Master in Artificial Intelligence

POS tagging
POS Taggers

Introduction to Human Language Technologies 4. POS tagging





Outline

POS tagging
POS Taggers

- POS tagging
 - Goal and motivation
 - Part of Speech categories
- 2 POS Taggers
 - Stochastic taggers
 - Hidden Markov Model
 - Viterbi algorithm

Outline

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Goal and motivation
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Goal

POS tagging Goal and motivation

POS Taggers

- Morphological analysis provides lexical information related to forms (POS, num, gen, tense, . . .)
- Multiple analyses can result (POS tags from Penn Treebank tagset)

form	analyses	example of use
fish	NNS	'Cats eat fish'
	VBG	'I am fishing'
bass	NN	'I saw you play the bass'
	JJ	'Bass clarinets sound good'

■ Goal: disambiguate POS of word forms occurring in text

Motivation

Examples of applications of POS tagging:

 Syntactic parsing: words with the same POS tag play a similar syntactic role

Ex: a determiner followed by a common noun is a noun phrase

Machine translation

Ex: (POS tags from Penn Treebank tagset)

'El hombre	bajo	toca el	bajo	bajo	el puente'
POS	NN		NN	NN	
tagging	JJ		JJ	JJ	
	IN		IN	IN	
	VB		VB	VB	
possible	low		bass	under	
English	small			below	
words	short				
	poor				
'The	small	man plays the	bass	under	the bridge'

POS tagging
Goal and motivation
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POS tagging Part of Speech categories

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Open class vs. Closed class

- General classes:
 - Closed class: never invent new closed items (functional words)

Usual subclasses for indo-european languages:

prepositions, conjunctions, determiners, pronouns,
auxiliary verbs or particles (prepositions or adverbs in
phrasal verbs)

- Open class: new open items can be invented Usual subclasses for indo-european languages: nouns, non-auxiliary verbs, adjectives and adverbs
- Each language defines its particular set of subclasses
- Subclasses can be represented with a particular granularity by a set of categories

Ex: Brown corpus: annotated with 87 different POS tags
Ex: Penn Treebank corpus: with 48 different POS tags

POS tagging Part of Speech

POS Taggers

Penn Treebank tagset

POS tagging Part of Speech categories

POS Taggers

CC	Coordinating conjunction	PP	Possessive pronoun
CD	Cardinal number	RB	Adverb
DT	Determiner	RBR	Adverb, comparative
EX	Existential there	RBS	Adverb, superlative
FW	Foreign word	RP	Particle
IN	Preposition	SYM	Symbol
JJ	Adjective	TO	to
JJR	Adjective, comparative	UH	Interjection
JJS	Adjective, superlative	VB	Verb, base form
LS	List item marker	VBD	Verb, past tense
MD	Modal	VBG	Verb, gerund
NN	Noun, singular	VBN	Verb, past participle
NNP	Proper noun, singular	VBP	Verb, non-3rd ps. sing. present
NNS	Noun, plural	VBZ	Verb, 3rd ps. sing. present
NNPS	Proper noun, plural	WDT	wh-determiner
PDT	Predeterminer	WP	wh-pronoun
POS	Posessive ending	WP\$	Possessive wh-pronoun
PRP	Personal pronoun	WRB	wh-adverb

12 categories more related to punctuation marks

Ex: to/TO give/VB priority/NN to/IN teacher/NN pay/NN rises/NNS

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POS tagging methods

POS tagging
POS Taggers

Frequently used methods:

- Rule-based methods:
 - Rules built manually are not frequently used. High production cost
 - Rules learnt automatically from training corpus.
 - Ex: Brill's tagger.
- Stochastic methods:
 - Based on Hidden Markov Models learnt automatically from training corpus.

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POS Taggers Stochastic taggers

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Stochastic taggers

Goal: Assign the most likely POS-tag sequence to a word sequence.

$$W = w_1 \dots w_n$$
 (a word sequence)
 $T = t_1 \dots t_n$ (a POS-tag sequence)

Tagger result:
$$\hat{T} = \underset{T}{\operatorname{argmax}} P(T|W)$$

- I How is P(T|W) computed? Apply a Hidden Markov Model
- 2 How is \hat{T} found?

 Apply Viterbi algorithm

POS tagging

POS Taggers Stochastic taggers

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Preliminaries: Markov model

■ $X = (X_1, ..., X_T)$ sequence of random variables taking values in observed states $S = \{s_1, ..., s_N\}$

- Inference: Sequence probability P(X)?
- Markov Properties
 - Limited Horizon: $P(X_{t+1} = s_k \mid X_1, \dots, X_t) = P(X_{t+1} = s_k \mid X_t)$
 - Time Invariant (Stationary): $P(X_{t+1} = s_i \mid X_t = s_j) = P(X_2 = s_i \mid X_1 = s_j)$
- Transition matrix: $a_{ij} = P(X_{t+1} = s_j \mid X_t = s_i); \quad \forall i, j \ a_{ij} \ge 0; \quad \forall i \ \sum_{j=1}^N a_{ij} = 1$
- Initial probabilities (or extra state s_0): $\pi_i = P(X_1 = s_i); \quad \sum_{i=1}^N \pi_i = 1$

POS tagging

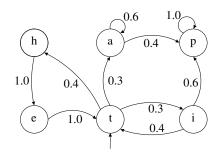
Preliminaries: Markov model

POS tagging

POS Taggers Hidden Markov Model

Sequence probability: (Bayesian rule+limited horizon)
$$P(X_1,...,X_T) = \\ = P(X_1)P(X_2 \mid X_1)P(X_3 \mid X_1X_2)\dots P(X_T \mid X_1...X_{T-1}) \\ = P(X_1)P(X_2 \mid X_1)P(X_3 \mid X_2)\dots P(X_T \mid X_{T-1}) \\ = \pi_{X_1} \prod_{t=2}^T a_{X_{t-1}X_t}$$

Example:



$$P(t, h, e, t, i, p, p) = 1 \cdot (0.4 \cdot 1 \cdot 1 \cdot 0.3 \cdot 0.6 \cdot 1) = 0.072$$

Hidden Markov model

■ $X = (X_1, ..., X_T)$ sequence of random variables taking values in unobserved [hidden] states $S = \{s_1, ..., s_N\}$ given a sequence of observations $O = (O_1, ..., O_T)$

- Inference: Probability of . . .
 - \blacksquare a process: P(O) ?
 - the state of a process at the end: $P(X_T \mid O)$?
 - the explanation of a process: $P(X_1, ..., X_T \mid O)$? POS tagging: X = POS tags; O = words
- Transition matrix:

$$a_{ij} = P(X_{t+1} = s_j \mid X_t = s_i); \quad \forall i, j \ a_{ij} \geqslant 0; \quad \forall i \ \sum_{j=1}^N a_{ij} = 1$$

Initial probabilities (or extra state s_0): $\pi_i = P(X_1 = s_i)$: $\sum_{i=1}^{N} \pi_i = 1$

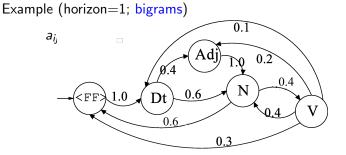
■ Emission Probability:

$$b_{ik} = P(O_t = k \mid X_t = s_i) \quad \forall i, k \ b_{ik} \geqslant 0; \quad \forall i \ \sum_{k=1}^N b_{ik} = 1$$

POS tagging
POS Taggers
Hidden Markov

Hidden Markov model

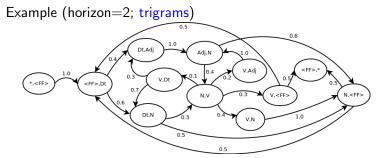
POS tagging



b_{ik}			this	cat	kid	eats	runs	fish	fresh	little	big
<ff></ff>	1.0										
Dt		0.6	0.4								
Ν				0.3	0.1		0.1	0.3	0.2		
V					0.1	0.5	0.3	0.1			
Adj					0.1				0.2	0.3	0.4

Hidden Markov model

POS tagging



b_{ik}		the	this	cat	kid	eats	runs	fish	fresh	little	big
?, <ff></ff>	1.0										
?,Dt		0.6	0.4								
?,N				0.3	0.1		0.1	0.3	0.2		
?,V					0.1	0.5	0.3	0.1			
?,Adj					0.1				0.2	0.3	0.4

Learning of parameters

POS tagging

- Parameters a_{ij} , b_{ik} and π_i can be estimated over a training corpus C
- Use smoothing techniques
- Use Baum-Welch algorithm
- learning of parameters will be studied in AHLT

