Orbital Constraints on Exoplanet Habitability

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A 1-D energy balance climate model is developed in order to investigate how changing certain orbital parameters can result in changes to a planet's habitability. Theoretical relationships between temperature, semimajoraxis, and eccentricity are derived from a simple 0-D energy balance model and are tested against the 1-D model and are found to be correct. A qualitative analysis of obliquity shows that there are optimal obliquities to minimise and maximise global temperature. The climates of exomoons orbiting gas giants are also investigated, including reflected light from the gas giant, eclipsing, and tidal heating. It is expected that these additional sources of heat move the habitable zones for the planet outwards.

CONTENTS

1. INTRODUCTION

$$\pi r^2 S(1-A) = 4\pi r^2 \sigma T^4, \tag{1}$$

$$C\frac{\partial T(t,x)}{\partial t} - D\frac{\partial}{\partial x}\left((1-x^2)\frac{\partial T(x,t)}{\partial x}\right) + I - S(1-A) = 0 \tag{2}$$

2. 1-D ENERGY BALANCE CLIMATE MODEL

The EBCM can be derived from the standard heat equation given by

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T, \tag{3}$$

where $T(t, r, \theta, \phi)$ is the temperature at time t, radius r, co-latitude θ, and longitude ϕ. The constant α is related to the heat capacity and diffusion rate of the system. Expanding the lapacian in spherical coordinates the equation becomes

$$\frac{\partial T}{\partial t} = \alpha\left[\frac{1}{r}\frac{\partial^2}{\partial r^2}(rT) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial\theta^2}\right]. \tag{4}$$

The EBCM is arrived at by first letting $T(r, \theta, \phi) = T(\lambda)$, with $\lambda = \pi - \theta$. Thus the equation simplifies to

$$\begin{aligned}\frac{\partial T}{\partial t} &= \frac{\alpha}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right) & (5)\\ &= \frac{\alpha}{r^2}\left(\tan\lambda\frac{\partial T}{\partial\lambda} + \frac{\partial^2 T}{\partial\lambda^2}\right). & (6)\end{aligned}$$

The original equation can be recovered by defining $\alpha/r^2 \equiv D/C$ for diffusion constant D and heat capacity C. Then adding incoming solar radiation S (insolation), which is reduced by planetary albedo A, and outgoing IR-emission I to the PDE. Thus the form of the EBCM in terms of latitude (instead of $x = \sin\lambda$) is given by

$$C\frac{\partial T}{\partial t} = D\left(\tan\lambda\frac{\partial T}{\partial\lambda} + \frac{\partial^2 T}{\partial\lambda^2}\right) + S(1-A) - I. \tag{7}$$

A. Characterising Model Parameters

In this analysis the forms of the model parameters are taken from Williams and Kastings (WK97) [1].

The diffusion varies with orbital and atmospheric parameters as,

$$\frac{D}{D_0} = \frac{p}{p_0}\frac{c_p}{c_{p,0}}\left(\frac{m}{28}\right)^{-2}\left(\frac{\Omega}{1\mathrm{day}^{-1}}\right)^{-2} \tag{8}$$

where $D_0 = 0.56$ Wm^{-2}K^{-1} is from fitting to the time averaged Earth model from North and Coakley 1979 [2]. p is the atmospheric pressure relative to $p_0 = 101$ kPa. c_p is the heat capacity of the atmosphere, relative to $c_{p,0} = 1 \times 10^3$ g^{-1}K^{-1}. m is the (average) mass of the particles in the atmosphere, relative to the Nitrogen molecule. Ω is the rotation rate of the planet, relative to Earth's 1 rotation per day. This can be extended to be time variable, such as having CO_2 emissions increase pressure, change heat capacity, and increase mass of particles. However this paper only considers varying the rotation rate of the planet.

