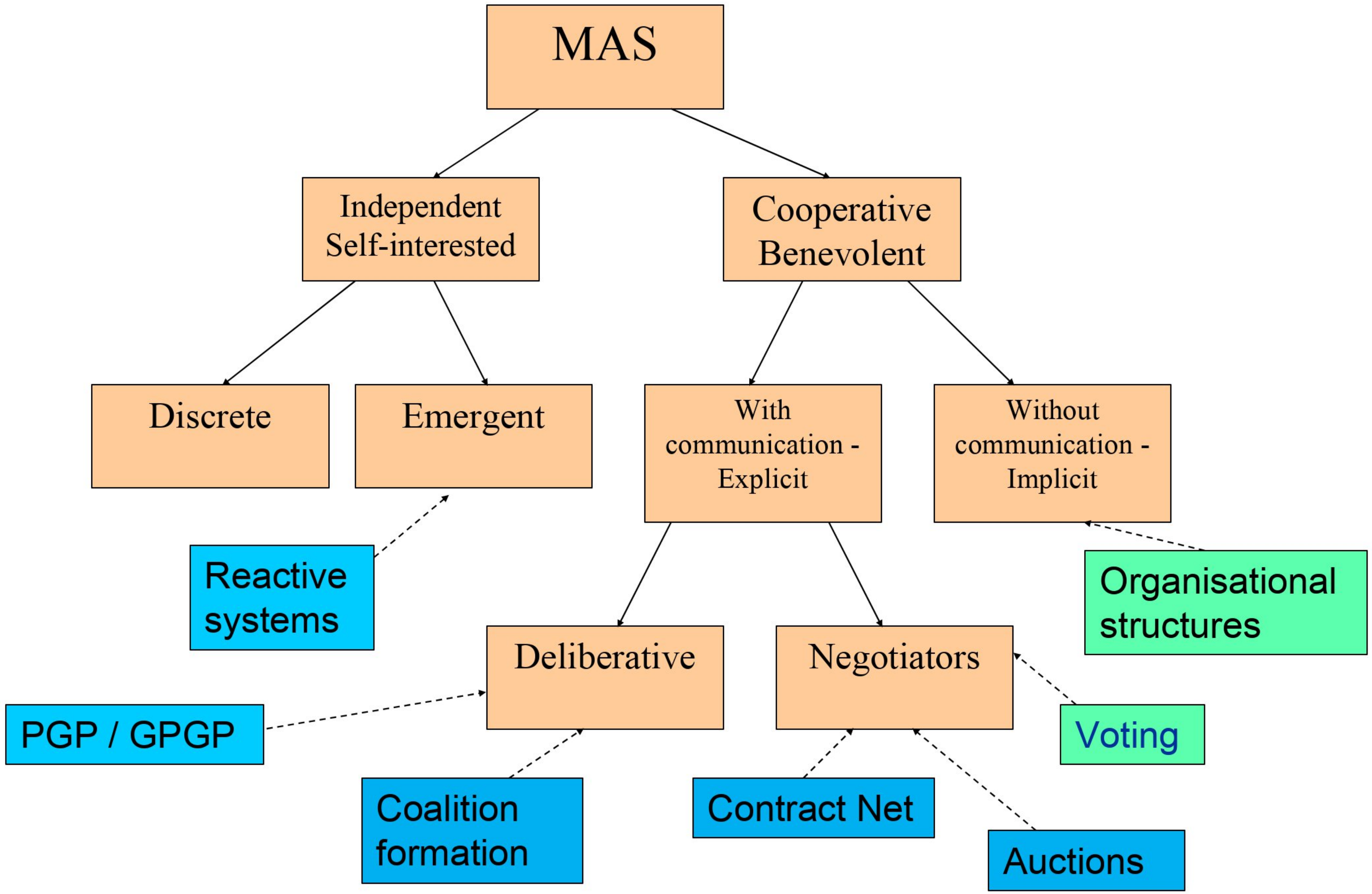
Lecture 8: Cooperation in MAS (IV) – Voting protocols

Multi-Agent Systems

Universitat Rovira i Virgili

MESIIA – Master’s Degree in Computer Security Engineering and Artificial Intelligence
MAI – Master's Degree in Artificial Intelligence

Cooperation hierarchy [last lectures]



Outline

- Voting protocols
  - Social choice
  - Properties of social choice rules
  - Simple voting protocols
    - Plurality / Anti-plurality / Best-Worst / Approval
    - Total order protocols: Binary / Borda / Condorcet
  - Complex voting mechanisms
    - Linguistic votes
    - Uncertain opinions

Distributed Decision Making – Voting

- Mechanism which chooses the outcome of a negotiation based on the inputs (votes) given by all agents to a set of competing options
- We will assume that voters are truthful (they vote for the candidate they think is best)

Insincere (Strategic) Voters

- Self-interested agents can benefit from *insincerely* declaring their preferences
  - Suppose your choice will likely come in second place. If you rank the first choice of the rest of the group (insincerely) very low, you may lower that choice enough so yours is first
  - "*Useful vote*" effect in politics: more than two candidates, and you are quite certain that your preferred candidate doesn't have any chance
- However, knowledge of the true preferences of all voters is rarely available

Basic elements in a voting protocol

- Aim of the negotiation: rank a set of alternatives based on the individual ranking of those options by each agent
  - A - set of *n* agents
  - O - set of *m* alternatives
  - Each agent *i* has a preference relation

$$<_i : O \times O$$

Preference relations

- **Weak order**: complete and transitive

R_1
x_1
x_2 x_3
x_4

R_2
x_2 x_4
x_3
x_1

- **Linear order**: weak order + anti-symmetric

R_3
x_3
x_1
x_2
x_4

Social choice rule

- Input: the agents preference relations

  $(<_1, \ldots, <_n)$
- Output: elements of O sorted according to the input (social preference relation $<^*$ of the agent group)
- In other words, it creates an ordering of the group of alternatives, so that the most (socially) preferred alternative is chosen
  - This order may be partial

Social choice rule: Desirable properites (I)

- Calculability

  A social preference ordering $<^*$ should exist for all possible inputs
- Completeness

  $<^*$ should be defined for every pair of alternatives $(o, o') \in O$
- Linearity

  $<^*$ should be antisymmetric and transitive over O

Social choice rule: Desirable properties (II)

- Anonymity / No dictatorship
  The outcome of the social choice rule depends on the set of opinions, but not on which agents have these opinions.
  No agent *i* should be a *dictator* in the sense that $o <_i o'$ implies $o <^* o'$ regardless of the preferences of the other agents

Social choice rule: Desirable properties (III)

- Unanimity / Pareto efficiency
  If $\forall i \in A\ (o <_i o')$, then $(o <^* o')$
  Do not misorder the options if all agents agree.
  If everybody thinks that A is better than B, A should be preferred to B in the aggregated order

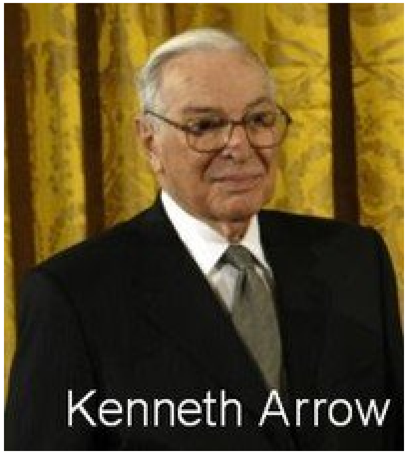
Social choice rule: Desirable properties (IV)

- Neutrality
  The outcome of the social choice rule should not depend on how alternatives are named or ordered
- Independence of irrelevant alternatives
  Removing / Adding an irrelevant alternative should not affect the winner of the vote

Arrow's impossibility theorem - 1951

- No social choice rule satisfies all of these conditions
  - E.g., in many cases an irrelevant alternative affects the outcome of the voting protocol
- Some of the constraints must be relaxed
  - We may not require $<^*$ to be always defined
  - We may not require that $<^*$ is asymmetric and transitive

Economy
Nobel Prize
1972
(youngest, 51)



Simple voting mechanisms

- Some basic voting mechanisms
  - Plurality / Anti-plurality / Best-Worst / Approval
- Protocols based on total orders
  - Binary protocol (series of votes of 2 options each)
  - Borda protocol (sum of all the preferences of the agents)
  - Condorcet protocol (pairwise comparison of options, given full preference ordering of each agent)
- All the protocols are problematic in one sense or another

Plurality voting

- Each agent can give 1 vote to 1 of the alternatives
- The alternative with the highest number of votes wins

Plurality voting: Problems (I)

- *Useful* vote
  - Quite common effect of political polls
  - 45 % option A, 40% option B, 15% option C
  - The most socially preferred option is A => it should be the winner if all agents vote truthfully
  - The agents that prefer option C know that they have no possibility to win, and most of them (80%) prefer option B to option A => 80% vote for B and 20% for A
  - A: 48% B: 52%, and B finally wins

Plurality voting: Problems (II)

- Huge effect of irrelevant alternative
  - 2 basic options, A and B, around 50% each
  - Another –very minoritary option- C appears, attracting 1% of the voters of A
  - A: 49% B:50% C:1% and B wins
- Scarce information about the preferences of each voter
  - Each agent can only give 1 vote, even if it considers 2-3-4 “good” alternatives

Plurality voting: Problems (III)

- Strange effects

If we look at the rankings made by the agents internally, we could have:

  - 42% A C B D
  - 26% B C D A
  - 15% C D B A
  - 17% D C B A
  - A wins with 42% of the votes
    - 58% of the voters preferred other options
    - 58% consider A the worst option
    - 100% consider C the 1st-2nd best option

Plurality voting: Advantages

- Most simple voting mechanism
- Very efficient from the computational point of view
- Equality principle, as it preserves the idea of 1 agent = 1 vote

Plurality voting: Anti-Plurality

- Each voter gives a negative vote to the alternative he considers the worst
- The option with *less* votes wins
- In the previous example, A would have 58 negative votes!

Plurality voting: Anti-Plurality – Example

- 30% CBDA
- 30% CADB
- 20% ABDC
- 20% BADC

Last: C (40% negative votes) – but also first option for 60%
A and B get 30% negative votes
D is the winner with 0 negative votes – but it was not the first or second option for anyone

Best-worst voting

- Each agent gives a positive vote to his best alternative and a negative vote to his worst alternative
- Each alternative receives $\alpha > 0$ points for each positive vote and $-\delta < 0$ points for each negative vote
- The option with more points wins

Best-worst voting: Example

	1	2	3	4	5	6	7
	C	C	B	B	A	A	A
	B	B	C	C	B	C	C
	A	A	A	A	C	B	B

A $\rightarrow 3\alpha - 4\delta$ points B $\rightarrow 2\alpha - 2\delta$ points C $\rightarrow 2\alpha - \delta$ points

Plurality $(\delta = 0)$	A $\succ$ B $\sim$ C
Antiplurality $(\alpha = 0)$	C $\succ$ B $\succ$ A
$\alpha = \delta = 1$	C $\succ$ B $\succ$ A
$\alpha = 2$ and $\delta = 1$	C $\succ$ A $\sim$ B
$\alpha = 4$ and $\delta = 1$	A $\succ$ C $\succ$ B

Approval voting

- Each voter selects a subset of the candidates
- The candidate with most votes wins
- k-approval voting
  - Each voter selects a subset of *k* candidates
    k=1: plurality
    k= n-1: anti-plurality

Protocols based on linear orders

- Each voter gives a full list of the options, ordered according to his preferences (from best to worst)
- A voter prefers option *i* to option *j* if option *i* appears before option *j* in his list

Binary voting

- All the options are ordered and then evaluated in pairs (options 1 and 2, the winner with option 3, the winner with option 4, etc.)
- Simple majority: option A is better than option B if and only if the number of voters that prefer A to B is greater than the number of voters that prefer B to A
- The option that wins the last evaluation is the overall winner

$$\mathrm{win}(o_5, \mathrm{win}\ (o_4, \mathrm{win}\ (o_3, \mathrm{win}(o_2,o_1))))$$

Binary voting: The ordering problem

- x > z > y (35%)
  y > x > z (33%)
  z > y > x (32%)
- Note that y is preferred to x (65 - 35), x is preferred to z (68 - 32), and z is preferred to y (67-33)
  - win(x, win(y,z)) = x
  - win(y, win(x,z)) = y
  - win(z, win(x,y)) = z
- The order of the pairings affects the outcome!
  - The voter organiser may influence the result
  - The last options have more chances of winning
- No Neutrality

Binary voting: Another problematic example

35% of agents have preferences c > d > b > a
33% of agents have preferences a > c > d > b
32% of agents have preferences b > a > c > d

Evaluation in the order *acbd*:
Win(a, c) = a Win(a, b) = b Win(b, d) = d => d Wins

- d was the worst alternative for 32%
- d was not the best alternative for anyone
- Everybody prefers c to d (!) – No Unanimity

