Pattern Recognition

Homework 2

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0.1 Find the maximum likelihood estimate of θ

Ans:

$$p(x|\theta) = \frac{\theta^x e^{-\theta}}{x!}, \quad x = 0, 1, 2...$$

$$p(D|\theta) = \prod_{k=1}^{n} p(x_k|\theta)$$

$$l(\theta) = \ln P(D|\theta) = \sum_{k=1}^{n} p(x_k|\theta)$$

$$\frac{\partial}{\partial\theta} l(\theta) = 0$$

$$\frac{\partial l(\theta)}{\partial\theta} = \frac{1}{\sum_{k=1}^{n} k!}\left[\frac{(n+1)n}{2}\theta^{\frac{n^2+n-2}{2}} e^{-\theta n} + \theta^{\frac{n^2+n}{2}}(-n)e^{-\theta n}\right] = 0$$

$$= \frac{1}{\sum_{k=1}^{n} k!}\left(\frac{(n+1)n}{2\theta} - n\right)\theta^{\frac{n^2+n}{2}} e^{-\theta n} = 0$$

$$\frac{(n+1)n}{2\theta} = n$$

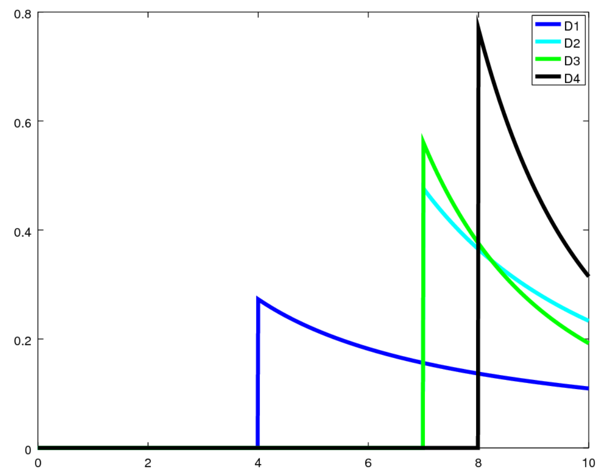
$$\frac{n+1}{2} = \theta$$

0.2 Recursive Bayes Learning

(a) Calculate and plot the posterior densities $p(\theta|D^1)$, $p(\theta|D^2)$, $p(\theta|D^3)$, $and\ p(\theta|D^4)$
Ans :

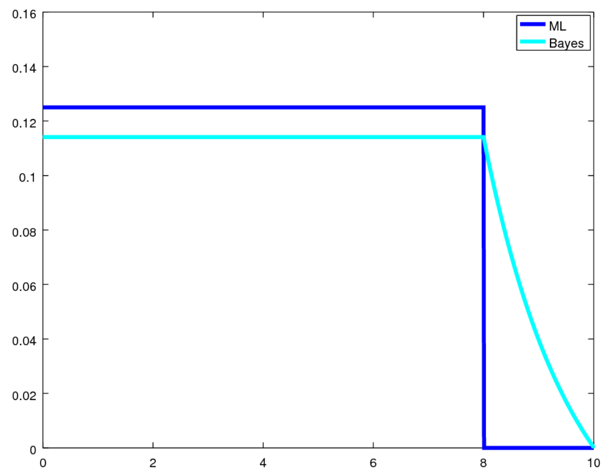
$$P(\theta|D^0) = p(\theta) = U(0, 10)$$

$$p(\theta|D^n) = \frac{p(x_n|\theta)p(\theta|D^{n-1})}{\int p(x_n|\theta)p(\theta|D^{n-1})d\theta}$$



(b) Calculate and plot the desired density $p(\theta|D^4)$
Ans :

$$p(x|D) = \int p(x|\theta)p(\theta|D)d\theta$$



0.3 Principal Component Analysis

(a) Determine the scatter matrix
Ans :

$$mean = \frac{1}{30}\sum_{k=1}^{30} x_k = \begin{bmatrix} 0.0304 \\ 0.0418 \\ -0.0762 \end{bmatrix}$$

$$S = \sum_{k=1}^{n}(x_k - m)(x_k - m)^t$$

$$s = \begin{bmatrix} 13.5236 & 10.3097 & 4.6601 \\ 10.3097 & 55.2695 & 13.0659 \\ 4.6601 & 13.0659 & 70.8436 \end{bmatrix}$$

