MODERN ROBOTICS

MECHANICS, PLANNING, AND CONTROL

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Foreword by Roger Brockett

In the 1870s, Felix Klein was developing his far-reaching Erlangen Program, which cemented the relationship between geometry and group theoretic ideas. With Sophus Lie's nearly simultaneous development of a theory of continuous (Lie) groups, important new tools involving infinitesimal analysis based on Lie algebraic ideas became available for the study of a very wide range of geometric problems. Even today, the thinking behind these ideas continues to guide developments in important areas of mathematics. Kinematic mechanisms are, of course, more than just geometry; they need to accelerate, avoid collisions, etc., but first of all they are geometrical objects and the ideas of Klein and Lie apply. The groups of rigid motions in two or three dimensions, as they appear in robotics, are important examples in the work of Klein and Lie.

In the mathematics literature the representation of elements of a Lie group in terms of exponentials usually takes one of two different forms. These are known as exponential coordinates of the first kind and exponential coordinates of the second kind. For the first kind one has $X = e^{(A_1x_1+A_2x_2\cdots)}$. For the second kind this is replaced by $X = e^{A_1x_1}e^{A_2x_2}\cdots$. Up until now, the first choice has found little utility in the study of kinematics whereas the second choice, a special case having already shown up in Euler parametrizations of the orthogonal group, turns out to be remarkably well-suited for the description of open kinematic chains consisting of the concatenation of single degree of freedom links. This is all nicely explained in Chapter 4 of this book. Together with the fact that $Pe^AP^{-1} = e^{PAP^{-1}}$, the second form allows one to express a wide variety of kinematic problems very succinctly. From a historical perspective, the use of the product of exponentials to represent robotic movement, as the authors have done here, can be seen as illustrating the practical utility of the 150-year-old ideas of the geometers Klein and Lie.

In 1983 I was invited to speak at the triennial Mathematical Theory of Networks and Systems Conference in Beer Sheva, Israel, and after a little thought I decided to try to explain something about what my recent experiences had taught me. By then I had some experience in teaching a robotics course that discussed kinematics, including the use of the product of exponentials representation of kinematic chains. From the 1960s onward e^{At} had played a central role in system theory and signal processing, so at this conference a familiarity, even an affection, for the matrix exponential could be counted on. Given this, it was natural for me to pick something e^{Ax}-related for the talk. Although I had no reason to think that there would be many in the audience with an interest in kinematics, I still hoped I could say something interesting and maybe even inspire further developments. The result was the paper referred to in the preface that follows.

In this book, Frank and Kevin have provided a wonderfully clear and patient explanation of their subject. They translate the foundation laid out by Klein and Lie 150 years ago to the modern practice of robotics, at a level appropriate for undergraduate engineers. After an elegant discussion of fundamental properties of configuration spaces, they introduce the Lie group representations of rigid-body configurations, and the corresponding representations of velocities and forces, used throughout the book. This consistent perspective is carried through foundational robotics topics including forward, inverse, and differential kinematics of open chains, robot dynamics, trajectory generation, and robot control, and more specialized topics such as kinematics of closed chains, motion planning, robot manipulation, planning and control for wheeled mobile robots, and control of mobile manipulators.

I am confident that this book will be a valuable resource for a generation of students and practitioners of robotics.

Roger Brockett
Cambridge, Massachusetts, USA
November, 2016

Foreword by Matthew Mason

Robotics is about turning ideas into action. Somehow, robots turn abstract goals into physical action: sending power to motors, monitoring motions, and guiding things towards the goal. Every human can perform this trick, but it is nonetheless so intriguing that it has captivated philosophers and scientists, including Descartes and many others.

What is the secret? Did some roboticist have a eureka moment? Did some pair of teenage entrepreneurs hit on the key idea in their garage? To the contrary, it is not a single idea. It is a substantial body of scientific and engineering results, accumulated over centuries. It draws primarily from mathematics, physics, mechanical engineering, electrical engineering, and computer science, but also from philosophy, psychology, biology and other fields.

Robotics is the gathering place of these ideas. Robotics provides motivation. Robotics tests ideas and steers continuing research. Finally, robotics is the proof. Observing a robot's behavior is the nearly compelling proof that machines can be aware of their surroundings, can develop meaningful goals, and can act effectively to accomplish those goals. The same principles apply to a thermostat or a fly-ball governor, but few are persuaded by watching a thermostat. Nearly all are persuaded by watching a robot soccer team.

