Orbital Constraints on Exoplanet Habitability

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A 1-D energy balance climate model is developed in order to investigate how changing certain orbital parameters can result in changes to a planet's habitability. Theoretical relationships between temperature, semimajoraxis, and eccentricity are derived from a simple 0-D energy balance model and are tested against the 1-D model and are found to be correct. A qualitative analysis of obliquity shows that there are optimal obliquities to minimise and maximise global temperature. The climates of exomoons orbiting gas giants are also investigated, including reflected light from the gas giant, eclipsing, and tidal heating. It is expected that these additional sources of heat move the habitable zones for the planet outwards.

CONTENTS

1. INTRODUCTION

$$\pi r^2 S(1-A) = 4\pi r^2 \sigma T^4, \tag{1}$$

$$C(\lambda,T)\frac{\partial T(t,\lambda)}{\partial t} = D\left[\frac{\partial^2 T(t,\lambda)}{\partial \lambda^2} - \tan\lambda \frac{\partial T(t,\lambda)}{\partial \lambda}\right] + S(\lambda,t)(1-A(T)) - I(T), \tag{2}$$

2. 1-D ENERGY BALANCE CLIMATE MODEL

The EBCM can be derived from the standard heat equation given by

$$\frac{\partial T}{\partial t} = \alpha \nabla^2 T, \tag{3}$$

where $T(t,r,\theta,\phi)$ is the temperature at time t, radius r, co-latitude θ, and longitude ϕ. The constant α is related to the heat capacity and diffusion rate of the system. Expanding the laplacian in spherical coordinates the equation becomes

$$\frac{\partial T}{\partial t} = \alpha\left[\frac{1}{r}\frac{\partial^2}{\partial r^2}(rT) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 T}{\partial \theta^2}\right]. \tag{4}$$

The EBCM is arrived at by first letting $T(t,r,\theta,\phi) = T(t,\lambda)$, with latitude $\lambda = \pi - \theta$. Thus the equation simplifies to

$$\frac{\partial T}{\partial t} = \frac{\alpha}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right) \tag{5}$$

$$= \frac{\alpha}{r^2}\left(\frac{\partial^2 T}{\partial\lambda^2} - \tan\lambda\frac{\partial T}{\partial\lambda}\right). \tag{6}$$

The original equation can be recovered by defining $\alpha/r^2 \equiv D/C$ for diffusion constant D and heat capacity C. Then adding incoming solar radiation S (insolation), which is reduced by planetary albedo A, and outgoing IR-emission I to the PDE. Thus the original form of the 1D EBCM in eqn. (2) is recovered.

A. Discretisation of the climate model

Numerically integrating the EBCM requires the derivatives to be discretised. Spatially the planet can be split into S latitude bands, separated by

$$\Delta\lambda = \frac{\pi^{\text{rad}}}{S-1} = \frac{180^\circ}{S-1}, \tag{7}$$

with spatial indexing of each band from $m = 0, 1, \ldots, S-1$. Similarly, a temporal indexing of $n = 0, 1, \ldots$ is used to discretise time in steps of Δt. Thus T_n^m is the temperature at the m$^{\text{th}}$ timestep for the n$^{\text{th}}$ latitude band.

The spatial derivatives can then be approximated by the central difference and second order central difference:

$$\frac{\partial T_n^m}{\partial \lambda} = \frac{T_n^{m+1} - T_n^{m-1}}{2\Delta\lambda}, \tag{8}$$

$$\frac{\partial^2 T_n^m}{\partial \lambda^2} = \frac{T_n^{m+2} - 2T_n^m + T_n^{m-2}}{(2\Delta\lambda)^2}, \tag{9}$$

and the temporal derivative can be approximated as a forward difference,

$$\frac{\partial T_n^m}{\partial t} = \frac{T_{n+1}^m - T_n^m}{\Delta t}, \tag{10}$$

with numerical stability analysed in appendix B. Evolving the EBCM is performed by solving eqn. (10) for T_{n+1}^m in terms of the parameter and temperature values at timestep n.

However, a problem arises at the edges of the model as $m = -2, -1, S, S+1$ are not defined. To fix this the derivatives at $m = 0$ ($m = S-1$) are discretised as forward then backward (backward then forward) derivatives. By imposing that $\partial T_n^{m=0,S-1}/\partial\lambda = 0$, these second order derivatives reduce to

$$\frac{\partial^2 T_n^{m=0}}{\partial \lambda^2} = \left(\frac{\partial T_n^{m=1}}{\partial \lambda} - \frac{\partial T_n^{m=0}}{\partial \lambda}\right)/\Delta\lambda \qquad = \frac{T_n^{m=1} - T_n^{m=0}}{(\Delta\lambda)^2} \tag{11}$$

$$\frac{\partial^2 T_n^{m=S-1}}{\partial \lambda^2} = \left(\frac{\partial T_n^{m=S-1}}{\partial \lambda} - \frac{\partial T_n^{m=S-2}}{\partial \lambda}\right)/\Delta\lambda = \frac{T_n^{m=S-2} - T_n^{m=S-1}}{(\Delta\lambda)^2}. \tag{12}$$

Furthermore, the treatment imposed for the $m = 1$ and $m = S-2$ second order derivatives is much the same, using central-backward and central-forward derivatives respectively.

