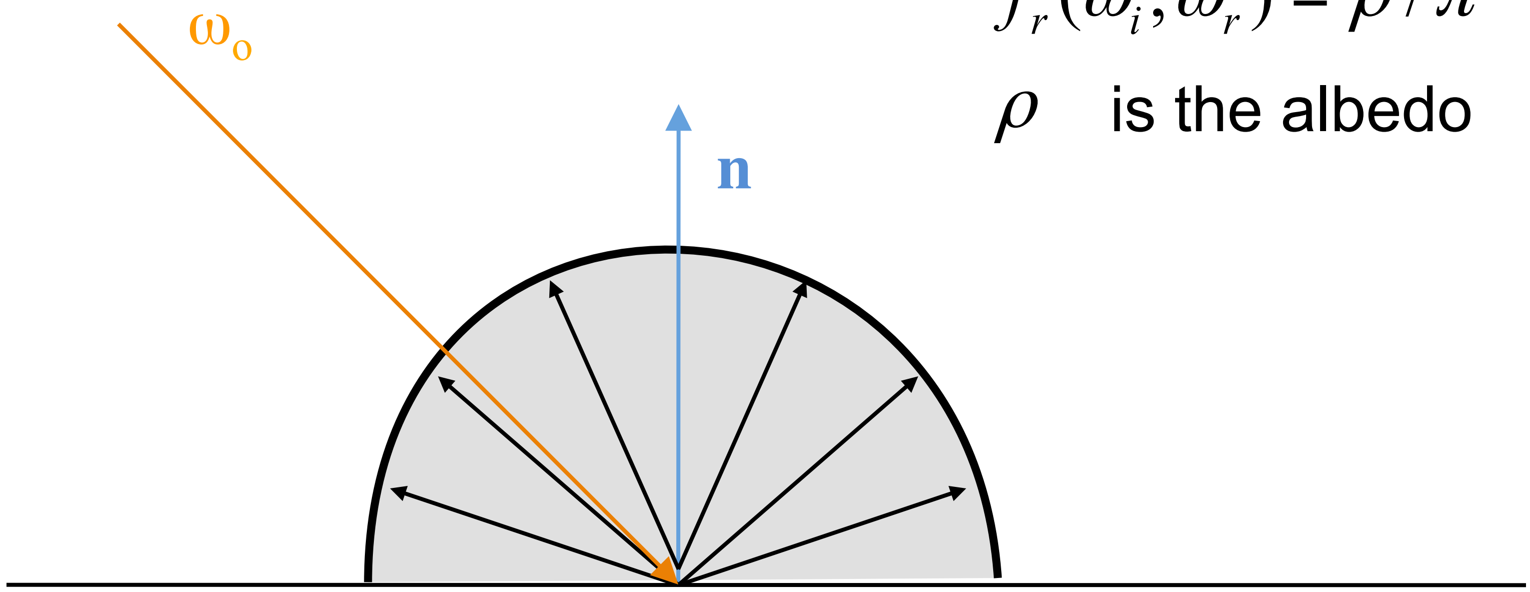
- Translucent materials
  - Skin
  - Wax
  - Marble
  - Paper

Describing the Reflectance

- The full BRDF is a 4D function
- Can sample and store
- Can find more compact BRDF **models**
  - Phong
  - Ward
  - Lafortune
  - etc.

Perfectly diffuse

Radiance reflected equally in every direction independently of the incoming direction