The heart of robotics is motion – controlled programmable motion – which brings us to the present text. *Modern Robotics* imparts the most important insights of robotics: the nature of motion, the motions available to rigid bodies, the use of kinematic constraint to organize motions, the mechanisms that enable general programmable motion, the static and dynamic character of mechanisms, and the challenges and approaches to control, programming, and planning motions. *Modern Robotics* presents this material with a clarity that makes it accessible to undergraduate students. It is distinguished from other undergraduate texts in two important ways.

First, in addressing rigid-body motion, *Modern Robotics* presents not only the classical geometrical underpinnings and representations, but also their expression using modern matrix exponentials, and the connection to Lie algebras. The rewards to the students are two-fold: a deeper understanding of motion, and better practical tools.

Second, *Modern Robotics* goes beyond a focus on robot mechanisms to address the interaction with objects in the surrounding world. When robots make contact with the real world, the result is an *ad hoc* kinematic mechanism, with associated statics and dynamics. The mechanism includes kinematic loops, unactuated joints, and nonholonomic constraints, all of which will be familiar concepts to students of *Modern Robotics*.

Even if this is the only robotics course students take, it will enable them to analyze, control, and program a wide range of physical systems. With its introduction to the mechanics of physical interaction, *Modern Robotics* is also an excellent beginning for the student who intends to continue with advanced courses or with original research in robotics.

Matthew T. Mason
Pittsburgh, PA, USA
November, 2016

Preface

It was at the IEEE International Conference on Robotics and Automation in Pasadena in 2008 when, over a beer, we decided to write an undergraduate textbook on robotics. Since 1996, Frank had been teaching robot kinematics to Seoul National University undergraduates using his own lecture notes; by 2008 these notes had evolved to the kernel around which this book was written. Kevin had been teaching his introductory robotics class at Northwestern University from his own set of notes, with content drawn from an eclectic collection of papers and books.

We believe that there is a distinct and unifying perspective to mechanics, planning, and control for robots that is lost if these subjects are studied independently, or as part of other more traditional subjects. At the 2008 meeting, we noted the lack of a textbook that (a) treated these topics in a unified way, with plenty of exercises and figures, and (b), most importantly, was written at a level appropriate for a first robotics course for undergraduates with only freshman-level physics, ordinary differential equations, linear algebra, and a little bit of computing background. We decided that the only sensible recourse was to write such a book ourselves. (We didn't know then that it would take us more than eight years to finish the project!)

通常情况下，机器人最突出的物理特征最好由几何描述来体现。

A second motivation for this book, and one that we believe sets it apart from other introductory treatments on robotics, is its emphasis on modern geometric techniques. Often the most salient physical features of a robot are best captured by a geometric description. The advantages of the geometric approach have been recognized for quite some time by practitioners of classical screw theory. What has made these tools largely inaccessible to undergraduates—the primary target audience for this book—is that they require an entirely new language of notations and constructs (screws, twists, wrenches, reciprocity, transversality, conjugacy, etc.), and their often obscure rules for manipulation and transformation. On the other hand, the mostly algebraic alternatives to screw theory often mean that students end up buried in the details of calculation, losing the simple and elegant geometric interpretation that lies at the heart of what they are calculating.

The breakthrough that makes the techniques of classical screw theory accessible to a more general audience arrived in the early 1980's, when Roger Brockett showed how to mathematically describe kinematic chains in terms of the Lie group structure of the rigid-body motions [20]. This discovery allowed one, among other things, to re-invent screw theory simply by appealing to basic linear algebra and linear differential equations. With this "modern screw theory" the powerful tools of modern differential geometry can be brought to bear on a wide-ranging collection of robotics problems, some of which we explore here, others of which are covered in the excellent but more advanced graduate textbook by Murray, Li and Sastry [122].

As the title indicates, this book covers what we feel to be the fundamentals of robot mechanics, together with the basics of planning and control. A thorough treatment of all the chapters would likely take two semesters, particularly when coupled with programming assignments or experiments with robots. The contents of Chapters 2-6 constitute the minimum essentials, and these topics should probably be covered in sequence.

The instructor can then selectively choose content from the remaining chapters. At Seoul National University, the undergraduate course M2794.0027 Introduction to Robotics covers, in one semester, Chapters 2-7 and parts of Chapters 10, 11, and 12. At Northwestern, ME 449 Robotic Manipulation covers, in an 11-week quarter, Chapters 2-6 and 8, then touches on different topics in Chapters 9-13 depending on the interests of the students and instructor. A course focusing on the kinematics of robot arms and wheeled vehicles could cover chapters 2-7 and 13, while a course on kinematics and motion planning could additionally include Chapters 9 and 10. A course on the mechanics of manipulation would cover Chapters 2-6, 8, and 12, while a course on robot control would cover Chapters 2-6, 8, 9, and 11. If the instructor prefers to avoid dynamics (Chapter 8), the basics of robot control (Chapters 11 and 13) can be covered by assuming control of velocity at each actuator, not forces and torques. A course focusing only on motion planning could cover Chapters 2 and 3, Chapter 10 in depth (possibly supplemented by research papers or other references cited in that chapter), and Chapter 13.

To help the instructor choose which topics to teach and to help the student keep track of what she has learned, we have included a summary at the end of each chapter and a summary of important notation and formulas used throughout the book (Appendix A). For those whose primary interest in this text is as an introductory reference, we have attempted to provide a reasonably comprehensive, though by no means exhaustive, set of references and bibliographic notes at the end of each chapter. Some of the exercises provided at the end of each chapter extend the basic results covered in the book, and for those who wish to probe further, these should be of some interest in their own right. Some of the more advanced material in the book can be used to support independent study projects.

Another important component of the book is the software, which is written to reinforce the concepts in the book and to make the formulas operational. The software was developed primarily by Kevin's ME 449 students at Northwestern and is freely downloadable from `http://modernrobotics.org`. Video lectures that accompany the textbook will also be available at the website. The intent of the video content is to "flip" the classroom. Students watch the brief lectures on their own time, rewinding and rewatching as needed, and class time is focused more on collaborative problem-solving. This way, the professor is present when the students are applying the material and discovering the gaps in their understanding, creating the opportunity for interactive mini-lectures addressing the concepts that need most reinforcing. We believe that the added value of the professor is greatest in this interactive role, not in delivering a lecture the same way it was delivered the previous year. This approach has worked well for Kevin's introduction to mechatronics course, `http://nu32.org`.

Video content is generated using the Lightboard, `http://lightboard.info`, created by Michael Peshkin at Northwestern University. We thank him for sharing this convenient and effective tool for creating instructional videos.

We have also found the V-REP robot simulation software to be a valuable supplement to the book and its software. This simulation software allows students to interactively explore the kinematics of robot arms and mobile manipulators and to animate trajectories that are the result of exercises on kinematics, dynamics, and control.

While this book presents our own perspective on how to introduce the fundamental topics in first courses on robot mechanics, planning, and control, we acknowledge the excellent textbooks that already exist and that have served our field well. Among these, we would like to mention as particularly influential the books by Murray, Li, and Sastry [122]; Craig [32]; Spong, Hutchinson, and Vidyasagar [177]; Siciliano, Sciavicco, Villani, and Oriolo [171]; Mason [109]; Corke [30]; and the motion planning books by Latombe [80], LaValle [83], and Choset, Lynch, Hutchinson, Kantor, Burgard, Kavraki, and Thrun [27]. In addition, the Handbook of Robotics [170], edited by Siciliano and Khatib with a multimedia extension edited by Kröger (`http://handbookofrobotics.org`), is a landmark in our field, collecting the perspectives of hundreds of leading researchers on a huge variety of topics relevant to modern robotics.

