Homework 2.1

• We are going to examine the effects of grain fracture on the Burn profiles for our familiar AMW L777 Rocket

• Let's develop a simple "fracture model" for 0, 1, 2, 3, and 4 "whole length" grain fractures .. Use Integrator of your choice to integrate the chamber pressure equation .. Assume isentropic nozzle

$$\frac{\partial P_0}{\partial t} = \frac{A_{burn} a P_o^{\,n}}{V_c}\left[\rho_p R_g T_0 - P_0\right] - P_0\left[\frac{A^*}{V_c}\sqrt{\gamma R_g T_0\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\right]$$

$$\dot{r} = a \cdot P_0^{\,n} \longrightarrow \dot{m}_{propellant} = \rho_p \cdot A_{burn} \cdot \dot{r}$$

Homework 2.1 (2)

Cylindrical port

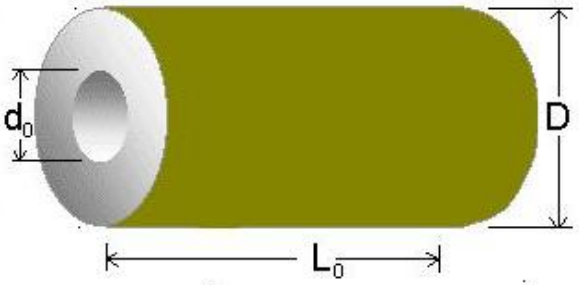
Fuel Grain Geometry

$L_0 = 35$ cm

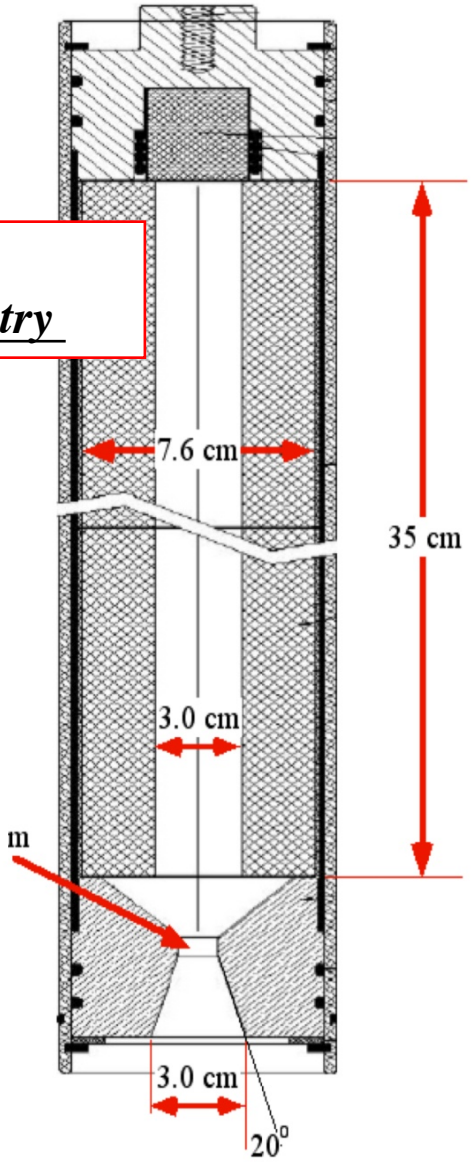
$D_0 = 7.6$ cm

$D_0 = 3$ cm

$\rho_{propellant} = 1260\ kg/M^3$



Animal Works™, L700 Motor Geometry



Nozzle Geometry

$A^* = 1.887\ cm^2$

$A_{exit}/A^* = 3.746$

$\theta_{exit} = 20$ deg.

Assume ends are burn inhibited, single segment grain

Homework 2.1 (3)

Combustion Gas Properties

$\gamma = 1.18$

$M_W = 23$ *kg/kg-mol*

$T_0 = 2900$ *K*

Burn Parameters

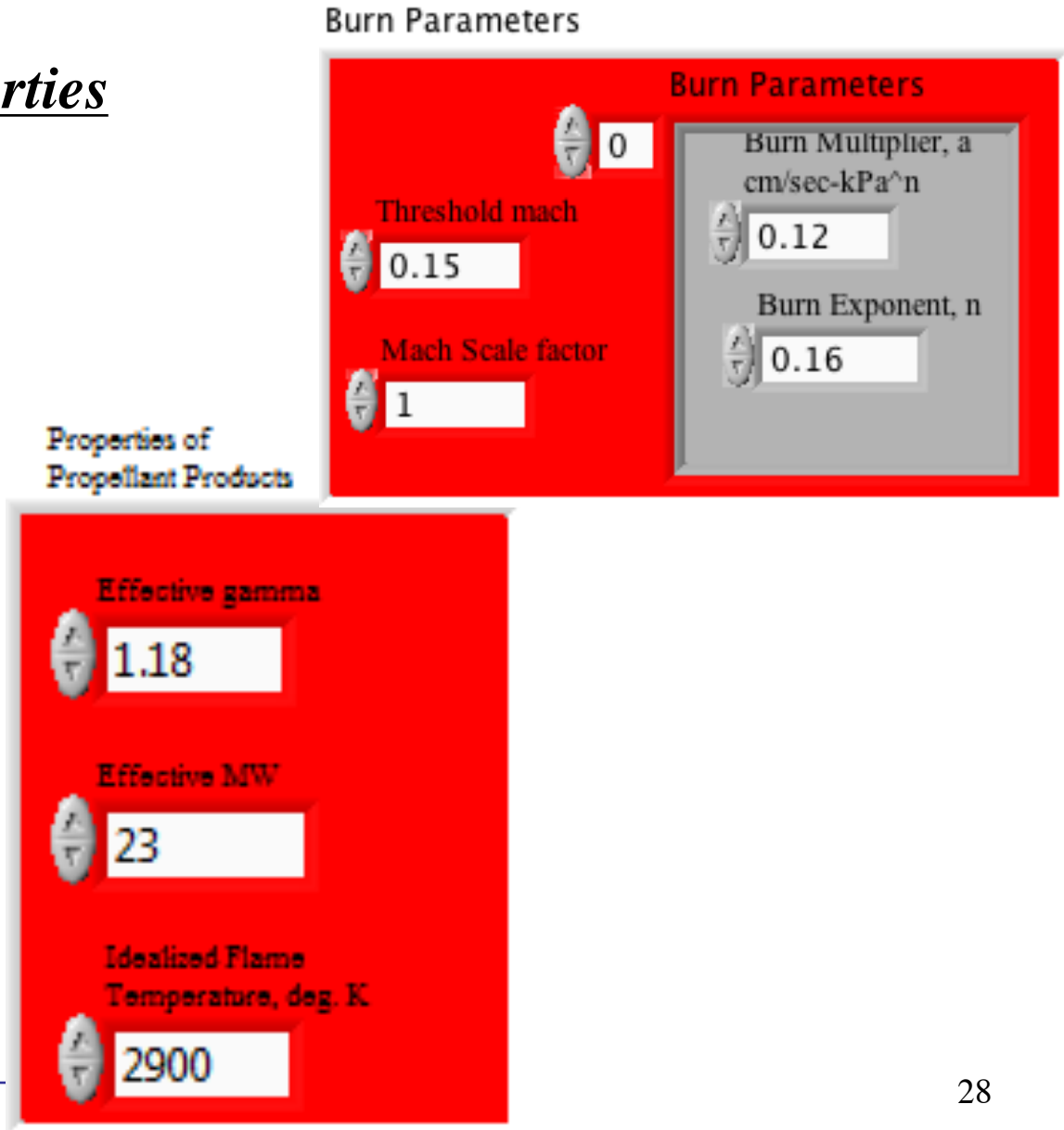
$a = 0.12$ *cm/(sec-kPa ^{n})*

$n = 0.16$

$M^{crit} = 0.15$

$k = 1.0$

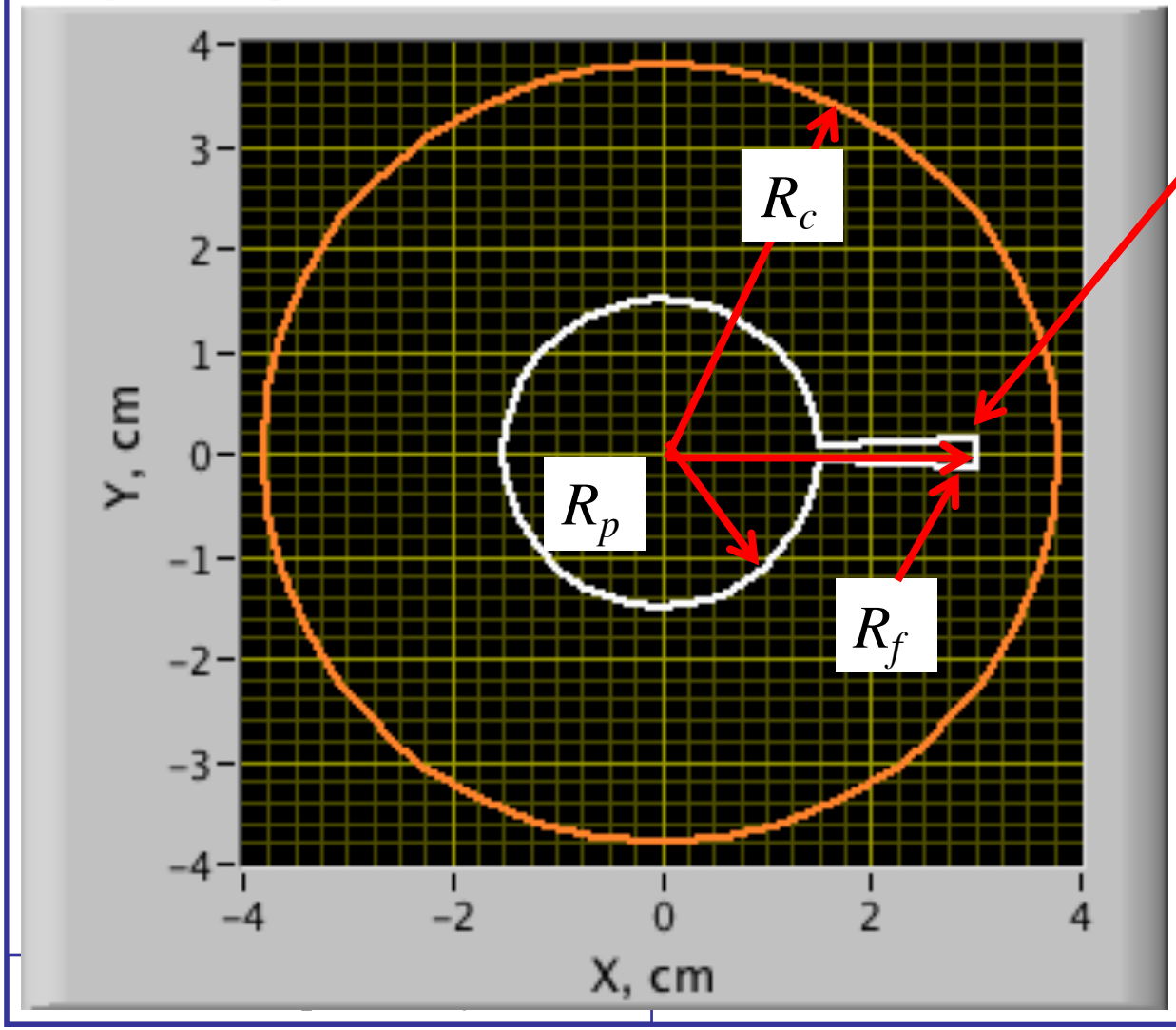
(cylindrical port only)



Homework 2.1 (4)

Simple. Linear Regression, Grain Fracture Model

Single Crack port cross section



Grain Fracture

R_p (Port radius)

R_f (Fracture radius)

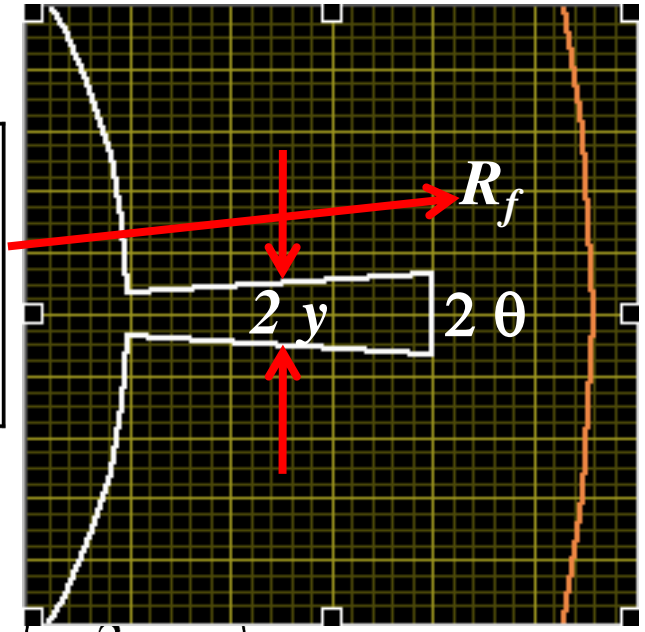
R_c (Motor case radius)