Learning of parameters

Example: MLE estimator; assume u, v, w are different POS tags in the training corpus

bigram-based HMM

POS tagging POS Taggers

Hidden Markov

$$a(u, v) \approx P_{MLE}(v \mid u) = \frac{c(u, v)}{c(u)}$$
$$b(O_i, u) \approx P_{MLE}(O_i \mid u) = \frac{c(u, O_i)}{c(u)}$$
$$\pi(u) \approx P_{MLE}(u \mid *) = \frac{c(*, u)}{c(*)}$$

trigram-based HMM

$$a(uv, vw) \approx P_{MLE}(vw \mid uv) = \frac{c(u, v, w)}{c(u, v)}$$

$$b(O_i, uv) = b(O_i, v) \approx P_{MLE}(O_i \mid v) = \frac{c(v, O_i)}{c(v)}$$

$$\pi(*u) \approx P_{MLE}(*u \mid **) = \frac{c(*, *, u)}{c(**)} \quad \pi(uv) \approx P_{MLE}(uv \mid *u) = \frac{c(*, u, v)}{c(*u)}$$

$$b(O_i, u) pprox P_{MLE}(v \mid u) = rac{c(u)}{c(u)}$$
 $b(O_i, u) pprox P_{MLE}(O_i \mid u) = rac{c(u, O_i)}{c(u)}$
 $\pi(u) pprox P_{MLE}(u \mid *) = rac{c(*, u)}{c(*)}$

Given the following corpus,

```
horse/NN flies/NNS time/VBP morning/NN rays/NNS ./. eat/VB breakfast/NN at/IN morning/NN time/NN ./. take/VB time/NN with/IN arrow/NN projects/NNS ./. dinner/NN time/NN goes/VBZ before/IN sleep/NN ./. flies/NNS smell/VBP an/DT arrow/NN drink/NN ./. bees/NNS sting/VBP like/IN some/DT flies/NNS ./.
```

apply MLE to estimate the non-zero parameters for the POS-tags involved in the sentence using bigrams:

"time flies like horse flies ."

POS tagging

horse/NN flies/NNS time/VBP morning/NN rays/NNS ./. eat/VB breakfast/NN at/IN morning/NN time/NN ./. take/VB time/NN with/IN arrow/NN projects/NNS ./. dinner/NN time/NN flies/VBZ before/IN sleep/NN ./. flies/NNS smell/VBP an/DT arrow/NN drink/NN ./. bees/NNS sting/VBP like/IN some/DT flies/NNS ./.

"time flies like horse flies ."

POS tagging

POS Taggers Hidden Markov Model

c(u)	c(u,v) NN NNS VBP VBZ IN .
NN NNS VBP VBZ IN	* NN NNS VBP VBZ IN

POS tagging

POS Taggers Hidden Markov Model

- * horse/NN flies/NNS time/VBP morning/NN rays/NNS ./. * eat/VB breakfast/NN at/IN morning/NN time/NN ./.
- * take/VB time/NN with/IN arrow/NN projects/NNS ./.
- * dinner/NN time/NN flies/VBZ before/IN sleep/NN ./.
- * flies/NNS smell/VBP an/DT arrow/NN drink/NN ./.
- * bees/NNS sting/VBP like/IN some/DT flies/NNS ./.

Joint transition matrix A and initial probabilities π :

c(u)	6	_ ` ' /	NN NNS VBP VBZ IN	١.
NN NNS VBP VBZ IN		* NN NNS VBP VBZ IN		
	ı		•	

"time flies like horse flies ."

```
* horse/NN flies/NNS time/VBP morning/NN rays/NNS ./. eat/VB breakfast/NN at/IN morning/NN time/NN ./. take/VB time/NN with/IN arrow/NN projects/NNS ./. * dinner/NN time/NN flies/VBZ before/IN sleep/NN ./. flies/NNS smel/VBP an/DT arrow/NN drink/NN ./. bees/NNS sting/VBP like/IN some/DT flies/NNS ./.
```

"time flies like horse flies ."

POS tagging

```
Joint transition matrix A and initial probabilities \pi \! :
```

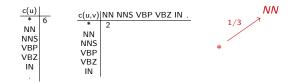
c(u)		c(u,v)	NN I	NNS	VBP	VBZ	ZIN .
* NN NNS VBP VBZ IN	6	* NN NNS VBP VBZ IN	2	1113	<u> </u>	V.D.2	

horse/NN flies/NNS time/VBP morning/NN rays/NNS ./. eat/VB breakfast/NN at/IN morning/NN time/NN ./. take/VB time/NN with/IN arrow/NN projects/NNS ./. dinner/NN time/NN flies/VBZ before/IN sleep/NN ./. flies/NNS smell/VBP an/DT arrow/NN drink/NN ./. bees/NNS sting/VBP like/IN some/DT flies/NNS ./.

"time flies like horse flies ."

POS tagging

POS Taggers Hidden Markov Model

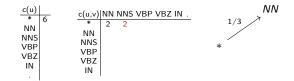


```
horse/NN flies/NNS time/VBP morning/NN rays/NNS ./.
eat/VB breakfast/NN at/IN morning/NN time/NN ./.
take/VB time/NN with/IN arrow/NN projects/NNS ./.
dinner/NN time/NN flies/VBZ before/IN sleep/NN ./.
#flies/NNS smell/VBP an/DT arrow/NN drink/NN ./.
bees/NNS sting/VBP like/IN some/DT flies/NNS ./.
```

"time flies like horse flies ."

POS tagging

POS Taggers Hidden Markov Model

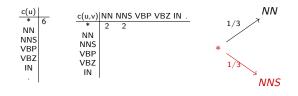


horse/NN flies/NNS time/VBP morning/NN rays/NNS ./. eat/VB breakfast/NN at/IN morning/NN time/NN ./. take/VB time/NN with/IN arrow/NN projects/NNS ./. dinner/NN time/NN flies/VBZ before/IN sleep/NN ./. flies/NNS smell/VBP an/DT arrow/NN drink/NN ./. bees/NNS sting/VBP like/IN some/DT flies/NNS ./.

"time flies like horse flies ."

POS tagging

POS Taggers Hidden Markov Model

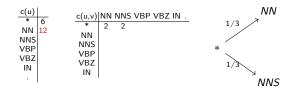


```
horse/NN flies/NNS time/VBP morning/NN rays/NNS ./. eat/VB breakfast/NN at/IN morning/NN time/NN ./. take/VB time/NN with/IN arrow/NN projects/NNS ./. dinner/NN time/NN flies/VBZ before/IN sleep/NN ./. flies/NNS smell/VBP an/DT arrow/NN drink/NN ./. bees/NNS sting/VBP like/IN some/DT flies/NNS ./.
```

"time flies like horse flies ."

POS tagging

POS Taggers Hidden Markov Model

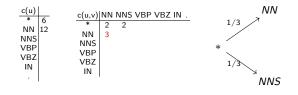


horse/NN flies/NNS time/VBP morning/NN rays/NNS ./. eat/VB breakfast/NN at/IN morning/NN time/NN ./. take/VB time/NN with/IN arrow/NN projects/NNS ./. dinner/NN time/NN flies/VBZ before/IN sleep/NN ./. flies/NNS smell/VBP an/DT arrow/NN drink/NN ./. bees/NNS sting/VBP like/IN some/DT flies/NNS ./.

"time flies like horse flies ."

POS tagging

POS Taggers Hidden Markov Model

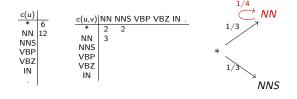


```
horse/NN flies/NNS time/VBP morning/NN rays/NNS ./. eat/VB breakfast/NN at/IN morning/NN time/NN ./. take/VB time/NN with/IN arrow/NN projects/NNS ./. dinner/NN time/NN flies/VBZ before/IN sleep/NN ./. flies/NNS smell/VBP an/DT arrow/NN drink/NN ./. bees/NNS sting/VBP like/IN some/DT flies/NNS ./.
```

"time flies like horse flies ."

POS tagging

POS Taggers Hidden Markov Model

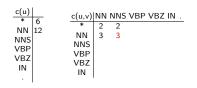


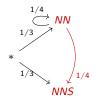
```
horse/NN flies/NNS time/VBP morning/NN rays/NNS ./. eat/VB breakfast/NN at/IN morning/NN time/NN ./. take/VB time/NN with/IN arrow/NN projects/NNS ./. dinner/NN time/NN flies/VBZ before/IN sleep/NN ./. flies/NNS smell/VBP an/DT arrow/NN drink/NN ./. bees/NNS sting/VBP like/IN some/DT flies/NNS ./.
```

"time flies like horse flies ."

POS tagging

POS Taggers Hidden Markov Model



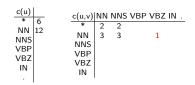


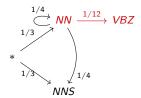
```
horse/NN flies/NNS time/VBP morning/NN rays/NNS ./. eat/VB breakfast/NN at/IN morning/NN time/NN ./. take/VB time/NN with/IN arrow/NN projects/NNS ./. dinner/NN time/NN flies/VBZ before/IN sleep/NN ./. flies/NNS smell/VBP an/DT arrow/NN drink/NN ./. bees/NNS sting/VBP like/IN some/DT flies/NNS ./.
```

"time flies like horse flies ."

POS tagging

POS Taggers Hidden Markov Model



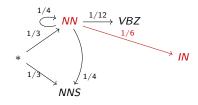


POS tagging

POS Taggers Hidden Markov Model

```
horse/NN flies/NNS time/VBP morning/NN rays/NNS ./.
eat/VB breakfast/NN at/IN morning/NN time/NN ./.
take/VB time/NN with/IN arrow/NN projects/NNS ./.
dinner/NN time/NN flies/VBZ before/IN sleep/NN ./.
flies/NNS smell/VBP an/DT arrow/NN drink/NN ./.
bees/NNS sting/VBP like/IN some/DT flies/NNS ./.
```

c(u)		c(u,v)	NN	NNS	VBP	VBZ	IN .
NN	12	*	2	2			
NNS VBP VBZ IN	12	NN NNS VBP VBZ IN	3	3		1	2

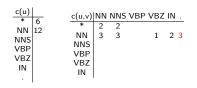


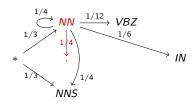
```
horse/NN flies/NNS time/VBP morning/NN rays/NNS ./. eat/VB breakfast/NN at/IN morning/NN time/NN ./. take/VB time/NN with/IN arrow/NN projects/NNS ./. dinner/NN time/NN flies/VBZ before/IN sleep/NN ./. flies/NNS smell/VBP an/DT arrow/NN drink/NN ./. bees/NNS sting/VBP like/IN some/DT flies/NNS ./.
```

"time flies like horse flies ."

POS tagging

POS Taggers Hidden Markov Model





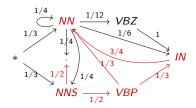
horse/NN flies/NNS time/VBP morning/NN rays/NNS ./. eat/VB breakfast/NN at/IN morning/NN time/NN ./. take/VB time/NN with/IN arrow/NN projects/NNS ./. dinner/NN time/NN flies/VBZ before/IN sleep/NN ./. flies/NNS smell/VBP an/DT arrow/NN drink/NN ./. bees/NNS sting/VBP like/IN some/DT flies/NNS ./.

"time flies like horse flies ."

POS tagging

POS Taggers Hidden Markov Model

c(u) * NN NNS VBP	6 12 6 3	c(u,v) * NN NNS	NN 2 3	NNS 2 3	VBP	VBZ 1	1N 2	_
	3	VBP	1				1	
VBZ	1	VBZ	-				ī	
IN	6	IN	3					



horse/NN flies/NNS time/VBP morning/NN rays/NNS ./. eat/VB breakfast/NN at/IN morning/NN time/NN ./. take/VB time/NN with/IN arrow/NN projects/NNS ./. dinner/NN time/NN flies/VBZ before/IN sleep/NN ./. flies/NNS smell/VBP an/DT arrow/NN drink/NN ./. bees/NNS sting/VBP like/IN some/DT flies/NNS ./.

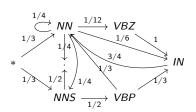
"time flies like horse flies ."

POS tagging

POS Taggers Hidden Markov Model

IN 3

Joint transition matrix A and initial probabilities π :



Emission matrix B:

6

IN

c(u)	6	c(u, <i>O</i> _i)	time	flies	like	horse	_
NN	12	NN	3			1	
NNS		NNS		3			
	6	VBP	1				
VBP	3	VBZ		1			
VBZ	1	IN		-	1		
IN	4	IIV			1		
	6						О

	time	flies	like	horse.	
NN NNS	1/4			1/12	
		1/2			
VBP VBZ	1/3				
VBZ		1			
IN			1/4		
					1

How is the prob. of a POS-tag sequence computed?

POS tagging

POS Taggers Hidden Markov Model

Explanation probability:

Generative model (joint probabilities) instead of conditional model

$$P(X \mid O) = \frac{P(X,O)}{P(O)} \approx P(X,O)$$
 $P(O)$ constant

$$P(X_1, ..., X_T, O) = P(X_1, ..., X_T) \cdot P(O \mid X_1 ... X_T)$$

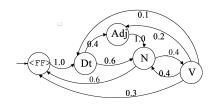
$$P(X_1, ..., X_T) = \pi_{X_1} \prod_{t=2}^{T} a_{X_{t-1}X_t}$$

 $P(O \mid X_1 ... X_T) = \prod_{t=1}^{T} b_{O.X_t}$

$$P(X_1, ..., X_T, O) = \pi_{X_1} \cdot b_{O_1 X_1} \cdot \prod_{t=2}^T a_{X_{t-1} X_t} \cdot b_{O_t X_t}$$

How is the prob. of a POS-tag sequence computed?

Following the previous example



POS tagging

POS Taggers Hidden Markov

b_{ik}	.	this	cat	eats	fish	
<ff></ff>	1.0					
Dt		0.4				
N			0.3		0.3	
V				0.5	0.1	
Adj						

$$P(X, O) = P(X, ., this, cat, eats, fish)$$
? 7 possible X sequences

```
X = \langle FF \rangle, Dt, Adj, N, \langle FF \rangle

X = \langle FF \rangle, Dt, Adj, N, V

X = \langle FF \rangle, Dt, N, \langle FF \rangle, Dt

X = \langle FF \rangle, Dt, N, V, \langle FF \rangle

X = \langle FF \rangle, Dt, N, V, N

P(X,O) = (1 \cdot 1) \cdot (1 \cdot 0.4) \cdot (0.6 \cdot 0.3) \cdot (0.4 \cdot 0.5) \cdot (0.4 \cdot 0.3) = 0.001728

X = \langle FF \rangle, Dt, N, V, Adj

X = \langle FF \rangle, Dt, N, V, Dt
```

How is the best POS-tag sequence found?

POS tagging

POS Taggers Hidden Markov Model We want to find

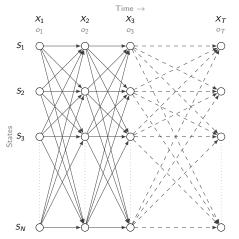
$$\hat{X} = \operatorname*{argmax}_{X} P(X \mid O) \approx \operatorname*{argmax}_{X} P(X, O)$$

- Brute force, $O(N^T)$
 - N states (POS tags) and T observations (word sequence length)
- Viterbi algorithm, dinamic programming, $O(T * N^2)$

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- 1 POS tagging
 - Goal and motivation
 - Part of Speech categories
- 2 POS Taggers
 - Stochastic taggers
 - Hidden Markov Model
 - Viterbi algorithm



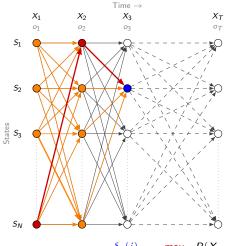
Auxiliary structure:

TRELLIS of a fully connected HMM

Node (S_i, X_t) stores information related to state S_i about the partial sequence from X_1 to X_t : $\delta_t(i)$ and $\phi_t(i)$

POS tagging
POS Taggers
Viterbi algorithm





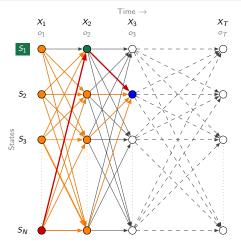
Auxiliary structure:

TRELLIS of a fully connected HMM

Node (S_i, X_t) stores information related to state S_i about the partial sequence from X_1 to X_t : $\delta_t(i)$ and $\phi_t(i)$

$$\delta_t(i) = \max_{X_1, \dots, X_{t-1}} P(X_1, \dots, X_{t-1}, X_t = S_i, O)$$





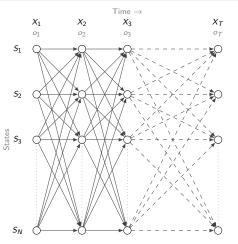
Auxiliary structure:

TRELLIS of a fully connected HMM

Node (S_i, X_t) stores information related to state S_i about the partial sequence from X_1 to X_t : $\delta_t(i)$ and $\varphi_t(i)$

$$\delta_{t}(i) = \max_{X_{1},...,X_{t-1}} P(X_{1},...,X_{t-1},X_{t} = S_{i},O)$$

$$\varphi_{t}(i) = last(\underset{X_{1},...,X_{t-1}}{\operatorname{argmax}} P(X_{1},...,X_{t-1},X_{t} = S_{i},O)$$



Algorithm:

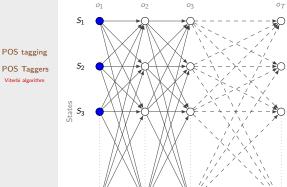
POS tagging
POS Taggers
Viterbi algorithm

 X_1

 S_N

 $\mathsf{Time} \to$

 X_3



 X_2

Algorithm:

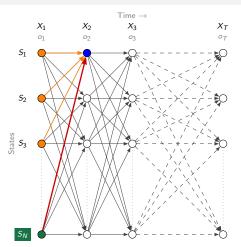
 X_T

1. Initialization step:

$$\forall j = 1 \dots N :$$
 $\delta_1(i) = \pi_i * b_{i,o_1}$

Viterbi algorithm





Algorithm:

1. Initialization step:

$$\forall j = 1 \dots N :$$
 $\delta_1(i) = \pi_i * b_{i,o_1}$

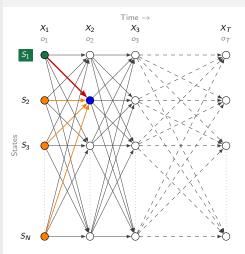
2. Inference step:

$$\delta_{t}(j) = \max_{1 \le i \le N} (\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$$

 $\forall t = 2 \dots T : \forall j = 1 \dots N :$

$$\varphi_t(j) = \underset{1 \leqslant i \leqslant N}{\operatorname{argmax}} (\delta_{t-1}(i) * a_{i,j})$$

POS tagging
POS Taggers
Viterbi algorithm



Algorithm:

1. Initialization step:

$$\forall j = 1 \dots N :$$

 $\delta_1(i) = \pi_i * b_{i,o_1}$

2. Inference step: $\forall t = 2 \dots T$: $\forall j = 1 \dots N$:

$$\delta_t(j) = \max_{1 \leqslant i \leqslant N} (\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$$

$$\varphi_t(j) = \underset{1 \leq i \leq N}{\operatorname{argmax}} (\delta_{t-1}(i) * a_{i,j})$$

 X_2

02

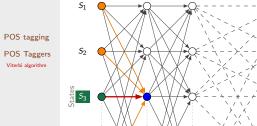
 X_1

 S_N

Time \rightarrow

 X_3

03



Algorithm:

 X_T

1. Initialization step:

$$\forall j = 1 \dots N :$$
 $\delta_1(i) = \pi_i * b_{i,o_1}$

2. Inference step: $\forall t = 2 \dots T$: $\forall j = 1 \dots N$:

$$\delta_t(j) = \max_{1 \le i \le N} (\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$$

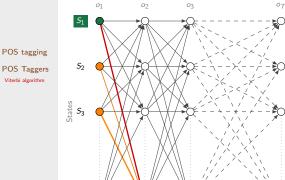
$$\varphi_t(j) = \underset{1 \leq i \leq N}{\operatorname{argmax}} (\delta_{t-1}(i) * a_{i,j})$$

 X_1

 S_N

Time \rightarrow

 X_3



 X_2

Algorithm:

 X_T

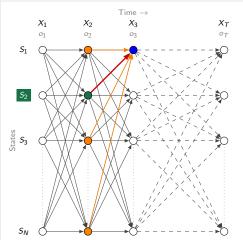
1. Initialization step:

$$\forall j = 1 \dots N :$$
 $\delta_1(i) = \pi_i * b_{i,o_1}$

2. Inference step: $\forall t = 2 \dots T : \forall j = 1 \dots N :$

$$\delta_t(j) = \max_{1 \le i \le N} (\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$$

$$\varphi_t(j) = \underset{1 \leq i \leq N}{\operatorname{argmax}} (\delta_{t-1}(i) * a_{i,j})$$



Algorithm:

1. Initialization step:

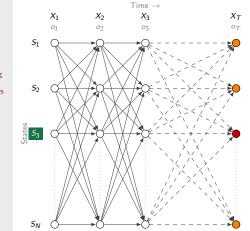
$$\forall j = 1 \dots N :$$
 $\delta_1(i) = \pi_i * b_{i,o_1}$

2. Inference step: $\forall t = 2 \dots T$: $\forall j = 1 \dots N$:

$$\delta_t(j) = \max_{1 \leqslant i \leqslant N} (\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$$

$$\varphi_t(j) = \underset{1 \leqslant i \leqslant N}{\operatorname{argmax}} (\delta_{t-1}(i) * a_{i,j})$$

POS tagging
POS Taggers
Viterbi algorithm



Algorithm:

1. Initialization step:

$$\forall j = 1 \dots N :$$
 $\delta_1(i) = \pi_i * b_{i,o_1}$

2. Inference step: $\forall t = 2 \dots T$: $\forall j = 1 \dots N$:

$$\delta_t(j) = \max_{1 \leqslant i \leqslant N} (\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$$

 $\varphi_t(j) = \operatorname*{argmax}_{1 \leqslant i \leqslant N} (\delta_{t-1}(i) * a_{i,j})$

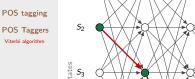
3. Termination step:

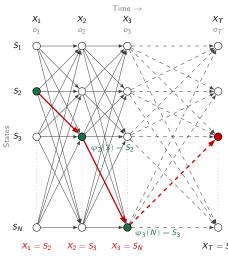
$$X_T = \operatorname{argmax}_i(\delta_T(i))$$

$$X_T = S_3$$

POS tagging POS Taggers

Viterbi algorithm





Algorithm:

1. Initialization step:

$$\forall j = 1 \dots N :$$

 $\delta_1(i) = \pi_i * b_{i,o_1}$

2. Inference step: $\forall t = 2 \dots T : \forall j = 1 \dots N :$

$$\delta_t(j) = \max_{1 \leqslant i \leqslant N} (\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t}$$

 $\varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i) * a_{i,j})$ $1 \le i \le N$

3. Termination step:

$$X_T = \operatorname{argmax}_i(\delta_T(i))$$

4. Backward path readout step:

$$X_T = S_3$$
 $\forall t = 1 \dots T - 1:$ $X_t = \varphi_{t+1}(X_{t+1})$

Apply Viterbi algorithm using the following HMM to

The	kid	fishes	fish
DT	NN	NNS	NN
			NNS
	JJ	VBZ	VBP

POS tagging

	D.T.			N.I.N.I.C	\	1 (0.0
Α	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

В	the	big	kid	fish	time	fishes	times
DT	1						
JJ		0.8	0.2				
NN			0.3	0.4	0.3		
NNS				0.3		0.4	0.3
VBZ						0.6	0.4
VBP				0.7	0.3		

π	
DT	0.4
IJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

Α	DT	IJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
IJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

O-kid

В	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

O. -fish

POS tagging

POS Taggers Viterbi algorithm

Trellis from Viterbi:

 O_{\bullet} —the

O	X_1	X_2			X_3	25				<i>Ο</i> ₄	X_4	511			
$\bar{s}_1 = \bar{DT}$		 	 	 	 		-	 	-	-	-	-	-	-	-
		 	 	 	 		-	 	-	-	-	-	-	-	-
$\bar{s}_3 = N\bar{N}$		 	 	 	 		-	 	-	-	-	-	-	-	-
		 	 	 	 		-	 	-	-	-	-	-	-	-
$\bar{s}_5 = \bar{V} \bar{B} \bar{Z}$		 	 _	 -	 		-	 	-	-	-	-	_	-	-
		 	 	 	 		-	 	-	-	-	-	-	-	-
		 	 	 	 		_	 	_	_	_	_	_	_	_

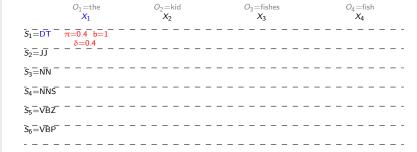
O-fishes

π	
DT	0.4
IJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

_	БТ		NINI	NINIC	1/07	VDD
Α	DT	IJ	NN	NNS	VBZ	ARA
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1			
IJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

Trellis from Viterbi: Initialization step: $\delta_1(i) = \pi_i * b_{i,o_1}$



POS tagging

π	
DT	0.4
IJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

Α	DT	IJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1			
IJ	0	0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

Trellis from Viterbi: Initialization step: $\delta_1(i) = \pi_i * b_{i,o_1}$

POS tagging

	π	
ſ	DT	0.4
ſ	IJ	0.2
Ī	NN	0
	NNS	0.3
Ī	VBZ	
[VBP	0.1
ı		_

_	БТ		NINI	NINIC	1/07	VDD
Α	DT	IJ	NN	NNS	VBZ	ARA
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1			
IJ		0.2		
NN	0	0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS Taggers

Viterbi algorithm

POS tagging

Trellis from Viterbi: Initialization step: $\delta_1(i) = \pi_i * b_{i,o_1}$

$$O_1=$$
the X_1

$$C_2$$
=kid X_2

$$O_3$$
=fishes X_3

$$O_4$$
=fish X_4

 $\overline{S}_1 = \overline{D}T - \overline{\pi} = \overline{0}.4 \text{ b} = 1$ $\delta = 0.4$

$$\bar{S}_2 = J\bar{J}$$

$$\bar{S}_3 = N\bar{N} - \pi = 0 \ \bar{b} = 0$$

$$\bar{S}_4 = \bar{NNS}$$

$$\bar{S}_5 = \bar{V} \bar{B} \bar{Z}$$

$$S_6 = VBP$$

π	
DT	0.4
IJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

DT	IJ	NN	NNS	VBZ	VBP
	0.2	0.5	0.3		
		0.8	0.2		
			0.1	0.9	
					1
		0.2	0.3		
0.4		0.4	0.2		
		0.2	0.2 0.5 0.8	0.2 0.5 0.3 0.8 0.2 0.1	0.2 0.5 0.3 0.8 0.2 0.1 0.9 0.5 0.2 0.3

