Programming Club

Ray Tracing

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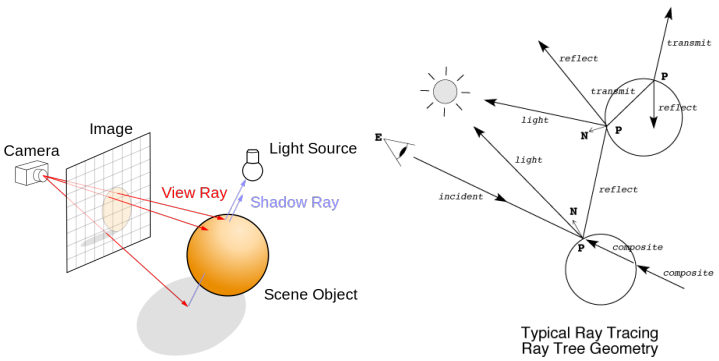
1 Problem description

Build a simple ray tracer in any language you like.

2 Introduction

Reverse ray tracing



3 Some maths

3.1 Dot product

$$\begin{aligned}\mathbf{a} \cdot \mathbf{b} &= (a_x, a_y, a_z) \cdot (b_x, b_y, b_z) \\ &= a_x b_x + a_y b_y + a_z b_z \\ &= \|\mathbf{a}\| \|\mathbf{b}\| cos\theta\end{aligned}$$

3.2 Length of vector

$\|\mathbf{x}\| = \sqrt{\mathbf{x} \cdot \mathbf{x}}$

3.3 Reflection

Incident vector, $\mathbf{x}$, surface normal, $\mathbf{n}$.

Reflected vector is $\mathbf{r} = \mathbf{x} - 2\mathbf{n}(\mathbf{x} \cdot \mathbf{n})$

3.4 Refraction

Incident vector, $\mathbf{x}$, surface normal, $\mathbf{n}$. Refractive index of original medium, γ_1, of new medium, γ_2.

Let

$$\begin{aligned} \gamma &= \gamma_1/\gamma_2 \\ c_1 &= \mathbf{x} \cdot \mathbf{n} \\ c_2 &= \sqrt{1 - \gamma^2 * (1 - c_1^2)} \end{aligned}$$

Refracted vector is $\mathbf{r} = (\gamma * \mathbf{x}) + (\gamma * c_1 - c_2) * \mathbf{n}$

3.5 Intersection of ray and sphere

Sphere is centred at $\mathbf{p_c}$, radius r. Ray origin at $\mathbf{p_0}$, direction $\mathbf{d}$.

Let:

$$\begin{aligned} a &= \mathbf{d} \cdot \mathbf{d} \\ b &= 2\mathbf{d} \cdot (\mathbf{p_0} - \mathbf{p_c}) \\ c &= (\mathbf{p_0} - \mathbf{p_c}) \cdot (\mathbf{p_0} - \mathbf{p_c}) - r^2 \end{aligned}$$

Intersection at $t = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

3.6 Intersection of ray and plane

Plane has normal $\mathbf{n}$ and c (gives implict eqn $\mathbf{n} \cdot \mathbf{p} + c = 0$)

Ray origin at $\mathbf{p_0}$, direction $\mathbf{d}$.

Intersection at $t = -\dfrac{c + \mathbf{n} \cdot \mathbf{p_0}}{\mathbf{n} \cdot \mathbf{d}}$

3.7 Lighting and shading

Energy from point source is inversely proportional to distance squared. Tends to be very dark, consider lower exponent than 2.

Energy per unit area proportional to $cos\theta$, where θ is angle between surface normal and light source.

Note, this simple model will be a bit rubbish. Consider adding ambient light

Shadows if object intersects ray from object to light.

Reflection and refraction can spawn a new ray each - have a depth threshold.

Note simple checkerboard patterns are easy, based on where the point is in space.

4 Output

The simplest picture format you might use is PPM. You can, however use any way to show the ray traced image you like.

The PPM format is:

- A "magic number" for identifying the file type. A ppm image's magic number is the two characters "P6".
- Whitespace (blanks, TABs, CRs, LFs).
- A width, formatted as ASCII characters in decimal.
- Whitespace.
- A height, again in ASCII decimal.
- Whitespace.
- The maximum colour value (Maxval), again in ASCII decimal. Must be less than 65536 and more than zero.
- A single whitespace character (usually a newline).
- A raster of Height rows, in order from top to bottom. Each row consists of Width pixels, in order from left to right. Each pixel is a triplet of red, green, and blue samples, in that order. Each sample is represented in pure binary by either 1 or 2 bytes. If the Maxval is less than 256, it is 1 byte. Otherwise, it is 2 bytes. The most significant byte is first.
- A row of an image is horizontal. A column is vertical. The pixels in the image are square and contiguous.