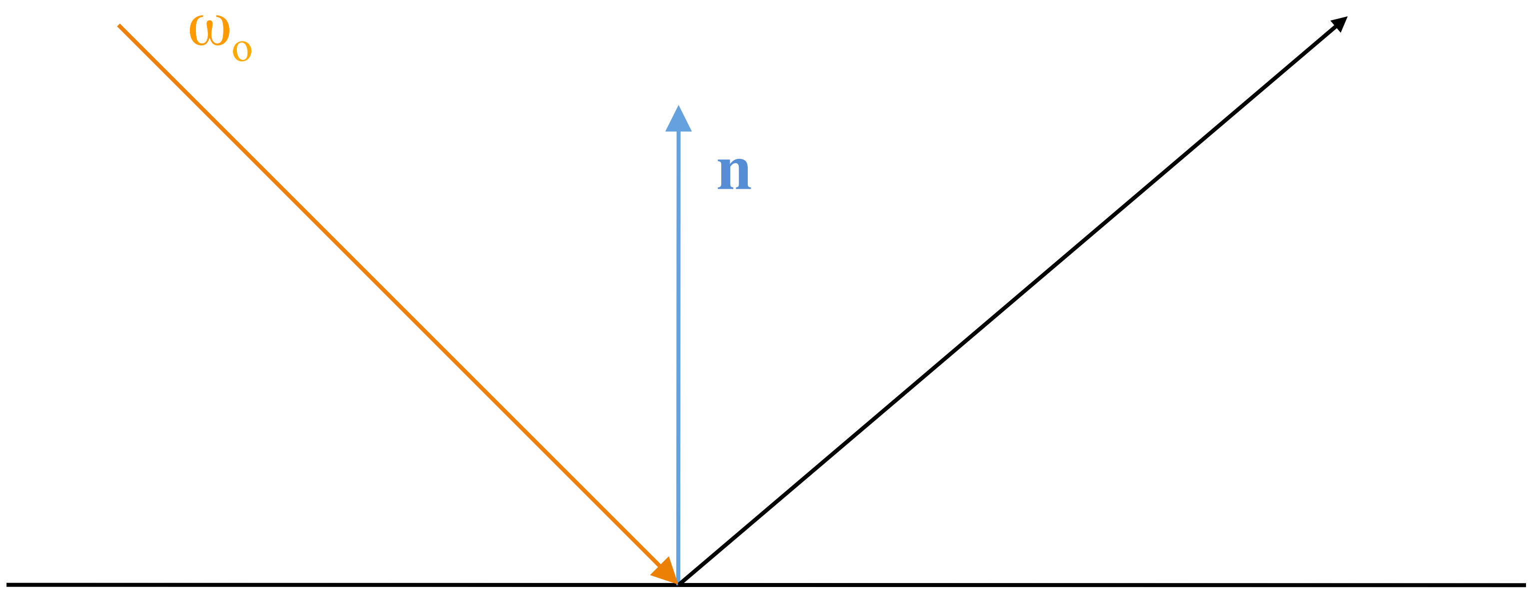


$$f_r(\omega_i, \omega_r) = \rho / \pi$$

ρ is the albedo

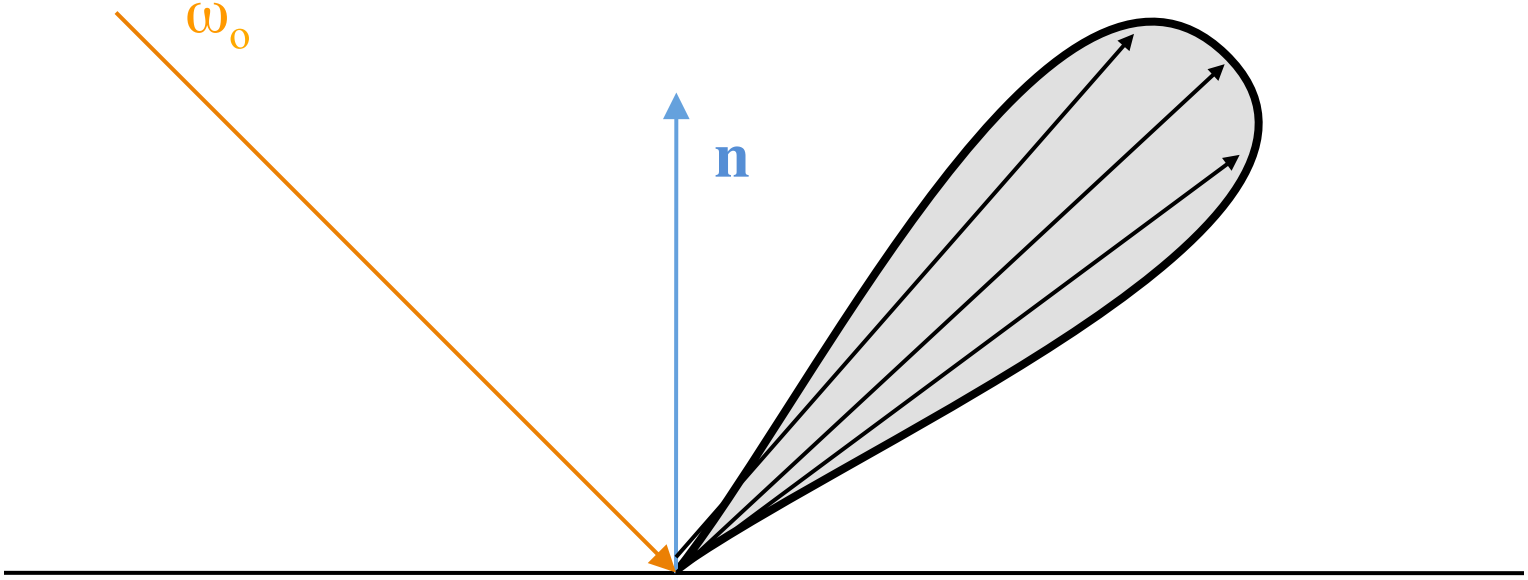
Perfectly specular

- Reflected dependently of the incident light
- What’s its BRDF? Dirac.
- Not physically possible