3. EARTH-LIKE MODEL

A. Characterising Model Parameters

In this analysis the forms of the model parameters are taken from Williams and Kastings (WK97) [1].

The diffusion varies with orbital and atmospheric parameters as,

$$\frac{D}{D_0} = \frac{p}{p_0}\frac{c_p}{c_{p,0}}\left(\frac{m}{28}\right)^{-2}\left(\frac{\Omega}{1\text{day}^{-1}}\right)^{-2} \tag{13}$$

where $D_0 = 0.56$ Wm^{-2}K^{-1} is from fitting to the time averaged Earth model from North and Coakley 1979 [2]. p is the atmospheric pressure relative to $p_0 = 101$ kPa. c_p is the heat capacity of the atmosphere, relative to $c_{p,0} = 1 \times 10^3$ g^{-1}K^{-1}. m is the (average) mass of the particles in the atmosphere, relative to the Nitrogen molecule. Ω is the rotation rate of the planet, relative to Earth's 1 rotation per day. This can be extended to be time variable, such as having CO_2 emissions increase pressure, change heat capacity, and increase mass of particles. However this paper only considers varying the rotation rate of the planet.

Heat capacity, $C(\lambda, t)$, varies with latitude through the ocean-land fraction, $f_o(\lambda)$, and with temperature through the ice-ocean fraction, $f_i(T)$, as

$$C(\lambda, T) = (1 - f_o(\lambda))C_{land} + f_o(\lambda)((1 - f_i(T))C_{ocean} + f_i(T)C_{ice}(T)), \tag{14}$$

Semimajoraxis a_{gas}, au	Eccentricity e_{gas}	Obliquity δ, deg	No. spatial nodes S	Timestep Δt, days	Land fraction type
1	0.0167	23.5	61	1	Uniform 70% Ocean

TABLE I: A summary of the default parameters for the Earth-like model. A 'Uniform' land fraction indicates that the model has the same ratio of land to ocean across the entire planet. The odd number of spatial nodes means there is a true equator with $\lambda = 0$ as well as poles with $\lambda = \pm 90°$

Where $C_{\text{land}} = 5.25 \times 10^6$ Jm^{-2}K^{-1} and $C_{\text{ocean}} = 40 \times C_{\text{land}}$ are constant, and

$$C_{\text{ice}}(T) = \begin{cases} 9.2C_{\text{land}} & T \geq 263\text{K} \\ 2.0C_{\text{land}} & T < 263\text{K}, \end{cases} \tag{15}$$

WK97 provides three sets of IR-emission and Albedo functions. Following the example of SMS08 and Dressing et al 2010 (here on Dressing10) [3] the second set of IR and Albedo functions which are given by

$$I(T) = I_2(T) = \frac{\sigma T^4}{1 + 0.5925(T/273\text{K})^3} \tag{16}$$

$$A(T) = A_2(T) = 0.525 - 0.245 \tanh\left(\frac{T - 268\text{K}}{5}\right), \tag{17}$$

are used in all models. This IR-emission is a blackbody radiation term (numerator) damped by the optical thickness of the atmosphere (denominator) which is roughly equivalent to a greenhouse gas effect. The albedo function is a smooth scaling from low to high reflectivity due to snow and water-vapour reflectance.

The insolation function, S, is defined in WK97 as the day averaged incident (based on latitude) radiation from the sun,

$$S(\lambda, t) = \frac{q_0}{\pi}\left(\frac{1\text{ au}}{a}\right)^2 (H(t)\sin\lambda\sin\delta(t) + \cos\lambda\cos\delta(t)\sin H(t))$$

where $q_0 = 1360$ Wm^{-2} is the insolation from the Sun, a is the distance from the Sun, $\cos H(t) = -\tan\lambda\tan\delta(t)$ is the radian half-day length with $0 < H < \pi$, and $\delta(t)$ is the solar declination defined by

$$\sin\delta(t) = -\sin\delta_0\cos(L_s(t) + \pi/2)$$

where δ_0 is the obliquity of the planet and $L_s(t) = \omega t$ is orbital longitude from an orbital angular velocity found by Kepler's laws.

4. EARTH-LIKE EXOPLANETS

A. Investigating time-averaged solar flux

General temperature relations for a planet can be found from the 0D EBCM. Time averaged insolation of an planet in an elliptical orbit is given by

