Project 2

Build Unsteady Model of "Pike" .. Use Integrator of your choice

Calculate:

Chamber pressure profile
Regression rate profile
Massflow rate (compare to choking massflow)
Mass depletion vs time
plot Thrust profile
plot Total Impulse profile
Effective Mean Specific Impulse

Allow:

St. Robert's Parameter Input
Variable Step Size
Variable Thermodynamic Properties ***(as inputs to the problem)***
Erosive burn model for cylindrical port (Not Bates grain)

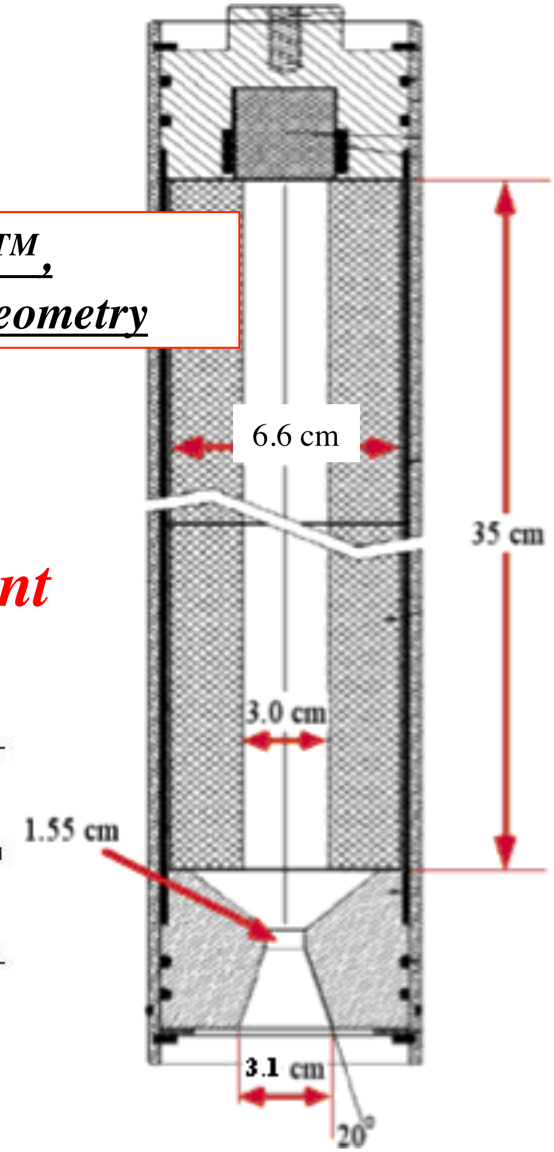
Project 2 (2)

Part 1, cylindrical port

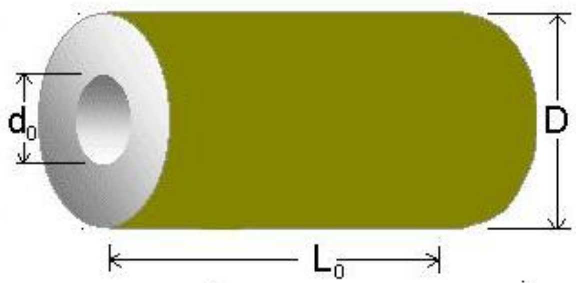
Fuel Grain Geometry

$L_0 = 35$ cm
$D_0 = 6.6$ cm
$D_0 = 3$ cm
$\rho_{propellant} = 1260$ kg/M^3

Animal Works™, L700 Motor Geometry



Single propellant segment



Nozzle Geometry

$A^* = 1.887$ cm^2
$A_{exit}/A^* = 4.0$
$\theta_{exit} = 20$ deg.

Assume ends are burn inhibited

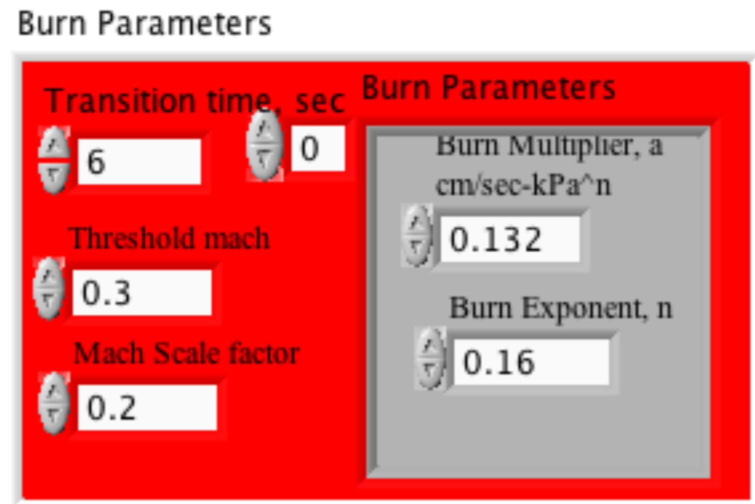
Project 2 (3)