В	the	kid	fish	fishes
DT	1			
IJ		0.2		
NN		0.3	0.4	
NNS	0		0.3	0.4
VBZ				0.6
VBP			0.7	

POS Taggers Viterbi algorithm

POS tagging

Trellis from Viterbi: Initialization step:
$$\delta_1(i) = \pi_i * b_{i,o_1}$$

$${O_1}=$$
the $oldsymbol{X_1}$

$$C_2=$$
kid X_2

$$O_3$$
=fishes X_3

$$_{X_4}^{O_4=\mathrm{fish}}$$

$$S_1 = DT - \pi = 0.4 b = 1$$

$$\bar{S}_2 = J\bar{J} - - \frac{\delta = 0.4}{-}$$

$$\bar{S}_3 = N\bar{N}$$

$$S_4 = NNS \pi = 0.3 b = 0$$

$$\overline{S_5} = \overline{VBZ} - \frac{\delta = 0}{2}$$

$$\overline{S}_6 = \overline{VBP}$$

π	
DT	0.4
IJ	0.2
NN	
NNS	0.3
VBZ	0
VBP	0.1

Α	DT	IJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
IJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ	0			0.6
VBP			0.7	

POS Taggers

Viterbi algorithm

POS tagging

Trellis from Viterbi: Initialization step: $\delta_1(i) = \pi_i * b_{i,o_1}$

$$O_1=$$
the X_1

$$C_2$$
=kid X_2

$$O_3$$
=fishes X_3

$$C_4=$$
fish X_4

 $\bar{S}_1 = \bar{D}\bar{T} - \bar{\pi} = \bar{0}.4 b = 1$

$$\bar{S}_2 = J\bar{J} - - \frac{\delta = 0.4}{2}$$

$$\bar{S}_3 = N\bar{N}$$

$$\bar{S}_4 = N\bar{N}\bar{S}$$

$$S_5 = VBZ \pi = 0 b = 0$$

П	π	
	-/-	
	DT	0.4
	IJ	0.2
	NN	
	NNS	0.3
	VBZ	
	VBP	0.1

Α	DT	IJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1			
IJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP	0		0.7	

POS tagging POS Taggers

Viterbi algorithm

Trellis from Viterbi: Initialization step:
$$\delta_1(i) = \pi_i * b_{i,o_1}$$

$${\scriptstyle O_1= ext{the} \ X_1}$$

$$C_2$$
=kid X_2

$$O_3$$
=fishes X_3

$$O_4$$
=fish X_4

$$S_1 = DT - \pi = 0.4 b = 1$$

$$\bar{S}_2 = J\bar{J} - - \frac{\delta = 0.4}{2}$$

$$\bar{S}_3 = N\bar{N}$$

$$\bar{S}_4 = N\bar{N}\bar{S}$$

$$\overline{S}_5 = \overline{VBZ}$$

$$\overline{S}_6 = \overline{VBP} \quad \overline{\pi} = \overline{0}.1 \quad b = 0$$

$$S_6 = VBP$$
 $\pi = 0.1$ b= $\delta = 0$

π	
DT	0.4
IJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

Α	DT	IJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

Α	DT	IJ	NN	NNS	VBZ	VBP
DT	0	0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1	0		
IJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

```
Trellis from Viterbi: Inference step: \delta_t(j) = \max(\delta_{t-1}(i)*a_{i,j})*b_{j,o_t} \quad \varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i)*a_{i,j}) \begin{array}{cccc} O_1 = \operatorname{the} & O_2 = \operatorname{kid} & O_3 = \operatorname{fishes} & O_4 = \operatorname{fish} \\ X_1 & X_2 & X_3 & X_4 \\ \hline S_1 = \overline{D1} & \overline{\pi} = \overline{0.4} \quad b = 1 \quad a = 0 \quad & \overline{max} = 0 \quad b = 0 \\ \hline S_2 = JJ & - & - & - & - & - \\ \hline S_3 = \overline{NN} & - & - & - & - & - \\ \hline S_4 = \overline{NNS} & - & - & - & - & - \\ \hline S_5 = \overline{VB}Z & - & - & - & - & - \\ \hline S_6 = \overline{VB}P & - & - & - & - & - \\ \hline \end{array}
```

POS tagging

π	
DT	0.4
IJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

Α	DT	IJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
IJ	0		0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1	0		
IJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

π	
DT	0.4
IJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

_	DT		NINI	NINIC	1/07	VDD
Α	DT	IJ	NN	MN2	VBZ	ARA
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN	0			0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1	0		
IJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

```
\textbf{Trellis from Viterbi:} \ \ \text{Inference step:} \ \ \delta_t(j) = \max(\delta_{t-1}(i)*a_{i,j})*b_{j,o_t} \quad \  \phi_t(j) = \operatorname{argmax}(\delta_{t-1}(i)*a_{i,j})
                        O_1=the
                                                              O_2=kid X_2
                                                                                                         O_3=fishes
                                                                                                                                                          O_4=fish
                           X_1
                                                                                                               X_3
                                                                                                                                                              X_4
S_1 = DT - \pi = 0.4 \text{ b} = 1
                                                       _{\bowtie} max=0 b=0
                        \delta = 0.4
\overline{S}_2 = J\overline{J}
\bar{S}_3 = N\bar{N}
                         \delta = 0
 \overline{S_4} = \overline{NNS}
\overline{S}_5 = \overline{VBZ}
 S_6 = VBP
```

POS tagging

π	
DT	0.4
IJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

DT	IJ	NN	NNS	VBZ	VBP
	0.2	0.5	0.3		
		0.8	0.2		
			0.1	0.9	
0					1
0.5		0.2	0.3		
0.4		0.4	0.2		
	0.5	0.2 0 0 0.5	0.2 0.5 0.8 0 0.5 0.2	0.2 0.5 0.3 0.8 0.2 0.1 0.5 0.2 0.3	0.2 0.5 0.3 0.8 0.2 0.1 0.9 0 0.5 0.2 0.3

В	the	kid	fish	fishes
DT	1	0		
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

```
\textbf{Trellis from Viterbi:} \ \ \text{Inference step:} \ \ \delta_t(j) = \max(\delta_{t-1}(i)*a_{i,j})*b_{j,o_t} \quad \  \phi_t(j) = \operatorname{argmax}(\delta_{t-1}(i)*a_{i,j})
                        O_1=the
                                                            O_2=kid
X_2
                                                                                                     O_3=fishes
                                                                                                                                                      O_4=fish
                          X_1
                                                                                                            X_3
                                                                                                                                                          X_4
S_1 = DT = \pi = 0.4 \text{ b} = 1
                                                      → max=0 b=0
                        \delta = 0.4
\overline{S}_2 = J\overline{J}
\bar{S}_3 = N\bar{N}
\overline{S}_4 = \overline{NNS}
                         \delta = 0
 \overline{S}_5 = \overline{VBZ}
 S_6 = VBP
```

POS tagging

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

Α	DT	IJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
IJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1	0		
IJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

```
\textbf{Trellis from Viterbi:} \ \ \text{Inference step:} \ \ \delta_t(j) = \max(\delta_{t-1}(i)*a_{i,j})*b_{j,o_t} \quad \  \phi_t(j) = \operatorname{argmax}(\delta_{t-1}(i)*a_{i,j})
                          O_1=the
                                                                  O_2=kid
X_2
                                                                                                                O_3=fishes
                                                                                                                                                                     O_4=fish
                             X_1
                                                                                                                       X_3
                                                                                                                                                                          X_4
 S_1 = DT = \pi = 0.4 \text{ b} = 1
                                                            \frac{1}{\sqrt{max}} max=0 b=0
                          \delta = 0.4
 \overline{S}_2 = J\overline{J}
 \bar{S}_3 = N\bar{N}
 \overline{S}_4 = \overline{NNS}
 \overline{S}_5 = \overline{VBZ}
                           \delta = 0
 \overline{S}_6 = \overline{VBP}
```

POS tagging

π	
DT	0.4
IJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

Α	DT	IJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
IJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1	0		
IJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

```
\textbf{Trellis from Viterbi:} \ \ \text{Inference step:} \ \ \delta_t(j) = \max(\delta_{t-1}(i)*a_{i,j})*b_{j,o_t} \quad \  \phi_t(j) = \operatorname{argmax}(\delta_{t-1}(i)*a_{i,j})
                         O_1=the
                                                               O_2=kid
X_2
                                                                                                         O_3=fishes
                                                                                                                                                            O_4=fish
                           X_1
                                                                                                                 X_3
                                                                                                                                                                X_4
S_1 = DT = \pi = 0.4 \text{ b} = 1
                                                        \frac{1}{\sqrt{max}} max=0 b=0
                         \delta = 0.4
\overline{S}_2 = J\overline{J}
\bar{S}_3 = N\bar{N}
 \overline{S_4} = \overline{NNS}
\overline{S}_5 = \overline{VBZ}
 S_6 = VBP
```

POS tagging

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

Α	DT	IJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

Α	DT	IJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
IJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