$$\langle F\rangle = \frac{q_0}{a^2\sqrt{1 - e^2}}, \tag{18}$$

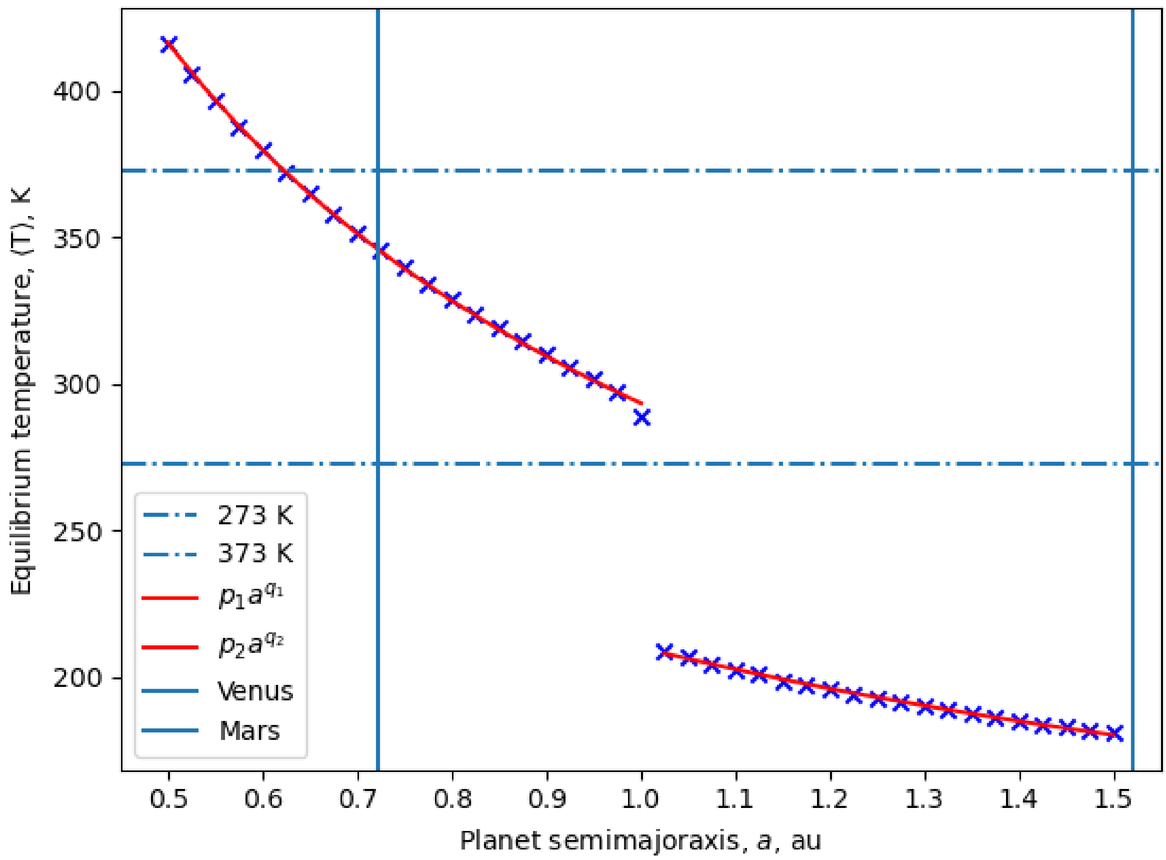

FIG. 1: A plot of the global temperature of the planet when varying its semimajor axis. Overlaid on the plot are two curves which are fitted to the data by a least squares regression. The form of the curve is $\langle T \rangle = p_i a^{q_i}$. It is expected from a 0D EBCM (see eq. (19)) that $q_i = -0.5$. The first zone obeys the expected powerlaw nicely, with $q_1 = -0.505 \pm 0.004$. The other free parameter for the first zone is $p_1 = 293.5 \pm 0.4$. The "snowball" zone after 1 au represents a sudden drop in temperature due to ice-albedo feedback, and follows a very different powerlaw to the first zone. Free parameters for this zone are $p_2 = 210.2 \pm 0.2$ and $q_2 = -0.378 \pm 0.003$ Also shown are Venus and Mars to highlight the range of values considered.

where $q_0 = L_{\text{Sun}}/4\pi a_{\text{Earth}}^2 \approx 1360\ \text{Wm}^{-2}$ is the solar flux for Earth, a and e are the semimajor axis and eccentricity respectively [4].

By subsituting this relation into equation (1), the temperature of a planet can be related to semimajor axis and eccentricity through

$$T \propto a^{-1/2}(1 - e^2)^{-1/8}, \tag{19}$$

with proportionality constant $(q_0(1 - A)/4\sigma)^{1/4} = 255$ K for an Earth-like albedo of 0.3.

By keeping $e = 0.0167$ constant and varying a from 0.5 au to 1.5 au the validity of this proportionality can be investigated. As seen in Figure 1 there are three main zones to consider. The first zone with $a < 0.65$ au has temperatures too high to sustain life due to being too close to the Sun. The second zone with $0.65 < a < 1$ au is much more temperate, and is able to sustain liquid water on the planet's surface. There is a small dip at 1 au where the planet is