Binary voting: Problems summary

- Decisive role of the ordering of the alternatives
- An alternative *x* may win even if there is another alternative *x'* which is preferred to *x* by all agents
  - Alternatives may be misordered
- Temporal cost of the voting process (sequence of pairwise eliminative votes)

Borda voting (1770)

- For each voter, we assign |O| points to the preferred option, |O|-1 points to the second, and so on
  - Example: 4 possibilities, one agent considers the order ABCD
    A: 4 points, B: 3 points, C: 2 points, D: 1 point

Borda voting: Result

- The points are added across the voters and the alternative with the highest count becomes the social choice
  [similar to Eurovision song contest]

Borda voting: The Borda paradox

- a > b > c > d
- b > c > d > a
- c > d > a > b
- a > b > c > d
- b > c > d > a
- c > d > a > b
- a > b > c > d

a=18, b=19, c=20, d=13

If the worst alternative –d- is removed

- a > b > c
- b > c > a
- c > a > b
- a > b > c
- b > c > a
- c > a > b
- a > b > c

a=15, b=14, c=13

Even if we keep the relative preferences between a, b and c, the final result changes completely

Borda voting: Inverted-order paradox

- *Borda protocol* with 4 alternatives
  x=22, a=17, b=16, c=15
  Order: xabc

1. x > c > b > a
2. a > x > c > b
3. b > a > x > c
4. x > c > b > a
5. a > x > c > b
6. b > a > x > c
7. x > c > b > a

- If we remove x:
  c=15, b=14, a=13

  The second one is c, not a

1. c > b > a
2. a > c > b
3. b > a > c
4. c > b > a
5. a > c > b
6. b > a > c
7. c > b > a

Borda voting: Problems

- Most computationally expensive
  - The Eurovision Song Contest is endless…
- Eliminating (or adding) one irrelevant alternative may totally change the outcome of the protocol
  - Winner => Last
  - Second worst => Winner
- Total order changes if options are removed one by one

Borda with weak orders

- The Borda protocol has been extended to manage weak orders in different ways
- A simple one: an option *o* receives from a voter *v* as many points as the number of options that are considered worst than *o* by *v*

R_1	R_2	R_3
x_1	$x_2 \quad x_4$	x_3
$x_2 \quad x_3$	x_3	x_1
x_4	x_1	x_2
		x_4

$$x_1 \rightarrow 3+0+2=5 \qquad x_2 \rightarrow 1+2+1=4$$

$$x_3 \rightarrow 1+1+3=5 \qquad x_4 \rightarrow 0+2+0=2$$

$$x_1 \sim x_3 \succ x_2 \succ x_4$$

Condorcet voting (1785)

- Each voter ranks the candidates in order of preference
- Each candidate is compared to each other
- If a candidate wins all the comparisons, it is the winner of the election
- In the event of a tie, use another resolution method (e.g., Borda count)

Condorcet voting: Example (I)

- Election of the capital city of Tennessee
- Everybody prefers to have the capital as close as possible

42% of voters (close to Memphis)	26% of voters (close to Nashville)	15% of voters (close to Chattanooga)	17% of voters (close to Knoxville)
1. **Memphis**	1. **Nashville**	1. **Chattanooga**	1. **Knoxville**
2. Nashville	2. Chattanooga	2. Knoxville	2. Chattanooga
3. Chattanooga	3. Knoxville	3. Nashville	3. Nashville
4. Knoxville	4. Memphis	4. Memphis	4. Memphis

Condorcet voting: Example (II)

Pair	Winner
Memphis (42%) vs. Nashville (58%)	Nashville
Memphis (42%) vs. Chattanooga (58%)	Chattanooga
Memphis (42%) vs. Knoxville (58%)	Knoxville
Nashville (68%) vs. Chattanooga (32%)	Nashville
Nashville (68%) vs. Knoxville (32%)	Nashville
Chattanooga (83%) vs. Knoxville (17%)	Chattanooga

		A			
		Memphis	**Nashville**	**Chattanooga**	**Knoxville**
B	**Memphis**		[A] 58% [B] 42%	[A] 58% [B] 42%	[A] 58% [B] 42%
	Nashville	[A] 42% [B] 58%		[A] 32% [B] 68%	[A] 32% [B] 68%
	Chattanooga	[A] 42% [B] 58%	[A] 68% [B] 32%		[A] 17% [B] 83%
	Knoxville	[A] 42% [B] 58%	[A] 68% [B] 32%	[A] 83% [B] 17%	
Ranking:		4th	1st	2nd	3rd