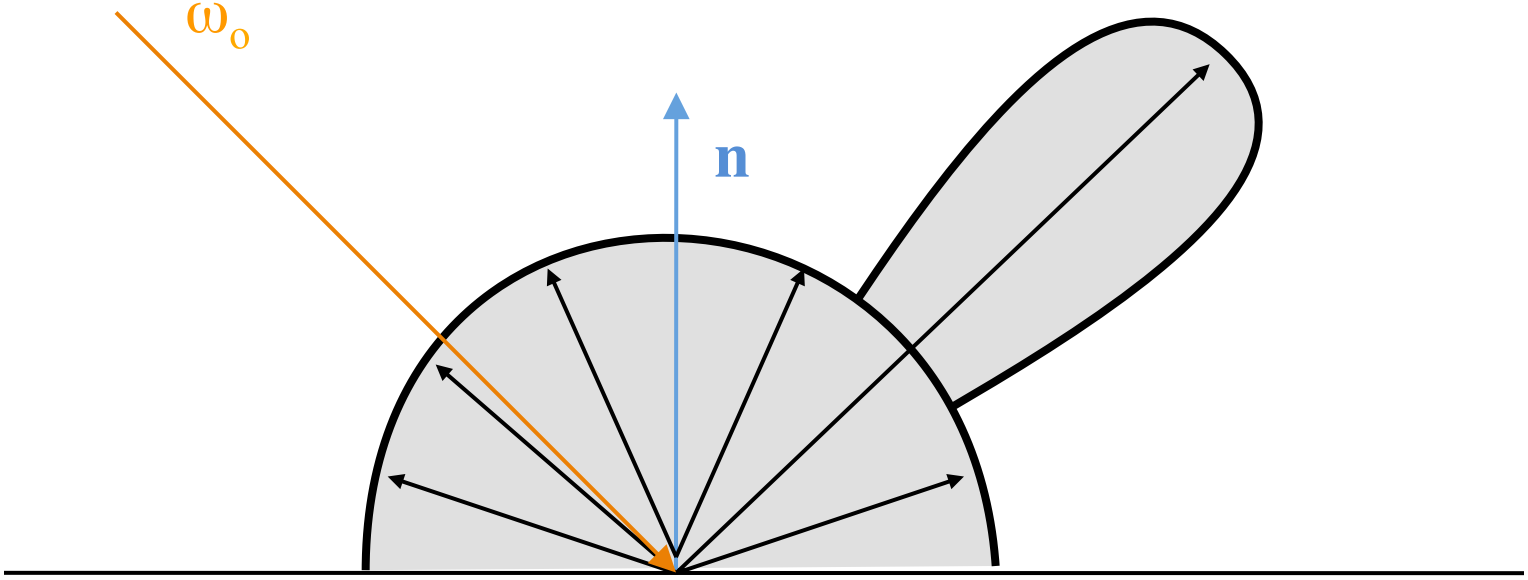
Glossy BRDF

Glossy is a blurry mirror



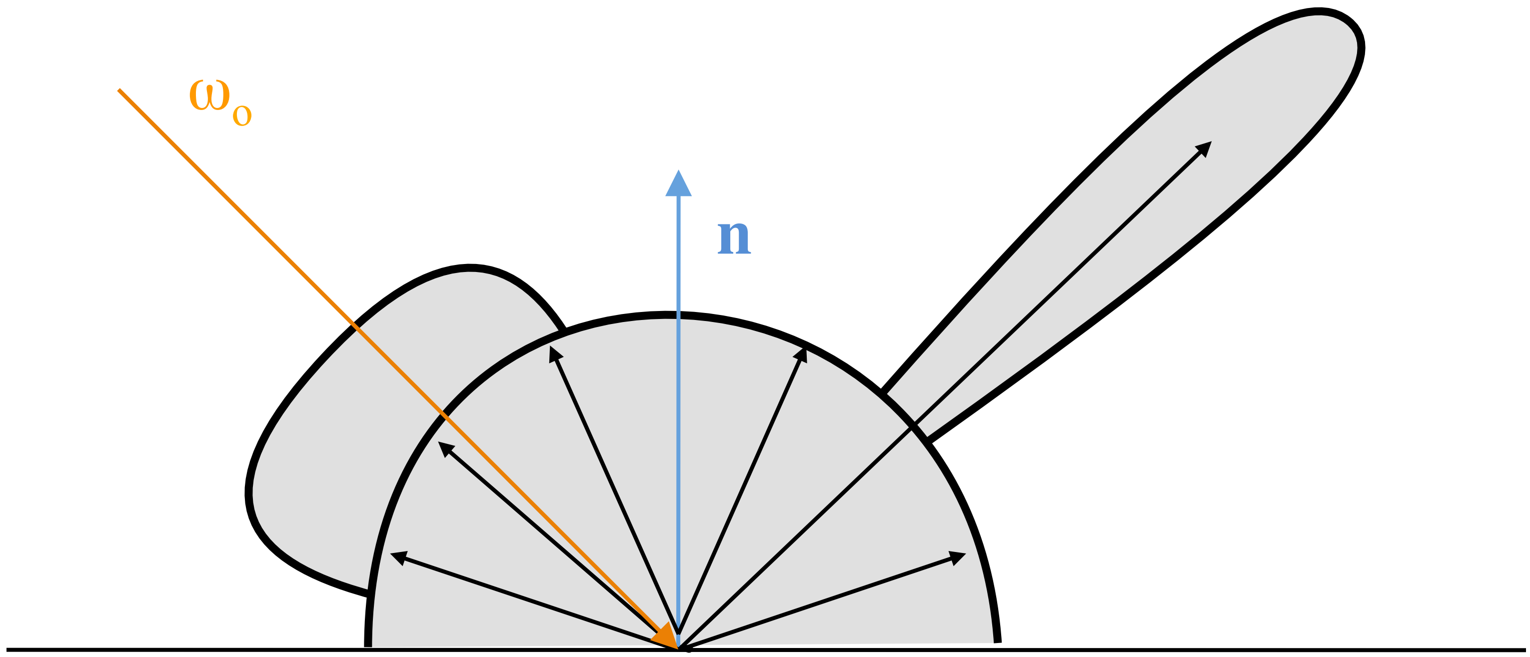
Diffuse and glossy BRDF

Diffuse and glossy



Multiple specular peaks

Multiple specular peaks, e.g. retroreflective

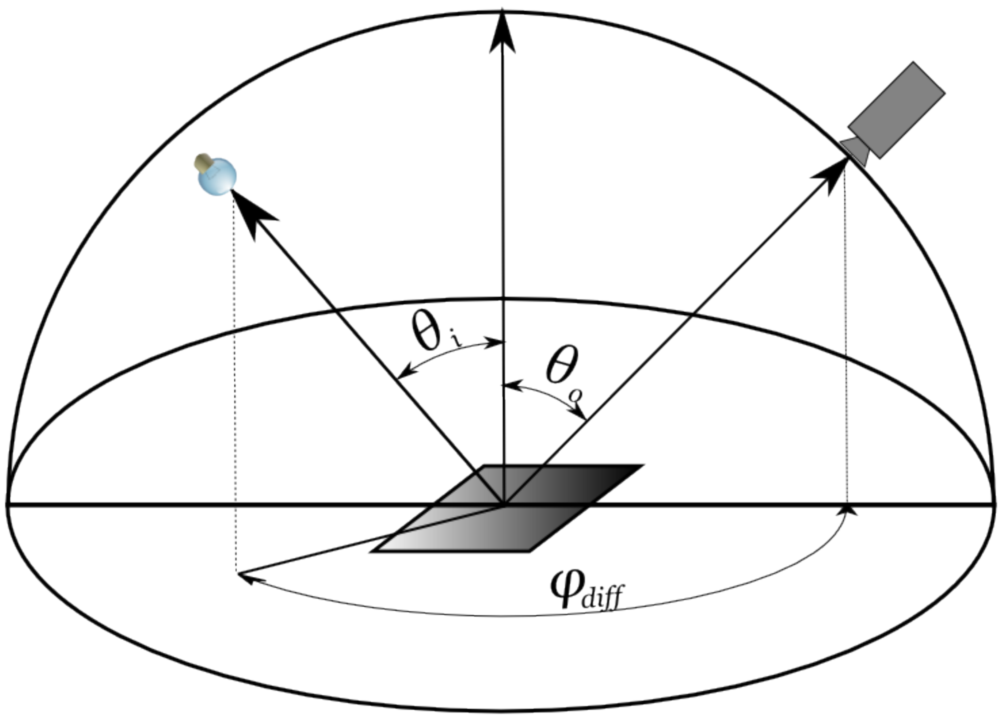


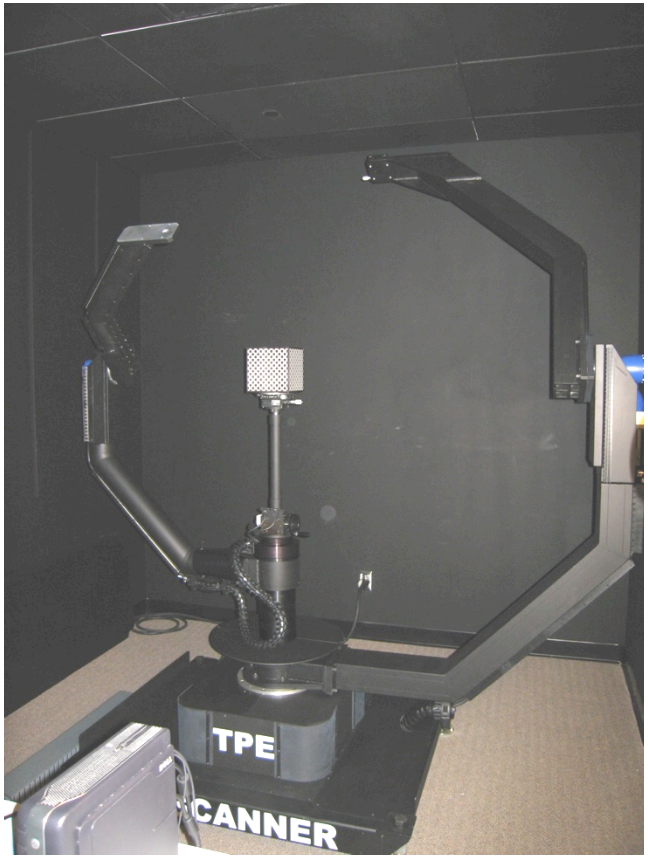
How to define a BRDF

- Three main options
  - Choose model and select parameters
  - Measure
  - Estimate from photographs (inverse illumination)

BRDF Measurment

- There are numerous devices for measuring reflectance
- Gonioreflectometer

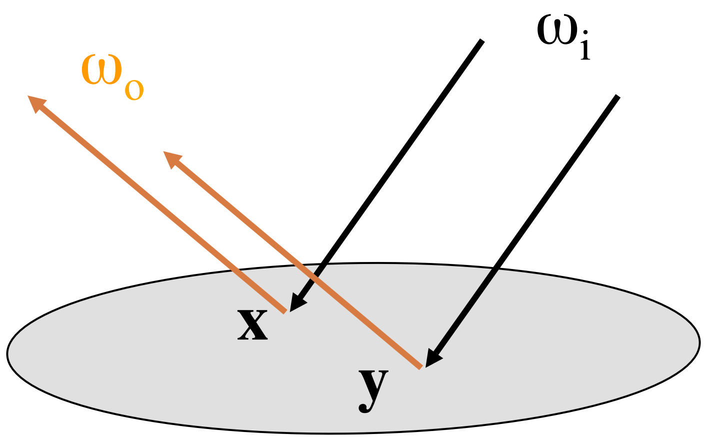




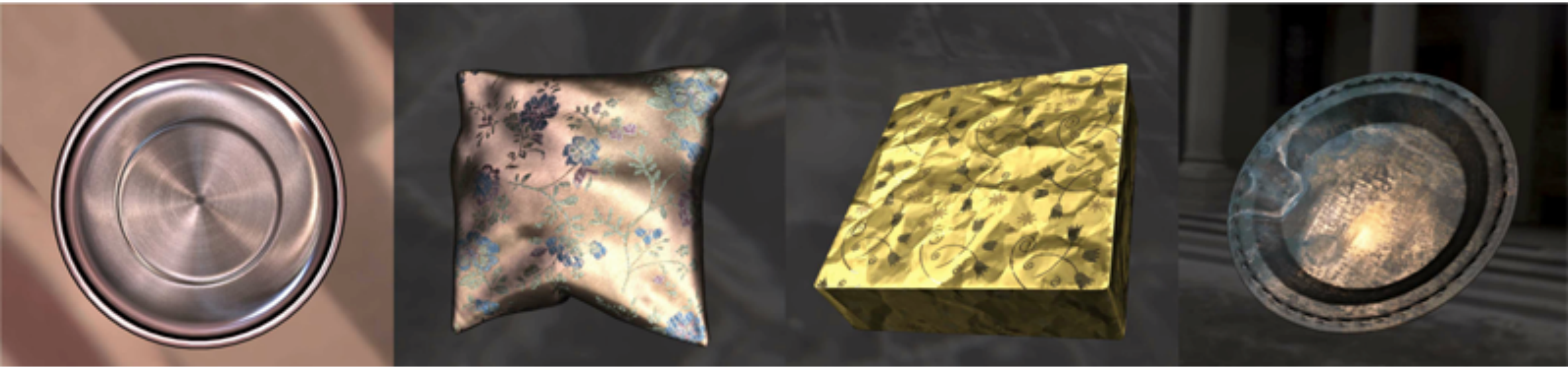
svBRDF

- Spatially-varying BRDF
  $f_r(\mathbf{x}, \omega_i, \omega_r)$

- The reflection might change from location to location



svBRDF Examples



Why is this hard to solve?