y (fracture half-width at midpoint)

Homework 2.1 (5)

- **Fracture Dimensions**

$$\left.\begin{aligned} R_f &= R_{f_0} + s \\ R_p &= R_{p_0} + s \\ y &= y_0 + s \end{aligned}\right| \rightarrow \boxed{\begin{aligned} s &= \int_r \dot{r} \cdot dt \\ \text{"0"} &= \textit{Initial values} \end{aligned}}$$



- **Fracture angular half width**

$$\theta = \tan^{-1}\left(\frac{y}{\frac{R_f - R_p}{2} + R_p}\right) = \tan^{-1}\left(\frac{y}{\frac{R_f + R_p}{2}}\right) = \tan^{-1}\left(\frac{2 \cdot y}{R_f + R_p}\right)$$

- **Port Cross Section Area and Internal Volume**

$$A_c = \pi \cdot R_p^{\,2} + \theta \cdot \left(R_f^{\,2} - R_p^{\,2}\right)$$

$$V_{port} = A_c \cdot L_{port}$$

Homework 2.1 (6)

- **Port Total Perimeter and Surface Burn Area**

$$P_{port} = 2 \cdot (\pi - \theta) \cdot R_p + 2 \cdot \left[\left(R_c - R_p \right) \right] + 2 \cdot \theta \cdot R_c =$$

$$2 \cdot \pi \cdot R_p + 2 \cdot \left[\left(R_c - R_p \right) \right] + 2 \cdot \theta \cdot \left(R_c - R_p \right)$$

