Homework 3

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Problem 1

Problem Statement: Determine weather Picard's Theorem can be used to show the existence of a unique solution in an open interval containing t = 0 **Solution:** Picard's theorem states:

(a) $y' = ty^{\frac{4}{3}}, y(0) = 0$

Problem 2

Problem Statement: Find the order, linearity, homogeneity, and the variablity of the coefficients of the following:

Problem	**Diff Eq**	**Order**	**Linearity**	**Homogeneity**	**coefficients**
(a)	$y'' + 2y' + y = 0$	2	Linear	Homogeneous	Constant
(b)	$\ddot{x} + 2\dot{x} + tx = \sin(t)$	2	Linear	Non-Homogeneous	Variable
(c)	$\cos(y') + ty = 0$	1	Non-Linear	Homogeneous	Variable
(d)	$y'' + ey' + \pi y = 0$	2	Linear	Homogeneous	Constant
(e)	$y' + \frac{1}{1+t^2}y = 7$	1	Linear	Non-Homogeneous	Variable

Problem 6

Problem Statement: Find the general solution to the non homogenoues ODE $y' + \frac{1}{t+1}y = 2$
Solution: We will solve using the superposition principal. $y_{G,NH} = y_{G,H} + y_{P,NH}$

(a)

Problem Satement: Show that $y_p = \frac{t^2+2t}{t+1}$ is a particular solution to the Non-Homogeneous ODE $y' + \frac{1}{t+1}y = 2$ **Solution:** We will start by finding the derivative of y_p

$$\begin{aligned} y_p &= \frac{t^2+2t}{t+1} \\ y_p' &= \frac{(t+1)(2t+2)-(t^2+2t)}{(t+1)^2} \\ y_p' &= \frac{2t^2+2t+2t+2-t^2-2t}{(t+1)^2} \\ y_p' &= \frac{t^2+4t+2}{(t+1)^2} \end{aligned}$$

Now we will plug y_p and y_p' into the Non-Homogeneous ODE

$$\begin{aligned} y_p' + \frac{1}{t+1}y_p &= 2 \\ \frac{t^2+4t+2}{(t+1)^2} + \frac{t^2+2t}{(t+1)^2} &= 2 \\ \frac{t^2+4t+2}{t^2+2t+1} + \frac{t^2+2}{t^2+2t+1} &= 2 \\ \frac{2t^2+4t+2}{t^2+2t+1} &= 2 \\ \frac{2(t^2+2t+1)}{t^2+2t+1} &= 2 \\ 2 &= 2 \end{aligned}$$

(b)

Problem Statement: Find the general solution to the Non-Homogeneous ODE $y' + \frac{1}{t+1}y = 2$ **Solution:** We will use the superposition principal to find the general solution to the Non-Homogeneous ODE

$$y_{G,NH} = y_{G,H} + y_{P,NH}$$

We will start by finding the general solution to the Homogeneous ODE

$$\begin{aligned} y' + \frac{1}{t+1}y &= 0 \\ \frac{1}{y}dy &= -\frac{1}{t+1}dt \\ \ln|y| &= -\ln|t+1| + c \\ y &= \frac{c}{t+1} \end{aligned}$$

Now we will find the general solution to the Non-Homogeneous ODE

$$y_{G,NH} = \frac{c}{t+1} + \frac{t^2+2t}{t+1}$$

Problem 8

Problem Statement: Solve the differential equation $(x+1)\frac{dy}{x} + y = \ln(t)$

Solution: We will solve using the integrating factor method We will start by rewriting the equation

$$\frac{dy}{dt} + \frac{y}{t+1} = \frac{lnt}{t+1}$$

Set right side equal to zero

$$\frac{dy}{dt} + \frac{y}{t+1} = 0$$

Solve for y

$$\frac{dy}{dt} = -\frac{y}{t+1}$$
$$\frac{dy}{y} = -\frac{dt}{t+1}$$

Integrate both sides

$$\int \frac{1}{y}dy = -\frac{1}{t+1}dt$$
$$\ln|y| = -ln|t+1| + c$$
$$y = e^{-\ln|t+1|+c}$$
$$y = \frac{c}{t+1}$$

Solve for integrating factor μ

$$\mu = \frac{1}{y_{G,H}}$$
$$\mu = t+1$$

Multiply orignial equation by μ

$$\begin{aligned}
\mu\frac{dy}{dt} + \mu\frac{y}{t+1} &= \mu\frac{lnt}{t+1} \\
D[(t+1)y] &= \frac{\ln(t)}{t+1}(t+1) \\
(t+1)y &= \int \ln(t) \\
(t+1)y &= t\ln(t) - t + c \\
y &= \frac{t\ln(t) - t + c}{t+1}
\end{aligned}$$