(b) Determine the two largest eigenvalues of the scatter matrix and the corresponding eigenvectors. Ans:
criterion function :

$$J(x_0) = \sum_{k=1}^{n} \|x_0 - x_k\|^2$$

$$a_k = e^t(x_k - m) \qquad x_0 = \frac{1}{n}\sum_{k=1}^{n} x_k = m + ae$$

$$J(e) = \sum_{k=1}^{n} \|(m + a_k e) - x_k\|^2$$

$$= \sum_{k=1}^{n} \|a_k e - (x_k - m)\|^2$$

$$= \sum_{k=1}^{n} a_k^2 \|e\|^2 - 2\sum_{k=1}^{n} a_k e^t(x_k - m) + \sum_{k=1}^{n} \|x_k - m\|^2$$

$$= -\sum_{k=1}^{n}[e^t(x_k - m)]^2 + \sum_{k=1}^{n} \|X_k - m\|^2$$

$$= -\sum_{k=1}^{n} e^t(x_k - m)(x_k - m)^t e + \sum_{k=1}^{n} \|x_k - m\|^2$$

$$= -e^t S e + \sum_{k=1}^{n} \|x_k - m\|^2$$

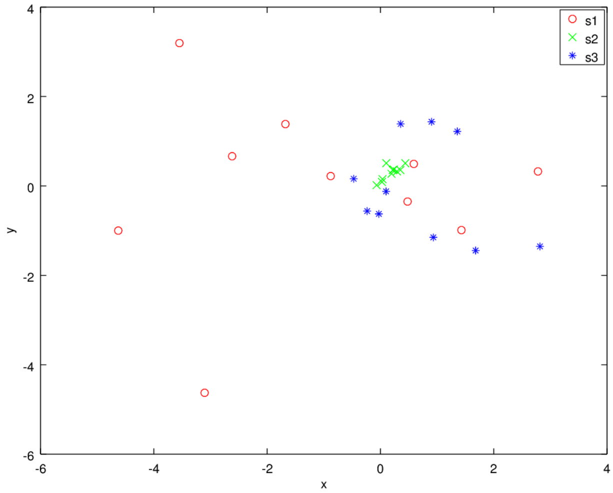
$$Assume : \|e\|^2 = 1$$

$$Se = \lambda e \quad , e^t S e = \lambda e^t e = \lambda$$

$$eigen\ vector = \begin{bmatrix} 0.1710 & 0.1400 \\ 0.8290 & 0.5145 \\ -0.5324 & 0.8460 \end{bmatrix}$$

$$eigen\ value = \begin{bmatrix} 49.0038 & 79.5607 \end{bmatrix}$$

(c) Plot the projected data points on the two-dimensional subspace
Ans:



(d) Assume that the prior probabilities are equal and the class-conditional probability density functions are normal. Calculate the percentage of misclassified samples in the two-dimensional subspace Ans :

$$Error\ rate = \frac{1 - correct\ sample}{total\ sample}$$

$$e_1 = 0.5 \qquad e_2 = 0 \qquad e_3 = 0$$

0.4 Fisher Linear Discriminant

(a) Determine the within-class scatter matrix S_W and the between-class scatter matrix S_B
Ans :

$$S_i = \sum_{x \in D_i}^{c} (x - m_i)(x - m_i)^t$$

$$S_w = \sum_{i=1}^{c} S_i$$

$$s_1 = \begin{bmatrix} 9.0618 & 5.6778 & 3.9408 \\ 5.6678 & 42.0071 & 7.3370 \\ 3.9408 & 7.3370 & 45.4195 \end{bmatrix}$$

$$s_2 = \begin{bmatrix} 0.5393 & -0.1465 & -0.0518 \\ -0.1465 & 0.4597 & 0.0851 \\ -0.0518 & 0.0851 & 0.0727 \end{bmatrix}$$

