Mathematics of the Simple Camera

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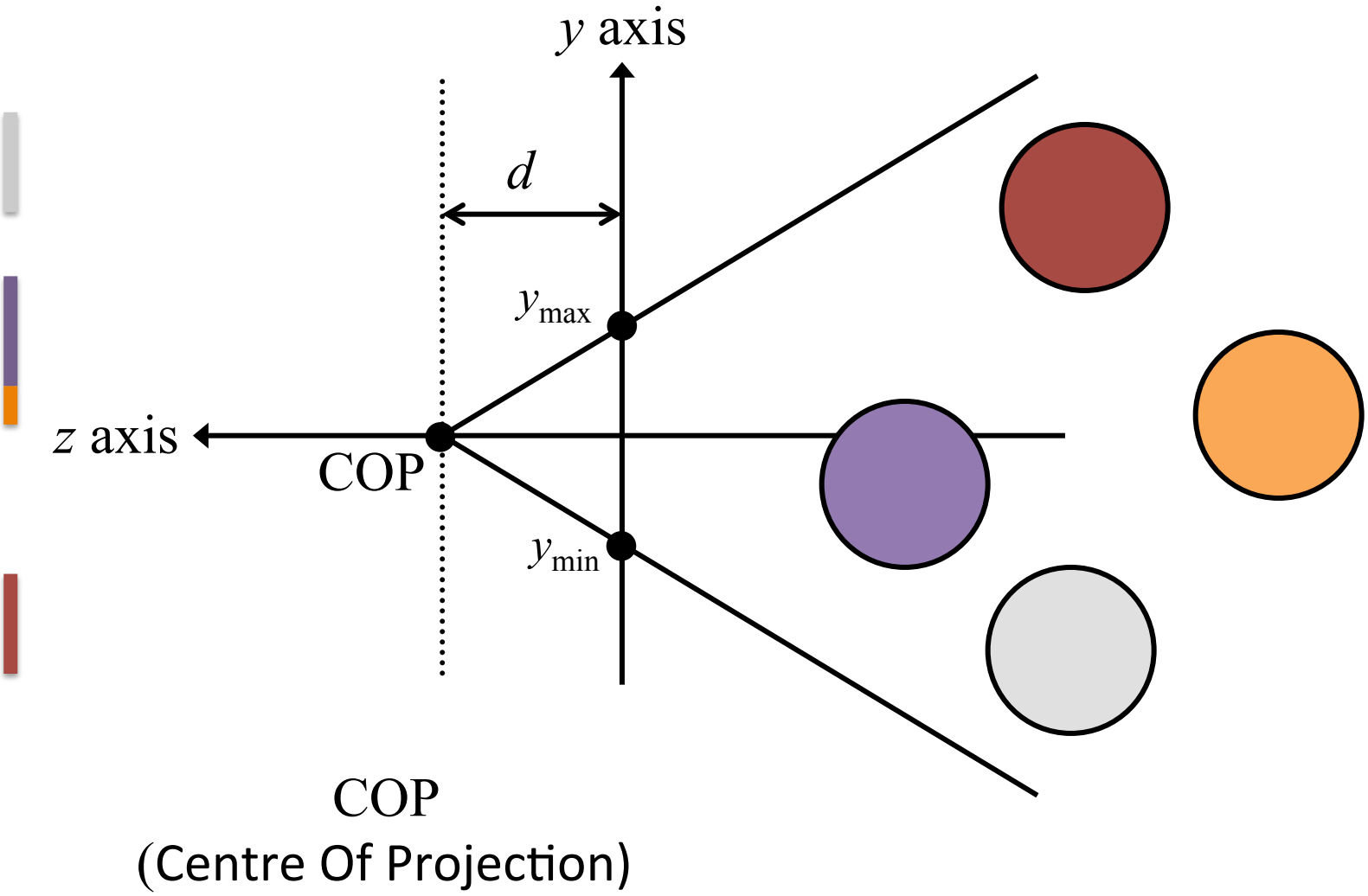
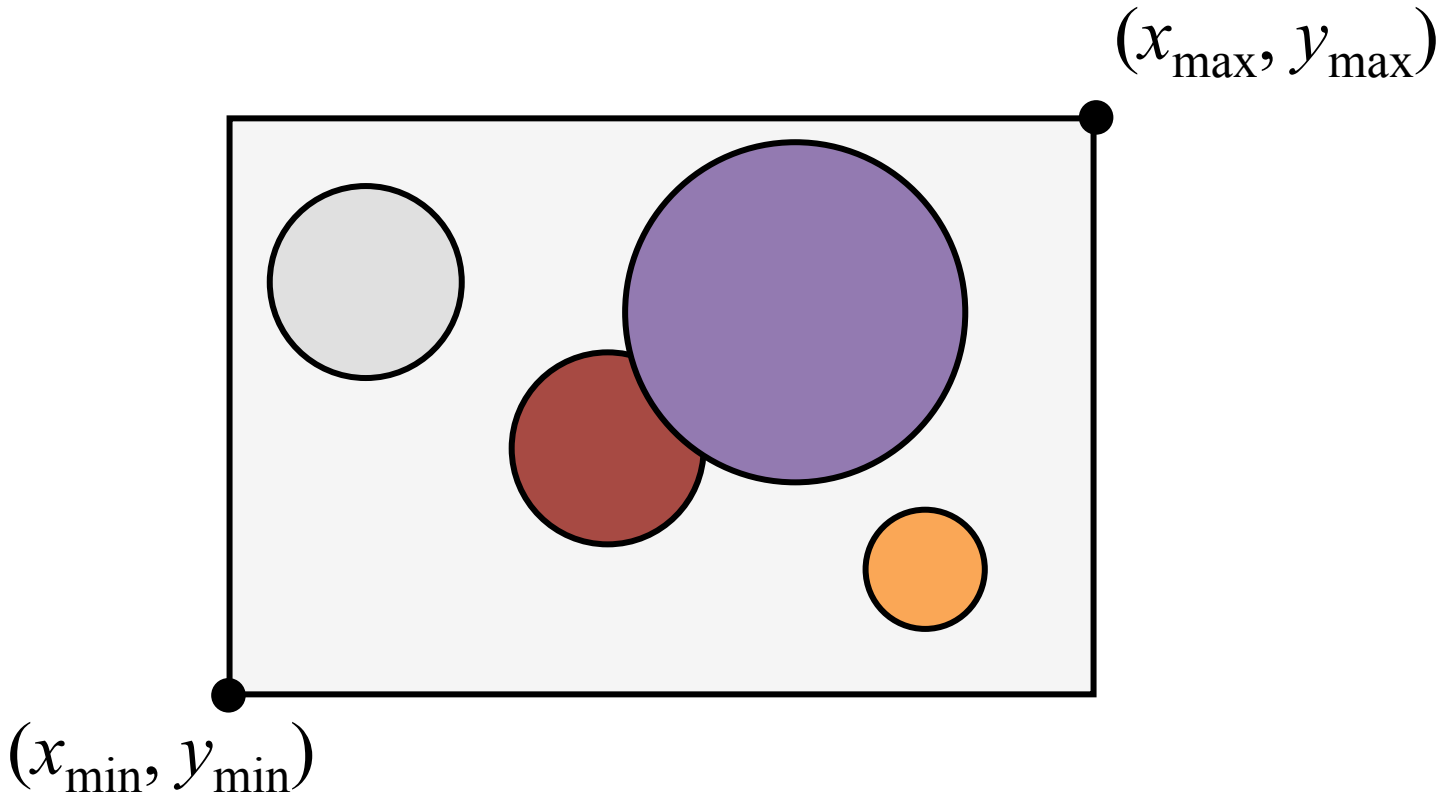
Overview