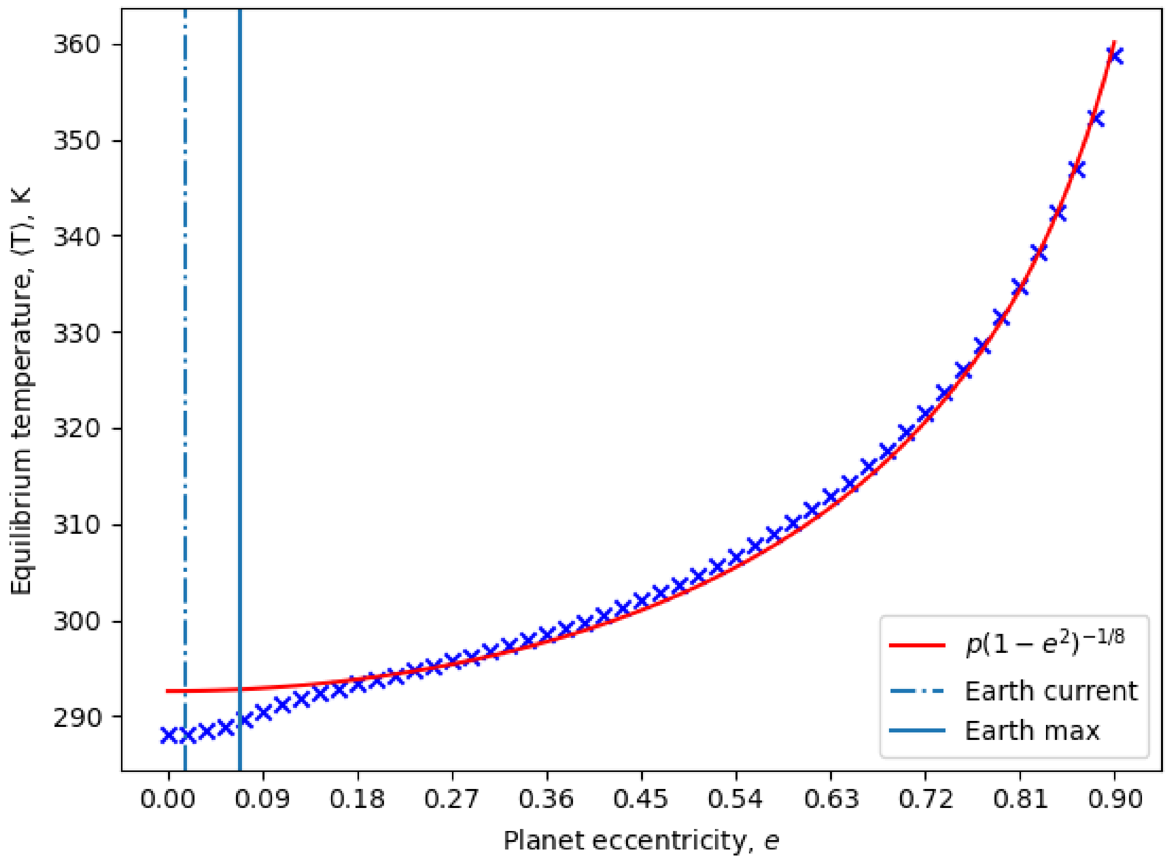

FIG. 2: $\langle T \rangle = p(1-e^2)^{-1/8}$ $p = 292.6 \pm 0.2$ Also shown are the current and maximum theoretical value of Earth's eccentricity. The minimum value is 0.

marginally colder than expected where the ice albedo feedback is starting to become stronger. Both the first and second zones are described by $\langle T \rangle = p_1 a^{q_1} = 293a^{-0.505}$ which is very close to the expected $a^{-0.5}$ powerlaw seen in eq. (19). The value of p_1 is 38 K higher than the proportionality constant above, most likely due to the additional greenhouse effect present in the 1D model.

The third zone with $a > 1$ au is a sudden departure from this expected powerlaw to $\langle T \rangle = p_2 a^{q_2} = 210.2a^{-0.378}$ due to ice-albedo feedback. As the planet cools, more ice forms with a higher albedo. This higher albedo means more light is reflected, thus the planet absorbs less heat, so cools more. At 1 au the planet is on a tipping point in terms of this feedback loop. This, along with the following analysis of eccentricity and obliquity, help show why the Earth has had many ice ages in the past [5].

Alternatively, a can be fixed at 1 au and the eccentricity can be varied from a perfect circle, $e = 0$, to a very eccentric ellipse, $e = 0.9$.