$$A_{burn} = P_{port} \cdot L_{port}$$

- **Motor case, and Fuel Grain Cross section area, volume**

$$A_{case} = 2 \cdot \pi \cdot R_g^{\,2}$$

$$A_{fuel} = A_{case} - A_c$$

$$V_{case} = 2 \cdot \pi \cdot R_g^{\,2} \cdot L_{port}$$

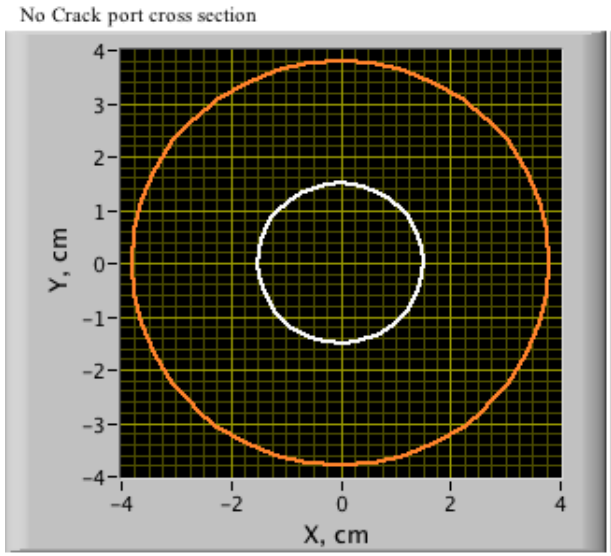
$$V_{fuel} = A_{fuel} \cdot L_{port}$$

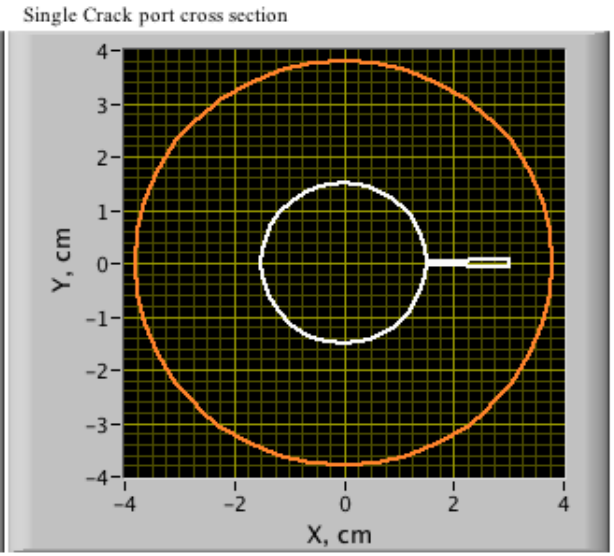
- **Total Propellant Mass**

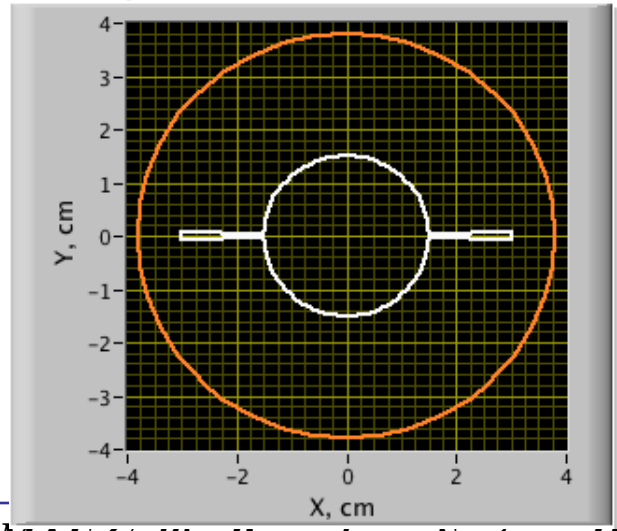
$$M_{propellant} = \rho_{propellant} \cdot A_{propellant} \cdot L_{port}$$

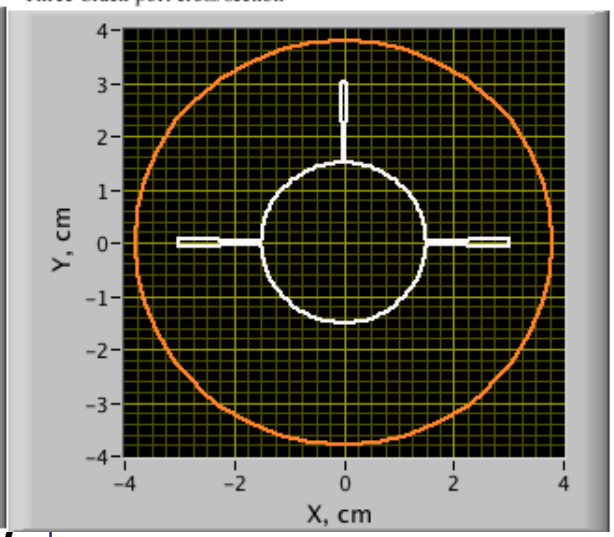
Homework 2.1 (7)

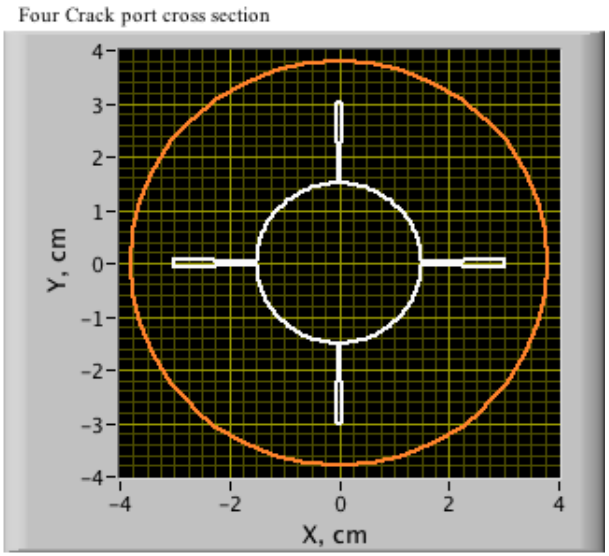
- Allow for 0, 1, 2, 3, or 4 symmetrical grain fractures











$y_0 = 0.4\ mm$

$R_{f0} = 3\ cm$

Homework 2.1 (8)