Nashville wins

Condorcet voting: Problem

- Possibility of circular ambiguities
  - No alternative wins to all the other alternatives
  - There are many ways to resolve them
    - Keep the candidate that wins more matches (Copeland)
    - Take into account the relative strengths of defeats (Minimax, Ranked Pairs, Schulze, …)
  - You can look at the winning votes or at the winning margin

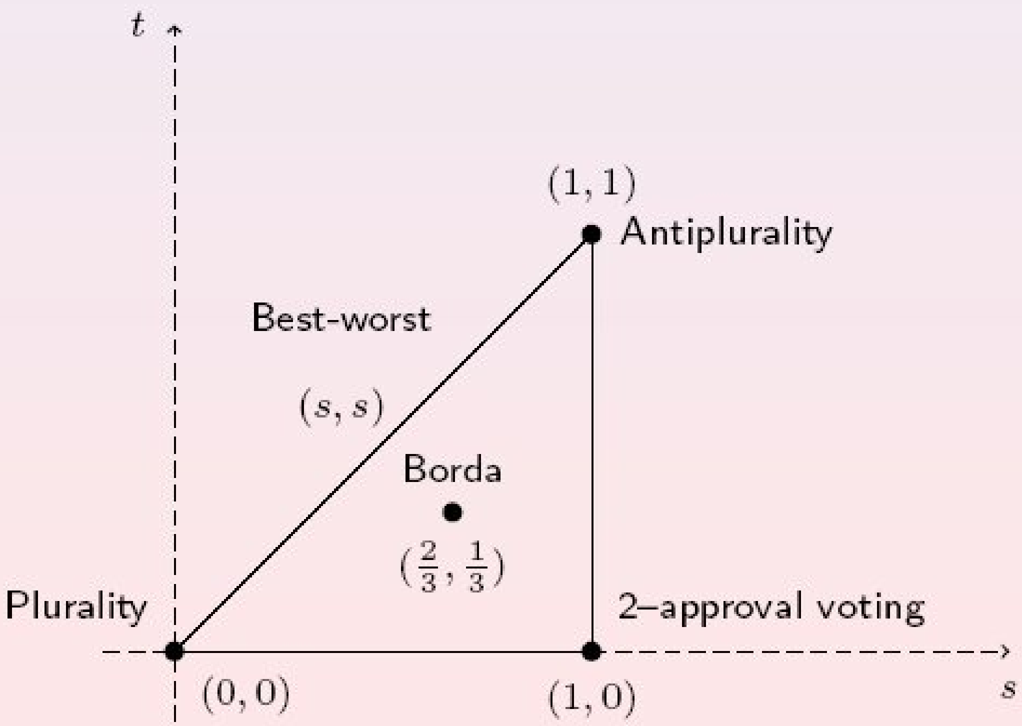
Generalization of voting protocols

- Scoring vector
- $S = (s_1, \ldots, s_n)$ with $s_i >= s_{i+1}$ and $s_1 > s_n$
- Each voter gives s_i points to his *i*-th option
- The option with more points wins
- Examples with 4 options
  - vector (4,3,2,1) => Borda protocol
  - vector (1,0,0,0) => Plurality protocol
  - vector (0,0,0,-1) => Anti-plurality protocol
  - vector (1,0,0,-1) => Best-worst

Normalisation of scoring rules

$$(s_1, \ldots, s_n) \equiv \left(1, \frac{s_2 - s_n}{s_1 - s_n}, \ldots, \frac{s_{n-1} - s_n}{s_1 - s_n}, 0\right)$$

$$n = 4: \ \{(1, s, t, 0) \mid 0 \leq t \leq s \leq 1\} \equiv \{(s, t) \in [0, 1]^2 \mid t \leq s\}$$



Example of the influence of the protocol (I)

- 45 ABCD, 25 CDBA, 20 DBCA, 10 DCBA
- Plurality protocol: A - 45   B - 0   C - 25   D - 30
  - A wins quite clearly, B has no votes
  - However, note that 55% say that A is the worst option (!)

