Differential Equations Homework 02

Your Name

Problem 3

Solve the following differential equations or initial value problems:

(a) Solve: $\frac{dy}{dt} = \frac{t^2+7}{y^4-4y^3}$, $y(1) = 2$

This equation is separable, so we can rewrite it as:

$$\frac{y^4 - 4y^3}{dy} = (t^2 + 7)dt$$

We then integrate both sides:

$$\int \frac{y^4 - 4y^3}{dy} = \int (t^2 + 7)dt$$

Performing the integration:

$$\int (y^4 - 4y^3)dy = \frac{y^5}{5} - y^4 \quad \text{and} \quad \int (t^2 + 7)dt = \frac{t^3}{3} + 7t$$

Thus, the implicit solution is:

$$\frac{y^5}{5} - y^4 = \frac{t^3}{3} + 7t + C$$

To find the constant C, use the initial condition $y(1) = 2$:

$$\frac{2^5}{5} - 2^4 = \frac{1^3}{3} + 7(1) + C$$

Simplifying:

$$\frac{32}{5} - 16 = \frac{1}{3} + 7 + C$$

$$\frac{32}{5} - \frac{80}{5} = \frac{1}{3} + 7 + C$$

$$\frac{-48}{5} = \frac{1}{3} + 7 + C$$

Solve for C:

$$C = -\frac{48}{5} - 7 - \frac{1}{3}$$

Simplifying further yields the value of C, giving the implicit solution.

(b) Solve: $\frac{dy}{dt} = \cos^2(y)\ln(t)$

This equation is separable, so we rewrite it as:

$$\frac{1}{\cos^2(y)}dy = \ln(t)dt$$

Integrating both sides:

$$\int \sec^2(y)dy = \int \ln(t)dt$$

The integrals are straightforward:

$$\tan(y) = \frac{t\ln(t) - t + C}{t}$$

We solve this equation for y to get the explicit solution.

(c) Solve: $(t^2 + t)y' + y^2 = ty^2$, $y(1) = -1$

First, simplify the equation:

$$(t^2 + t)y' = ty^2 - y^2$$

$$y' = \frac{y^2(t-1)}{t(t+1)}$$

This is also separable, so we rewrite it as:

$$\frac{dy}{y^2(t-1)} = \frac{dt}{t(t+1)}$$

We then integrate both sides and use the initial condition $y(1) = -1$ to find the constant and write the explicit solution for $y(t)$.