Mathematical Foundations of Computer Science

Constructor University Dr. Jürgen Schönwälder

Problem Sheet #3

Problem 3.1: proof by contrapositive

(4 points)

Module: CH-233

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Let $x, y \in \mathbb{R}$ be real numbers. If $y^3 + yx^2 \le x^3 + xy^2$, then $y \le x$.

Problem 3.2: proof by contradiction

(2 points)

The equation $2x^2 + 2x - 1 = 2y^2$ has no solution for integer numbers $x, y \in \mathbb{Z}$.

Problem 3.3: proof by induction

(4 points)

Let $n \in \mathbb{N}$ be a natural number with $n \ge 1$. Prove that the following holds:

$$1^{2} + 3^{2} + 5^{2} + \dots (2n-1)^{2} = \sum_{k=1}^{n} (2k-1)^{2} = \frac{2n(2n-1)(2n+1)}{6}$$