Volume and Surface Area

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The Function and Graph

$$f(x) = 5 + \frac{2}{3}x - \frac{1}{13}x^2 + \frac{1}{3000}x^3 + \frac{1}{8000}x^4 - \frac{1}{30000000}x^6 - \frac{1}{370000000}x^7 + \frac{1}{1200000000000}x^9$$

f(x) for $0 < x < 25$

Strategies

- Volume
  - Disk Method
- Surface Area
  - Integral for Arc Length
  - Integral for Surface Area using Arc Length
- We wrote a Java program which could brute force the surface area and volume to double check our calculations
- The program is open source and has been released on www.github.com/zachohara/pictographer

Surface Area Calculation

$$SA = 2\pi \int_0^{25} f(x)\sqrt{1+f'(x)^2}\,dx$$

$$SA = 2\pi \int_0^{25} \left[5+\frac{2}{3}x-\frac{1}{13}x^2+\frac{1}{3000}x^3+\frac{1}{8000}x^4-\frac{1}{30000000}x^6-\frac{1}{370000000}x^7+\frac{1}{1200000000000}x^9\right]\sqrt{1+\left[\frac{2}{3}-\frac{2}{13}x+\frac{1}{1000}x^2+\frac{1}{2000}x^3-\frac{1}{5000000}x^5-\frac{7}{370000000}x^6+\frac{3}{400000000000}x^8\right]^2}\,dx$$

$$SA = 888.046852 \ \ cm^2$$

Displacement (Outside) Volume

$$V = \pi \int_0^{25} f(x)^2 dx$$

$$V = \pi \int_0^{25} \left[5 + \frac{2}{3}x - \frac{1}{13}x^2 + \frac{1}{3000}x^3 + \frac{1}{8000}x^4 - \frac{1}{30000000}x^6 - \frac{1}{370000000}x^7 + \frac{1}{1200000000000}x^9 \right]^2 dx$$

$$V = 2393.54639 \ \ cm^3$$

Capacity (Inside) Volume

$$V = \pi \int_0^{25} (f(x) - .25)^2 dx$$

$$V = \pi \int_0^{25} \left[4.75 + \frac{2}{3}x - \frac{1}{13}x^2 + \frac{1}{3000}x^3 + \frac{1}{8000}x^4 - \frac{1}{30000000}x^6 - \frac{1}{370000000}x^7 + \frac{1}{1200000000000}x^9 \right]^2 dx$$

$$V = 2183.69903 \ cm^3$$