$$s_3 = \begin{bmatrix} 3.0186 & 4.0474 & -1.8042 \\ 4.4074 & 6.4496 & -2.0130 \\ -1.8042 & -2.0130 & 12.6214 \end{bmatrix}$$

$$s_w = \begin{bmatrix} 12.6196 & 9.5787 & 2.0848 \\ 9.5787 & 48.9165 & 5.4091 \\ 2.0848 & 5.4091 & 58.1136 \end{bmatrix}$$

$$S_{Bi} = n_i(m_i - m)(m_i - m)^t$$

$$S_B = S_{B1} + S_{B2} + S_{B3}$$

$$m_i = \frac{1}{10}\sum_{k=1}^{10} x_{ki} \quad , n_i = number\ of\ sample \quad , m = \sum_{i=1}^{3} m_i$$

$$S_{B1} = \begin{bmatrix} 0.1026 & 0.6549 & 0.8456 \\ 0.6549 & 4.1792 & 5.3965 \\ 0.8456 & 5.3965 & 6.9685 \end{bmatrix}$$

$$S_{B2} = \begin{bmatrix} 0.2045 & -0.5550 & -0.1143 \\ -0.5550 & 1.5065 & 0.3103 \\ -0.1143 & 0.3103 & 0.0639 \end{bmatrix}$$

$$S_{B3} = \begin{bmatrix} 0.5968 & 0.6311 & 1.8440 \\ 0.6311 & 0.6674 & 1.9500 \\ 1.8440 & 1.9500 & 5.6976 \end{bmatrix}$$

$$S_B = \begin{bmatrix} 0.9039 & 0.7309 & 2.5753 \\ 0.7309 & 6.3530 & 7.6568 \\ 2.5753 & 7.6568 & 12.7300 \end{bmatrix}$$

(b) (b) Determine the two largest eigenvalues of -1 and the corresponding eigenvectors.
Ans :

$$Assume\ f(w) = w^T S_B w\ , st\ g(w) = w^T S_w w = c$$

$$\frac{\partial}{\partial w}[f(w) - \lambda(g(w) - c)] = 0$$

$$\frac{\partial}{\partial w}[w^T S_B w - \lambda(w^T S_B w - c)] = (S_B + S_B^T)w - \lambda(S_w + S_w^T) = 0$$

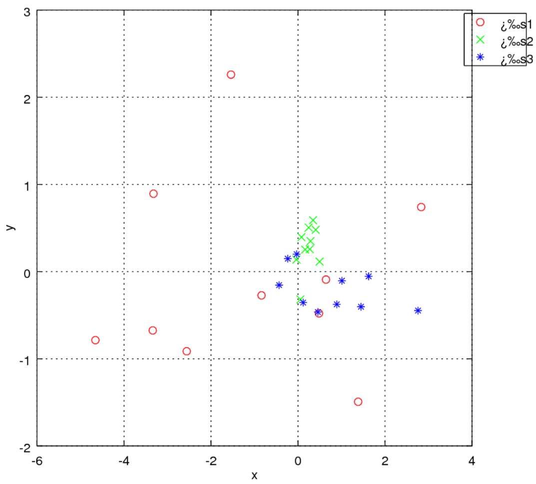
$$S_B w = \lambda S_w w$$

$$S_w^{-1} S_B w = \lambda S_w w = \lambda w$$

$$\max J(w) = \frac{w^T S_B w}{w^T S_w w} = \lambda_{max} = v_{max} = \begin{bmatrix} 0.1386 & -0.9081 \\ 0.5653 & 0.4046 \\ 0.8132 & -0.1079 \end{bmatrix}$$

$$Eigenvalues = \begin{bmatrix} 0.2972 & 0 \\ 0 & 0.1028 \\ 0 & 0 \end{bmatrix}$$

(c) Plot the projected data points on the two-dimensional subspace.
Ans :



(d) Assume that the prior probabilities are equal and the class-conditional probability density functions are normal. Calculate the percentage of misclassified samples in the two-dimensional subspace.
Ans :

$$Error\ rate = \frac{1 - correct\ sample}{total\ sample}$$

$$e_1 = 0.3 \qquad e_2 = 0.1 \qquad e_3 = 0.1$$