1 The Koch Snowflake

The *Koch snowflake*, one of the first fractals, is based on work by the Swedish mathematician Helge von Koch [1]. It is what we get if we start with an equilateral triangle

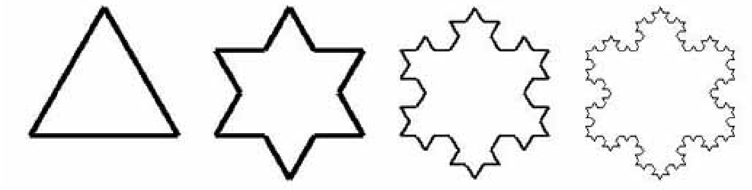

Figure 1: The Koch snowflake after 0, 1, 2, and 3 iterations.

and repeat the following an infinite number of times:

> *Divide all line segments into three segments of equal length. Then draw, for each middle line segment, an equilateral triangle that has the middle segment as its base and points outward. Finally, remove all middle segments.*

Figure 1 shows the first iterations in the construction. (Original)

2 Two properties

Theorem 1. *The Koch snowflake has infinite length.*

Proof. Let Δ denote a triangle, with side length s, on which we base the construction of a snowflake. Let N_i denote the number of line segments, and L_i the length of the segments, in iteration i of the construction. Then

$$N_n = \begin{cases} 3, & \text{if } n = 0 \text{ (i.e. before any iterations), and} \\ 4N_{n-1}, & \text{otherwise.} \end{cases}$$

This solves to

$$N_n = 3 \cdot 4^n, \tag{1}$$

while

$$L_n = \frac{L_{n-1}}{3} = \frac{L_{n-2}}{3^2} = \frac{L_{n-3}}{3^3} = \ldots = \frac{L_0}{3^n} = \frac{s}{3^n} \tag{2}$$

From Eqs. 1 and 2, the total length

$$N_n L_n = 3 \cdot 4^n \frac{s}{3^n} = 3s\frac{4^n}{3^n} = 3s\left(\frac{4}{3}\right)^n.$$

Since $4/3 > 1$, it follows that $N_n L_n$ tends to infinity as $n \to \infty$, which means the Koch snowflake indeed has infinite length. $\square$

The Koch snowflake has finite area.

In an iteration,, the number of new triangles T_n, Eq. 1, can be simplified to a_n

$$a_0 =$$

Δ, the initial equilateral triangle,, or

$$a_n = \frac{a_{n-1}}{9} = \ldots. \tag{3}$$

Eqs. 2 and 3

$$b_n == (\cdot 4^n)\,(a_0) = .$$

total area

$$\begin{aligned} A &= a + \sum_{k=1}^{n} b \\ &= a_0 \left(1 + ()^k\right) \\ &= . \end{aligned}$$

Now, since

$$\lim_{n} 3\,() = 0,$$

$\lim_{\to\infty} A_n$..

References

[1] Helge. *Sur une courbe continue sans tangente, obtenue par une construction géométrique élémentaire.*, Arkiv, Kungliga Vetenskapsakademien. **1**, 681-702,.