



System of ODEs

We define the parameter $\mu = d = b$.

$$\begin{aligned} \frac{d\mathbf{S}_1}{dt} = & \mu N_1 - \beta(\mathbf{I}_1/N_1)\mathbf{S}_1 \\ & - \mu\mathbf{S}_1 + \rho\mathbf{R}_1 \end{aligned} \quad (1.1)$$

$$\begin{aligned} \frac{d\mathbf{I}_1}{dt} = & \beta(\mathbf{I}_1/N_1)\mathbf{S}_1 - p\tau\mathbf{I}_1 \\ & - \mu\mathbf{I}_1 - (1-p)\gamma\mathbf{I}_1 \end{aligned} \quad (1.2)$$

$$\begin{aligned} \frac{d\mathbf{T}_1}{dt} = & p\tau\mathbf{I}_1 - r\mathbf{T}_1 - \mu\mathbf{T}_1 \end{aligned} \quad (1.3)$$

$$\begin{aligned} \frac{d\mathbf{R}_1}{dt} = & r\mathbf{T}_1 - \mu\mathbf{R}_1 - \rho\mathbf{R}_1 \\ & + (1-p)\gamma\mathbf{I}_1 \end{aligned} \quad (1.4)$$

$$\begin{aligned} \frac{d\mathbf{S}_2}{dt} = & \mu N_2 - \beta(\mathbf{I}_2/N_2)\mathbf{S}_2 \\ & - \mu\mathbf{S}_2 + \rho\mathbf{R}_2 \end{aligned} \quad (1.5)$$

$$\begin{aligned} \frac{d\mathbf{I}_2}{dt} = & \beta(\mathbf{I}_2/N_2)\mathbf{S}_2 - p\tau\mathbf{I}_2 \\ & - \mu\mathbf{I}_2 - (1-p)\gamma\mathbf{I}_2 \end{aligned} \quad (1.6)$$

$$\begin{aligned} \frac{d\mathbf{T}_2}{dt} = & p\tau\mathbf{I}_2 - r\mathbf{T}_2 - \mu\mathbf{T}_2 \end{aligned} \quad (1.7)$$

$$\begin{aligned} \frac{d\mathbf{R}_2}{dt} = & r\mathbf{T}_2 - \mu\mathbf{R}_2 - \rho\mathbf{R}_2 \\ & + (1-p)\gamma\mathbf{I}_2 \end{aligned} \quad (1.8)$$