Heat capacity, $C(\lambda, t)$, varies with latitude through the ocean-land fraction, $f_o(\lambda)$, and with temperature through the ice-ocean fraction, $f_i(T)$, as

$$C(\lambda, T) = (1 - f_o(\lambda))C_{land} + f_o(\lambda)((1 - f_i(T))C_{ocean} + f_i(T)C_{ice}(T)), \tag{9}$$

Where $C_{\text{land}} = 5.25 \times 10^6$ Jm^{-2}K^{-1} and $C_{\text{ocean}} = 40 \times C_{\text{land}}$ are constant, and

$$C_{\text{ice}}(T) = \begin{cases} 9.2C_{\text{land}} & T \geq 263\text{K} \\ 2.0C_{\text{land}} & T < 263\text{K}, \end{cases} \tag{10}$$

WK97 provides three sets of IR-emission and Albedo functions. Following the example of SMS08 and Dressing et al 2010 (here on Dressing10) [3] the second set of IR and Albedo functions which are given by

$$I(T) = I_2(T) = \frac{\sigma T^4}{1 + 0.5925(T/273\text{K})^3} \tag{11}$$

$$A(T) = A_2(T) = 0.525 - 0.245 \tanh\left(\frac{T - 268\text{K}}{5}\right), \tag{12}$$

are used in all models. This IR-emission is a blackbody radiation term (numerator) damped by the optical thickness of the atmosphere (denominator) which is roughly equivalent to a greenhouse gas effect. The albedo function is a smooth scaling from low to high reflectivity due to snow and water-vapour reflectance.

The insolation function, S, is defined in WK97 as the day averaged incident (based on latitude) radiation from the sun,

$$S(\lambda, t) = \frac{q_0}{\pi}\left(\frac{1\text{ au}}{a}\right)^2 (H(t)\sin\lambda\sin\delta(t) + \cos\lambda\cos\delta(t)\sin H(t))$$

where $q_0 = 1360$ Wm^{-2} is the insolation from the Sun, a is the distance from the Sun, $\cos H(t) = -\tan\lambda\tan\delta(t)$ is the radian half-day length with $0 < H < \pi$, and $\delta(t)$ is the solar declination defined by

$$\sin\delta(t) = -\sin\delta_0\cos(L_s(t) + \pi/2)$$

where δ_0 is the obliquity of the planet and $L_s(t) = \omega t$ is orbital longitude from an orbital angular velocity found by Kepler's laws.

B. Discretisation of the climate model

Numerically integrating the EBCM requires the derivatives to be discretised. Spatially the planet can be split into S nodes, separated by

$$\Delta\lambda = \frac{\pi^{\text{rad}}}{S - 1} = \frac{180^\circ}{S - 1}, \tag{13}$$

Semimajoraxis a_{gas}, au	Eccentricity e_{gas}	Obliquity δ, deg	No. spatial nodes S	Timestep Δt, days	Land fraction type
1	0.0167	23.5	61	1	Uniform 70% Ocean

TABLE I: A summary of the default parameters for the Earth-like model. A 'Uniform' land fraction indicates that the model has the same ratio of land to ocean across the entire planet. The odd number of spatial nodes means there is a true equator with $\lambda = 0$ as well as poles with $\lambda = \pm 90°$

with spatial indexing of each zone from $m = 0, 1, \ldots, S-1$. Similarly, a temporal indexing of $n = 0, 1, \ldots$ is used to discretise time in steps of Δt.

The spatial derivatives can then be approximated by the central difference and second order central difference:

$$\frac{\partial T_n^m}{\partial \lambda} = \frac{T_n^{m+1} - T_n^{m-1}}{2\Delta\lambda}, \tag{14}$$

$$\frac{\partial^2 T_n^m}{\partial \lambda^2} = \frac{T_n^{m+2} - 2T_n^m + T_n^{m-2}}{(2\Delta\lambda)^2}, \tag{15}$$

and the temporal derivative can be approximated as a forward difference,

$$\frac{\partial T_n^m}{\partial t} = \frac{T_{n+1}^m - T_n^m}{\Delta t}, \tag{16}$$

with numerical stability analysed in appendix B. Evolving the EBCM is performed by solving eqn. 16 for T_{n+1}^m in terms of the parameter and temperature values at time n.

