Problem Sheet #3

Problem 3.1: *proof by contrapositive* (4 points)

Let $x, y \in \mathbb{R}$ be real numbers. If $y^3 + yx^2 \leq x^3 + xy^2$, then $y \leq x$.

Problem 3.2: *proof by contradiction* (2 points)

The equation $2x^2 + 2x - 1 = 2y^2$ has no solution for integer numbers $x, y \in \mathbb{Z}$.

Problem 3.3: *proof by induction* (4 points)

Let $n \in \mathbb{N}$ be a natural number with $n \geq 1$. Prove that the following holds:

$$1^2 + 3^2 + 5^2 + \ldots (2n-1)^2 = \sum_{k=1}^{n} (2k-1)^2 = \frac{2n(2n-1)(2n+1)}{6}$$