- It involves an integral with no analytic solution
- It is an integral equation, so the RHS contains the LHS in an integrand

(Spherical) integration

$$L(\mathbf{x}, \omega) = L_e(\mathbf{x}, \omega) + \int f_r(\mathbf{x}, \omega_i, \omega)\, L(\mathbf{y}, -\omega_i) \cos\theta_i \, d\omega_i$$

Same thing!

How ray-tracing solves the RE

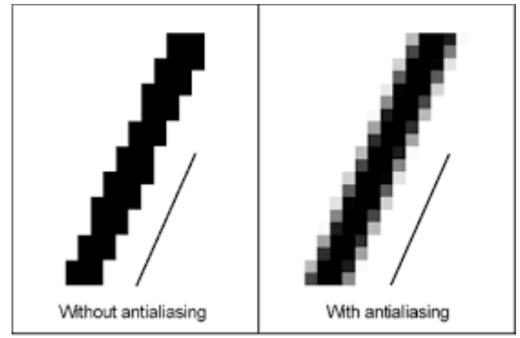
- Many got suspicious about ray-tracing
- Example: Does metal have finite gloss?
  - Yes, cause otherwise I cant see highlight!
  - No, as reflections are not blurry in steel balls!

How ray-tracing solves the RE

- The integral is split into a sum of two:
  - A Dirac-paths, solved by binary recusing
  - A path connecting to a point light, can evaluate without recursion