$$T \propto (1-e^2)^{-1/8} \tag{20}$$

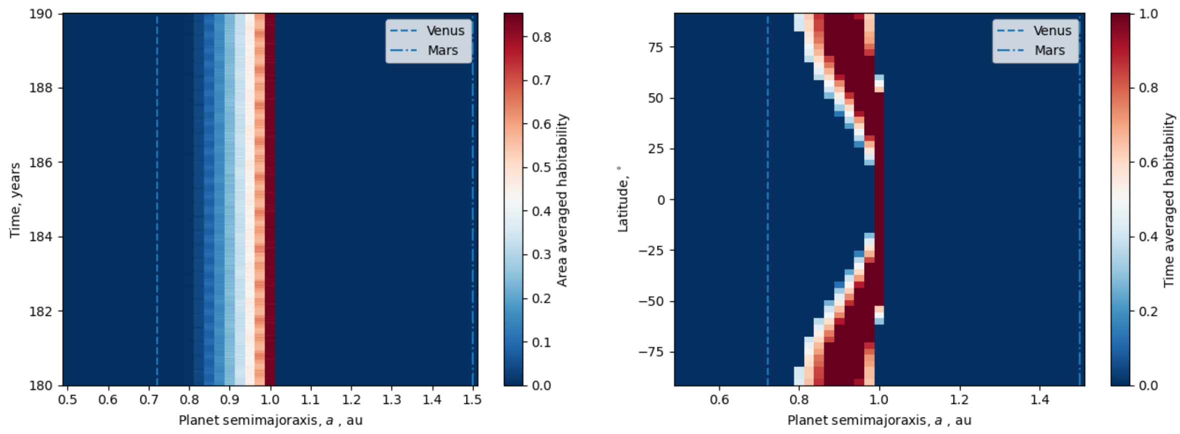

FIG. 3: Qualitative look at time and area habitability for variable semimajor axis