Α	DT	IJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
IJ		0	0.8	0.2		
NN		0		0.1	0.9	
NNS		0				1
	0.5	0	0.2	0.3		
VBP	0.4	0	0.4	0.2		

В	the	kid	fish	fishes
DT	1			
IJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

 $\textbf{Trellis from Viterbi:} \ \ \text{Inference step:} \ \ \delta_t(j) = \max(\delta_{t-1}(i)*a_{i,j})*b_{j,o_t} \quad \ \phi_t(j) = \operatorname{argmax}(\delta_{t-1}(i)*a_{i,j})$ O_1 =the O_3 =fishes O_4 =fish O_2 =kid X_1 X₂ X_3 X_4 $\pi = 0.4 \text{ b} = 1$ $S_1 = DT$ $\delta = 0.4$ $\overline{S}_2 = \overline{J}\overline{J}$ a=0 max=0.08 b=0.2 $\bar{S}_3 = N\bar{N}$ $\delta = 0$ $\overline{S_4} = \overline{NNS}$ $\delta = 0$ $\overline{S}_5 = \overline{VBZ}$ $\delta = 0$ $\overline{S}_6 = \overline{VBP}$ $\delta = 0$

POS tagging

π	
DT	0.4
IJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

Α	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

```
O_4=fish
              O_1=the
                                     O_2=kid
                                                              O_3=fishes
                 X<sub>1</sub>
                                        X_2
                                                                  X_3
                                                                                              X_4
S_1 = DT - \pi = 0.4 b = 1
               \delta = 0.4
\overline{S}_2 = \overline{J}\overline{J}
                                 max=0.08 b=0.2
                                \delta=0.016, \varphi=DT
\bar{S}_3 = N\bar{N}
\overline{S_4} = \overline{NNS}
\overline{S}_5 = \overline{VBZ}
S_6 = VBP
```

POS tagging

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

Α	DT	IJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1			
IJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

```
\textbf{Trellis from Viterbi:} \ \ \text{Inference step:} \ \ \delta_t(j) = \max(\delta_{t-1}(i)*a_{i,j})*b_{j,o_t} \quad \  \phi_t(j) = \operatorname{argmax}(\delta_{t-1}(i)*a_{i,j})
                                                                                                                                                             O_4=fish
                         O_1=the
                                                               O_2=kid
                                                                                                           O_3=fishes
                            X_1
                                                                                                                 X_3
                                                                                                                                                                 X_4
S_1 = DT - \pi = 0.4 \text{ b} = 1
                         \delta = 0.4
\overline{S}_2 = J\overline{J}
                                                        max=0.08 b=0.2
                                                      \delta = 0.016, \ \varphi = DT
max = 0.2 \ b = 0.3
\bar{S}_3 = N\bar{N}
 \overline{S_4} = \overline{NNS}
\overline{S}_5 = \overline{VBZ}
 S_6 = VBP
```

POS tagging

0.4
0.2
0.3
0.1

Α	DT	IJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1			
IJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

```
Trellis from Viterbi: Inference step: \delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t} \quad \varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i) * a_{i,j})
                       O_1=the
                                                            O_2=kid
                                                                                                    O_3=fishes
                                                                                                                                                    O_4=fish
                           X_1
                                                                X_2
                                                                                                           X_3
                                                                                                                                                        X_4
S_1 = DT - \pi = 0.4 b = 1
                        \delta = 0.4
 \overline{S}_2 = J\overline{J}
                                                     max=0.08 b=0.2
                                                   \delta = 0.016, \ \phi = DT
max=0.2 b=0.3
 \overline{S}_3 = \overline{NN}
                                                     \delta=0.06, \varphi=DT
 \overline{S_4} = \overline{NNS}
 \overline{S}_5 = \overline{VBZ}
 S_6 = VBP
```

POS tagging

π	
DT	0.4
IJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

Α	DT	IJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1			
IJ		0.2		
NN		0.3	0.4	
NNS		0	0.3	0.4
VBZ				0.6
VBP			0.7	

```
\textbf{Trellis from Viterbi: } \text{Inference step: } \delta_t(j) = \max(\delta_{t-1}(i)*a_{i,j})*b_{j,o_t} \quad \varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i)*a_{i,j})
                       O_1=the
                                                          O_2=kid
                                                                                                  O_3=fishes
                                                                                                                                                O_4=fish
                          X_1
                                                                                                        X_3
                                                                                                                                                    X_4
S_1 = DT - \pi = 0.4 \text{ b} = 1
                       \delta = 0.4
\overline{S}_2 = J\overline{J}
                                                   max=0.08 b=0.2
                                                  \delta=0.016, \phi=DT
\bar{S}_3 = N\bar{N}
 \overline{S_4} = \overline{NNS}
                                                     max = 0.12 b = 0
 \overline{S}_5 = \overline{VBZ}
 S_6 = VBP
```

POS tagging

π	
DT	0.4
IJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1
	JJ NN NNS VBZ

Α	DT	IJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
IJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

```
Trellis from Viterbi: Inference step: \delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t} \quad \varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i) * a_{i,j})
                     O_1=the
                                                       O_2=kid
                                                                                           O_3=fishes
                                                                                                                                      O_4=fish
                        X<sub>1</sub>
                                                          X2
                                                                                                 X_3
                                                                                                                                          X_4
                 \pi = 0.4 b=1
S_1 = DT
                     \delta = 0.4
 \overline{S}_2 = J\overline{J}
                                                max=0.08 b=0.2
                                               \delta=0.016, \phi=DT
 \bar{S}_3 = N\bar{N}
                                                 max=0.2 b=0.3
                                                \delta=0.06, \varphi=DT
 \overline{S_4} = \overline{NNS}
                                                 max=0.12 b=0
                                                  \delta = 0, \phi = DT
 \overline{S}_5 = \overline{VBZ}
 S_6 = VBP
```

POS tagging

π	
DT	0.4
IJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1
	JJ NN NNS VBZ

Α	DT	IJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3	0	
IJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1			
IJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ		0		0.6
VBP			0.7	

```
O_1=the
                                     O_2=kid
                                                              O_3=fishes
                                                                                           O_4=fish
                X_1
                                        X2
                                                                  X_3
                                                                                              X_4
S_1 = DT - \pi = 0.4 \text{ b} = 1
              \delta = 0.4
\overline{S}_2 = J\overline{J}
                                max=0.08 b=0.2
                                \delta=0.016, \phi=DT
\bar{S}_3 = N\bar{N}
                                 max=0.2 b=0.3
                                \delta=0.06, \varphi=DT
\overline{S_4} = \overline{NNS}
\overline{S_5} = \overline{VBZ}
                                  \frac{1}{4} max=0 b=0
S_6 = VBP
```

POS tagging

0.4
0.2
0.3
0.1

_						
Α	DT	IJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

```
Trellis from Viterbi: Inference step: \delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t} \quad \varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i) * a_{i,j})
                     O_1=the
                                                        O_2=kid
                                                                                             O_3=fishes
                                                                                                                                         O_4=fish
                        X<sub>1</sub>
                                                                                                   X_3
                                                                                                                                            X_4
                                                           X_2
                 \pi = 0.4 \text{ b} = 1
S_1 = DT
                      \delta = 0.4
 \overline{S}_2 = J\overline{J}
                                                 max=0.08 b=0.2
                                                \delta=0.016, \phi=DT
 \bar{S}_3 = N\bar{N}
                                                  max=0.2 b=0.3
                                                 \delta=0.06, \varphi=DT
 \overline{S_4} = \overline{NNS}
 \overline{S_5} = \overline{VBZ}
                                                    max=0 b=0
                                                   \delta=0, \phi=DT
 S_6 = VBP
```

POS tagging

0.4
0.2
0.3
0.1

Α	DT	IJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		0
IJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1			
IJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP		0	0.7	

```
O_4=fish
               O_1=the
                                        O_2=kid
                                                                   O_3=fishes
                 X<sub>1</sub>
                                          X2
                                                                       X_3
                                                                                                    X_4
S_1 = DT - \pi = 0.4 \text{ b} = 1
               \delta = 0.4
\overline{S}_2 = J\overline{J}
                                   max=0.08 b=0.2
                                  \delta=0.016, \phi=DT
\bar{S}_3 = N\bar{N}
                                   max=0.2 b=0.3
                                   \delta=0.06, \varphi=DT
\overline{S_4} = \overline{NNS}
\overline{S}_5 = \overline{VBZ}
S_6 = \overline{VBP}
                                    \sqrt{\text{max}=0} b=0
```

