



### System of ODEs

We define the parameter  $\mu = d = b$ .

$$\begin{aligned} \frac{d\mathbf{S}}{dt} = & \mu\mathbf{N} - p\beta\mathbf{I}\mathbf{S} - \mu\mathbf{S} + \rho\mathbf{R} \\ & - (1-p)\beta\mathbf{I}\mathbf{S} \end{aligned} \quad (1.1)$$

$$\frac{d\mathbf{I}}{dt} = (1-p)\beta\mathbf{I}\mathbf{S} - \gamma\mathbf{I} - \mu\mathbf{I} \quad (1.2)$$

$$\frac{d\mathbf{T}}{dt} = p\beta\mathbf{I}\mathbf{S} - r\mathbf{T} - \mu\mathbf{T} \quad (1.3)$$

$$\frac{d\mathbf{R}}{dt} = r\mathbf{T} - \mu\mathbf{R} - \rho\mathbf{R} + \gamma\mathbf{I} \quad (1.4)$$