Calculate, plot, and compare for 0, 1, 2, 3, 4 grain fracture models:

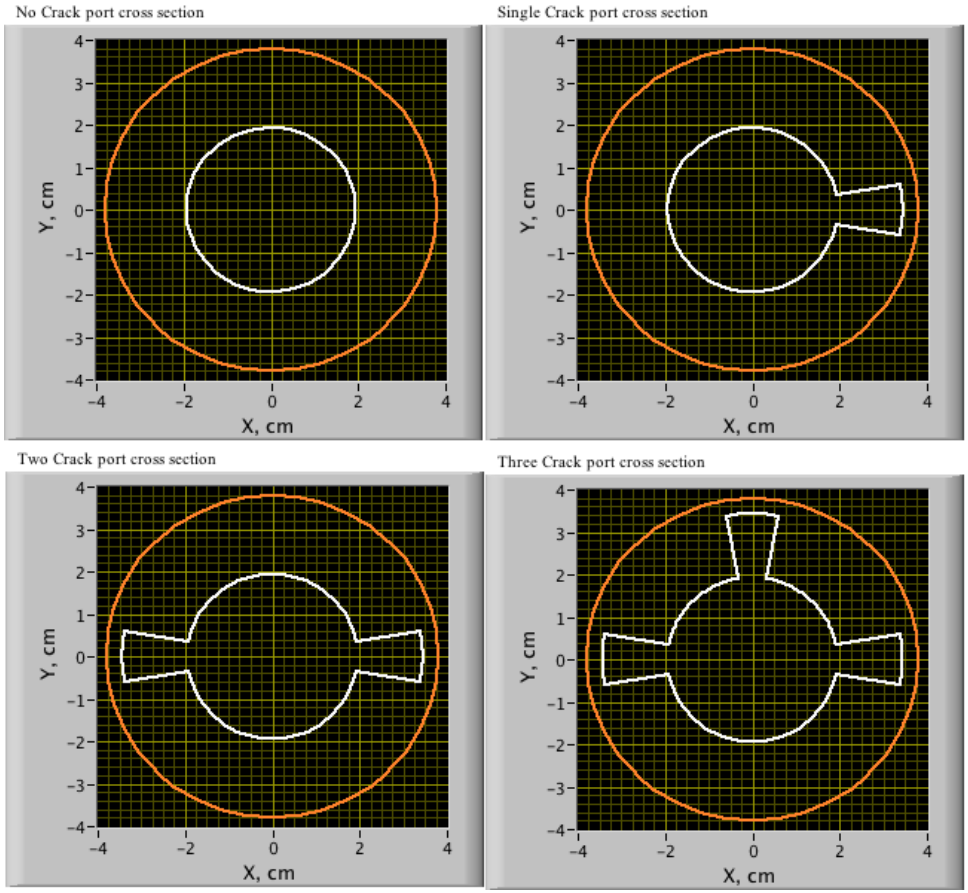
- *Chamber pressure profile*
- *Regression rate profile*
- *Massflow rate profile*
- *Thrust profile*
- *Total Impulse profile*
- *Ratio of Surface Burn Area to Port Volume*
- *Calculate Effective Mean Specific Impulse*

Allow:

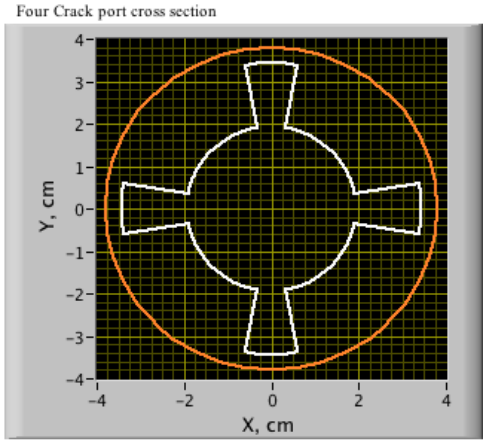
- *St. Robert's Parameter Input*
- *Variable Step Size*
- *Variable Thermodynamic Properties (as inputs to the problem)*
- *Erosive burn model for cylindrical port*

Homework 2.1 (9)

Plot grain cross section profiles at 0, 1, 2, 4, 6, and 8 seconds. Note Burn times when burn breaches outer motor case wall



Grain cross section after 1 sec burn



- Turn In Report describing your results