CSE 373

Graphs 3: Implementation

reading: Weiss Ch. 9

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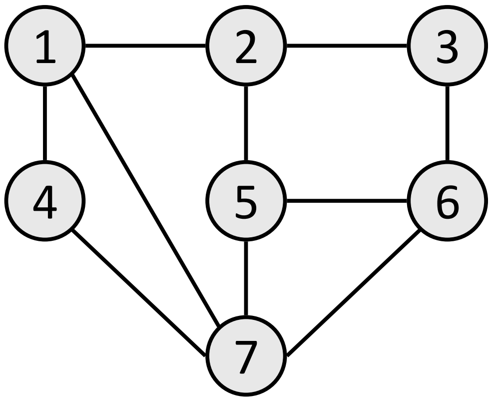
Implementing a graph

- If we wanted to program an actual data structure to represent a graph, what information would we need to store?
  - for each vertex? for each edge?

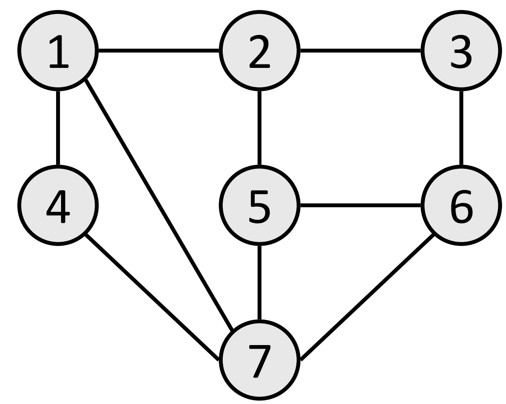


- What kinds of questions would we want to be able to answer quickly:
  - about a vertex?
  - about edges / neighbors?
  - about paths?
  - about what edges exist in the graph?

- We'll explore three common graph implementation strategies:
  - *edge list, adjacency list, adjacency matrix*

Edge list

- **edge list**: An unordered list of all edges in the graph.
  - an array, array list, or linked list



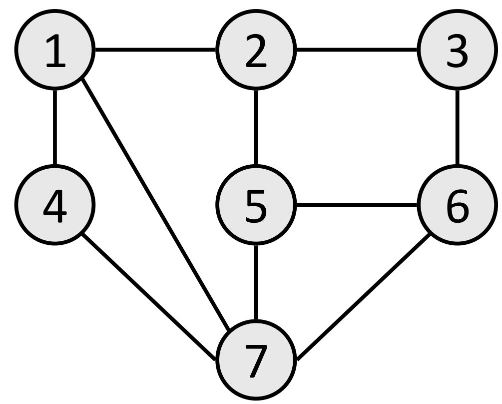
- *advantages*:
  - easy to loop/iterate over all edges

- *disadvantages*:
  - hard to quickly tell if an edge exists from vertex *A* to *B*
  - hard to quickly find the degree of a vertex (how many edges touch it)

0	1	2	3	4	5	6	7	8
(1, 2)	(1, 4)	(1, 7)	(2, 3)	2, 5)	(3, 6)	(4, 7)	(5, 6)	(6, 7)

Graph operations

- Using an edge list, how would you find:
  - all neighbors of a given vertex?
  - the degree of a given vertex?
  - whether there is an edge from *A* to *B*?
  - whether there are any loops (self-edges)?
    - What is the Big-Oh of each operation?

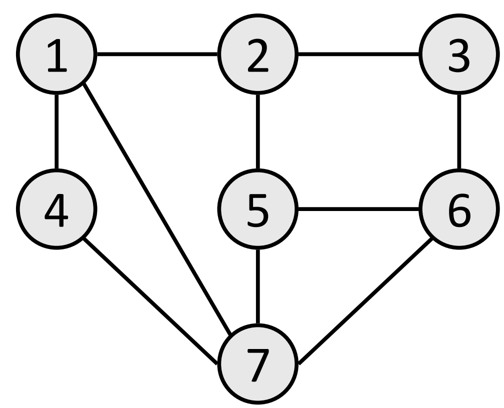


0	1	2	3	4	5	6	7	8
(1, 2)	(1, 4)	(1, 7)	(2, 3)	2, 5)	(3, 6)	(4, 7)	(5, 6)	(6, 7)

Adjacency matrix

- **adjacency matrix**: An $N \times N$ matrix where:
  - the non-diagonal entry $a[i,j]$ is the number of edges joining vertex i and vertex j (or the weight of the edge joining vertex i and vertex j).
  - the diagonal entry $a[i,i]$ corresponds to the number of loops (self-connecting edges) at vertex i *(often disallowed)*.
  - in an undirected graph, $a[i,j]$ = $a[j,i]$ for all i, j. *(diagonally symmetric)*

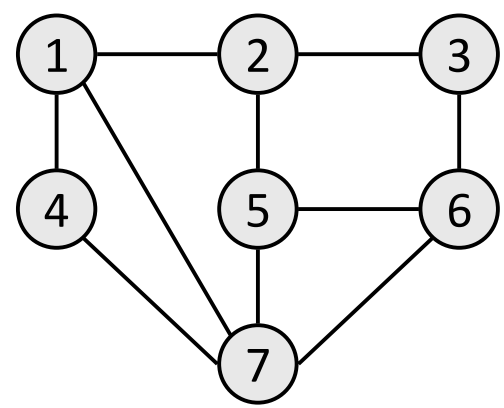
	1	2	3	4	5	6	7
1	0	**1**	0	**1**	0	0	**1**
2	**1**	0	**1**	0	**1**	0	0
3	0	**1**	0	0	0	**1**	0
4	**1**	0	0	0	0	0	**1**
5	0	**1**	0	0	0	**1**	**1**
6	0	0	**1**	0	**1**	0	**1**
7	**1**	0	0	**1**	**1**	**1**	0



Graph operations

- Using an *adjacency matrix*, how would you find:
  - all neighbors of a given vertex?
  - the degree of a given vertex?
  - whether there is an edge from *A* to *B*?
  - whether there are any loops (self-edges)?
    - What is the Big-Oh of each operation?

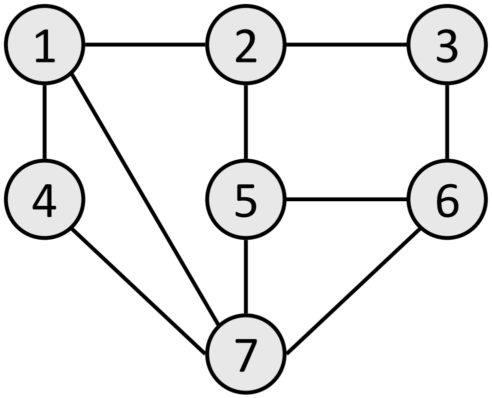
	1	2	3	4	5	6	7
1	0	**1**	0	**1**	0	0	**1**
2	**1**	0	**1**	0	**1**	0	0
3	0	**1**	0	0	0	**1**	0
4	**1**	0	0	0	0	0	**1**
5	0	**1**	0	0	0	**1**	**1**
6	0	0	**1**	0	**1**	0	**1**
7	**1**	0	0	**1**	**1**	**1**	0