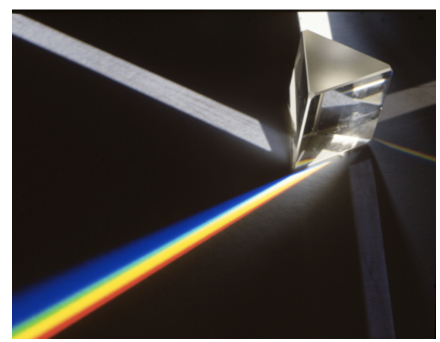
More general image formation

Motion blur



Anti-aliasing

Depth of field



Spectral effects

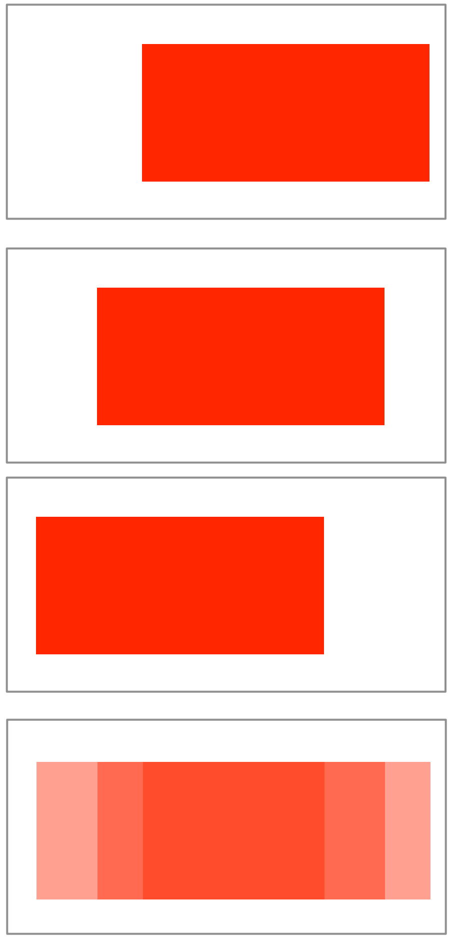
Motion blur: intuition

$t = 0.0$

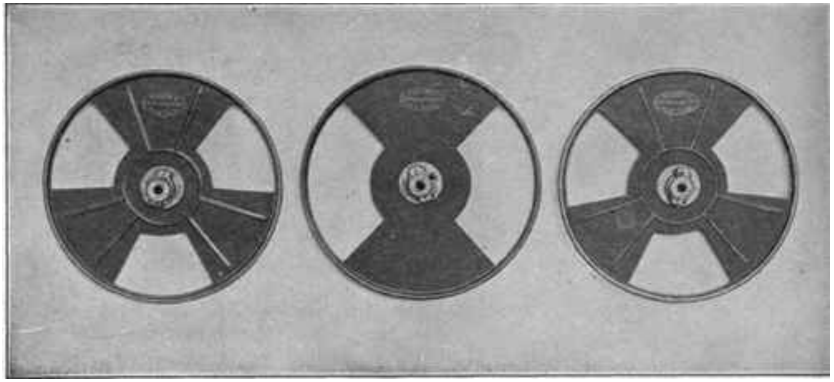
$t = 0.5$

$t = 1.0$

Result



Time-generalized RE

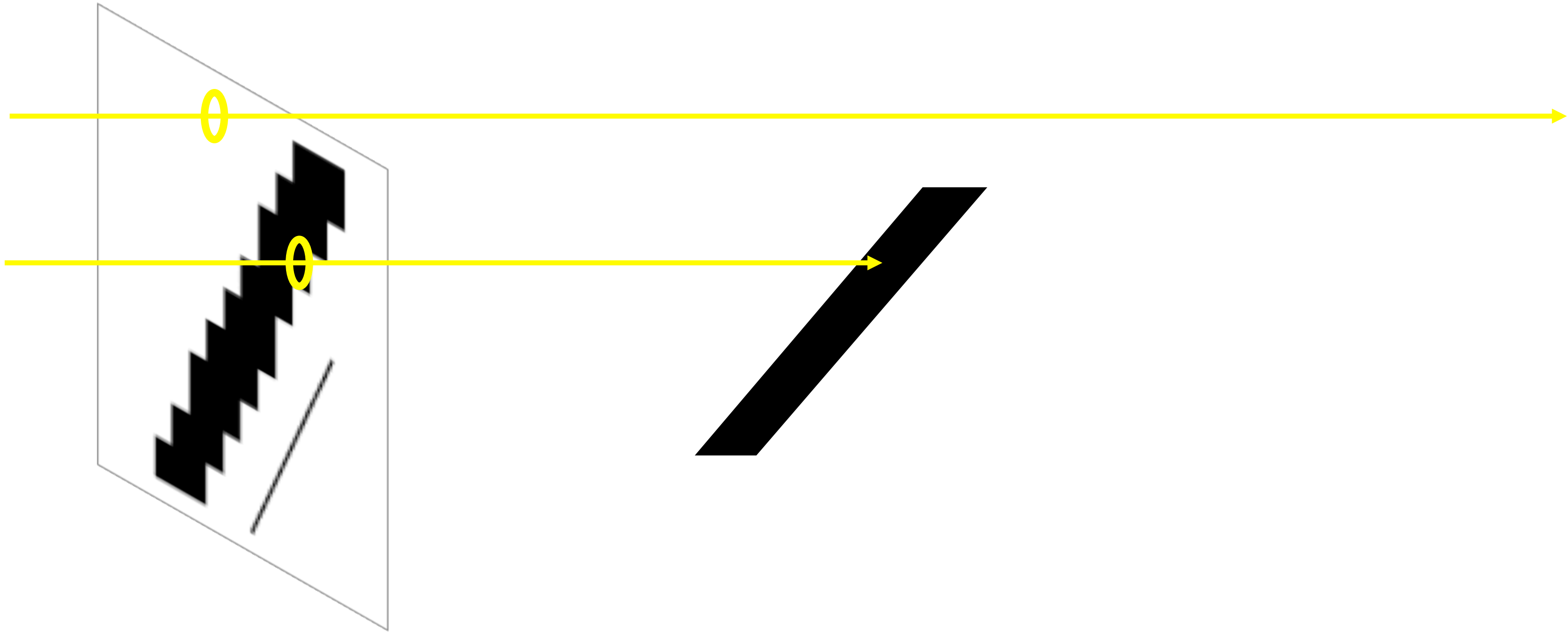


$$L(\mathbf{x}, \omega) = \int s(t) L_e(\mathbf{x}, \omega, t) + \int f_r(\mathbf{x}, \omega_I, \omega, t)\, L(\mathbf{y}, -\omega_i, t) \cos\theta_i \, d\omega_i dt$$

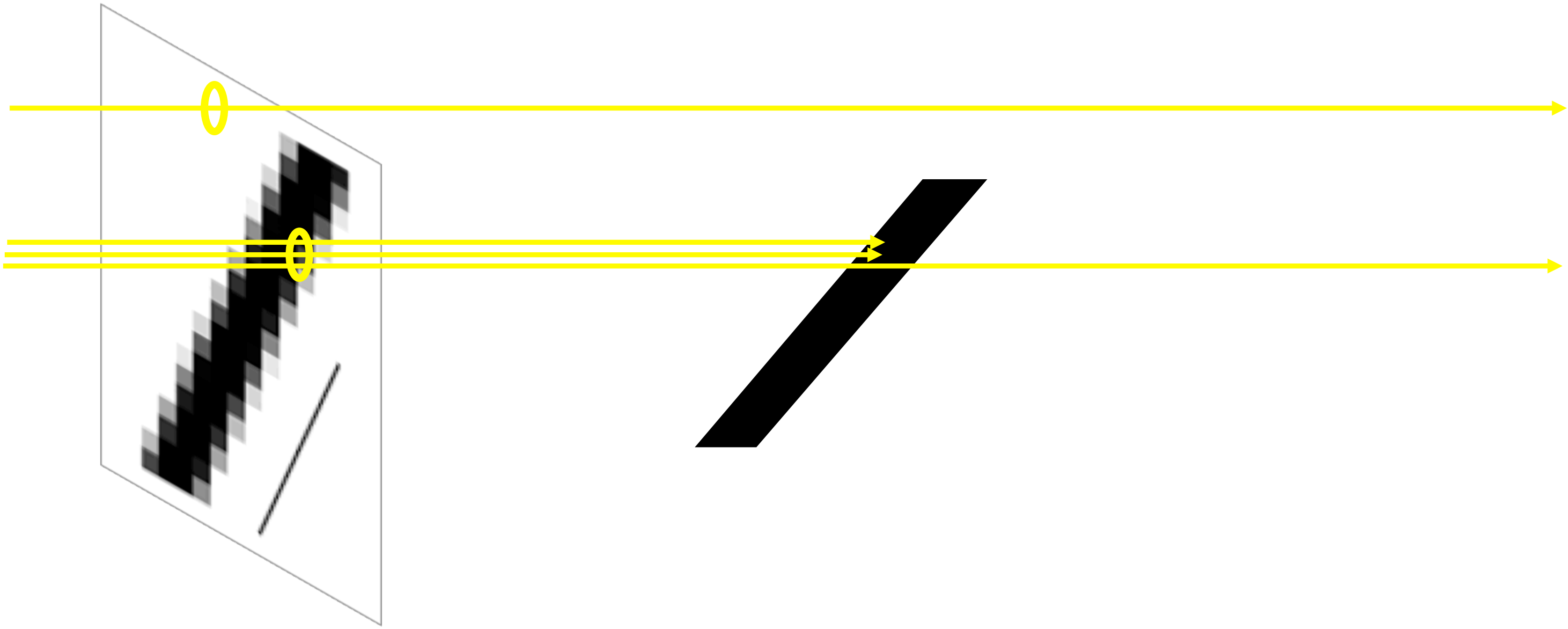
t is time coordinate

$s(t)$ is the shutter function. Different for each frame.

