Chapter 18: SIMULTANEOUS EQUATION MODELS

EXAMPLE

Consider the Keynesian National Income model (structural form)

Consumption function	$C_t = \beta_0 + \beta_1 Y_t + u_t$	$0 < \beta_1 < 1$
Income identity	$Y_t = C_t + I_t$	

where

$$\begin{aligned} C &= \text{consumption expenditure} \\ Y &= \text{income} \\ I &= \text{exogenous investment} \\ t &= \text{time} \\ u &= \text{stochastic disturbance term} \\ \beta_0 \text{ and } \beta_1 &= \text{parameters} \end{aligned}$$

Assumptions as before

The **reduced form** is: (any one of the following)

$$
\begin{aligned}
Y_t &= \beta_0 + \beta_1 Y_t + u_t + I_t \\
Y_t &= \frac{\beta_0}{1-\beta_1} + \frac{1}{1-\beta_1} I_t + \frac{1}{1-\beta_1} u_t \\
Y_t &= \pi_1 + \pi_2 I_t + u_t^* \qquad - A
\end{aligned}
$$

with

$$
\begin{aligned}
\pi_1 &= \frac{\beta_0}{1-\beta_1} \\
\pi_2 &= \frac{1}{1-\beta_1} \\
u_t^* &= \frac{1}{1-\beta_1} u_t
\end{aligned}
$$

and

$$
\begin{aligned}
C_t &= \beta_0 + \beta_1 (C_t + I_t) + u_t \\
C_t &= \frac{\beta_0}{1-\beta_1} + \frac{\beta_1}{1-\beta_1} I_t + \frac{1}{1-\beta_1} u_t \\
C_t &= \pi_3 + \pi_4 I_t + u_t^* \qquad - B
\end{aligned}
$$

with

$$
\begin{aligned}
\pi_3 &= \frac{\beta_0}{1-\beta_1} \\
\pi_4 &= \frac{\beta_1}{1-\beta_1} \\
u_t^* &= \frac{1}{1-\beta_1} u_t
\end{aligned}
$$

Consider the following data (40 observations - only 5 shown):

Obs	i	y	c
1	1004.59	2259.47	1254.88
2	1008.47	2312.56	1304.09
3	1039.48	2258.15	1218.66
4	1053.86	2302.86	1248.99
5	1076.32	2301.93	1225.62

In order to estimate without the problem of simultaneous equation bias we estimate one of the equations A or B above

NB - In this example only **one equation** is sufficient - not always the case

The regression results for both equations A and B follow:

```
                    The REG Procedure
                      Model: MODEL1
                  Dependent Variable: y

         Number of Observations Read          40
         Number of Observations Used          40

                   Analysis of Variance

                              Sum of           Mean
Source                 DF    Squares         Square  F Value  Pr > F

Model                   1    2664409        2664409  1131.84  <.0001
Error                  38      89454     2354.05133
```

Corrected Total	39	2753863				

Root MSE	48.51857	R-Square	0.9675
Dependent Mean	2689.62995	Adj R-Sq	0.9667
Coeff Var	1.80391		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	160.41607	75.56880	2.12	0.0403
i	1	2.03311	0.06043	33.64	<.0001

The REG Procedure
Model: MODEL2
Dependent Variable: c

Number of Observations Read	40
Number of Observations Used	40

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Value	Pr > F
Model	1	687971	687971	292.25	<.0001
Error	38	89454	2354.05133		
Corrected Total	39	777425			

Root MSE	48.51857	R-Square	0.8849
Dependent Mean	1445.61473	Adj R-Sq	0.8819
Coeff Var	3.35626		

Parameter Estimates

Variable	DF	Parameter Estimate	Standard Error	t Value	Pr > |t|
Intercept	1	160.41607	75.56880	2.12	0.0403
i	1	1.03311	0.06043	17.10	<.0001

Yielding the following estimated eqiations:

$$
\begin{aligned}
Y_t &= \pi_1 + \pi_2 I_t + u_t^* && - A \\
\hat{Y}_t &= 160.41607 + 2.03311 I_t && - A \\
&\text{with} \\
\pi_1 &= \frac{\beta_0}{1-\beta_1} \\
\pi_2 &= \frac{1}{1-\beta_1} \\
u_t^* &= \frac{1}{1-\beta_1} u_t \\
&\text{and} \\
C_t &= \pi_3 + \pi_4 I_t + u_t^* && - B \\
\hat{C}_t &= 160.41607 + 1.03311 I_t && - B \\
&\text{with} \\
\pi_3 &= \frac{\beta_0}{1-\beta_1} \\
\pi_4 &= \frac{\beta_1}{1-\beta_1} \\
u_t^* &= \frac{1}{1-\beta_1} u_t
\end{aligned}
$$

Remember: the objective is to estimate the unknown structural regression parameters

From the estimate of equation A:

$$\begin{aligned}
\hat{\pi}_2 &= \frac{1}{1-\hat{\beta}_1} \\
1-\hat{\beta}_1 &= \frac{1}{\hat{\pi}_2} \\
\hat{\beta}_1 &= 1-\frac{1}{\hat{\pi}_2} \\
\hat{\beta}_1 &= 1-\frac{1}{2.03311} \\
\hat{\beta}_1 &= 0.50814 \\
\hline
\hat{\pi}_1 &= \frac{\hat{\beta}_0}{1-\hat{\beta}_1} \\
\hat{\beta}_0 &= (1-\hat{\beta}_1)\hat{\pi}_1 \\
\hat{\beta}_0 &= (1-0.50814)160.41607 \\
\hat{\beta}_0 &= 78.90182
\end{aligned}$$

From the estimate of equation B:

$$
\begin{aligned}
\hat{\pi}_4 &= \frac{\hat{\beta}_1}{1-\hat{\beta}_1} \\
(1-\hat{\beta}_1)\hat{\pi}_4 &= \hat{\beta}_1 \\
\hat{\pi}_4 - \hat{\beta}_1\hat{\pi}_4 &= \hat{\beta}_1 \\
\hat{\beta}_1 + \hat{\beta}_1\hat{\pi}_4 &= \hat{\pi}_4 \\
\hat{\beta}_1(1+\hat{\pi}_4) &= \hat{\pi}_4 \\
\hat{\beta}_1 &= \frac{\hat{\pi}_4}{(1+\hat{\pi}_4)} \\
\hat{\beta}_1 &= \frac{1.03311}{(1+1.03311)} \\
\hat{\beta}_1 &= 0.50814 \\
\hline
\hat{\pi}_3 &= \frac{\hat{\beta}_0}{1-\hat{\beta}_1} \\
\hat{\beta}_0 &= (1-\hat{\beta}_1)\hat{\pi}_3 \\
\hat{\beta}_0 &= (1-0.50814)160.41607 \\
\hat{\beta}_0 &= 78.90182
\end{aligned}
$$

The final estimated structural model is:

Consumption function	$\hat{C}_t$	$= 78.90182 + 0.50814Y_t$
Income identity	$\hat{Y}_t$	$= \hat{C}_t + I_t$