Mathematical Foundations of Computer Science
Constructor University
Dr. Jürgen Schönwälder

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Mock Exam Mathematical Foundations of Computer Science

Fill in your name (please write legible) and sign below. The university's Code of Academic Integrity applies to this examination. Furthermore, by signing below you accept that this exam does not count in case it is decided that you were not qualified to sit for the exam.

First Name: 

Last Name:

Signature: 

This exam is **closed book**. You are not allowed to use any electronic devices, **no computers, smart phones, or calculators**. However, you are allowed to use an **A4 (single-sided) or A5 (double-sided) handwritten cheat sheet**. Please answer the questions on the problem sheets. If you need more space, feel free to write on the back of the pages. Write with a **pen (black or blue ink)**. Please keep the papers stapled. 100 points correspond to 100%.

No	Problem	Points	Score	Grader
1.1	mixed questions	10		
1.2	equivalence and partial orders	12		
1.3	utopia number system	10		
1.4	number representations	16		
1.5	quine mccluskey algorithm	18		
1.6	first order logic formulas	10		
	Total	76		

Problem 1.1: *mixed questions* (10 points) / 10

Indicate which of the following statements are correct or incorrect by marking the appropriate boxes. For every correctly marked box, you will earn two points. For every incorrectly marked box, you will loose one point. Statements which are not marked or which are marked as both true and false will be ignored. The minimum number of points you can achieve is zero.

true	false		
☑	☐	A computer program is a collection of instructions that perform a specific calculation when executed by a computer. The underlying method used for the calculation is known as an algorithm.	/ 2
☑	☐	The time complexity of an algorithm describes its computational effort while the space complexity of an algorithm describes the memory space needed to store the instructions of an algorithm.	/ 2
☐	☑	A deterministic algorithm always terminates and hence a deterministic algorithm is totally correct if it is partially correct.	/ 2
☑	☐	A spanning tree T of a connected graph G is a cycle-free connected subgraph of G that includes all vertices (nodes) of G and a subset of all edges of G.	/ 2
☐	☑	The Boyer-Moore string search algorithm uses the good character rule to skip efficiently over matching alignments and the bad suffix rule to remove suffixes from the pattern that can never be found in the text.	/ 2

Problem 1.2: *equivalence and partial order relations* (4+2+4+2 = 12 points) / 12

Let $\mathbb{N}_0 = (\mathbb{N} \cup \{0\})$ be the set of natural numbers including 0. Let $M = \mathbb{N}_0^3 = (\mathbb{N}_0 \times \mathbb{N}_0 \times \mathbb{N}_0)$. The relation $\sim$ on $M \times M$ is defined as

$$x \sim y \iff (a,b,c) \sim (d,e,f) \iff a+b+c = d+e+f$$

a) Prove that $\sim$ is an equivalence relation. / 4

b) Determine all elements of the equivalence class $(1,0,1)$. / 2

c) Let $x = (a,b,c) \in M$ and $y = (d,e,f) \in M$. The relation $\preceq$ on $M \times M$ is defined as follows: / 4

$$x \preceq y \iff (a,b,c) \preceq (d,e,f) \iff a \leq d \wedge b \leq e \wedge c \leq f$$

Prove the $\preceq$ is a partial order.

d) Is $\preceq$ also a linear order? Explain why or why not. / 2

Problem 1.3: *utopia number system* (3+5+2 = 10 points) / 10

You have found a calculator from the planet Utopia. You do not know the number system used on planet utopia, Utopians likely do not use decimal numbers. After playing with the calculator for a while, you have observed the following:

$$\begin{aligned} \alpha^2 &= \alpha \\ \alpha + \alpha &= \beta \\ \gamma^2 &= \gamma \\ \gamma + \gamma &= \gamma \\ \delta^2 &= \beta\beta \\ \delta + \delta &= \alpha\alpha \end{aligned}$$

$X^2 = 22$

$X + X = 11$

Note that $\alpha\alpha$ means a number represented by the two digits α and α (not the multiplication of α with itself). The same applies to $\beta\beta$.

a) What is the decimal value of α, β, and γ? / 3

b) What is the decimal value of δ? What is the base used by the number system? / 5

c) How is the decimal number 99 written on planet Utopia? / 2

49 71

202

a). Let $\alpha = 1$

- $1^2 = 1$ ✓
- $1 + 1 = 2$ ✓
- $X^2 = 22$
- $X = \sqrt{22}$
- $\delta = \sqrt{22}$

$\delta^2 = 22_3 = 8_{10}$ $\sqrt{8} = 2$

$\delta = 2_{10}$ or 2_3

$\gamma = 1$

δ

10 11 12 13 14 15
9, a, b, c, d, e, F

Problem 1.4: *number representations* (2+2+2+10 = 16 points) / 16

a) Represent the hexadecimal number 0xf0 as an octal number. / 2

b) Consider a fixed size integer numeral system with the base $b = 6$ and $n = 3$ digits using the b-complement notation for negative integers. What is the representation of -20_{10} in this numeral system? / 2

c) Calculate $12 + (-6)$ using 8-bit binary numbers and the b-complement representation of negative numbers. (The numbers in the expression are decimal numbers.) / 2

d) Convert the decimal fraction -35.75 into IEEE 754 single precision floating point format: / 10

$5_{10} = +$
$1 = -$

```
 0                   1                   2                   3
 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1 2 3 4 5 6 7 8 9 0 1
+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+
|S|    exponent   |          mantissa (23 bits)                   |
+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+-+
```

What is the hexadecimal representation of the IEEE 754 single precision floating point number representation? (Recall that the bias for the exponent is 127.)

bias = 127

a). 0xF0 =

b). ~~$20_{10} = 0$~~

020

$b = 20 \bmod 6 = 2$

$n = 20 // 6 = 3$

$b = 3 \bmod 6 = 2$

$n = 3 // 6 = 0$

$20_{10} = 032$

032_6

$-20_{10} = 523_6 + 1 = (524)$

c). $12_{10} = 00000100\ 00001100$

$b = 00000110$

$-b = 11111001 + 1$

$= 11111010$

```
  1111
  00001100
+ 11111010
-----------
  00000110
```

$= 00000110$

d). $35_{10} = X_2$

$X = \_\_\_\_\_$

$= 100011.1100000\ldots$

0.75×2	1.5	1
0.5×2	1	1
0		

$5_{10} \xrightarrow{+127} X_2 = 1 = 132_{10} = X_2$

$X =$

132	0
66	0
33	1
16	0
8	0
4	0
2	0
1	1

$N = 132$

$132 \% 2 = 0$

$66 \% = 0$

$33 \% 2 = 1$

$16 \% 2 = 0$

$8 \% 2 = 0$

$4 \% 2 = 0$

$2 \% = 0$

$1 \% 2 = 1$

1 10000100 00011110000000

Problem 1.5: *quine mccluskey algorithm* (2+9+4+1+2 = 18 points) / 18

The Boolean function $F(a, b, c, d)$ is defined by the following truth table.

	a	b	c	d	F
	0	0	0	0	0
m_1	0	0	0	1	1
	0	0	1	0	0
	0	0	1	1	0
m_2	0	1	0	0	1
m_3	0	1	0	1	1
	0	1	1	0	0
	0	1	1	1	0
	1	0	0	0	0
m_4	1	0	0	1	1
m_5	1	0	1	0	1
	1	0	1	1	0
	1	1	0	0	0
m_6	1	1	0	1	1
m_7	1	1	1	0	1
m_8	1	1	1	1	1

You can use the engineering notation where * denotes a logical and, + denotes a logical or, and the negation of a variable or an expression enclosed in parenthesis is indicated by appending a ' character, like for example: `a + b * c' + (a * d)'`.

a) Define the function F as a sum of minterms. (QM-0) / 2

b) Classify and sort the minterms and determine the set of prime implicants. (QM-1) / 9

c) Determine a minimum sum of prime implicants that defines the function. (QM-2) / 4

d) Define the Boolean function F using the minimal Boolean expression. / 1

e) Calculate the cost of the original sum of minterms and the cost of the minimal expression. Assume that the cost of an expression is determined by the number of ∧ and ∨ operators. / 2

a). $(a' * b' * c' * d) +$
$(a' * b * c' * d') +$
$(a' * b * c' * d) +$
$(a * b' * c' * d) +$
$(a * b' * c * d') +$
$(a * b * c' * d) +$
$(a * b * c * d') +$
$(a * b * c * d)$

b). Sort by Num of 1 Vars

1⊕). $m_1 * m_2$

2⊕). m_3, m_4, m_5

3⊕). m_6, m_7, m_8

b). $m_1 + m_3 = (a' * c' * d)$ — combine to $(c' * d)$ m_1, m_3, m_4, m_6
$m_1 + m_4 = (b' * c' * d)$
$m_2 + m_3 = (a' * b * c')$
$m_3 + m_6 = (b' * c * d')$
$m_4 + m_6 = (a * c' * d)$
$m_5 + m_7 = (a * c * d')$
$m_6 + m_8 = (a * b * d)$ — combine to $(a * b)$ m_6, m_7, m_8
$m_7 + m_8 = (a * b * c)$

Problem 1.6: *formalize logic puzzle in first order logic* ((2+2+2+2)+2 = 10 points) / 10

The following logic puzzle was published by George J. Summers in his book "50 Puzzles in Deductive Reasoning":

> Two women, Alice and Carol, and two men, Brian and David, are athletes. One is a swimmer, a second is a skater, a third is a gymnast, and a fourth is a tennis player. On a day, they were seated around a square table:
>
> 1. The swimmer sat on Alice's left.
> 2. The gymnast sat across from Brian.
> 3. Carol and David sat next to each other.
> 4. A woman set on the skater's left.
>
> Who was the tennis player?

Translate the puzzle into first-order logic.

a) Formalize the four statements into first order logic formulas. / 8

b) Define the domain of discourse $\mathcal{D}$ and the semantics of any predicates or functions you use. / 2

a).