

Homework 2 writeup

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Section: B

Problem 1

```
In [ ]: #y1
y_1 = 0
for k in range(100000) :
    y_1 += 0.1

A6 = np.absolute(10000 - y_1)

#y2
y_2 = 0
for k in range(100000000) :
    y_2 += 0.1

A7 = np.absolute(y_2 - 1000000)

#y3
y_3 = 0
for k in range(100000000) :
    y_3 += 0.25

A8 = np.absolute(25000000 - y_3)

#y4
y_4 = 0
for k in range(100000000) :
    y_4 += 0.5

A9 = np.absolute(y_4 - 50000000)
```

Part a

In Problem 2 of the coding portion of the homework, I found the following values for x_1 , x_2 , x_3 , and x_4 .

x_1	x_2	x_3	x_4
0.0000000188483	0.0188705	0.0	0.0

Part b

From the answers in part (a), we can see that floating point roundoff error has occurred. Theoretically, all four of these numbers should be 0, however experimentally, they are not. I would guess that x_1 and x_2 have the error because .1 or $1/10$ cannot be written as a base 2 exponent easily. x_2 is larger than x_1 because it is compounded 1,000,000 times rather than 10,000 times. x_3 has no error because we are adding .25, which can be represented as 2^{-2} by our machine. No matter how many times we add it together, the answer will be exact (unlike x_2).

Part c

As explained in part (b), I would guess x_3 and x_4 show no error because both .25 and .5 can be represented exactly in binary (2^{-2} and 2^{-1} , respectively). This means that no matter how many times we compute repetitive addition, the computed answer will always align exactly with the theoretical answer.

Problem 2

```
In [3]: import numpy as np
import matplotlib.pyplot as plt
import math

x = np.linspace(-np.pi, np.pi, 100)
y = np.cos(x)
plt.plot(x, y, color = "black", linewidth = 2, label = "cos(x)")

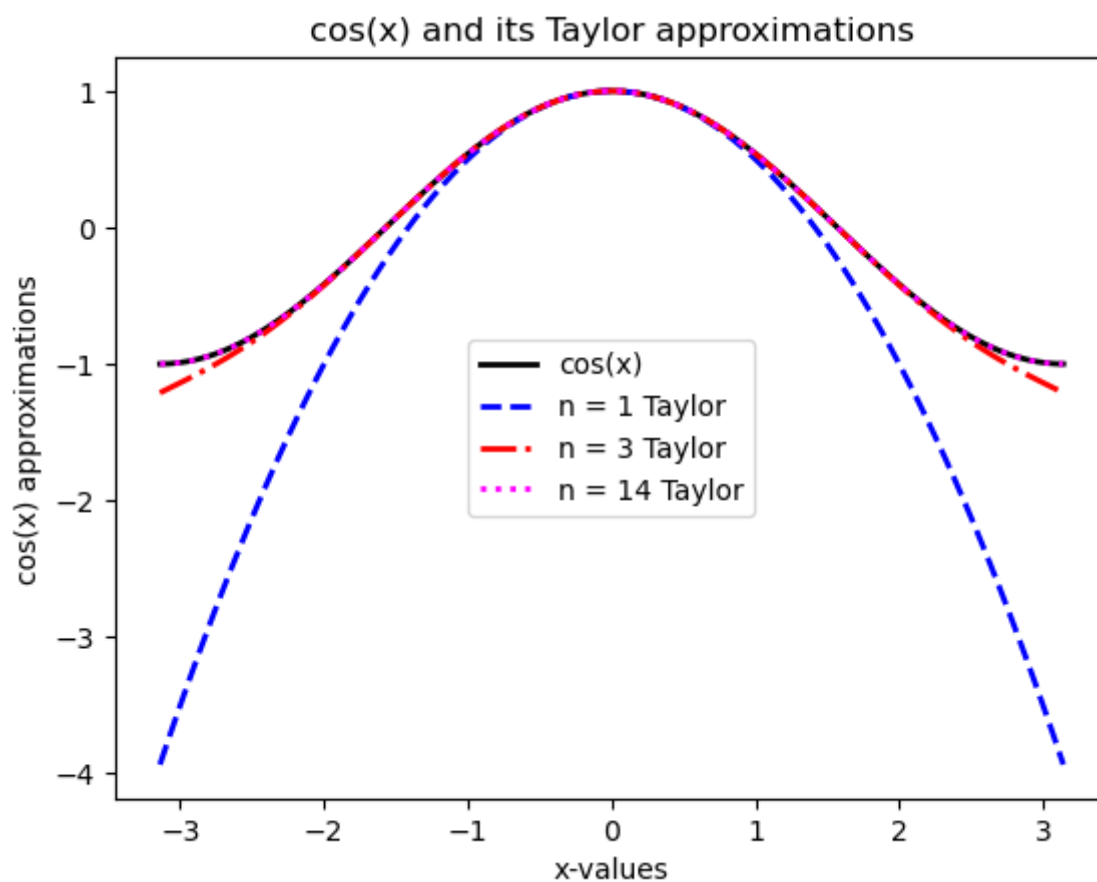
#Plot first order Taylor polynomial
def taylor_func(x, dim):
    temp = 0
    for l in range(dim + 1):
        temp += (((-1)**l)/(math.factorial(2*l))) * (x**(2*l))
    return temp

#Plot first order Taylor polynomial
z = taylor_func(x, 1)
plt.plot(x, z, color = "blue", linestyle = "dashed", linewidth = 2, label = "n = 1")

#Plot third order Taylor polynomial
p = taylor_func(x, 3)
plt.plot(x, p, color = "red", linestyle = "dashdot", linewidth = 2, label = "n = 3")

#Plot fourteenth order Taylor polynomial
q = taylor_func(x, 14)
plt.plot(x, q, color = "magenta", linestyle = "dotted", linewidth = 2, label = "n = 14")

plt.title("cos(x) and its Taylor approximations")
plt.xlabel("x-values")
plt.ylabel("cos(x) approximations")
plt.legend(loc="center")
plt.show()
```



In []: