Major Project 1 …

- **Look at problem of transferring satellite to MEO (GPS) from Initial LEO Orbit**
  - Code Continuous Thrust Example

- $a_{LEO} = 8530$km, $a_{MEO} = 13{,}200$ km

• You are going to compare (non-impulsive) low thrust, high I_{sp} transfer to high thrust, Low I_{sp} Hohmann transfer .. Both impulse and non-impulsive calculations

•a) Low thrust (EP) transfer, Thrust F=10 N, I_{sp} = 2000 sec
- Low Thrust final kick motor, Thrust F=2000 N, I_{sp} = 270 sec
- Assume final kick is performed impulsively
- Calculate consumed mass for each system burn and total consumed mass
- Accumulated ΔV for each burn, total delta V

b) High Thrust (Hohmann) transfer, Thrust F=2000 N, I_{sp} = 270 sec
- DV maneuvers are performed impulsively
- Calculate consumed ΔV, mass for each burn and total for both burns
- Compare to low thrust maneuver

Part a)

• *Continuous Small Thrust Problem*

• For Part a)... assume final Orbit insertion *ΔV* is delivered impulsively with Apogee Kick Motor Isp = 270 sec ….. Ignore atmospheric drag

Terminate thrust when

$$R_{apogee} = a \cdot (1 + e)$$

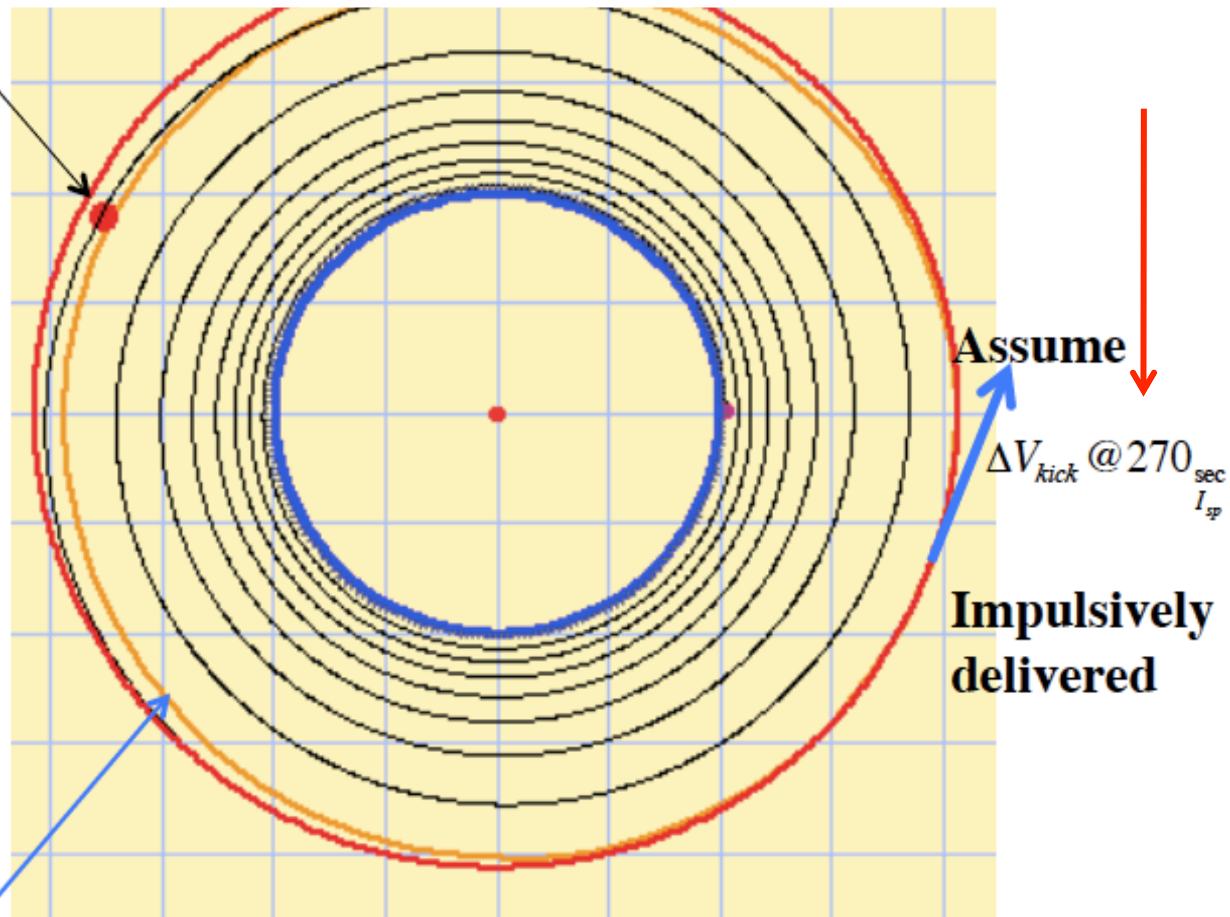
$$= 13{,}200\, km$$

Calculate:

1) Propellant mass req. $+\Delta V$
For continuous transfer

2) Propellant mass req. $+\Delta V$
For kick delta *V (impulsive)*
(orbit circularization)

3) Final mass = 1000 kg



• ***Calculate Mass Consumed Including Kick***

Part a)

• *Continuous Small Thrust Problem*

… compare continuous thrust propellant mass calculations against Hohmann transfer calculations .. Assuming impulsively delivered Delta V for each burn

Burn 1: Isp = 2000 sec

Burn 2: Isp = 270 sec

… what can you conclude about the accuracy of the rocket equations and the impulsive Delta V assumption when applied to a long duration non-impulsive burn?

Part a) Continuous Small Thrust

… Implement *both* Trapezoidal and Runge-Kutta Integration schemes

… Assume continuous thrust transfer to transfer orbit apogee using EP device, final orbit insertion using high thrust kick motor

… compare algorithm performance as
Time interval ΔT becomes progressively larger

… Is there a point where algorithm blows up?

MEO Orbit

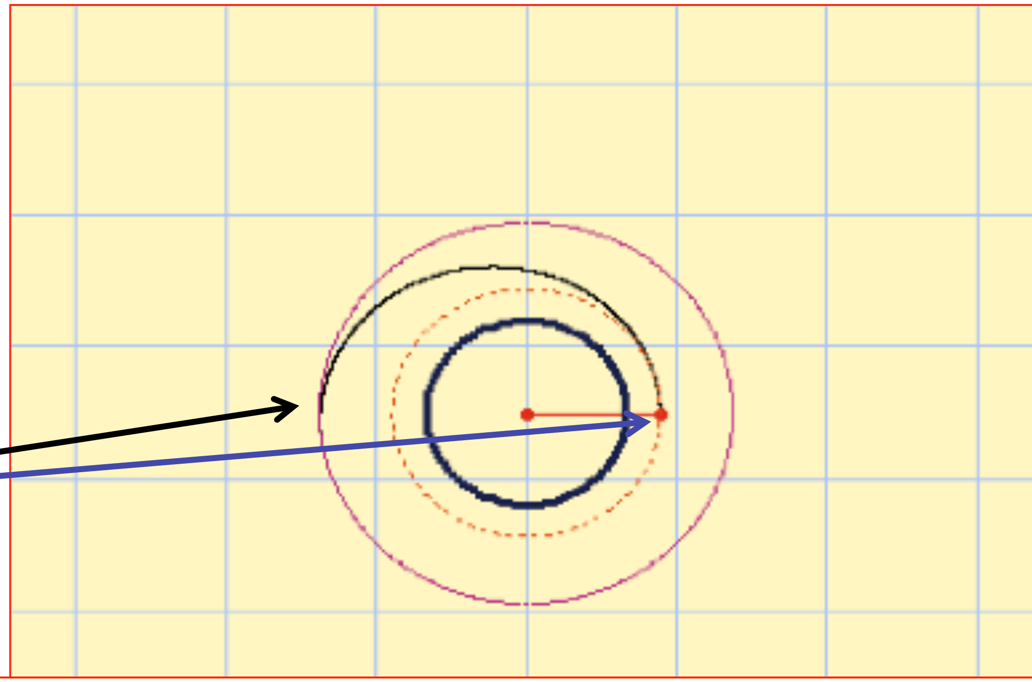
• Part b, Hohmann Transfer Calculations

Hohmann Transfer:

I_{sp}=270 sec

F_{thrust}=2000 Nt

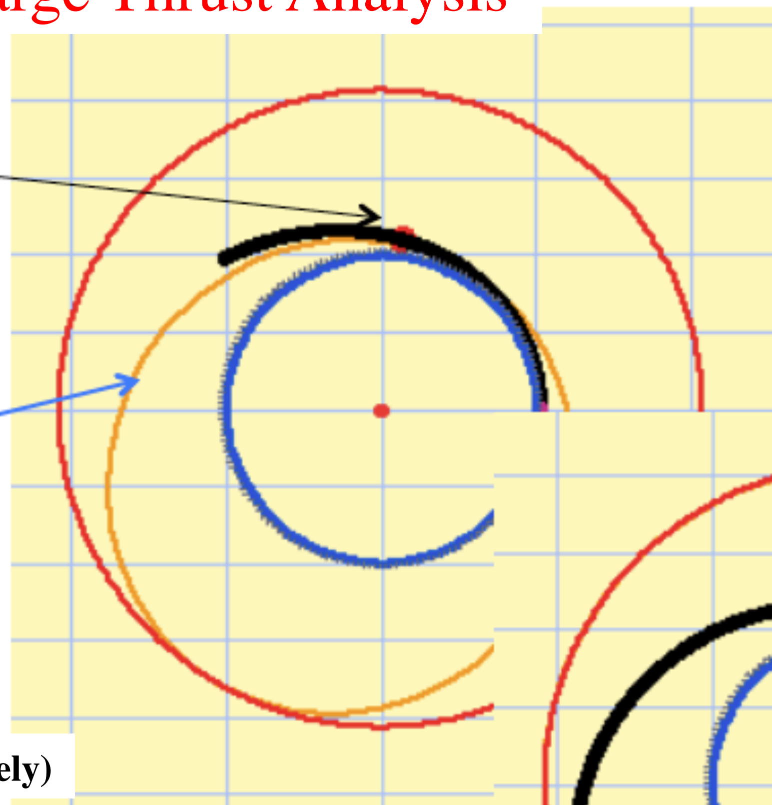
• ***Impulsive Burn Calculations***

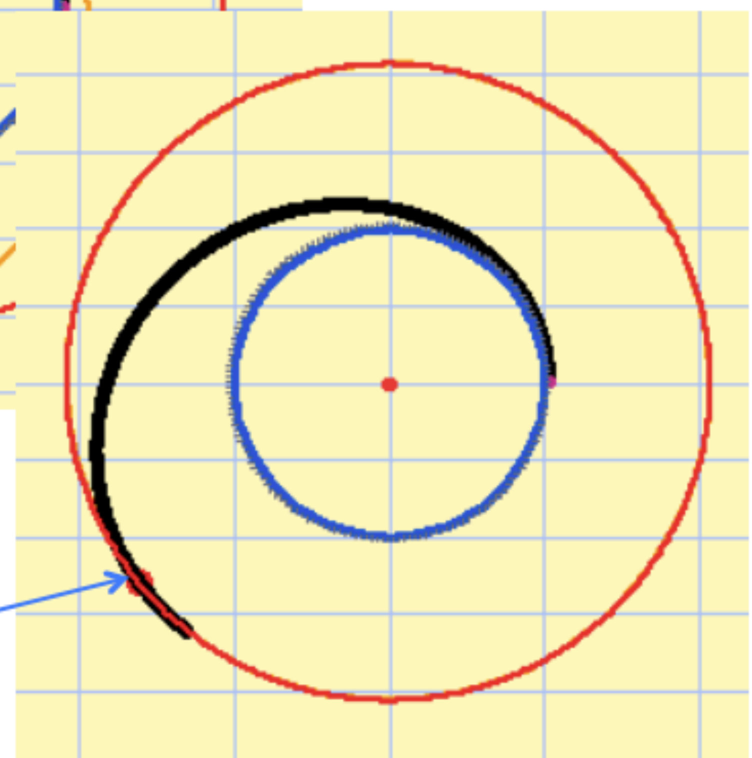


• Calculate *ΔV*, Mass Consumed Including Kick

• Compare to Continuous Low Thrust Transfer Results

Part c) Continuous Large Thrust Analysis





Terminate thrust when

$$R_{apogee} = a \cdot (1 + e)$$
$$= 13,200\,km$$