POS tagging

0.4
0.2
0.3
0.1

Α	DT	IJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
IJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

```
Trellis from Viterbi: Inference step: \delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t} \quad \varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i) * a_{i,j})
                      O_1=the
                                                         O_2=kid
                                                                                               O_3=fishes
                                                                                                                                            O_4=fish
                         X<sub>1</sub>
                                                                                                     X_3
                                                                                                                                                X_4
                                                             X_2
                  \pi = 0.4 \text{ b} = 1
S_1 = DT
                      \delta = 0.4
 \overline{S}_2 = J\overline{J}
                                                  max=0.08 b=0.2
                                                 \delta=0.016, \phi=DT
 \bar{S}_3 = N\bar{N}
                                                   max=0.2 b=0.3
                                                  \delta=0.06, \varphi=DT
 \overline{S_4} = \overline{NNS}
 \overline{S}_5 = \overline{VBZ}
 S_6 = \overline{VBP}
                                                     max=0 b=0
                                                     \delta=0, \phi=DT
```

POS tagging

π	
DT	0.4
IJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

DT	IJ	NN	NNS	VBZ	VBP
	0.2	0.5	0.3		
		0.8	0.2		
			0.1	0.9	
					1
		0.2	0.3		
0.4		0.4	0.2		
		0.2	0.2 0.5 0.8	0.2 0.5 0.3 0.8 0.2 0.1	0.2 0.5 0.3 0.8 0.2 0.1 0.9 0.5 0.2 0.3

В	the	kid	fish	fishes
DT	1			0
IJ		0.2		0
NN		0.3	0.4	0
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	0

```
POS Taggers
Viterbi algorithm
```

POS tagging

```
O_1=the
                                          O_2=kid
                                                                      O_3=fishes
                                                                                                       O_{\Lambda}=fish
                   X_1
                                             X_2
                                                                           X_3
                                                                                                          X_4
\bar{S}_1 = \bar{DT}
             \pi = 0.4 \text{ b} = 1
                                                                         b=0
                \delta = 0.4
                                                                     \delta=0, \phi=DT
\overline{S}_2 = J\overline{J}
                                     max=0.08 b=0.2
                                                                        b=0
                                    \delta=0.016, \varphi=DT
                                                                     \frac{\delta=0, \ \phi=DT}{b=0}
\overline{S}_3 = \overline{NN}
                                     max=0.2 b=0.3
                                     \delta=0.06, \varphi=DT
                                                                     \delta=0, \phi=DT
\bar{S}_4 = N\bar{N}\bar{S}
\overline{S}_5 = \overline{VBZ}
S_6 = \overline{VBP}
                                                                         b=0
                                                                     \delta=0, \phi=DT
```

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

Α	DT	IJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1			
IJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

```
O_1=the
                                       O_2=kid
                                                                  O_3=fishes
                                                                                                 O_4=fish
                 X_1
                                                                      X_3
                                                                                                    X_4
            \pi = 0.4 \text{ b} = 1
S_1 = DT
               \delta = 0.4
\overline{S}_2 = J\overline{J}
                                   max=0.08 b=0.2
                                  \delta = 0.016, \ \varphi = DT
max = 0.2 \ b = 0.3 \ a = 0.2
\bar{S}_3 = N\bar{N}
                                   \delta = 0.06, \varphi = DT = 0
\overline{S_4} = \overline{NNS}
                                                            max=0.006 b=0.4
\overline{S}_5 = \overline{VBZ}
S_6 = VBP
```

POS tagging

π	
DT	0.4
IJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

Α	DT	IJ		NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

```
Trellis from Viterbi: Inference step: \delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t} \quad \varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i) * a_{i,j})
                     O_1=the
                                                      O_2=kid
                                                                                           O_3=fishes
                                                                                                                                    O_{\Lambda}=fish
                        X_1
                                                                                                X_3
                                                                                                                                        X_4
                                                         X_2
S_1 = DT
                 \pi = 0.4 \text{ b} = 1
                     \delta = 0.4
 \overline{S}_2 = J\overline{J}
                                               max=0.08 b=0.2
                                              \delta=0.016, \phi=DT
 \bar{S}_3 = N\bar{N}
                                                max=0.2 b=0.3
                                               \delta=0.06, \phi=DT =0.1
 \overline{S}_4 = \overline{NNS}
                                                                                  ≯max=0.006 b=0.4
                                                                                    \delta=0.0024, \phi=NN
 \overline{S}_5 = \overline{VBZ}
 S_6 = VBP
```

POS tagging

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

Α	DT	IJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2	0	
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1			
IJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

```
Trellis from Viterbi: Inference step: \delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t} \quad \varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i) * a_{i,j})
                      O_1=the
                                                        O_2=kid
                                                                                               O_3=fishes
                                                                                                                                          O_4=fish
                         X_1
                                                            X_2
                                                                                                    X_3
                                                                                                                                              X_4
S_1 = DT
                 \pi = 0.4 \text{ b} = 1
                      \delta = 0.4
 \overline{S}_2 = J\overline{J}
                                                 max=0.08 b=0.2
                                                \delta = 0.016, \ \varphi = DT
max=0.2 b=0.3
 \bar{S}_3 = N\bar{N}
                                                 \delta = 0.06, \phi = DT
 \overline{S_4} = \overline{NNS}
                                                                                       max=0.006 b=0.4
                                                                                        \delta=0.0024, \phi=NN
 \overline{S_5} = \overline{VBZ}
                                                                                      max=0.054 b=0.6
 S_6 = VBP
```

POS tagging

0.4
0.2
0.3
0.1

Α	DT	JJ	NN	NINIS	VBZ	V/RP
	01	•			VDZ	VDI
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1			
IJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

```
Trellis from Viterbi: Inference step: \delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t} \quad \varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i) * a_{i,j})
                     O_1=the
                                                     O_2=kid
                                                                                          O_3=fishes
                                                                                                                                   O_{\Lambda}=fish
                        X_1
                                                         X_2
                                                                                               X_3
                                                                                                                                       X_4
S_1 = DT
                 \pi = 0.4 \text{ b} = 1
                     \delta = 0.4
 \overline{S}_2 = J\overline{J}
                                               max=0.08 b=0.2
                                              \delta=0.016, \varphi=DT
 \bar{S}_3 = N\bar{N}
                                                max=0.2 b=0.3
                                               \delta=0.06, \varphi=DT
 \overline{S_4} = \overline{NNS}
                                                                           ₹0.9 max=0.006 b=0.4
                                                                                    \delta=0.0024, \phi=NN
 \overline{S_5} = \overline{VBZ}
                                                                                 max=0.054 b=0.6
                                                                                    \delta=0.0324 \varphi=NN
 S_6 = VBP
```

POS tagging

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

Α	DT	IJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
IJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1		0	
IJ		0.2	0	
NN		0.3	0.4	
NNS			0.3	0.4
VBZ			0	0.6
VBP			0.7	

```
Trellis from Viterbi: Inference step: \delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t} \quad \varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i) * a_{i,j})
                     O_1=the
                                                     O_2=kid
                                                                                          O_3=fishes
                                                                                                                                   O_{\Lambda}=fish
                        X_1
                                                         X_2
                                                                                               X_3
                                                                                                                                       X_4
                 \pi = 0.4 \ b=1
 \bar{S}_1 = \bar{DT}
                                                                                                                                     b=0
                     \delta = 0.4
                                                                                                                               \delta=0, \phi=DT
 \overline{S}_2 = J\overline{J}
                                               max=0.08 b=0.2
                                                                                                                                     b=0
                                              \delta=0.016, \varphi=DT
                                                                                                                               \delta = 0, \omega = DT
 \bar{S}_3 = N\bar{N}
                                                max=0.2 b=0.3
                                               \delta=0.06, \varphi=DT
 \bar{S}_4 = N\bar{N}\bar{S}
                                                                                   max=0.006 b=0.4
                                                                                    \delta=0.0024, \phi=NN
 \overline{S_5} = \overline{VBZ}
                                                                                    max=0.054 b=0.6
                                                                                                                                     b=0
                                                                                    \delta=0.0324 \phi=NN
                                                                                                                               \delta = 0, \varphi = DT
 S_6 = VBP
```

POS tagging

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

Α	DT	IJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
IJ			0.8	0.2		
NN				0.1	0.9	
NNS			0			1
	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

```
Trellis from Viterbi: Inference step: \delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t} \quad \varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i) * a_{i,j})
                     O_1=the
                                                       O_2=kid
                                                                                            O_3=fishes
                                                                                                                                      O_{\Lambda}=fish
                        X_1
                                                                                                 X_3
                                                          X_2
                                                                                                                                          X_4
 \bar{S}_1 = \bar{DT}
                 \pi = 0.4 \text{ b} = 1
                     \delta = 0.4
 \overline{S}_2 = J\overline{J}
                                                max=0.08 b=0.2
                                               \delta=0.016, \varphi=DT
                                                                                                                    a=0.0065 b=0.4
 \bar{S}_3 = N\bar{N}
                                                 max=0.2 b=0.3
                                                \delta=0.06, \varphi=DT
 \bar{S}_4 = N\bar{N}\bar{S}
                                                                                     max=0.006 b=0.4
                                                                                      \delta=0.0024, \phi=NN
 \overline{S}_5 = \overline{VBZ}
                                                                                      max=0.054 b=0.6
                                                                                      \delta = 0.0324 \phi = NN
 \overline{S}_6 = \overline{VBP}
```