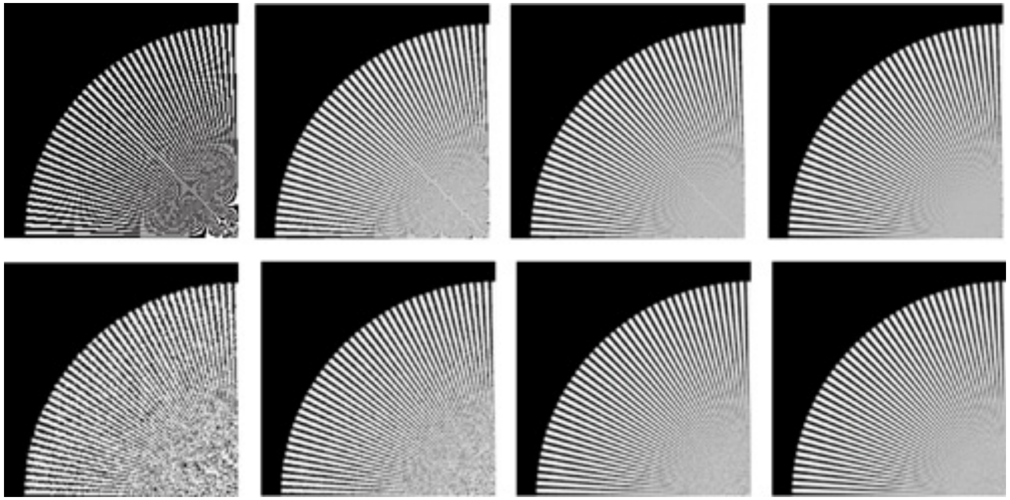
Anti-aliasing



Anti-aliasing



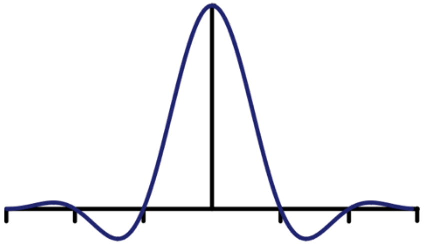
Anti-aliasing-generalized RE



$$L(\mathbf{p}) = \int r(\mathbf{p}) L(\mathbf{x}(\mathbf{p}), \omega)\, d\mathbf{p}$$