It is our pleasure to acknowledge the many people who have been the sources

of help and inspiration in writing this book. In particular, we would like to thank our Ph.D. advisors, Roger Brockett and Matt Mason. Brockett laid down much of the foundation for the geometric approach to robotics employed in this book. Mason's pioneering contributions to analysis and planning for manipulation form a cornerstone of modern robotics. We also thank the many students who provided feedback on various versions of this material, in M2794.0027 at Seoul National University and in ME 449 at Northwestern University. In particular, Frank would like to thank Seunghyeon Kim, Keunjun Choi, Jisoo Hong, Jinkyu Kim, Youngsuk Hong, Wooyoung Kim, Cheongjae Jang, Taeyoon Lee, Soocheol Noh, Kyumin Park, Seongjae Jeong, Sukho Yoon, Jaewoon Kwen, Jinhyuk Park, and Jihoon Song, as well as Jim Bobrow and Scott Ploen from his time at UC Irvine. Kevin would like to thank Matt Elwin, Sherif Mostafa, Nelson Rosa, Jarvis Schultz, Jian Shi, Mikhail Todes, Huan Weng, and Zack Woodruff.

Finally, and most importantly, we thank our wives and families, for putting up with our late nights and our general unavailability, and for supporting us as we made the final push to finish the book. Without the love and support of Hyunmee, Shiyeon, and Soonkyu (Frank) and Yuko, Erin, and Patrick (Kevin), this book would not exist. We dedicate this book to them.

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Frank C. Park
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November, 2016

Publication note. The authors consider themselves to be equal contributors to this book. Author order is alphabetical.

Chapter 1

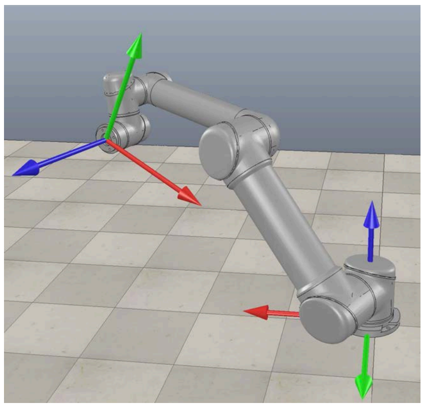
Preview

As an academic discipline, robotics is a relatively young field with highly ambitious goals, the ultimate one being the creation of machines that can behave and think like humans. This attempt to create intelligent machines naturally leads us first to examine ourselves – to ask, for example, why our bodies are designed the way they are, how our limbs are coordinated, and how we learn and perform complex tasks. The sense that the fundamental questions in robotics are ultimately questions about ourselves is part of what makes robotics such a fascinating and engaging endeavor. 努力

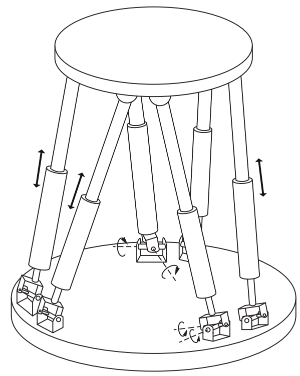
Our focus in this book is on mechanics, planning, and control for **robot mechanisms**. Robot arms are one familiar example. So are wheeled vehicles, as are robot arms mounted on wheeled vehicles. Basically, a mechanism is constructed by connecting rigid bodies, called **links**, together by means of **joints**, so that relative motion between adjacent links becomes possible. **Actuation** of the joints, typically by electric motors, then causes the robot to move and exert forces in desired ways.

The links of a robot mechanism can be arranged in serial fashion, like the familiar open-chain arm shown in Figure 1.1(a). Robot mechanisms can also have links that form closed loops, such as the Stewart–Gough platform shown in Figure 1.1(b). In the case of an open chain, all the joints are actuated, while in the case of mechanisms with closed loops, only a subset of the joints may be actuated.

Let us examine more closely the current technology behind robot mechanisms. The links are moved by actuators, which typically are electrically driven (e.g., by DC or AC motors, stepper motors, or shape memory alloys) but can also be driven by pneumatic or hydraulic cylinders. In the case of rotating



(a) An open-chain industrial manipulator, visualized in V-REP [154].



(b) Stewart–Gough platform. Closed loops are formed from the base platform, through the legs, through the top platform, and through the legs back to the base platform.

Figure 1.1: Open-chain and closed-chain robot mechanisms.

electric motors, these would ideally be lightweight, operate at relatively low rotational speeds (e.g., in the range of hundreds of RPM), and be able to generate large forces and torques. Since most currently available motors operate at low torques and at up to thousands of RPM, speed reduction and torque amplification are required. Examples of such transmissions or transformers include gears, cable drives, belts and pulleys, and chains and sprockets. These speed-reduction devices should have zero or low slippage and **backlash** (defined as the amount of rotation available at the output of the speed-reduction device without motion at the input). Brakes may also be attached to stop the robot quickly or to maintain a stationary posture.