Adj matrix pros / cons

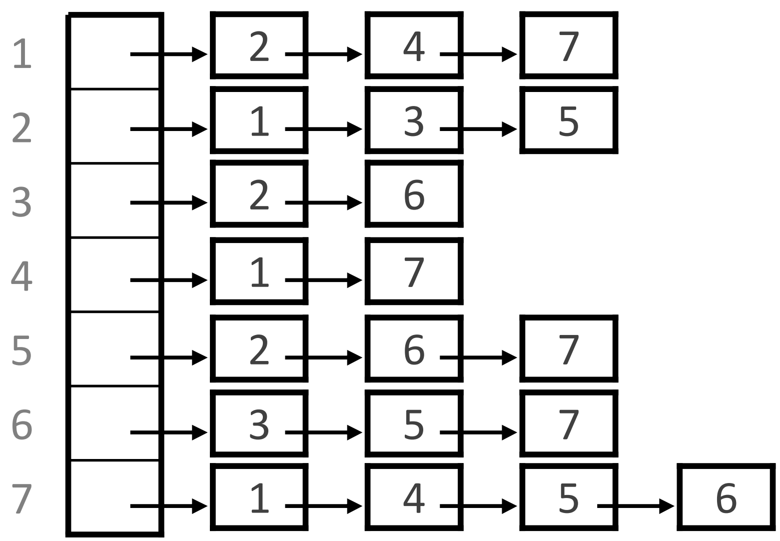
- *advantages*:
  - fast to tell whether an edge exists between any two vertices *i* and *j* (and to get its weight)
- *disadvantage*:
  - consumes a lot of memory on sparse graphs (ones with few edges)

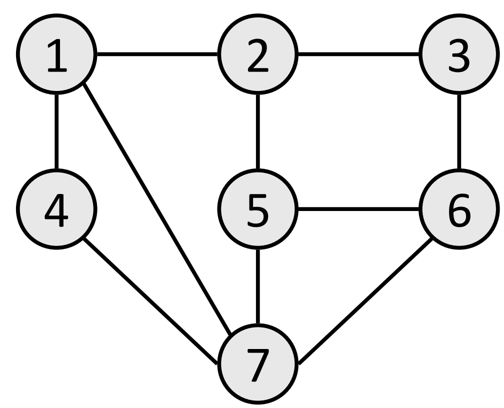
	1	2	3	4	5	6	7
1	0	**1**	0	**1**	0	0	**1**
2	**1**	0	**1**	0	**1**	0	0
3	0	**1**	0	0	0	**1**	0
4	**1**	0	0	0	0	0	**1**
5	0	**1**	0	0	0	**1**	**1**
6	0	0	**1**	0	**1**	0	**1**
7	**1**	0	0	**1**	**1**	**1**	0



Adjacency list

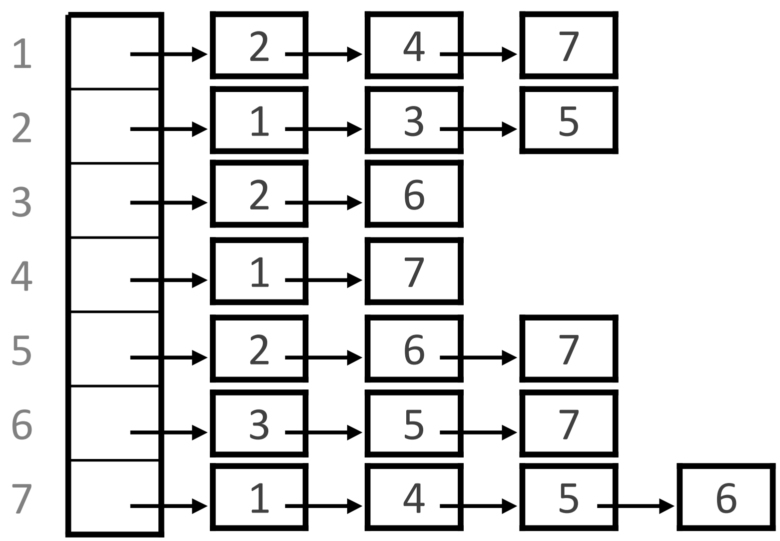
- **adjacency list**: Stores edges as individual linked lists of references to each vertex's neighbors.
  - in unweighted graphs, the lists can simply be references to other vertices and thus use little memory
  - in undirected graphs, edge (*i*, *j*) is stored in both *i*'s and *j*'s lists