Orbit coast

Calculate:

1) Propellant mass req. ***+ΔV***
For continuous transfer

2) Propellant mass req. ***+ΔV***
For kick delta V **(Non-impulsively)**
(orbit circularization)

3) Final mass = 1000 kg

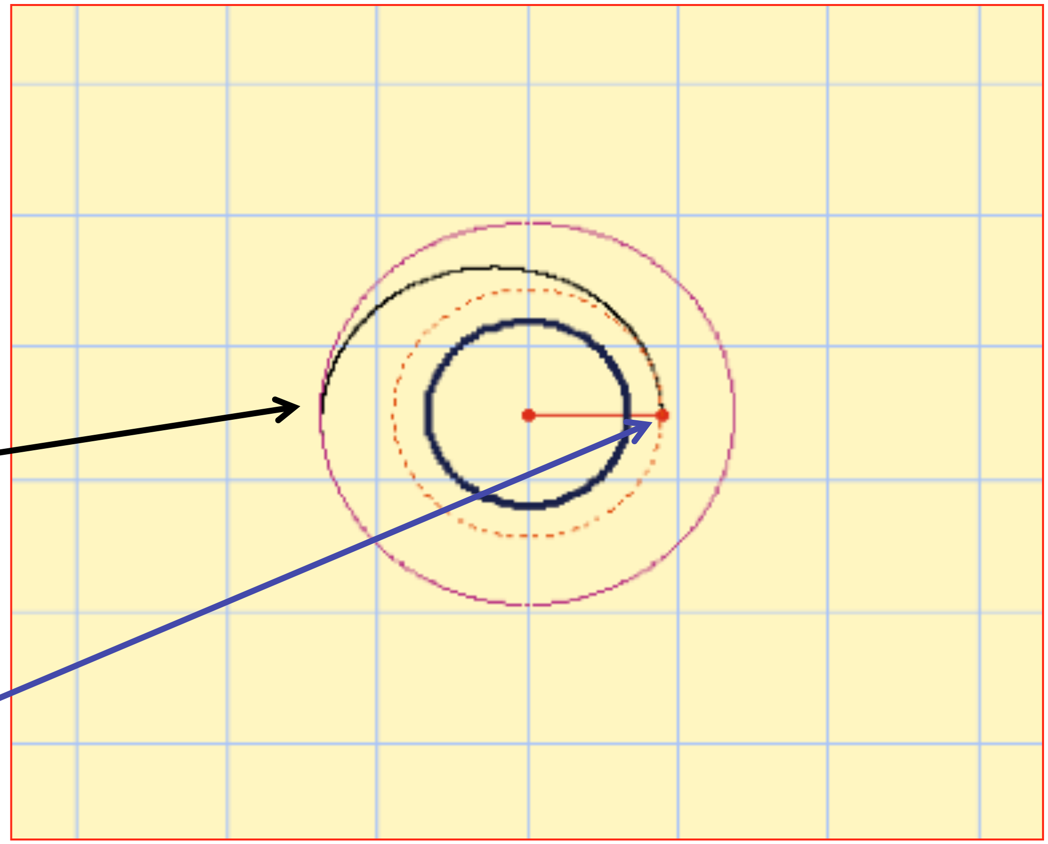
Final Delta V delivered Non-impulsively

• Part d) … Work continuous large
Thrust problem with ***non-impulsive burns*** at both ends