Example of the influence of the protocol (II)

- 45 ABCD, 25 CDBA, 20 DBCA, 10 DCBA

- Pairwise comparisons
  - A loses with B,C and D
  - B wins C, C wins D, D wins B
- Binary protocol
  - A can never win (!)
  - Order ABCD: D wins
  - Order BDAC: C wins
  - Order CDBA: B wins

Example of the influence of the protocol (III)

- 45 ABCD, 25 CDBA, 20 DBCA, 10 DCBA

- Condorcet protocol: no option beats all the others, and B/C/D tied with 2 victories
- Borda protocol
  - A: 235   B: 265 C: 260 D: 240
  - B wins (although all the options get similar values)
  - B is not the first option for anyone !!!
  - B is a good consensual option (2nd for 65%, 3rd for 35%), whereas A was very problematic (45% very positive support, but 55% very negative support)
  - Recall that B had 0 votes in the plurality protocol (!)

More complex voting mechanisms

- Use of linguistic information to represent the opinion of each voter with respect to each alternative
- Management of uncertainty in the voter's opinions

Use of linguistic information

- Each voter can use a linguistic label from a predetermined set to evaluate each alternative

R	To Reject
P	Poor
A	Acceptable
G	Good
VG	Very Good
E	Excellent

	1	2	3	4	5	6	7
Alan	A	R	VG	E	A	P	A
Benny	VG	E	G	VG	G	G	G
Charles	G	P	E	R	G	G	VG

Use of linguistic information: Evaluation

- The evaluation of each of the alternatives is also a linguistic label, obtained with an aggregation of the voter's opinions
- Aggregation function
  - Idempotent
  - Monotonic
- Example: the result of the aggregation of $A = \{a_1, \ldots, a_n\}$ can be the label l_i that minimizes the distance$(A, \{l_i, \ldots, l_i\})$

Use of linguistic information: Distance measure

- The distance between two vectors of linguistic labels may be defined as the addition of the distances between the labels in each position
  - Distance(Poor, Poor) = 0
  - Distance(Poor, Good) = 2
- Distance(Reject, Excellent) = 5

R	To Reject
P	Poor
A	Acceptable
G	Good
VG	Very Good
E	Excellent

Use of linguistic information: Example

R	To Reject
P	Poor
A	Acceptable
G	Good
VG	Very Good
E	Excellent

	1	2	3	4	5	6	7
Alan	A	R	VG	E	A	P	A
Benny	VG	E	G	VG	G	G	G
Charles	G	P	E	R	G	G	VG

	R	*P*	*A*	*G*	*VG*	*E*	***Global evaluation***
Alan	16	11	8	11	14	19	Acceptable
Benny	25	18	11	4	5	10	Good (Very Good)
Charles	19	14	11	8	11	16	Good

Use of linguistic information: Uncertain preferences

- Each voter represents the opinion on every alternative with an interval defined over an ordered set of linguistic labels