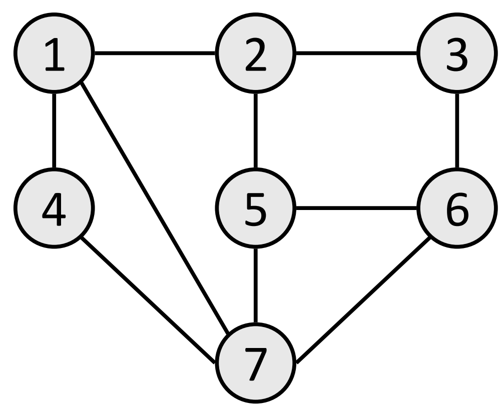




Graph operations

- Using an *adjacency list*, how would you find:
  - all neighbors of a given vertex?
  - the degree of a given vertex?
  - whether there is an edge from *A* to *B*?
  - whether there are any loops (self-edges)?
    - What is the Big-Oh of each operation?





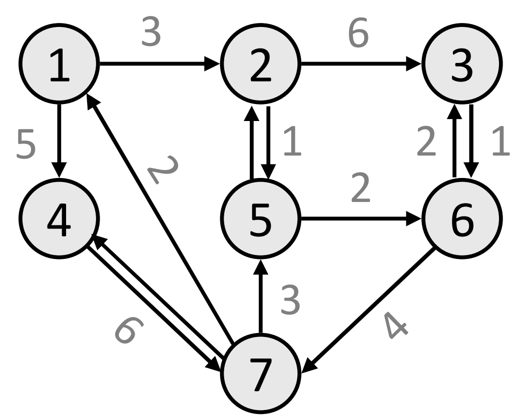
Adj list pros / cons

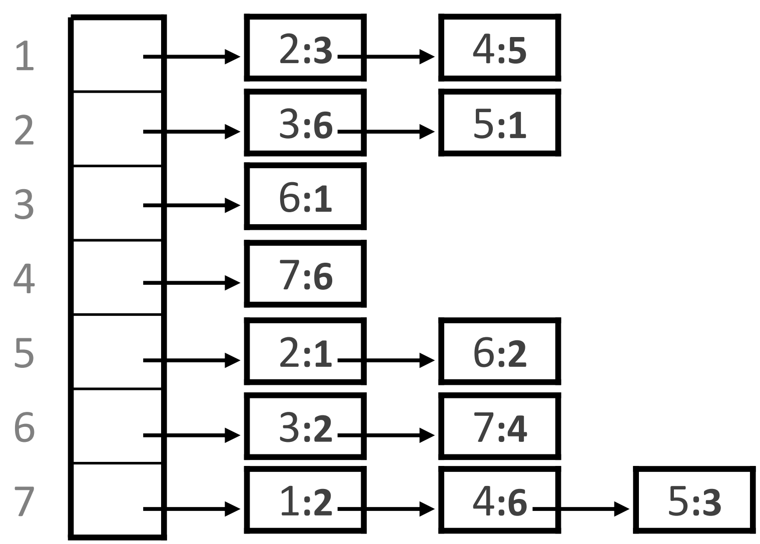
- *advantages*:
  - new vertices can be added to the graph easily, and they can be connected with existing nodes simply by adding elements to the appropriate arrays;
  - easy to find all neighbors of a given vertex (and its degree)

- *disadvantages*:
  - determining whether an edge exists between two vertices requires O(N) time, where N is the average number of edges per node

Weighted/directed graphs

- *weighted*:
  - *adj. list*: store weight in each edge node
  - *adj. matrix*: store weight in each matrix box
- directed:
  - *adj. list*: edges appear only in start vertex's list
  - *adj. matrix*: no longer diagonally symmetric