Combustion Gas Properties

$\gamma = 1.18$

$M_W = 23$ kg/kg-mol

$T_0 = 2900$ K



Burn Parameters

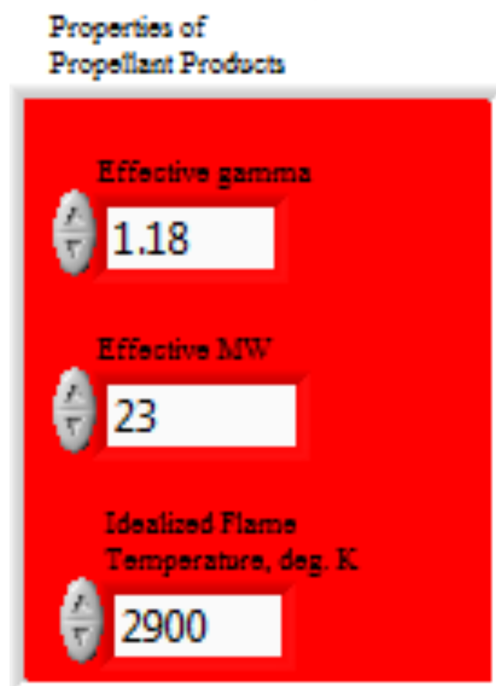
$a = 0.132$ cm/(sec-kPa ^{n})

$n = 0.16$

$M^{crit} = 0.3$

$k = 0.2$

(cylindrical port only)



Utah State University

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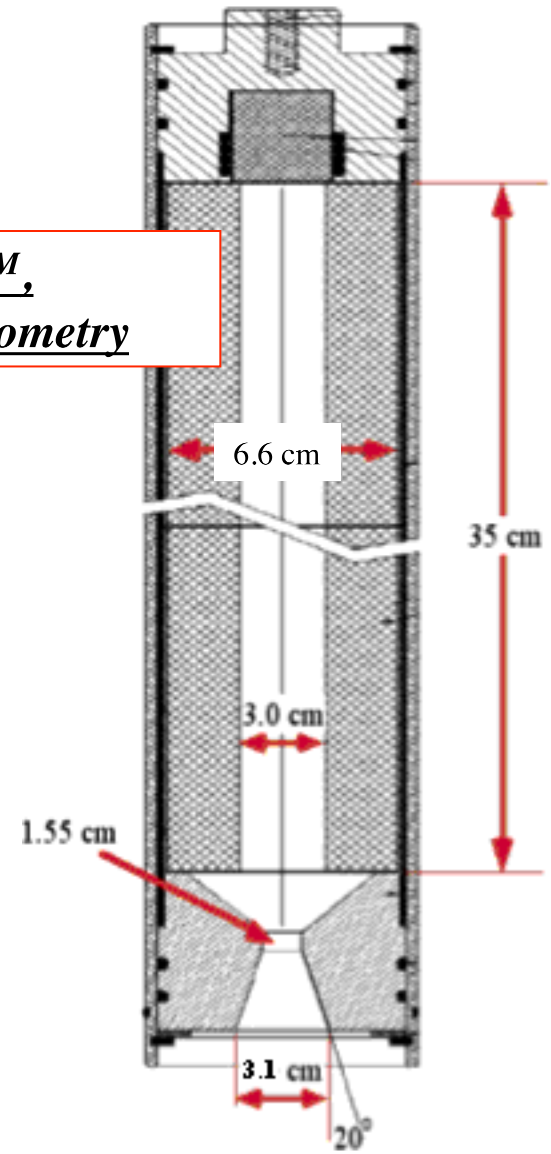
Project 2 (4)

Part 1, cylindrical port

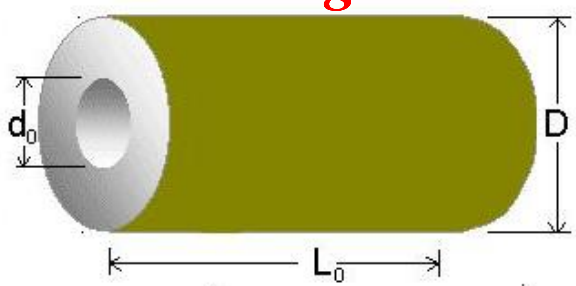
Fuel Grain Geometry

$L_0 = 35$ cm
$D_0 = 6.6$ cm
$D_0 = 3$ cm
$\rho_{propellant} = 1260$ kg/M^3

Animal Works™, L700 Motor Geometry



Repeat results Using bates grain With 3 segments



Nozzle Geometry

$A^* = 1.887$ cm ^{2}
$A_{exit}/A^* = 4.0$
$\theta_{exit} = 20$ deg.