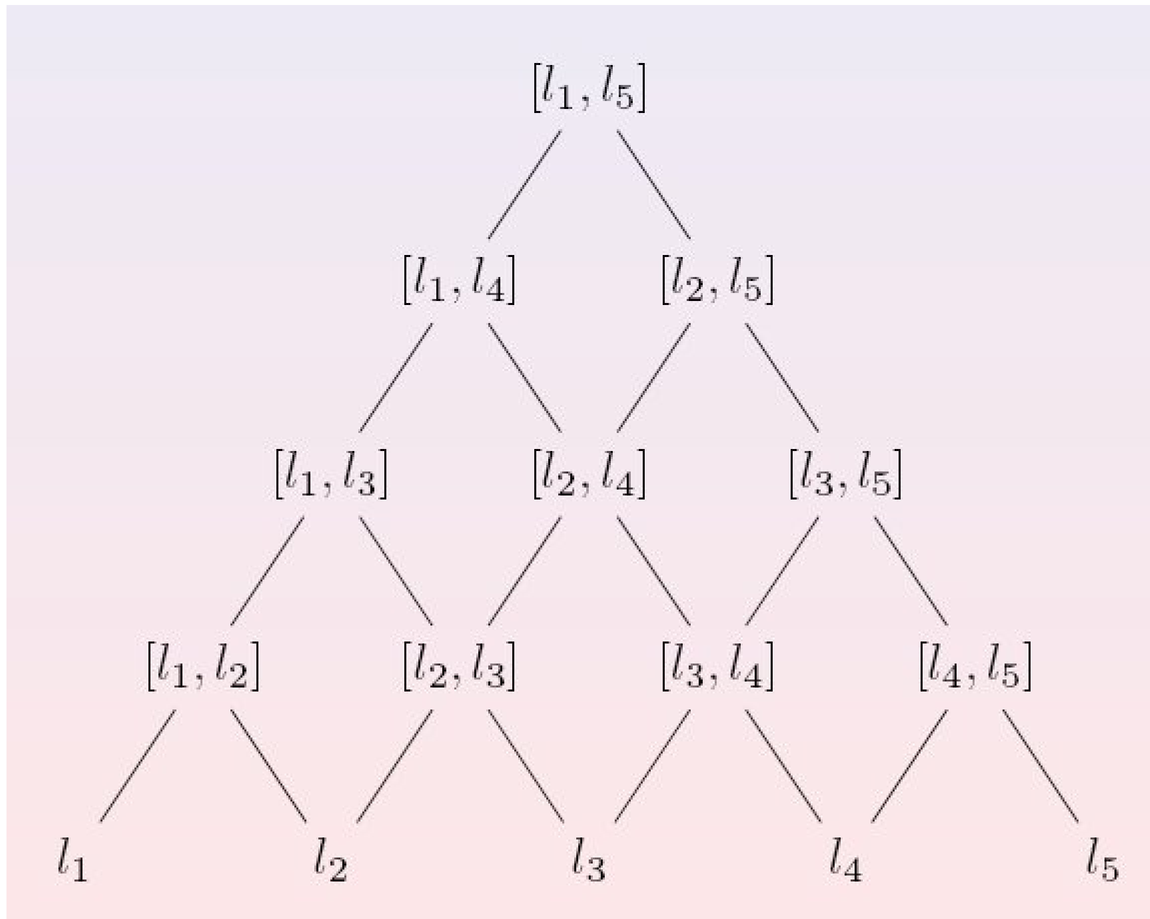
l_1	l_2	l_3	l_4	l_5
very bad	bad	acceptable	good	very good

$[l_4, l_5]$ means *between good and very good*

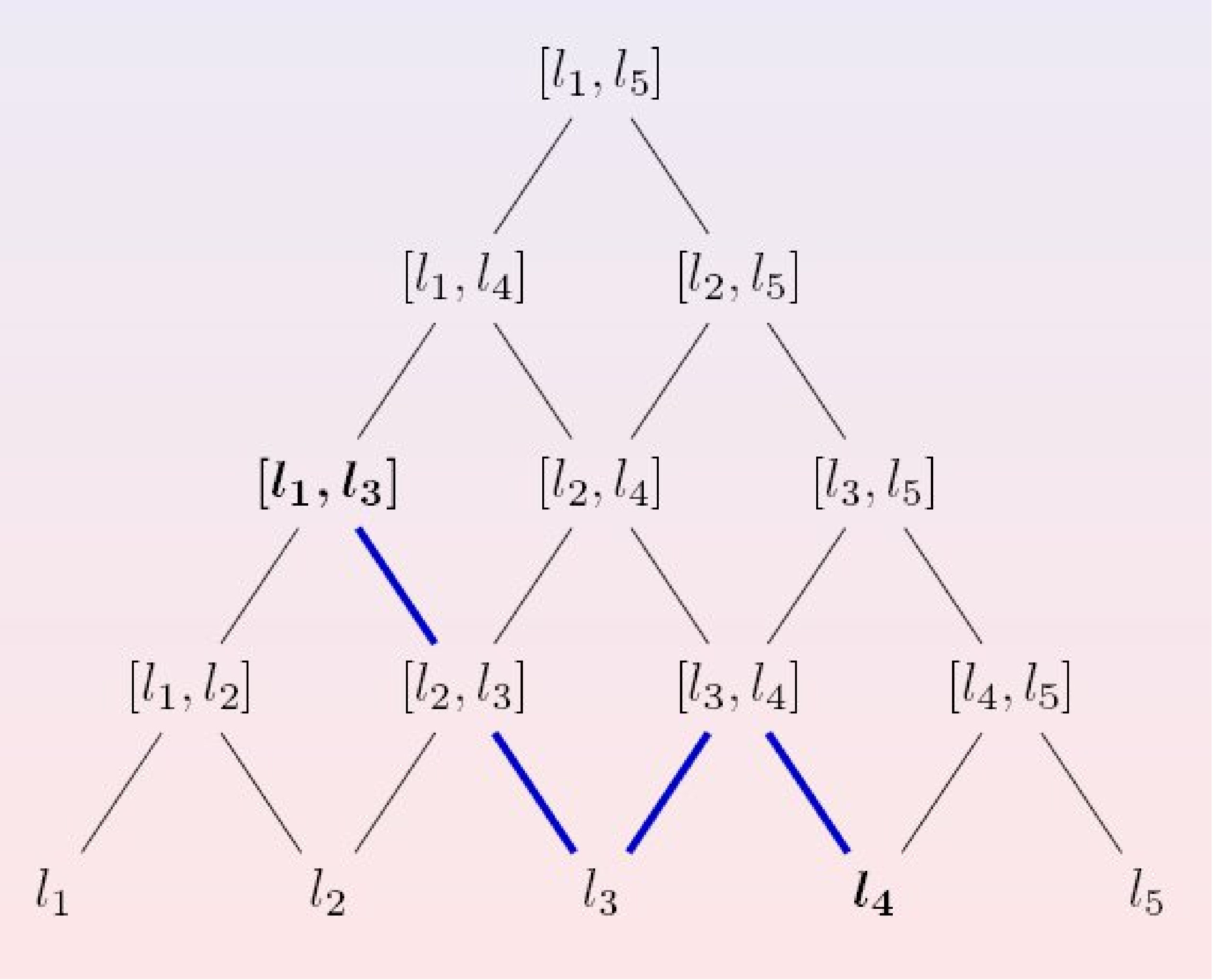
Use of linguistic information: Graph of intervals (5 labels)