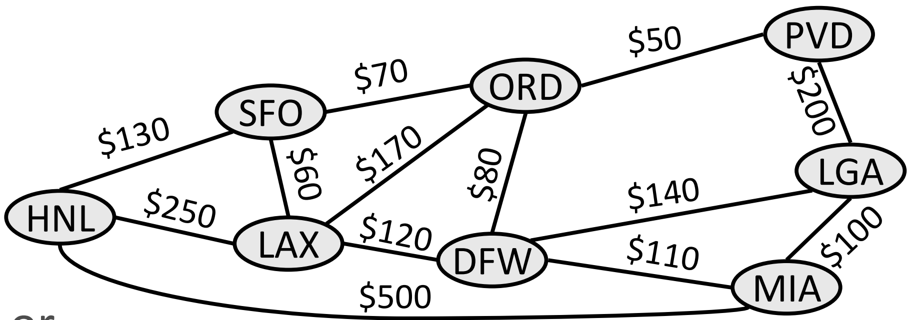
	1	2	3	4	5	6	7
1	0	**3**	0	**5**	0	0	0
2	0	0	**6**	0	**1**	0	0
3	0	0	0	0	0	**1**	0
4	0	0	0	0	0	0	**6**
5	0	**1**	0	0	0	**2**	0
6	0	0	**2**	0	0	0	**4**
7	**2**	0	0	**6**	**3**	0	0

Runtime comparison

▪ *|V| vertices, |E| edges* ▪ *no parallel edges* ▪ *no self-loops*	Edge List	Adjacency List	Adjacency Matrix
Memory usage	$\|V\| + \|E\|$	$\|V\| + \|E\|$	$\|V\|^2$
Find all neighbors of ***v***	$\|E\|$	degree(***v***)	$\|V\|$
Is ***v*** a neighbor of ***w***?	$\|E\|$	degree(***v***)	1
add a vertex	1	1	$\|V\|^2$
add an edge	1	1	1
remove a vertex	$\|E\|$	1	$\|V\|^2$
remove an edge	$\|E\|$	deg(***v***)	1

Representing vertices

- Not all graphs have vertices/edges that are easily "numbered".
  - How do we represent lists or matrices of vertex/edge relationships?
  - How do we quickly look up edges or vertices near a given vertex?



  - edge list:
    - `List<Edge>`
  - adjacency list:
    - `Map<Vertex, List<Edge>>` or
    - `Multimap<Vertex, Edge>`
  - adjacency matrix:
    - `Map<Vertex, Map<Vertex, Edge>>` or
    - `Table<Vertex, Vertex, Edge>`

A graph ADT

- As with other ADTs, we can create a Graph ADT interface:

```
public interface Graph<V, E> {
    void addEdge(V v1, V v2, E e, int weight);
    void addVertex(V v);
    void clear();
    boolean containsEdge(E e);
    boolean containsEdge(V v1, V v2);
    boolean containsVertex(V v);
    int cost(List<V> path);
    int degree(V v);
    E edge(V v1, V v2);
    int edgeCount();
    Set<E> edges();
    int edgeWeight(V v1, V v2);
```

A graph ADT, cont'd.

```
// public interface Graph<V, E> {
    ...
    boolean isDirected();
    boolean isEmpty();
    boolean isReachable(V v1, V v2);   // DFS
    boolean isWeighted();
    List<V> minimumWeightPath(V v);    // Dijkstra's
    Set<V> neighbors(V v);
    int outDegree(V v);
    void removeEdge(V v1, V v2);
    void removeVertex(V v);
    List<V> shortestPath(V v1, V v2);  // BFS
    String toString();
    int vertexCount();
    Set<V> vertices();
}
```

Info about vertices

- Information stored in each vertex (for internal use):
  - can store various flags and fields for use by path search algorithms

```
public class Vertex<V> {
    public int cost() {...}
    public int number() {...}
    public V previous() {...}
    public boolean visited() {...}

    public void setCost(int cost) {...}
    public void setNumber(int number) {...}
    public void setPrevious(V previous) {...}
    public void setVisited(boolean visited) {...}
    public void clear() {...}    // reset dist,prev,visited
}
```

Info about edges

- Information stored in each edge (for internal use):

```
public class Edge<V, E> {
    public boolean contains(V vertex) {...}
    public E edge() {...}
    public V end() {...}
    public V start() {...}
    public int weight() {...}    // 1 if unweighted
}
```