Robots are also equipped with sensors to measure the motion at the joints. For both revolute and prismatic joints, encoders, potentiometers, or resolvers measure the displacement and sometimes tachometers are used to measure velocity. Forces and torques at the joints or at the end-effector of the robot can be measured using various types of force–torque sensors. Additional sensors may be used to help localize objects or the robot itself, such as vision-only cameras, RGB-D cameras which measure the color (RGB) and depth (D) to each pixel, laser range finders, and various types of acoustic sensor.

The study of robotics often includes artificial intelligence and computer perception, but an essential feature of any robot is that it moves in the physical world. Therefore, this book, which is intended to support a first course in robotics for undergraduates and graduate students, focuses on mechanics, motion planning, and control of robot mechanisms.

In the rest of this chapter we provide a preview of the rest of the book.

Chapter 2: Configuration Space

As mentioned above, at its most basic level a robot consists of rigid bodies connected by joints, with the joints driven by actuators. In practice the links may not be completely rigid, and the joints may be affected by factors such as elasticity, backlash, friction, and hysteresis. In this book we ignore these effects for the most part and assume that all links are rigid.

With this assumption, Chapter 2 focuses on representing the **configuration** of a robot system, which is a specification of the position of every point of the robot. Since the robot consists of a collection of rigid bodies connected by joints, our study begins with understanding the configuration of a rigid body. We see that the configuration of a rigid body in the plane can be described using three variables (two for the position and one for the orientation) and the configuration of a rigid body in space can be described using six variables (three for the position and three for the orientation). The number of variables is the number of **degrees of freedom** (dof) of the rigid body. It is also the dimension of the **configuration space**, the space of all configurations of the body.

The dof of a robot, and hence the dimension of its configuration space, is the sum of the dof of its rigid bodies minus the number of constraints on the motion of those rigid bodies provided by the joints. For example, the two most popular joints, revolute (rotational) and prismatic (translational) joints, allow only one motion freedom between the two bodies they connect. Therefore a revolute or prismatic joint can be thought of as providing five constraints on the motion of one spatial rigid body relative to another. Knowing the dof of a rigid body and the number of constraints provided by joints, we can derive **Grübler's formula** for calculating the dof of general robot mechanisms. For **open-chain** robots such as the industrial manipulator of Figure 1.1(a), each joint is independently actuated and the dof is simply the sum of the freedoms provided by each joint. For **closed chains** like the Stewart–Gough platform in Figure 1.1(b), Grübler's formula is a convenient way to calculate a lower bound on the dof. Unlike open-chain robots, some joints of closed chains are not actuated.

Apart from calculating the dof, other configuration space concepts of interest include the **topology** (or "shape") of the configuration space and its **representation**. Two configuration spaces of the same dimension may have different shapes, just like a two-dimensional plane has a different shape from the two-dimensional surface of a sphere. These differences become important when determining how to represent the space. The surface of a unit sphere, for example, could be represented using a minimal number of coordinates, such as latitude and longitude, or it could be represented by three numbers (x, y, z) subject to the constraint $x^2 + y^2 + z^2 = 1$. The former is an **explicit parametrization** of the space and the latter is an **implicit parametrization** of the space. Each type of representation has its advantages, but in this book we will use implicit representations of configurations of rigid bodies.

A robot arm is typically equipped with a hand or gripper, more generally called an **end-effector**, which interacts with objects in the surrounding world. To accomplish a task such as picking up an object, we are concerned with the configuration of a reference frame rigidly attached to the end-effector, and not necessarily the configuration of the entire arm. We call the space of positions and orientations of the end-effector frame the **task space** and note that there is not a one-to-one mapping between the robot's configuration space and the task space. The **workspace** is defined to be the subset of the task space that the end-effector frame can reach.

Chapter 3: Rigid-Body Motions

This chapter addresses the problem of how to describe mathematically the motion of a rigid body moving in three-dimensional physical space. One convenient way is to attach a reference frame to the rigid body and to develop a way to quantitatively describe the frame's position and orientation as it moves. As a first step, we introduce a 3×3 matrix representation for describing a frame's orientation; such a matrix is referred to as a **rotation matrix**.