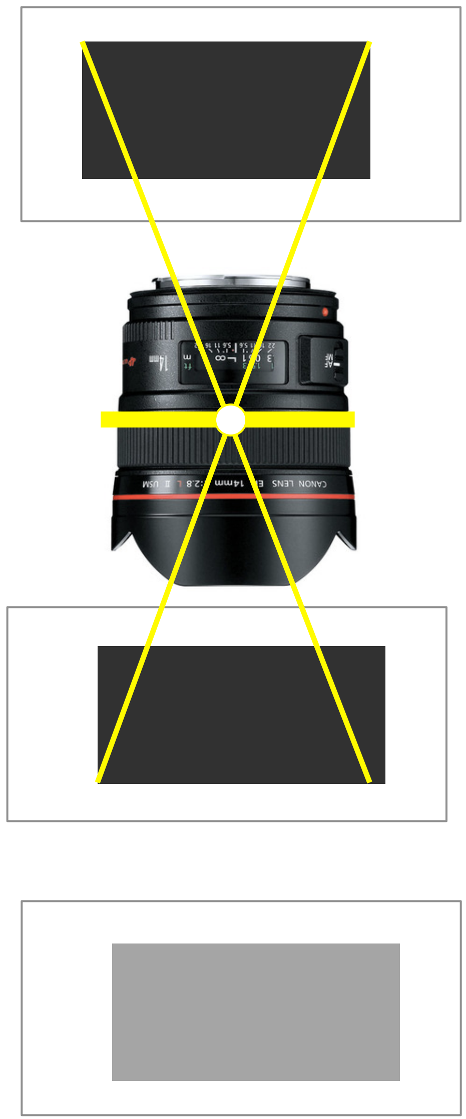
$\mathbf{p}$ is pixel coordinate

$r(\mathbf{p})$ is the pixel reconstruction function



Example for r: Lanczos

Depth of field: intuition



$l = 0.0$

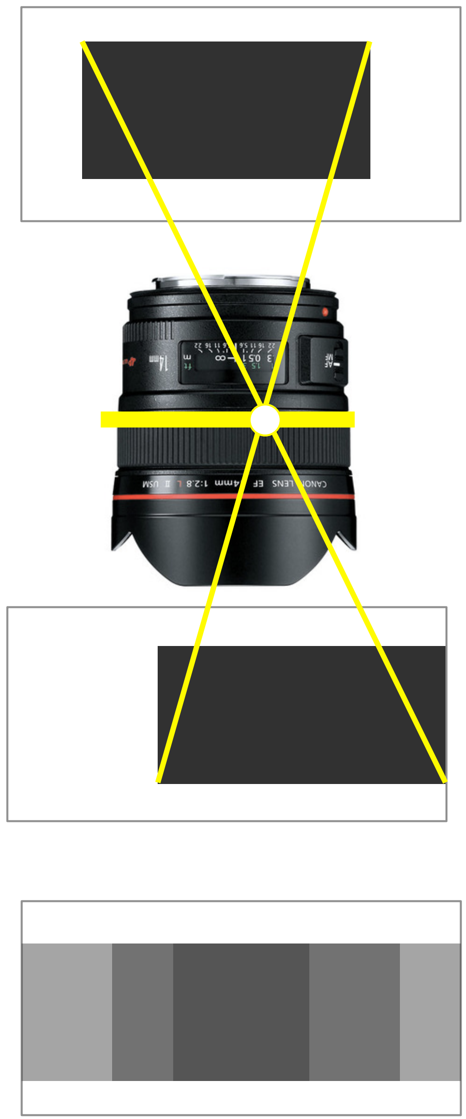
Result

Depth of field: intuition

$l = 0.5$

Result

Depth of field: intuition



$l = -0.5$

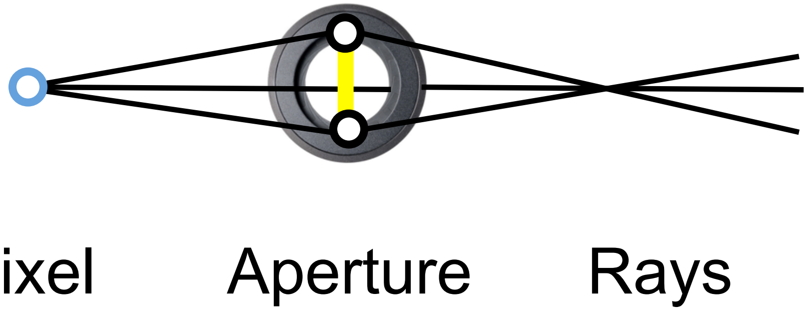
Result

Lens-generalized RE

$$L(\mathbf{p}) = \int a(\mathbf{l}) L(\mathbf{x}(\mathbf{l}), \omega(\mathbf{l}))\, \mathrm{d}\mathbf{l}$$

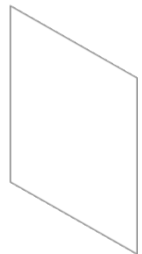
$\mathbf{l}$ is the 2D lens coordinate

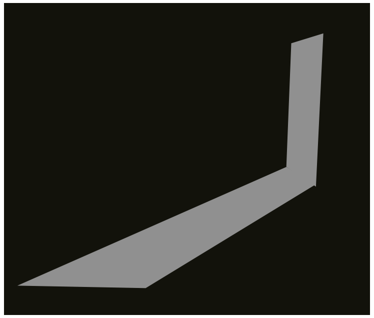
$a(\mathbf{l})$ is the aperture function



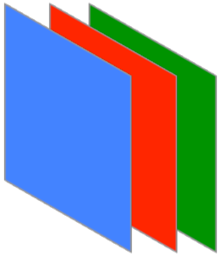
Spectral rendering

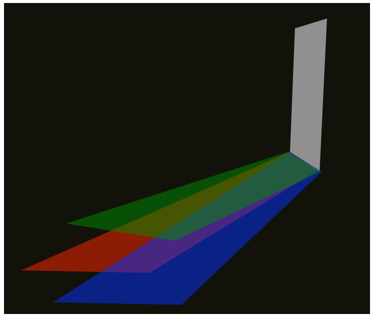
Single wavelength



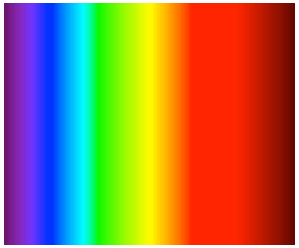


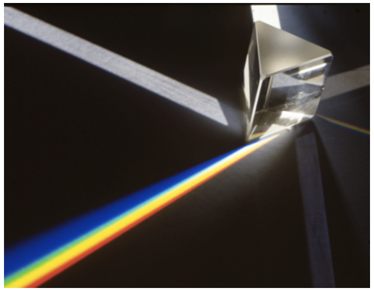
Three wavelengths



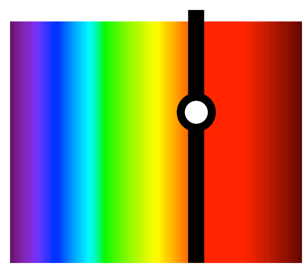


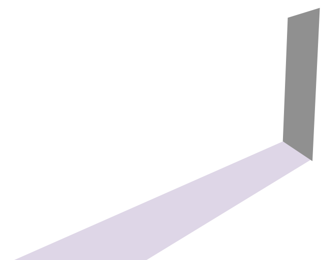
All wavelengths





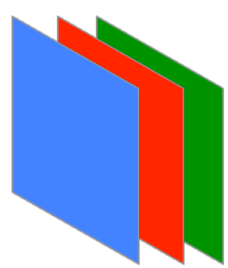
Spectral rendering





λ is spectral response

$r_c(\mathbf{l})$ is the channel basis function



$$L_r(\mathbf{p}) = \int r_r(\lambda) L(\mathbf{p}, \lambda)\, d\lambda$$
$$L_g(\mathbf{p}) = \int r_g(\lambda) L(\mathbf{p}, \lambda)\, d\lambda$$
$$L_b(\mathbf{p}) = \int r_b(\lambda) L(\mathbf{p}, \lambda)\, d\lambda$$

Big picture

Rendering is solving a very large integral equation

Pixel Color Aperture Time Paths

Recap

- Physically based lighting relies on solving the rendering equation
- Complete solutions to this equation are not tractable, so simple assumptions are made
- Need to be able to describe the reflectance properties of materials with a BRDF
- Can consider time, pixel, lens and spectrum too