

Give an analysis for the towers of hanoi function. Write a recurrence equation and then find a tight big-oh.

```
void moveDisks (int n, char fromPeg, char toPeg, char auxPeg) {
    if (n == 1) {
        cout << setw(6) << n
            << "          " << fromPeg << "          " << toPeg << endl;
    }
    else {
        moveDisks(n-1, fromPeg, auxPeg, toPeg);
        cout << setw(6) << n
            << "          " << fromPeg << "          " << toPeg << endl;
        moveDisks(n-1, auxPeg, toPeg, fromPeg);
    }
}
```

Recurrence equation:

$$T(n) = \begin{cases} O(1) & \text{if } n = 1 \\ O(1) + T(n-1) + T(n-1) & \text{if } n > 1 \end{cases}$$

Consider  $n > 1$ :

$$T(n) = 1 + \frac{T(n-1)}{1 + \frac{T(n-2)}{1 + \frac{T(n-3)}{1 + T(n-3) + T(n-3)}}} + \frac{T(n-1)}{1 + \frac{T(n-2)}{1 + \frac{T(n-3)}{1 + T(n-3) + T(n-3)}}} + \frac{T(n-2)}{1 + \frac{T(n-3)}{1 + T(n-3) + T(n-3)}} + \dots$$

Each time, at each level of replacement, we get a one (really  $O(1)$ ) appearing a power of 2 times. At the end,  $T(1)$  appears  $2^{(n-1)}$  times.

$$= 1 * (2^0 + 2^1 + 2^2 + 2^3 + \dots + 2^{n-2}) + 2^{n-1}$$

$$= \left[ \frac{2^{n-1} - 1}{2 - 1} \right] = 2^n - 1 \quad \text{So } T(n) = O(2^n).$$