Hohmann Transfer:
I_{sp}=270 sec
F_{thrust}=2000 Nt

• ***Non-Impulsive Burn Calculations***

• ***Continuous Thrust Burn Calculations***



• ***Use Hohmann Transfer for Guidance on Burns times, positions***

• Continuous Large Thrust Problem

• Assume BOTH burns are performed non-impulsively
Terminate burn thrust when

$$R_{apogee} = a \cdot (1 + e)$$
$$= 13{,}200\, km$$

• You decide when and how long to initiate the second burn to circularize the orbit

• Assume for large thrust …. 2000 Nt thrust (both burns) … Isp = 270 sec

• Calculate required propellant mass for Burn1, Burn2 (and Total)

• Use integrator of your choice … calculate actual delivered Delta V Based on consumed mass … using rocket equation

• Continuous Large Thrust Problem

… compare Hohmann Transfer for 2000 Nt Rocket (assuming impulsive thrust) Versus 2000 Nt rocket with Non Impulsive Thrust …. Also compare consumed masses to High I_{sp} Continuous Thrust transfer

… what can you conclude about the accuracy of the rocket equation and the impulsive Delta V assumption when applied to a short duration non-impulsive burn?

… what can you conclude about the effect of I_{sp} on required propellant mass?

Position within initial orbit:

$$\begin{bmatrix} r \\ \nu \end{bmatrix}_0 = \begin{bmatrix} \dfrac{a_0\left(1-e_0^{\ 2}\right)}{1+e_0\cos\left(\nu_0\right)} \\ \nu_0 \end{bmatrix} \rightarrow \begin{bmatrix} \text{circular orbit} \rightarrow e_0 = 0 \\ \text{can assume} \rightarrow \nu_0 = 0 \rightarrow a_0 = r_0 \end{bmatrix}$$

Angular velocity within initial orbit:

$$\omega_0 = \frac{\sqrt{\mu}\left[1+e_0\cos\left(\nu_0\right)\right]^2}{\left[a_0\left(1-e_0^{\ 2}\right)\right]^{3/2}} \rightarrow \begin{bmatrix} \text{circular orbit} \rightarrow e_0 = 0 \\ \text{can assume} \rightarrow \nu_0 = 0, a_0 = r_0 \end{bmatrix}$$

$$\omega_0 = \frac{\sqrt{\mu}\left[1+e_0\cos\left(\nu_0\right)\right]^2}{\left[a_0\left(1-e_0^{\ 2}\right)\right]^{3/2}} = \frac{1}{r_0}\sqrt{\frac{\mu}{r_0}}$$

Linear Velocity within initial orbit:

$$\begin{bmatrix} V_r \\ V_\nu \end{bmatrix}_0 = r_0\omega_0 \begin{bmatrix} \dfrac{e_0 \sin\left[\nu_o\right]}{\left[1+e_0\cos\left(\nu_0\right)\right]} \\ 1 \end{bmatrix} \rightarrow \begin{bmatrix} \text{circular orbit} \rightarrow e_0 = 0 \\ \text{can assume} \rightarrow \nu_0 = 0, a_0 = r_0 \end{bmatrix}$$

$$\begin{bmatrix} V_r \\ V_\nu \end{bmatrix}_0 = \begin{bmatrix} 0 \\ r_0\omega_0 \end{bmatrix} = \begin{bmatrix} 0 \\ \sqrt{\dfrac{\mu}{r_0}} \end{bmatrix}$$

Instantaneous $_{\text{(no-nonconservative foreces acting)}}$ Keplerian orbit $\rightarrow$ $given : \begin{bmatrix} V_r \\ V_\nu \end{bmatrix}, \begin{bmatrix} r \\ \nu \end{bmatrix}$

$$a = \frac{\mu}{\left[\frac{2\mu}{r} - \left[V_r^{\ 2} + V_\nu^{\ 2}\right]\right]}$$

$$e = \frac{r}{\mu}\sqrt{\left(V_\nu^{\ 2} - \frac{\mu}{r}\right)^2 + \left(V_r V_\nu\right)^2}$$

$$r_{perigee} = a(1 - e)$$

$$r_{apogee} = a(1 + e)$$