POS tagging

π	
DT	0.4
IJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

Α	DT	IJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
IJ			0.8	0.2		
NN				0.1	0.9	
						1
	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		
			0.2	0.1	0.9	1

В	the	kid	fish	fishes
DT	1			
IJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

```
Trellis from Viterbi: Inference step: \delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t} \quad \varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i) * a_{i,j})
                    O_1=the
                                                   O_2=kid
                                                                                      O_3=fishes
                                                                                                                              O_{\Lambda}=fish
                       X_1
                                                                                           X_3
                                                       X_2
                                                                                                                                 X_4
 \bar{S}_1 = \bar{DT}
                \pi = 0.4 \text{ b} = 1
                    \delta = 0.4
 \overline{S}_2 = J\overline{J}
                                             max=0.08 b=0.2
                                            \delta=0.016, \phi=DT
 \bar{S}_3 = N\bar{N}
                                              max=0.2 b=0.3
                                                                                                                     max=0.0065 b=0.4
                                             \delta=0.06, \varphi=DT
                                                                                                                      \delta = 0.0026, \omega = VBZ
 \bar{S}_4 = N\bar{N}\bar{S}
                                                                                max=0.006 b=0.4
                                                                                \delta=0.0024, \phi=NN
 S_5 = VBZ
                                                                                max=0.054 b=0.6
                                                                                \delta=0.0324 \phi=NN
 S_6 = VBP
```

POS tagging

0.4
0.2
0.3
0.1

Α	DT	IJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS				0		1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

```
Trellis from Viterbi: Inference step: \delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t} \quad \varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i) * a_{i,j})
                   O_1=the
                                                  O_2=kid
                                                                                    O_3=fishes
                                                                                                                           O_{\Lambda}=fish
                      X_1
                                                                                         X_3
                                                      X_2
                                                                                                                               X_4
                \pi = 0.4 b=1
 \bar{S}_1 = \bar{DT}
                    \delta = 0.4
 \overline{S}_2 = J\overline{J}
                                            max=0.08 b=0.2
                                           \delta=0.016, \varphi=DT
 \bar{S}_3 = N\bar{N}
                                             max=0.2 b=0.3
                                                                                                                   max=0.0065 b=0.4
                                            \delta=0.06, \varphi=DT
                                                                                                                   \delta=0.0026, \phi=VBZ
                                                                              max=0.006 b=0.4 a=0
 \overline{S}_4 = \overline{NNS}
                                                                                                                  max=0.0097 b=0.3
                                                                               \delta=0.0024, \phi=NN
                                                                               max=0.054 b=0.6 = 0.3
 \overline{S}_5 = \overline{VBZ}
                                                                               \delta=0.0324 \phi=NN
 S_6 = VBP
```

POS tagging

0.4
0.2
0.3
0.1

Α	DT	IJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1			
IJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

```
Trellis from Viterbi: Inference step: \delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t} \quad \varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i) * a_{i,j})
                   O_1=the
                                                 O_2=kid
                                                                                   O_3=fishes
                                                                                                                         O_{\Lambda}=fish
                      X_1
                                                     X_2
                                                                                        X_3
                                                                                                                            X_4
                \pi = 0.4 \ b=1
 \bar{S}_1 = \bar{DT}
                   \delta = 0.4
 \overline{S}_2 = J\overline{J}
                                           max=0.08 b=0.2
                                          \delta=0.016, \varphi=DT
 \bar{S}_3 = N\bar{N}
                                            max=0.2 b=0.3
                                                                                                                 max=0.0065 b=0.4
                                                                                                                 \delta=0.0026, \phi=VBZ
                                           \delta=0.06, \varphi=DT
 \overline{S}_4 = \overline{NNS}
                                                                             max=0.006 b=0.4
                                                                                                               max=0.0097 b=0.3
                                                                             \delta=0.0024, \phi=NN
                                                                                                                 \delta=0.0029, \phi=VBZ
                                                                             max=0.054 b=0.6 a=0.3
 S_5 = VBZ
                                                                             \delta=0.0324 \varphi=NN
 S_6 = VBP
```

POS tagging

0.4
0.2
0.3
0.1

Α	DT	IJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		0
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1			
IJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

```
Trellis from Viterbi: Inference step: \delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t} \quad \varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i) * a_{i,j})
                   O_1=the
                                                   O_2=kid
                                                                                     O_3=fishes
                                                                                                                            O_{\Lambda}=fish
                       X_1
                                                      X_2
                                                                                          X_3
                                                                                                                                X_4
                \pi = 0.4 \ b=1
 \bar{S}_1 = \bar{DT}
                    \delta = 0.4
 \overline{S}_2 = J\overline{J}
                                            max=0.08 b=0.2
                                            \delta=0.016, \phi=DT
 \bar{S}_3 = N\bar{N}
                                             max=0.2 b=0.3
                                                                                                                    max=0.0065 b=0.4
                                                                                                                    \delta=0.0026, \phi=VBZ
                                            \delta=0.06, \varphi=DT
 \bar{S}_4 = N\bar{N}\bar{S}
                                                                               max=0.006 b=0.4
                                                                                                                    max=0.0097 b=0.3
                                                                               \delta=0.0024, \phi=NN
                                                                                                                    \delta=0.0029, \varphi=VBZ
 \overline{S}_5 = \overline{VBZ}
                                                                               max=0.054 b=0.6
                                                                               \delta=0.0324 \varphi=NN
 S_6 = \overline{VBP}
                                                                                                            a=0 max=0.0024 b=0.7
```

POS tagging

π	
DT	0.4
IJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

Α	DT	IJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
IJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

```
Trellis from Viterbi: Inference step: \delta_t(j) = \max(\delta_{t-1}(i) * a_{i,j}) * b_{j,o_t} \quad \varphi_t(j) = \operatorname{argmax}(\delta_{t-1}(i) * a_{i,j})
                    O_1=the
                                                    O_2 = kid
                                                                                        O_3=fishes
                                                                                                                                O_{\Lambda}=fish
                       X_1
                                                        X_2
                                                                                             X_3
                                                                                                                                    X_4
 \bar{S}_1 = \bar{DT}
                 \pi = 0.4 \text{ b} = 1
                     \delta = 0.4
 \overline{S}_2 = J\overline{J}
                                              max=0.08 b=0.2
                                             \delta=0.016, \phi=DT
 \bar{S}_3 = N\bar{N}
                                               max=0.2 b=0.3
                                                                                                                        max=0.0065 b=0.4
                                                                                                                        \delta=0.0026, \phi=VBZ
                                              \delta=0.06, \varphi=DT
 \overline{S_4} = \overline{NNS}
                                                                                  max=0.006 b=0.4
                                                                                                                        max=0.0097 b=0.3
                                                                                  \delta=0.0024, \phi=NN
                                                                                                                        \delta=0.0029, \varphi=VBZ
 \overline{S}_5 = \overline{VBZ}
                                                                                  max=0.054 b=0.6
                                                                                  \delta=0.0324 \phi=NN
 S_6 = \overline{VBP}
                                                                                                                      \frac{1}{2} max=0.0024 b=0.7
                                                                                                                        \delta=0.0017, \phi=NNS
```

POS tagging

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

Α	DT	IJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
IJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1			
IJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

POS tagging POS Taggers

Viterbi algorithm

Trellis from Viterbi: Termination step: $X_4 = argmax_i(\delta_4(i))$

π	
DT	0.4
IJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

POS tagging POS Taggers

Viterbi algorithm

Α	DT	IJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
IJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1			
IJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

Trellis from Viterbi: Termination step: $X_4 = argmax_i(\delta_4(i))$

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

Α	DT	IJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

gers

POS Taggers Viterbi algorithm

POS tagging

Trellis from Viterbi: Backward path readout step: $X_t = \varphi(X_{t+1})$

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

POS tagging POS Taggers

Viterbi algorithm

Α	DT	IJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
VBZ	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

В	the	kid	fish	fishes
DT	1			
IJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

Trellis from Viterbi: Backward path readout step: $X_t = \varphi(X_{t+1})$

π	
DT	0.4
JJ	0.2
NN	
NNS	0.3
VBZ	
VBP	0.1

Α	DT	JJ	NN	NNS	VBZ	VBP
DT		0.2	0.5	0.3		
JJ			0.8	0.2		
NN				0.1	0.9	
NNS						1
	0.5		0.2	0.3		
VBP	0.4		0.4	0.2		

 $O_2 = kid$

 $X_2 = NN$

В	the	kid	fish	fishes
DT	1			
JJ		0.2		
NN		0.3	0.4	
NNS			0.3	0.4
VBZ				0.6
VBP			0.7	

 O_{Λ} =fish

 $X_4 = NNS$

POS tagging
POS Taggers

Viterbi algorithm

Trellis from Viterbi: Backward path readout step: $X_t = \varphi(X_{t+1})$

 O_1 =the

 $X_1 = DT$

 O_3 =fishes

 $X_3 = VBZ$