CSS 342  Notes to help with Big-O Problems

Best printed using a non-proportional font, e.g. courier-new 10pt.
Note:  ^ is used for exponentiation here (it is not valid in C++)

A few items to help with your homework on big-oh.  One of the problems ends up needing to know the following summation.

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

Also, all our summations start with 1.  What do you do if one doesn't? The following is a review of handling that as covered in class. Consider the code segment:

```
for (int i = 1; i <= n; i++)
    for (int j = i; j <= n; j++)
        something O(1);
```

The running time is big-oh of the following summations:

$$T(n) = O\left( \sum_{i=1}^{n} \sum_{j=i}^{n} 1 \right)$$

Look at just the inner summation, the j summation.  If it started at 1, then the sum would be n; or if we were summing up  i, we could use our beloved n(n+1)/2.  But it doesn't.  This is the tail end of the summation from 1 to n since  i  is somewhere between 1 and n.  Here's what we do.

Consider the whole sum from 1 to n, n terms in total.  Then consider the sum from 1 to i-1, the first (i-1) terms.  If we subtract these summations, what's left are the terms at the end of the summation, the terms from i to n, which are the ones we want.

$$\underbrace{\underbrace{1 + 2 + 3 + 4 + \ldots + i-1}_{\text{the sum from 1 to i-1}} + \underbrace{i + i+1 + \ldots + n-1 + n}_{\text{the sum of what we want}}}_{\text{the sum from 1 to n}}$$

$$= O\left( \sum_{i=1}^{n} \left( \sum_{j=1}^{n} 1 - \sum_{j=1}^{i-1} 1 \right) \right)$$ Now we can use our known formula.

$$= O\left( \sum_{i=1}^{n} (n - (i - 1)) \right) = O\left( \sum_{i=1}^{n} (n - i + 1) \right)$$

$$\sum_{i=1}^{n}$$

$$= O\left( \sum_{i=1}^{n} (n+1) - \sum_{i=1}^{n} i \right) = O( n(n+1) - n(n+1)/2 ) = O( (n^2 + n)/2 )$$

Thus the running time of our code segment is O(n^2).