More general, imprecise

More specific, precise



Use of linguistic information: Graph of intervals (5 labels)



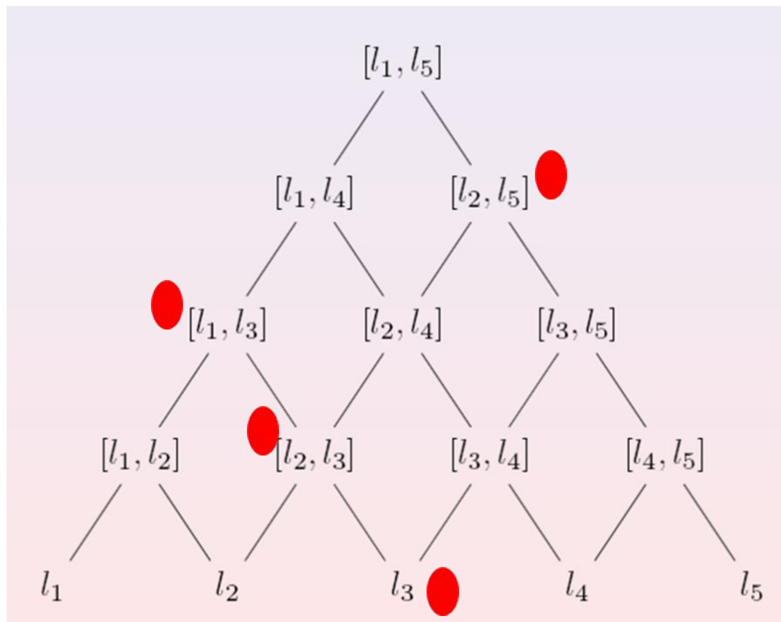
Use of linguistic information: Evaluation of an alternative

- Given *m* opinions (linguistic intervals) on a certain item x_i, the result of the aggregation of the *m* opinions may be defined as the interval *I* that minimizes the addition of the distances from *I* to each of the opinions

$$E(x_i) = \left\{ \mathcal{E} \in \mathbb{L} \mid \forall \mathcal{F} \in \mathbb{L} \ \sum_{p=1}^{m} d(\mathcal{E}, v_i^p) \leq \sum_{p=1}^{m} d(\mathcal{F}, v_i^p) \right\}$$

Use of linguistic information: Example

	Agent 1	*Agent 2*	*Agent 3*	*Agent 4*	*Joint opinion*
Option A	(A, VG)	(G, G)	(A, G)	(B,G)	(A, G)
Option B	(A, A)	(VB, A)	(B, A)	(B, VG)	(B, A)
Option C	(B, G)	(B, A)	(VG, VG)	(B,G)	(B, G)



l_1	l_2	l_3	l_4	l_5
very bad	bad	acceptable	good	very good

Proposed readings

- Chapter 12 of the book by M. Wooldridge “An introduction to Multi-Agent Systems” (2nd edition)
- Chapter 9 of the book by M. Fasli “Agent technology for e-commerce”