However, a problem arises at the edges of the model as $m = -2, -1, S, S+1$ are not defined. To fix this the derivatives at $m = 0$ ($m = S-1$) are discretised as forward then backward (backward then forward) derivatives. By imposing that $\partial T_n^{m=0,S-1}/\partial\lambda = 0$, these second order derivatives reduce to

$$\frac{\partial^2 T_n^{m=0}}{\partial \lambda^2} = \left(\frac{\partial T_n^{m=1}}{\partial \lambda} - \frac{\partial T_n^{m=0}}{\partial \lambda}\right)/\Delta\lambda = \frac{T_n^{m=1} - T_n^{m=0}}{(\Delta\lambda)^2} \tag{17}$$

$$\frac{\partial^2 T_n^{m=S-1}}{\partial \lambda^2} = \left(\frac{\partial T_n^{m=S-1}}{\partial \lambda} - \frac{\partial T_n^{m=S-2}}{\partial \lambda}\right)/\Delta\lambda = \frac{T_n^{m=S-2} - T_n^{m=S-1}}{(\Delta\lambda)^2}. \tag{18}$$

Furthermore, the treatment imposed for the $m = 1$ and $m = S-2$ second order derivatives is much the same, using central-backward and central-forward derivatives respectively.

3. EARTH-LIKE MODEL

4. EXOPLANETS

A. Investigating time-averaged solar flux

$$\langle S \rangle = \frac{L}{a^2\sqrt{1-e^2}} \tag{19}$$

$$\frac{L(1-A)}{a^2\sqrt{1-e^2}} = \sigma T^4 \tag{20}$$

$$T \propto a^{-1/2} \tag{21}$$

$$T \propto (1-e^2)^{-1/8} \tag{22}$$

$$a \propto (1-e^2)^{-1/4} \tag{23}$$

B. Investigating obliquity and rotation rate

C. Investigating of ocean fraction

5. EXOMOONS

A. Investigating eclipsing

B. Investigating tidal heating

C. Developing and investigating a depth model

6. DISCUSSION

7. CONCLUSION

REFERENCES

[1] Williams and Kasting, "Habitable planets with high obliquities," Icarus, vol. 129, no. 1, pp. 254–267, 1997.

[2] North and Coakley, "Differences between seasonal and mean annual energy balance model calculations of climate and climate sensitivity," J. Atmos. Sci., vol. 36, no. 7, pp. 1189–1204, 1979.

[3] C. D. D. et al, "Habitable climates: the influence of eccentricity," ApJ, vol. 721, no. 2, pp. 1295–1307, 2010.

Appendix A: Defining Equilibrium Temperature and Averages used

Appendix B: Numerical stability of the 1D EBCM

Appendix C: Tidal heating equations and method

SCIENTIFIC SUMMARY FOR A GENERAL AUDIENCE

Many interesting solar systems have been reported in the news, such as the Trappist-1 system which is filled with Earth-like planets. Simulations and models such as those in this paper are used to determine if a planet could be habitable. A habitable zone can be made by varying the parameters of the model to see where the model is habitable, partially habitable, or uninhabitable.

The main model in this paper takes a planet and divides it into a number of latitude bands which can have energy flow between them. Certain parameters, such as how the planet orbits around it's star and the angle the planet is tilted at, are varied to build this habitable zone. A result of this paper is if the Earth orbited slightly further away from the Sun then it is likely that it would fall into an ice age similar to what the Earth has experienced in the past. Another result found is that the tilt of the planet can affect how hot or cold it is, and indicates that the current tilt of the Earth gives a cold planet.

Another aspect of this paper's exoplanet research is exomoons orbiting a gas giant such as Jupiter. In certain configurations an exomoon can be heated not only from the host star, but also due to a process called tidal heating. Tidal heating is similar to stretching an elastic band. Stretching and relaxing an elastic band many times can cause the band to warm up. The moon of a gas planet is stretched slightly by unequal forces of gravity as one part of the moon is further away than the other. If the moon's orbit is not circular then the moon is stretched and relaxed, thus heats up in a similar way to the elastic band. Adding tidal heating to the model allows for investigations into how tidal heating can move, or change the shape of, the habitable zone.