Chapter 18: SIMULTANEOUS EQUATION MODELS

NATURE OF SIMULTANEOUS EQUATION SYSTEMS

- STK 310 / RAL 780 / EKT 713 - Single equation models
- Basic assumptions NB!!!!
- Prediction of Y conditional on fixed values of the X variables.
- Unidirectional causal realtionship
- EKT 720 - Part II - Simultaneous equation systems
- Two-way or simultaneous relationship between the X and Y variables
- Move away frome dependent and independent variables to endogenous and exogenous variables
- Do not estimate the parameters of a single equation without taking into account the information provided by the other equations of the system.
- OLS - ????

Crucial asssumption - the explanatory variables are non stochastic, or if stochastic are distributed independently from the stochastic disturbance term.

- If neither of these hold then - estimators are **biased** and **inconsistent**
- Examples of Simulataneous Equation models
  (a) Demand and Supply model
  (b) National Income model
  (c) Wage Price model
  (d) IS Model of macro-economics
  (e) LM Model of macro-economics

THE SIMULTANEOUS-EQUATION BIAS: INCONSISTENCY OF OLS ESTIMATORS

- Consider the Keynesian National Income model

Consumption function	$C_t = \beta_0 + \beta_1 Y_t + u_t$	$0 < \beta_1 < 1$
Income identity	$Y_t = C_t + I_t$	

where

$$\begin{aligned} C &= \text{consumption expenditure} \\ Y &= \text{income} \\ I &= \text{exogenous investment} \\ t &= \text{time} \\ u &= \text{stochastic disturbance term} \\ \beta_0 \text{ and } \beta_1 &= \text{parameters} \end{aligned}$$

Assume that

$$E(u_t) = 0$$

$$E(u_t^2) = \sigma^2$$

$$E(u_t; u_{t+j}) = 0 \quad for \quad i \neq j$$

$$Cov(I_t, u_t) = 0$$

Correlation between u_t and Y_t:

$$\begin{aligned}
Y_t &= \beta_0 + \beta_1 Y_t + u_t + I_t \\
Y_t &= \frac{\beta_0}{1-\beta_1} + \frac{1}{1-\beta_1} I_t + \frac{1}{1-\beta_1} u_t
\end{aligned}$$

Now

$$E(Y_t) = \frac{\beta_0}{1-\beta_1} + \frac{1}{1-\beta_1} I_t$$

Therefore

$$Y_t - E(Y_t) = \frac{u_t}{1-\beta_1}$$

and

$$u_t - E(u_t) = u_t$$

yielding

$$\begin{aligned}
Cov(Y_t, u_t) &= E[Y_t - E(Y_t)][u_t - E(u_t)] \\
&= E[\frac{u_t}{1-\beta_1}][u_t] \\
&= \frac{E(u_t^2)}{1-\beta_1} \\
&= \frac{\sigma^2}{1-\beta_1}
\end{aligned}$$

indicating a non-zero correlation between u_t and Y_t:

<u>Inconsistency of the estimator $\hat{\beta}_1$:</u>

$$\begin{aligned}\hat{\beta}_1 &= \frac{\Sigma(C_t - \overline{C})(Y_t - \overline{Y})}{\Sigma(Y_t - \overline{Y})^2} \\ &= \frac{\Sigma c_t y_t}{\Sigma y_t^2} \\ &= \frac{\Sigma C_t y_t}{\Sigma y_t^2}\end{aligned}$$

where the lowercase letters indicate deviation form

$$\begin{aligned}\hat{\beta}_1 &= \frac{\Sigma(\beta_0 + \beta_1 Y_t + u_t) y_t}{\Sigma y_t^2} \\ &= \beta_1 + \frac{\Sigma y_t u_t}{\Sigma y_t^2}\end{aligned}$$

If we take expectations on both sides:

$$E(\hat{\beta}_1) = \beta_1 + E\left(\frac{\Sigma y_t u_t}{\Sigma y_t^2}\right)$$

Unfortunately we cannot evaluate $E\left(\frac{\Sigma y_t u_t}{\Sigma y_t^2}\right)$ since the expectations operator is a linear operator.
Intuitively : If $\left(\frac{\Sigma y_t u_t}{\Sigma y_t^2}\right) = 0$ the the estimator is unbiased.
Compare - What did we prove in the previous section??????

$$Cov(Y_t, u_t) \neq 0$$

- sample versus population !!!!!!

But if the sample increases the sample measure will tend toward the population measure.

Use the large sample properties - probability limit:

$$\begin{aligned}
plim(\hat{\beta}_1) &= plim(\beta_1) + plim\left(\frac{\Sigma y_t u_t}{\Sigma y_t^2})\right) \\
&= plim(\beta_1) + plim\left(\frac{\Sigma y_t u_t/n}{\Sigma y_t^2/n}\right) \\
&= plim(\beta_1) + \left(\frac{plim(\Sigma y_t u_t)/n}{plim(\Sigma y_t^2)/n}\right) \\
&= \beta_1 + \left(\frac{\sigma^2/(1-\beta_1)}{\sigma_Y^2}\right) \\
&= \beta_1 + \frac{1}{1-\beta_1}\left(\frac{\sigma^2}{\sigma_Y^2}\right)
\end{aligned}$$

Yielding a biased and inconsitent estimator.