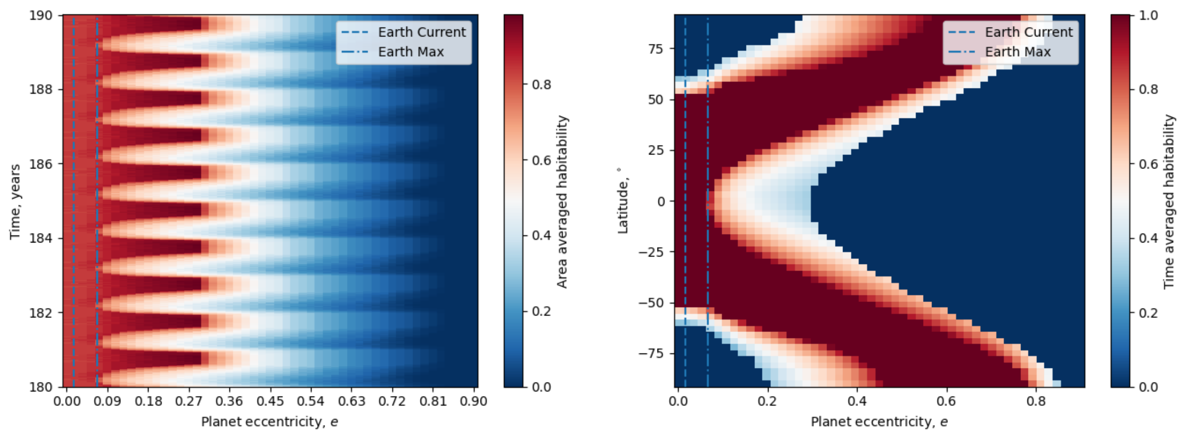

FIG. 4: Qualitative look at time and area habitability for variable eccentricity

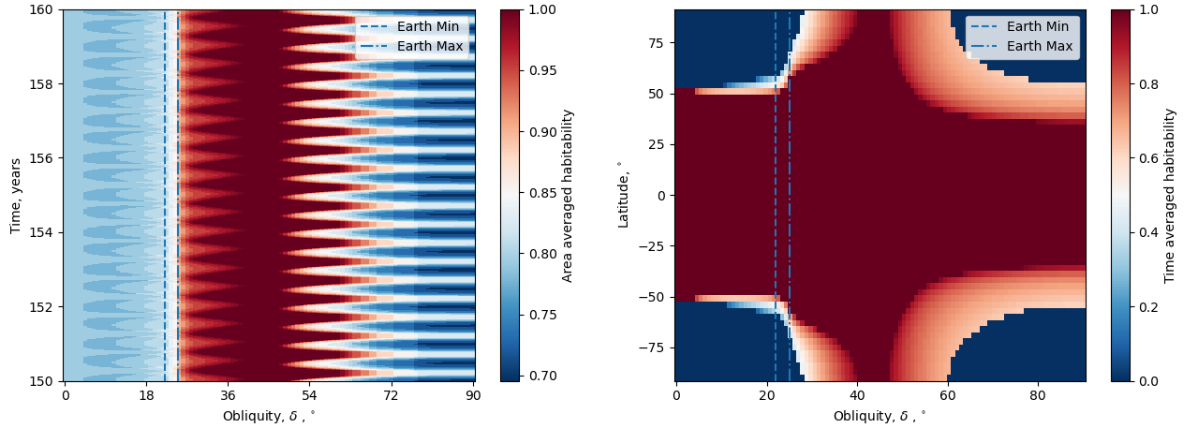

FIG. 5: Qualitative look at time and area habitability for variable eccentricity

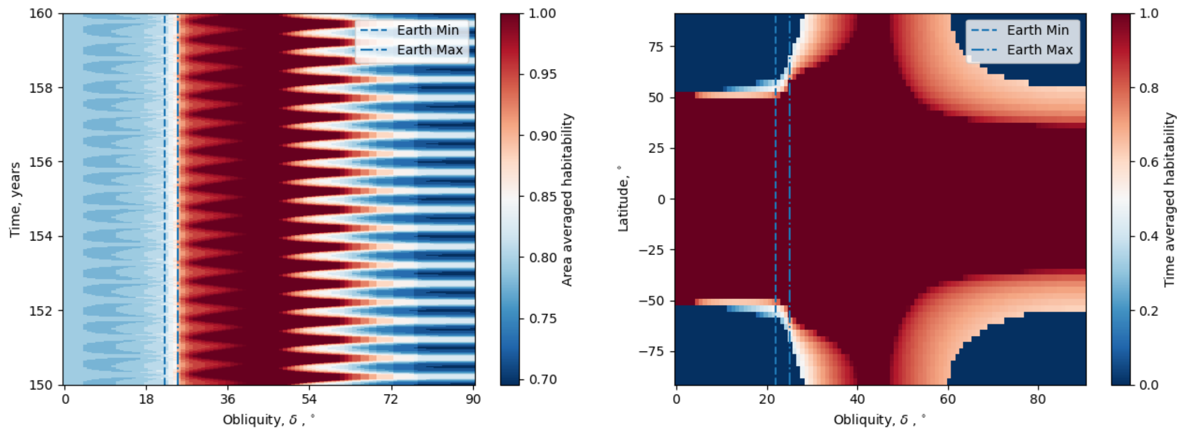

FIG. 6: Qualitative look at time and area habitability for variable eccentricity

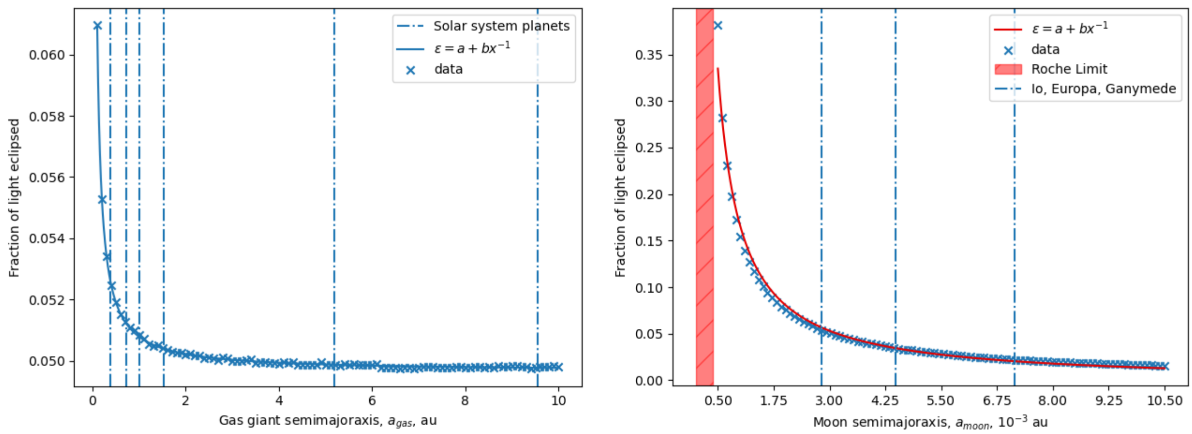

FIG. 7: Quantitative look at eclipsing fraction wrt semimajor axis of moon and gas giant

