Project One Template

MAT350: Applied Linear Algebra

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Problem 1

Develop a system of linear equations for the network by writing an equation for each router (A, B, C, D, and E). Make sure to write your final answer as A**x**=**b** where A is the 5x5 coefficient matrix, **x** is the 5x1 vector of unknowns, and **b** is a 5x1 vector of constants.

Solution:

Linear equations have been written for each node on the network. The unknown variables are demonstrated on the left side and the constants on the right. It is important to note here that the sum of the input equals the sum of output.

A: 2x1 + x2 = 100

B: x1 + x2 - x3 -x5 = 0

C: -x1 + x3 + x5 = 50

D: -x2 + x4 + x5 = 120

E: x2 + x3 - x4 + x5 = 0

The equations above can be rewritten into the following coefficient matrix A, with the unknowns as x and the vector of constants as b.

Ax = b

$$\begin{bmatrix} 2 & 1 & 0 & 0 & 0 \\ 1 & 1 & -1 & 0 & -1 \\ -1 & 0 & 1 & 0 & 1 \\ 0 & -1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 & 1 \end{bmatrix} \begin{bmatrix} \mathrm{x1} \\ \mathrm{x2} \\ \mathrm{x3} \\ \mathrm{x4} \\ \mathrm{x5} \end{bmatrix} = \begin{bmatrix} 100 \\ 0 \\ 50 \\ 120 \\ 0 \end{bmatrix}$$

Problem 2

Use MATLAB to construct the augmented matrix [A **b**] and then perform row reduction using the rref() function. Write out your **reduced matrix and identify the free and basic variables of the system**.

Solution:

```
%create coefficient matrix A and constants vector b
```

```
A = [2 1 0 0 0; 1 1 -1 0 -1; -1 0 1 0 1; 0 -1 0 1 1; 0 1 1 -1 1]
```

```
A = 5×5
     2     1     0     0     0
     1     1    -1     0    -1
    -1     0     1     0     1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
b = [100; 0; 50; 120; 0]
```

```
b = 5×1
   100
     0
    50
   120
     0
```

```
%create augmented matrix Ab
Ab = [A b]
```

```
Ab = 5×6
     2     1     0     0     0   100
     1     1    -1     0    -1     0
    -1     0     1     0     1    50
     0    -1     0     1     1   120
     0     1     1    -1     1     0
```

```
%row reduction on Ab to change data into echelon form.
[rowreducedAb, pivotvarsAb] = rref(Ab)
```

```
rowreducedAb = 5×6
     1     0     0     0     0    25
     0     1     0     0     0    50
     0     0     1     0     0    30
     0     0     0     1     0   125
     0     0     0     0     1    45
pivotvarsAb = 1×5
     1     2     3     4     5
```

```
%Use the size command to find the number of variables
%Store the following in numvars
[numeqns, numvars] = size(A)
```

```
numeqns = 5
numvars = 5
```

```
%Use the size command to find the number of pivot variables
%Store in numpivotvars
[numrows, numpivotvars] = size(pivotvarsAb)
```

```
numrows = 1
numpivotvars = 5
```

```
%Use subtraction to find the number of free variables
%Store the following number in numfreevars
numfreevars = numvars - numpivotvars
```

```
numfreevars = 0
```

```
%There are 5 basic variables and 0 free variables.
```

Problem 3

Use MATLAB to **compute the LU decomposition of A**, i.e., find A = LU. For this decomposition, find the transformed set of equations L**y** = **b**, where **y** = U**x**. Solve the system of equations L**y** = **b** for the unknown vector **y**.

Solution:

```
%We are finding the LU decomposition of A
[ L, U] = lu(A)
```

```
L = 5×5
    1.0000         0         0         0         0
    0.5000   -0.5000   -1.0000    1.0000         0
   -0.5000   -0.5000    1.0000         0         0
         0    1.0000         0         0         0
         0   -1.0000    1.0000   -0.5000    1.0000
U = 5×5
    2.0000    1.0000         0         0         0
         0   -1.0000         0    1.0000    1.0000
         0         0    1.0000    0.5000    1.5000
         0         0         0    1.0000    1.0000
         0         0         0         0    1.0000
```

```
%We are using LU decomposition to solve Ax=b
y = L\b
```

```
y = 5×1
   100
   120
   160
   170
    45
```

```
x = U\y
```

```
x = 5×1
    25
    50
    30
   125
    45
```

```
% or we can use the below command
x2 = U\(L\b)
```

```
x2 = 5×1
    25
    50
    30
   125
    45
```

Problem 4

Use MATLAB to **compute the inverse** of U using the inv() function.

Solution:

```
%compute the inverse
inv(U)
```

```
ans = 5×5
    0.5000    0.5000         0   -0.5000         0
         0   -1.0000         0    1.0000         0
         0         0    1.0000   -0.5000   -1.0000
         0         0         0    1.0000   -1.0000
         0         0         0         0    1.0000
```

Problem 5

Compute the solution to the original system of equations by transforming **y** into **x**, i.e., compute **x** = inv(U)**y**.

Solution:

```
%We are putting the inverse of U*y into the variable x for the following
%steps involving Cramer's Rule.
x = inv(U)*y
```

```
x = 5×1
    25
    50
    30
   125
    45
```

Problem 6

Check your answer for x_1 using Cramer's Rule. Use MATLAB to compute the required determinants using the det() function.

Solution:

```
%We are using cramers rule to check if x1 is correct
A1 = A
```

```
A1 = 5×5
     2     1     0     0     0
     1     1    -1     0    -1
    -1     0     1     0     1
     0    -1     0     1     1
     0     1     1    -1     1
```

```
%Replace the first column with the vector of constants b
A1(:,1)=b
```

```
A1 = 5×5
   100     1     0     0     0
```

```
     0     1    -1     0    -1
    50     0     1     0     1
   120    -1     0     1     1
     0     1     1    -1     1
```

```
%use the ratio of determinants to find the solution for x1
x = det(A1)/det(A)
```

```
x = 25.0000
```

Problem 7

The Project One Table Template, provided in the Project One Supporting Materials section in Brightspace, shows the recommended throughput capacity of each link in the network. Put your solution for the system of equations in the third column so it can be easily compared to the maximum capacity in the second column. In the fourth column of the table, provide recommendations for how the network should be modified based on your network throughput analysis findings. The modification options can be No Change, Remove Link, or Upgrade Link. In the final column, explain how you arrived at your recommendation.

Solution:

Network Link	Recommended Capacity (Mbps)	Solution	Recommendation	Explanation
x_1	60	25	Upgrade Link	The data rate on this link is less than the recommended and needs to be updated accordingly.
x_2	50	50	No Change	The data rate is equal, and no change is necessary
x_3	100	30	Upgrade Link	The data rate here is less than the recommended capacity and needs to be upgraded to meet the recommendation.
x_4	100	125	Remove Link	The data rate exceeds the recommended capacity and needs to be removed and updated because this could cause the network instability.
x_5	50	45	Upgrade Link	The data rate is less than the recommended capacity and needs to be upgraded.