A rotation matrix is parametrized by three independent coordinates. The most natural and intuitive way to visualize a rotation matrix is in terms of its **exponential coordinate** representation. That is, given a rotation matrix R, there exists some unit vector $\hat{\omega} \in \mathbb{R}^3$ and angle $\theta \in [0, \pi]$ such that the rotation matrix can be obtained by rotating the identity frame (that is, the frame corresponding to the identity matrix) about $\hat{\omega}$ by θ. The exponential coordinates are defined as $\omega = \hat{\omega}\theta \in \mathbb{R}^3$, which is a three-parameter representation. There are several other well-known coordinate representations, e.g., Euler angles, Cayley–Rodrigues parameters, and unit quaternions, which are discussed in Appendix B.

Another reason for focusing on the exponential description of rotations is that they lead directly to the exponential description of rigid-body motions. The latter can be viewed as a modern geometric interpretation of classical screw theory. Keeping the classical terminology as much as possible, we cover in detail the linear algebraic constructs of screw theory, including the unified description of linear and angular velocities as six-dimensional **twists** (also known as **spatial velocities**), and an analogous description of three-dimensional forces and moments as six-dimensional **wrenches** (also known as **spatial forces**).

Chapter 4: Forward Kinematics

For an open chain, the position and orientation of the end-effector are uniquely determined from the joint positions. The **forward kinematics** problem is to find the position and orientation of the reference frame attached to the end-effector given the set of joint positions. In this chapter we present the **product of exponentials (PoE)** formula describing the forward kinematics of open chains. As the name implies, the PoE formula is directly derived from the exponential coordinate representation for rigid-body motions. Aside from providing an intuitive and easily visualizable interpretation of the exponential coordinates as the twists of the joint axes, the PoE formula offers other advantages, like eliminating the need for link frames (only the base frame and end-effector frame are required, and these can be chosen arbitrarily).

In Appendix C we also present the Denavit–Hartenberg (D–H) representation for forward kinematics. The D–H representation uses fewer parameters but requires that reference frames be attached to each link following special rules of assignment, which can be cumbersome. Details of the transformation from the D–H to the PoE representation are also provided in Appendix C.

Chapter 5: Velocity Kinematics and Statics

Velocity kinematics refers to the relationship between the joint linear and angular velocities and those of the end-effector frame. Central to velocity kinematics is the **Jacobian** of the forward kinematics. By multiplying the vector of joint-velocity rates by this configuration-dependent matrix, the twist of the end-effector frame can be obtained for any given robot configuration. **Kinematic singularities**, which are configurations in which the end-effector frame loses the ability to move or rotate in one or more directions, correspond to those configurations at which the Jacobian matrix fails to have maximal rank. The **manipulability ellipsoid**, whose shape indicates the ease with which the robot can move in various directions, is also derived from the Jacobian.

By multiplying the vector of joint-velocity rates by this configuration-dependent matrix, the twist of the end-effector frame can be obtained for any given robot configuration.

Finally, the Jacobian is also central to static force analysis. In static equilibrium settings, the Jacobian is used to determine what forces and torques need to be exerted at the joints in order for the end-effector to apply a desired wrench.

The definition of the Jacobian depends on the representation of the end-effector velocity, and our preferred representation of the end-effector velocity is as a six-dimensional twist. We touch briefly on other representations of the end-effector velocity and their corresponding Jacobians.

Chapter 6: Inverse Kinematics

The **inverse kinematics** problem is to determine the set of joint positions that achieves a desired end-effector configuration. For open-chain robots, the inverse kinematics is in general more involved than the forward kinematics: for a given set of joint positions there usually exists a unique end-effector position and orientation but, for a particular end-effector position and orientation, there may exist multiple solutions to the jont positions, or no solution at all.

In this chapter we first examine a popular class of six-dof open-chain structures whose inverse kinematics admits a closed-form analytic solution. Iterative numerical algorithms are then derived for solving the inverse kinematics of general open chains by taking advantage of the inverse of the Jacobian. If the open-chain robot is **kinematically redundant**, meaning that it has more joints than the dimension of the task space, then we use the pseudoinverse of the Jacobian.