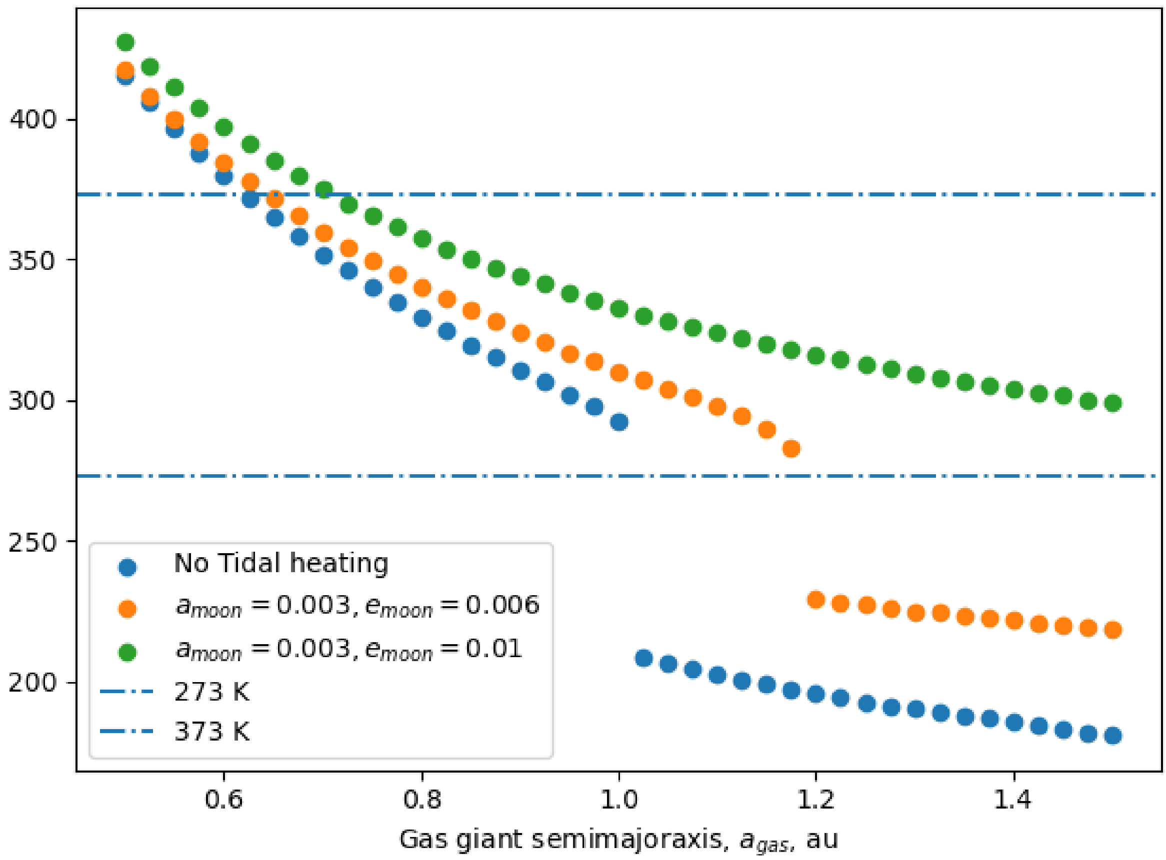

FIG. 8: Qualitative look at how tidal heating effects semimajoraxis temperature curve

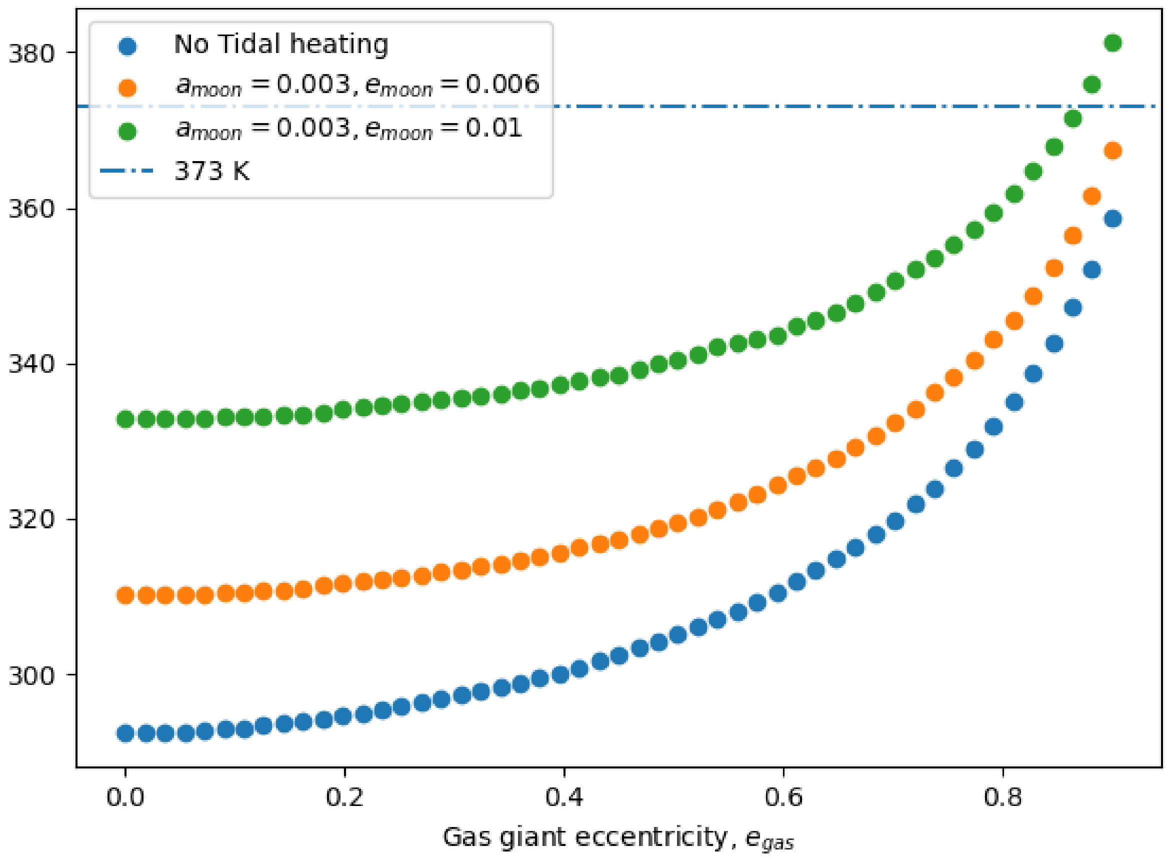

FIG. 9: Qualitative look at how tidal heating effects semimajoraxis temperature curve