- The simplest rendering possible
- Some spheres only
- Simple Camera
  COP (Centre Of Projection) on $+z$

Simple Camera (Cross Section)

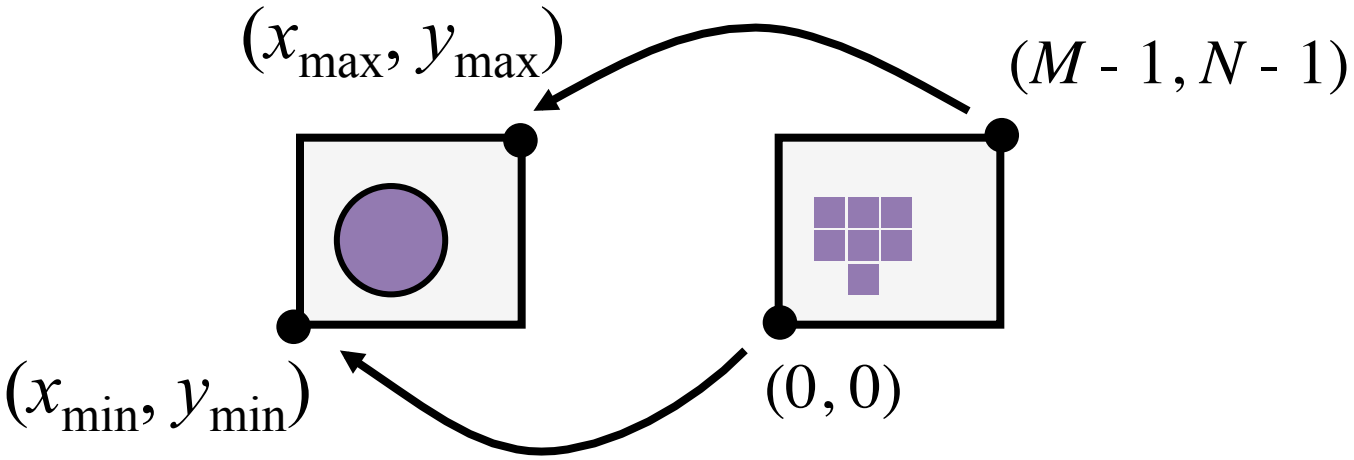


View From the Camera



Forming the Rays

- Map screen pixels (M by N pixel window) to points in camera view plane



Forming the Rays

- Consider pixel (i, j)
- It corresponds to a rectangle
  - $w = (x_{\max} - x_{\min}) / M$
  - $h = (y_{\max} - y_{\min}) / N$
- Our ray goes through the **centre** of the pixel
- Thus the ray goes through the 3D point

$$(x_{\min} + w \bullet (i + 0.5), y_{\min} + h \bullet (j + 0.5), 0)$$

Forming the Rays

- Thus the ray from the COP through pixel i, j is defined by

$$p(t) = (x(t), y(t), z(t)) = (\begin{array}{l} \mathrm{t} \bullet (x_{\min} + \mathrm{w} \bullet (i + 0.5)), \\ \mathrm{t} \bullet (y_{\min} + \mathrm{h} \bullet (j + 0.5)), \\ d - t \bullet d) \end{array}$$

Ray Casting

- Line-primitive intersection
- Simples variant:
  Line-sphere intersection
- Substitute the ray equation into the sphere equation and solve for t!

What is a 3D sphere?

Set of points (X, Y, Z), where distance to $\mathbf{P}$ is r

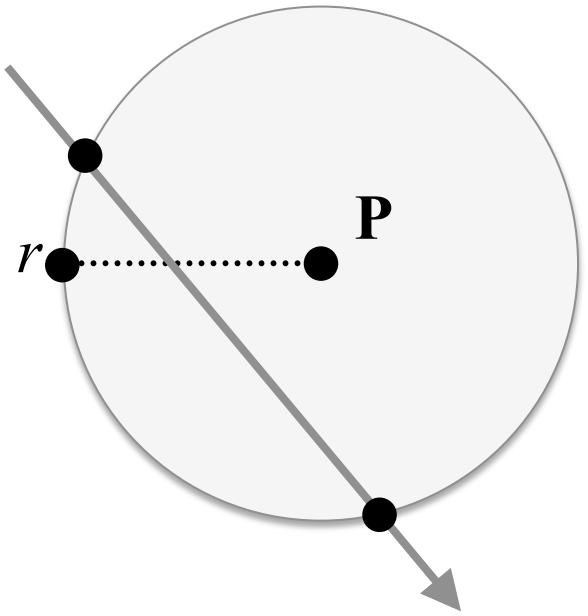
$$\sqrt{(X^2 + Y^2 + Z^2)} = r$$

$$X^2 + Y^2 + Z^2 = r^2$$

$$x(t)^2 + y(t)^2 + z(t)^2 = r^2$$

...

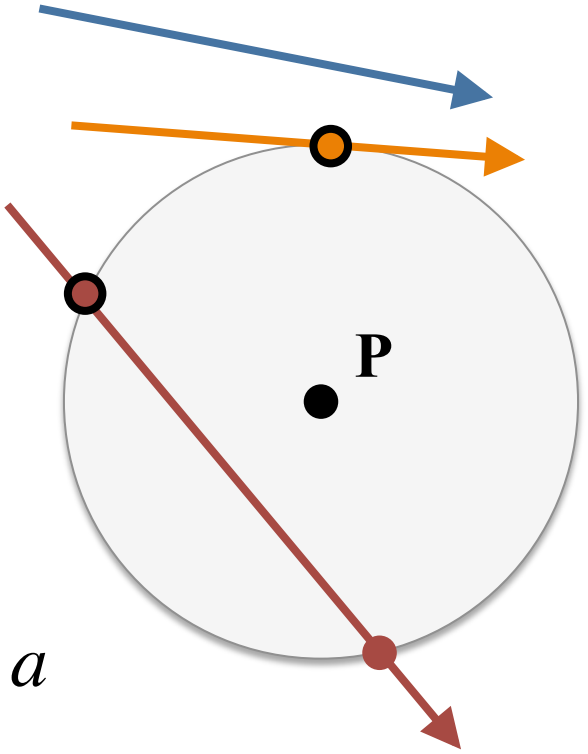
$$a\,t^2 + 2\,b\,t + c = 0$$



Sphere intersection

- If $b^2 - a\,c < 0$
  ray doesn't intersect the sphere
- If $b^2 - a\,c = 0$
  ray tangential to the sphere
- If $b^2 - a\,c > 0$
  two intersections given by