Chapter 7: Kinematics of Closed Chains

While open chains have unique forward kinematics solutions, closed chains often have multiple forward kinematics solutions, and sometimes even multiple solutions for the inverse kinematics as well. Also, because closed chains possess both actuated and passive joints, the kinematic singularity analysis of closed chains presents subtleties not encountered in open chains. In this chapter we study the basic concepts and tools for the kinematic analysis of closed chains. We begin with a detailed case study of mechanisms such as the planar five-bar linkage and the Stewart–Gough platform. These results are then generalized into a systematic methodology for the kinematic analysis of more general closed chains.

Chapter 8: Dynamics of Open Chains

Dynamics is the study of motion taking into account the forces and torques that cause it. In this chapter we study the dynamics of open-chain robots. In analogy to the notions of a robot's forward and inverse kinematics, the **forward dynamics** problem is to determine the resulting joint accelerations for a given set of joint forces and torques. The **inverse dynamics** problem is to determine the input joint torques and forces needed for desired joint accelerations. The dynamic equations relating the forces and torques to the motion of the robot's links are given by a set of second-order ordinary differential equations.

The dynamics for an open-chain robot can be derived using one of two approaches. In the Lagrangian approach, first a set of coordinates – referred to as generalized coordinates in the classical dynamics literature – is chosen to parametrize the configuration space. The sum of the potential and kinetic energies of the robot's links are then expressed in terms of the generalized coordinates and their time derivatives. These are then substituted into the **Euler–Lagrange equations**, which then lead to a set of second-order differential equations for the dynamics, expressed in the chosen coordinates for the configuration space.

The **Newton–Euler** approach builds on the generalization of $f = \mathfrak{m}a$, i.e., the equations governing the acceleration of a rigid body given the wrench acting on it. Given the joint variables and their time derivatives, the Newton–Euler approach to inverse dynamics is: to propagate the link velocities and accelerations outward from the proximal link to the distal link, in order to determine the velocity and acceleration of each link; to use the equations of motion for a rigid body to calculate the wrench (and therefore the joint force or torque) that must be acting on the outermost link; and to proceed along the links back toward the base of the robot, calculating the joint forces or torques needed to create the motion of each link and to support the wrench transmitted to the distal links. Because of the open-chain structure, the dynamics can be formulated recursively.

In this chapter we examine both approaches to deriving a robot's dynamic equations. Recursive algorithms for both the forward and inverse dynamics, as well as analytical formulations of the dynamic equations, are presented.

Chapter 9: Trajectory Generation

What sets a robot apart from an automated machine is that it should be easily reprogrammable for different tasks. Different tasks require different motions, and it would be unreasonable to expect the user to specify the entire time-history of each joint for every task; clearly it would be desirable for the robot's control computer to "fill in the details" from a small set of task input data.

This chapter is concerned with the automatic generation of joint trajectories from this set of task input data. Formally, a trajectory consists of a **path**, which is a purely geometric description of the sequence of configurations achieved by a robot, and a **time scaling**, which specifies the times at which those configurations are reached.

Often the input task data is given in the form of an ordered set of joint values, called control points, together with a corresponding set of control times. On the basis of this data the trajectory generation algorithm produces a trajectory for each joint which satisfies various user-supplied conditions. In this chapter we focus on three cases: (i) point-to-point straight-line trajectories in both joint space and task space; (ii) smooth trajectories passing through a sequence of timed "via points"; and (iii) time-optimal trajectories along specified paths, subject to the robot's dynamics and actuator limits. Finding paths that avoid collisions is the subject of the next chapter on motion planning.

Chapter 10: Motion Planning

This chapter addresses the problem of finding a collision-free motion for a robot through a cluttered workspace, while avoiding joint limits, actuator limits, and other physical constraints imposed on the robot. The **path planning** problem is a subproblem of the general motion planning problem that is concerned with finding a collision-free path between a start and goal configuration, usually without regard to the dynamics, the duration of the motion, or other constraints on the motion or control inputs.

There is no single planner applicable to all motion planning problems. In this chapter we consider three basic approaches: grid-based methods, sampling methods, and methods based on virtual potential fields.