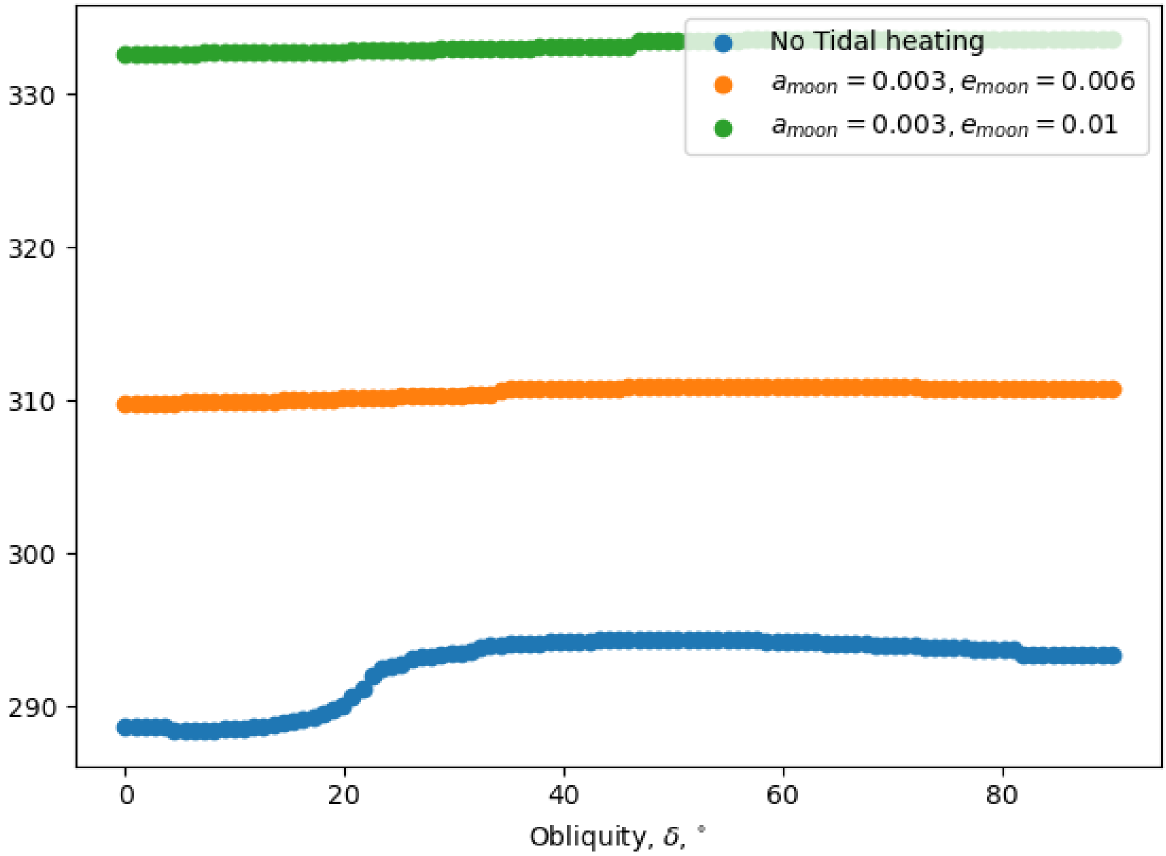

FIG. 10: Qualitative look at how tidal heating effects semimajoraxis temperature curve

B. Semimajor axis

C. Eccentricity

D. Obliquity

E. Ocean fraction

5. EXOMOONS

A. Eclipsing

B. Tidal heating

C. Developing and investigating a depth model

6. DISCUSSION

7. CONCLUSION

REFERENCES

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[2] North and Coakley, "Differences between seasonal and mean annual energy balance model calculations of climate and climate sensitivity," J. Atmos. Sci., vol. 36, no. 7, pp. 1189–1204, 1979.

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Appendix A: Defining Equilibrium Temperature and Averages used

Appendix B: Numerical stability of the 1D EBCM

Appendix C: Tidal heating equations and method

SCIENTIFIC SUMMARY FOR A GENERAL AUDIENCE

Many interesting solar systems have been reported in the news, such as the Trappist-1 system which is filled with Earth-like planets. Simulations and models such as those in this paper are used to determine if a planet could be habitable. A habitable zone can be made by varying the parameters of the model to see where the model is habitable, partially habitable, or uninhabitable.

The main model in this paper takes a planet and divides it into a number of latitude bands which can have energy flow between them. Certain parameters, such as how the planet orbits around its star and the angle the planet is tilted at, are varied to build this habitable zone. A result of this paper is if the Earth orbited slightly further away from the Sun then it is likely that it would fall into an ice age similar to what the Earth has experienced in the past. Another result found is that the tilt of the planet can affect how hot or cold it is, and indicates that the current tilt of the Earth gives a cold planet.

Another aspect of this paper's exoplanet research is exomoons orbiting a gas giant such as Jupiter. In certain configurations an exomoon can be heated not only from the host star, but also due to a process called tidal heating. Tidal heating is similar to stretching an elastic band. Stretching and relaxing an elastic band many times can cause the band to warm up. The moon of a gas planet is stretched slightly by unequal forces of gravity as one part of the moon is further away than the other. If the moon's orbit is not circular then the moon is stretched and relaxed, thus heats up in a similar way to the elastic band. Adding tidal heating to the model allows for investigations into how tidal heating can move, or change the shape of, the habitable zone.