Assume ends are not! burn inhibited

Project 2 (5)

Combustion Gas Properties

γ = 1.18

***M_W = 23** kg/kg-mol*

T_0= 2900 K

Burn Parameters

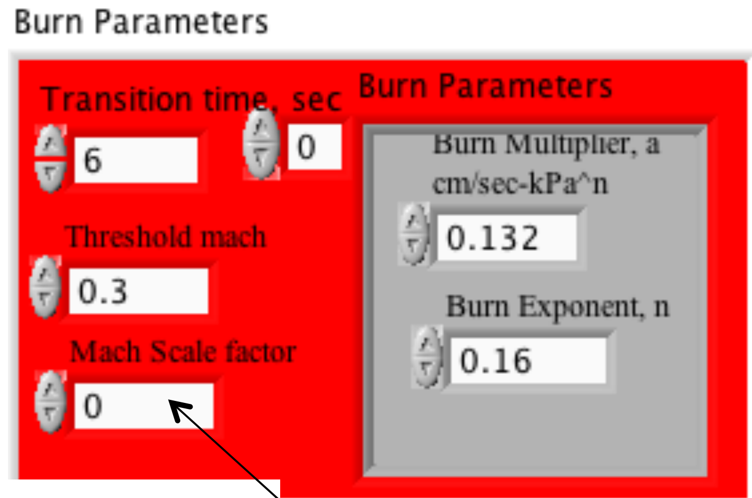
***a= 0.132** cm/(sec-kPa ^{n})*

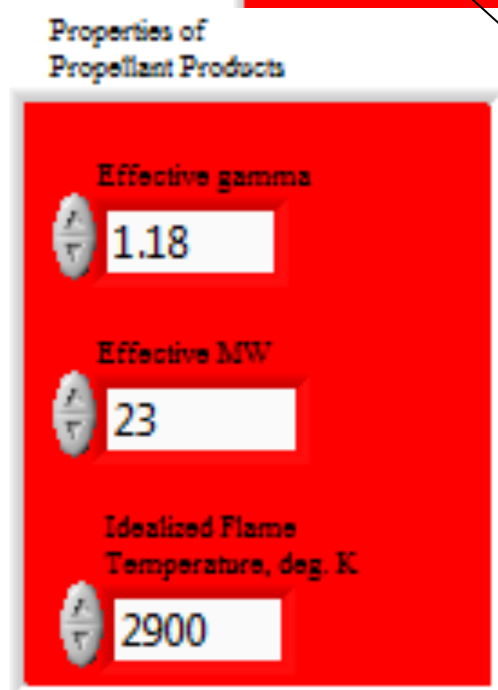
n=0.16

M^{crit} = 0.3

k = 0.0

(Bates grain only)





Set to Zero for Bates grain

Assume no erosive' burning

Project 2 (6)

Examine sensitivity of calculations to burn rate parameters, Critical Mach number (for erosion) … cylindrical port Only, Assume bates grain does not burn erosively

What is the effect of Flame temperature (T_0)

Plot Regression rate versus Chamber pressure

Prepare report stating your results and conclusions

State Equation Formulation of Problem

$$\begin{bmatrix} \dot{P}_0 \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \left(\frac{A_{burn} \cdot \dot{r}}{V_c}\right) \cdot \left(\rho_{propellant} \cdot R_g \cdot T_0 - P_0\right) - \left(\frac{A^*}{V_c}\right) \cdot P_0 \cdot \sqrt{\gamma \cdot R_g \cdot T_0 \cdot \left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{\gamma-1}\right)}} \\ a \cdot P_0^n \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ r \end{bmatrix}_{t=0} = \begin{bmatrix} P_{ambient} \\ \frac{d_0}{2} \end{bmatrix} \rightarrow \boxed{s(t) = \int_0^t \dot{r} \cdot dt \approx r(t) - \frac{d_0}{2}}$$

$$\begin{bmatrix} k \equiv Erosion\ Constant\ (GRAIN\ DEPENDENT) \\ M_{crit} \equiv Critical\ Port\ Mach\ Number \end{bmatrix}$$