Chapter 11: Robot Control

A robot arm can exhibit a number of different behaviors depending on the task and its environment. It can act as a source of programmed motions for tasks such as moving an object from one place to another, or tracing a trajectory for manufacturing applications. It can act as a source of forces, for example when grinding or polishing a workpiece. In tasks such as writing on a chalkboard, it must control forces in some directions (the force pressing the chalk against the board) and motions in other directions (the motion in the plane of the board). In certain applications, e.g., haptic displays, we may want the robot to act like a programmable spring, damper, or mass, by controlling its position, velocity, or acceleration in response to forces applied to it.

In each of these cases, it is the job of the robot controller to convert the task specification to forces and torques at the actuators. Control strategies to achieve the behaviors described above are known as **motion** (or **position**) **control**, **force control**, **hybrid motion–force control**, and **impedance control**. Which of these behaviors is appropriate depends on both the task and the environment. For example, a force-control goal makes sense when the end-effector is in contact with something, but not when it is moving in free space. We also have a fundamental constraint imposed by the mechanics, irrespective of the environment: the robot cannot independently control both motions and forces in the same direction. If the robot imposes a motion then the environment determines the force, and vice versa.

Most robots are driven by actuators that apply a force or torque to each joint. Hence, precisely controlling a robot requires an understanding of the relationship between the joint forces and torques and the motion of the robot; this is the domain of dynamics. Even for simple robots, however, the dynamic equations are complex and dependent on a precise knowledge of the mass and inertia of each link, which may not be readily available. Even if it were, the dynamic equations would still not reflect physical phenomena such as friction, elasticity, backlash, and hysteresis.

Most practical control schemes compensate for these uncertainties by using **feedback control**. After examining the performance limits of feedback control without a dynamic model of the robot, we study motion control algorithms, such as **computed torque control**, that combine approximate dynamic modeling with feedback control. The basic lessons learned for robot motion control are then applied to force control, hybrid motion–force control, and impedance control.

Chapter 12: Grasping and Manipulation

The focus of earlier chapters is on characterizing, planning, and controlling the motion of the robot itself. To do useful work, the robot must be capable of manipulating objects in its environment. In this chapter we model the contact between the robot and an object, specifically the constraints on the object motion imposed by a contact and the forces that can be transmitted through a frictional contact. With these models we study the problem of choosing contacts to immobilize an object by **form closure** and **force closure** grasping. We also apply contact modeling to manipulation problems other than grasping, such as pushing an object, carrying an object dynamically, and testing the stability of a structure.

Chapter 13: Wheeled Mobile Robots

The final chapter addresses the kinematics, motion planning, and control of wheeled mobile robots and of wheeled mobile robots equipped with robot arms. A mobile robot can use specially designed **omniwheels** or **mecanum wheels** to achieve omnidirectional motion, including spinning in place or translating in any direction. Many mobile bases, however, such as cars and differential-drive robots, use more typical wheels, which do not slip sideways. These no-slip constraints are fundamentally different from the loop-closure constraints found in closed chains; the latter are **holonomic**, meaning that they are configuration constraints, while the former are **nonholonomic**, meaning that the velocity constraints cannot be integrated to become equivalent configuration constraints.

Because of the different properties of omnidirectional mobile robots versus nonholonomic mobile robots, we consider their kinematic modeling, motion planning, and control separately. In particular, the motion planning and control of nonholonomic mobile robots is more challenging than for omnidirectional mobile robots.

Once we have derived their kinematic models, we show that the **odometry** problem – the estimation of the chassis configuration based on wheel encoder data – can be solved in the same way for both types of mobile robots. Similarly, for mobile manipulators consisting of a wheeled base and a robot arm, we show that feedback control for **mobile manipulation** (controlling the motion of the end-effector using the arm joints and wheels) is the same for both types of mobile robots. The fundamental object in mobile manipulation is the Jacobian mapping joint rates and wheel velocities to end-effector twists.

Each chapter concludes with a summary of important concepts from the chapter, and Appendix A compiles some of the most used equations into a handy reference. Videos supporting the book can be found at the book's website, `http://modernrobotics.org`. Some chapters have associated software, downloadable from the website. The software is meant to be neither maximally robust nor efficient but to be readable and to reinforce the concepts in the book. You are encouraged to read the software, not just use it, to cement your understanding of the material. Each function contains a sample usage in the comments. The software package may grow over time, but the core functions are documented in the chapters themselves.