$$t = (-b \pm \sqrt{(b^2 - a\,c)}) / a$$

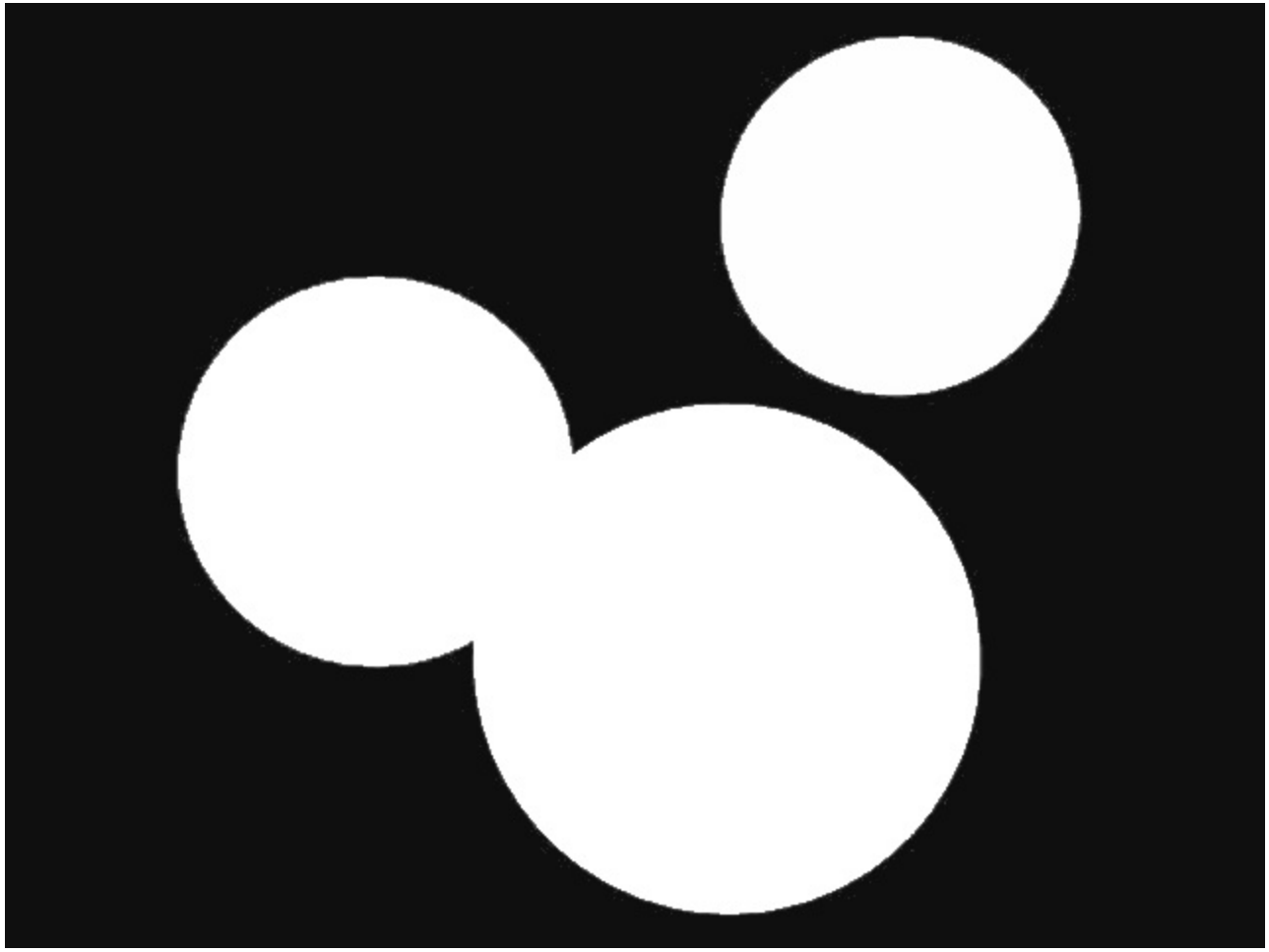


Choose the lowest value root (closer to the COP)

Ray Casting

- Intersection of sphere and line (general case)
  - Sphere is centred at (P_x, P_y, P_z)
  - Translate the start of the ray by $(-P_x, -P_y, -P_z)$
  - Proceed as before

Result – ”Sphere Detection“



Conclusions

- We can now draw images
  - Forming rays from the camera
  - Intersecting those rays with objects (spheres) in the scene
- But
  - No colour – merely binary detection operation
  - Camera is static - at the moment we must move the objects in front of the camera to be able to see them
  - Need more interesting scenes!