$\rightarrow$ *State Equations for Erosive Burning :*

$$\begin{bmatrix} \dot{P}_0 \\ \dot{r} \end{bmatrix} = \begin{bmatrix} \left(\frac{A_{burn} \cdot \dot{r}}{V_c}\right) \cdot \left(\rho_{propellant} \cdot R_g \cdot T_0 - P_0\right) - \left(\frac{A^*}{V_c}\right) \cdot P_0 \cdot \sqrt{\gamma \cdot R_g \cdot T_0 \cdot \left(\frac{2}{\gamma+1}\right)^{\left(\frac{\gamma+1}{\gamma-1}\right)}} \\ \left(1 + k \cdot \frac{M_{port}}{M_{crit}}\right) \cdot a \cdot P_0^n / \left(1+k\right) \end{bmatrix}$$

$$\begin{bmatrix} P_0 \\ r \end{bmatrix}_{t=0} = \begin{bmatrix} P_{ambient} \\ \frac{d_0}{2} \end{bmatrix} \rightarrow \boxed{s(t) = \int_0^t \dot{r} \cdot dt \approx r(t) - \frac{d_0}{2}}$$

State Equation Formulation of Problem (2)

$\rightarrow$ *Cylindrical Port :*

$$\boxed{\begin{aligned} A_{burn} &= 2\cdot\pi\cdot r\cdot L_{port} \\ V_c &= \pi\cdot r^2\cdot L_{port}\end{aligned}} \rightarrow \begin{bmatrix} r \equiv Port\ Radius \\ L_{port} \equiv Port\ Length \end{bmatrix}$$

$\rightarrow$ *Bates Grain :*

$$A_{burn} = N\cdot\pi\cdot\left\{ \left[\frac{D_0^2-(d_0+2\cdot s)^2}{2}\right] + (L_0-2\cdot s)\cdot(d_0+2\cdot s) \right\}$$

$$V_c = \frac{N\cdot\pi}{4}\cdot\left\{ (d_0+2\cdot s)^2\cdot(L_0-2\cdot s)+D_0^2\cdot 2s \right\}$$

Do NOT! Use Erosive Burning for Bates Grain

State Equation Formulation of Problem (3)

Calculating Chamber Mach Number

Erosive Burning

$$\rightarrow \frac{V_c / L_{port}}{A^*} = \frac{1}{M_{port}} \cdot \left[\left(\frac{2}{\gamma + 1} \right) \cdot \left(1 + \left(\frac{\gamma - 1}{2} \right) \cdot M_{port}^2 \right) \right]^{\left(\frac{\gamma+1}{2\cdot(\gamma-1)} \right)} .$$

$$\ldots \textit{Subsonic Branch Solution}!$$

Do NOT! Use Erosive Burning for Bates Grain

Cylindrical Port: Decoupled Model

- Use Trapezoidal rule or Runge-Kutta to integrate

$$\frac{\partial P_0}{\partial t} = \frac{A_{burn} a P_o^{\,n}}{V_c}\left[\rho_p R_g T_0 - P_0\right] - P_0\left[\frac{A^*}{V_c}\sqrt{\gamma R_g T_0 \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\right]$$

- Recursive propagation of chamber diameter

$$R_{burn_{\,k+1}} = R_{i_{\,initial}} + \int_0^{(k+1)\Delta t} \dot{r}\,dt = R_{i_{\,initial}} + \int_0^{(k)\Delta t} \dot{r}\,dt + \int_{(k)\Delta t}^{(k+1)\Delta t} \dot{r}\,dt \rightarrow$$

$$R_{burn_{\,k+1}} = R_{burn_{\,k}} + \int_{(k)\Delta t}^{(k+1)\Delta t} \dot{r}\,dt \approx R_{burn_{\,k}} + \dot{r}\,\Delta t = R_{burn_{\,k}} + a {P_o^{\,n}}_{k}\,\Delta t$$

Bates grain Port: Decoupled Model

- Use Trapezoidal rule or Runge-Kutta to integrate

$$\frac{\partial P_0}{\partial t} = \frac{A_{burn} a P_o^{\,n}}{V_c}\left[\rho_p R_g T_0 - P_0\right] - P_0\left[\frac{A^*}{V_c}\sqrt{\gamma R_g T_0\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{(\gamma-1)}}}\right]$$

$$\dot{r} = a \cdot P_o^{\,n}$$

$$s_{regression} = \int_t \dot{r} \cdot dt \quad \rightarrow$$

$$\left(A_{burn}\right)_{total} = N \cdot \pi \cdot \left[\frac{\left(D_0^{\,2} - \left(d_0 + 2 \cdot s\right)^2\right)}{2} + \left(L_0 - 2 \cdot s\right) \cdot \left(d_0 + 2 \cdot s\right)\right]$$

$$\left(V_{ol}\right)_{total} = \frac{N \cdot \pi}{4}\left[\left(d_0 + 2 \cdot s\right)^2 \cdot \left(L_0 - 2 \cdot s\right) + D_0^{\,2} \cdot \left(2 \cdot s\right)\right]$$