

Find recurrence equation and big-oh for the nodes at level n in binary tree
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Note - In this problem, n is being used differently from other problems in that it is not the number of nodes in the tree, it is the level (what previously was called k).

Recurrence equation:

$$T(n) = \begin{cases} O(1) & \text{if } n = 1 \\ O(1) + T(n-1) + T(n-1) & \text{if } n > 1 \end{cases}$$

Consider $n > 1$:

$$T(n) = 1 + \underbrace{T(n-1)}_{1 + \underbrace{T(n-2)}_{1 + \underbrace{T(n-3)}_{\dots}}} + \underbrace{T(n-1)}_{1 + \underbrace{T(n-2)}_{1 + \underbrace{T(n-3)}_{\dots}}} + \underbrace{T(n-2)}_{1 + \underbrace{T(n-3)}_{\dots}} + \underbrace{T(n-2)}_{1 + \underbrace{T(n-3)}_{\dots}}$$

Each time, at each level of replacement, we get a one (really $O(1)$) appearing a power of 2 times. At the end, $T(1)$ appears $2^{(n-1)}$ times.

$$= 1 * (2^0 + 2^1 + 2^2 + 2^3 + \dots + 2^{n-2}) + 2^{n-1}$$

$$= \sum_{i=0}^{n-1} 2^i = 2^n - 1$$

So $T(n) = O(2^n)$.