CS 3530: Assignment 1c

Fall 2015

Exercises

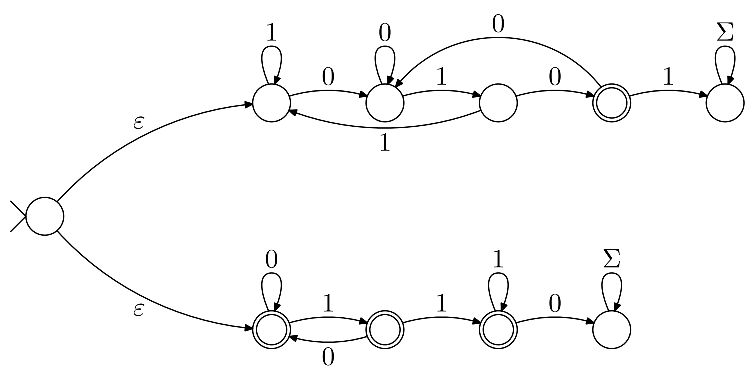
Exercise 1.8b (3 points)

Problem

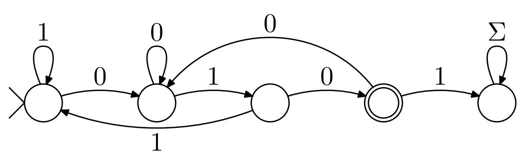
Use the construction given in the proof of Theorem 1.45 to give the state diagrams of NFAs recognizing the union of the languages given.

b. Language: $L_1 \cup L_2$ where L_1 is the language from 1.6c and L_2 is the language from 1.6f (note: both language are from assignment 1a)

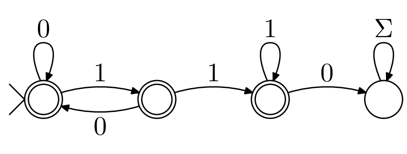
Solution



Language from 1.6c: $\{w : w \text{ contains the substring } 0101\text{, i.e., } w = x0101y \text{ for some } x \text{ and } y\}$



Language from 1.6f: $\{w : w \text{ doesn't contain the substring } 110\}$



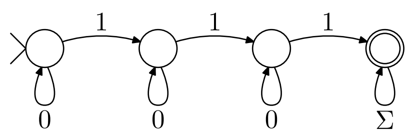
Exercise 1.9b (3 points)

Problem

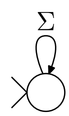
Use the construction given in the proof of Theorem 1.47 to give the state diagrams of NFAs recognizing the concatenation of the languages given.

b. Language: $L_1 \circ L_2$ where L_1 is the language from 1.6b and L_2 is the language from 1.6m (note: both language are from assignment 1a)

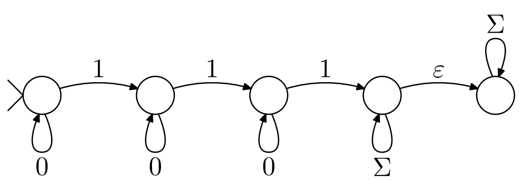
Language from 1.6b: $\{w : w \text{ contains at least three 1s}\}$



Language from 1.6m: The empty set



Solution



Exercise 1.15 (7 points)

Problem

Give a counterexample to show that the following construction fails to prove Theorem 1.49, the closure of the class of regular languages under the star operation.[1] Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize A_1. Construct $N = (Q_1, \Sigma, \delta, q_1, F)$ as follows. N is supposed to recognize A_1^*.

a The states of N are the states of N_1.

b The start state of N is the same as the start state of N_1.

c $F = \{q_1\} \cup F_1$.
The accept states F are the old accept states plus its start state.

d Define δ so that for any $q \in Q$ and any $a \in \Sigma_\varepsilon$,

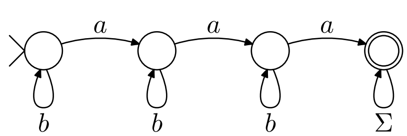
$$\delta(q, a) = \begin{cases} \delta_1(q, a) & q \notin F_1 \text{ or } a \neq \varepsilon \\ \delta_1(q, a) \cup \{q_1\} & q \in F_1 \text{ and } a = \varepsilon. \end{cases}$$

(Suggestion: Show this construction graphically, as in Figure 1.50.)

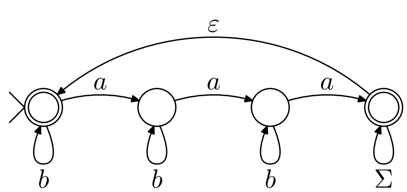
Solution

Language $\{w : w \text{ contains at least three } a\text{s}\}$

N_1



N



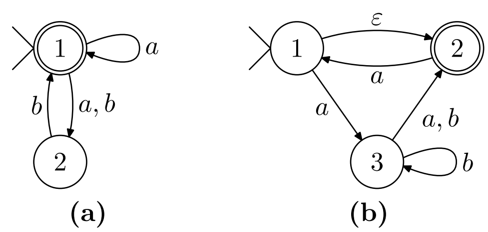
N fails on strings such as {b}, {b,b}, {b,b,b}.

Exercise 1.16 (7 points)

Problem

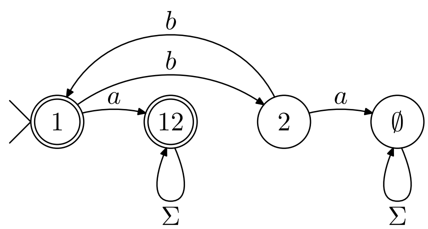
Use the construction given in Theorem 1.39 to convert the following two nondeterministic finite automata to equivalent deterministic finite automata.

[1] In other words, you must present a finite automaton, N_1, for which the constructed automaton N does not recognize the star of N_1's language.



Solution

(a)



(b)