# Skidbacken

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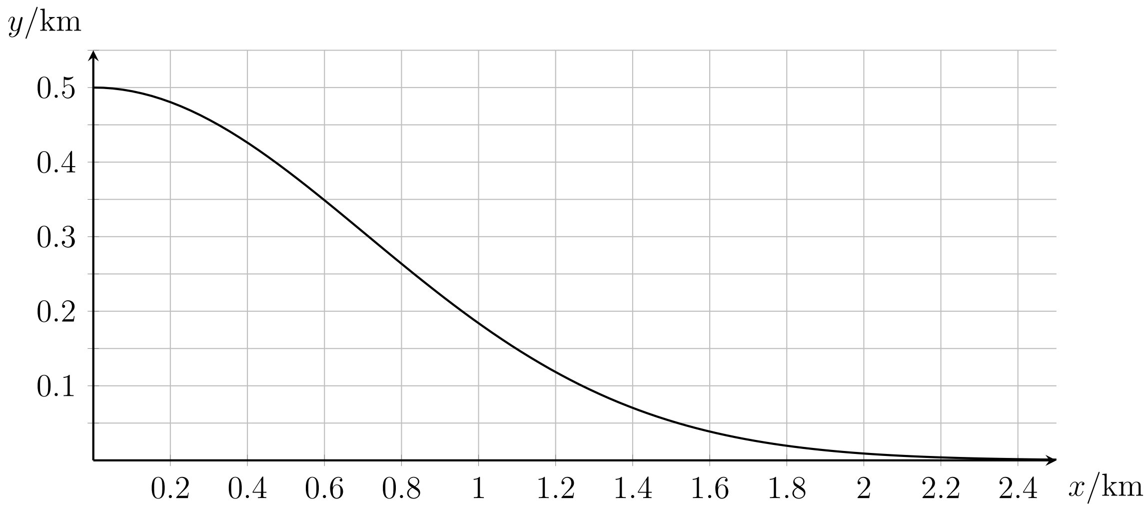
**Sammanfattning**

I denna rapport ska jag beskriva och förklara uppgifter angående en given funktion på en kurva

## 1 Introduktion

Rapporten kommer lösa och beskriva uppgifter kring en skidbacke som har fallhöjden 500 meter. Här ser du den grafiskt ritad:




Grafen är ritad med ett samband mellan y (höjden i km.) och x (längden i km.) som kan srivas som:
$y = 0,5e^{-x^2}$ där $0 \leq x \leq 2,5$

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# 2 The angle of a slope

First we solve:

> Determine the angle at which the curve of the function is inclined at the point $x = 0,8$.

To do this, we first need to calculate the derivative $\left(\frac{dy}{dx}\right)$ of the function, which will give us the slope of the tangent line at that point. From the slope, we can then calculate the angle using the arctangent function. Because the function involves an exponential expression, we will use the **chain rule**:

## 2.1 Splitting the function

The function $y = 0,5e^{-x^2}$ can be viewed as a composition of two functions:

- The outer function $f(u) = 0,5e^{u}$, where $u = -x^2$.
- The inner function $g(x) = -x^2$.

According to the chain rule, the derivative of the composite function is:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

## 2.2 Step 1: Derivative of the Outer Function

First, we differentiate the outer function $y = 0,5e^{u}$ with respect to $u$. The derivative of $e^{u}$ with respect to $u$ is simply $e^{u}$, and since there is a constant 0,5 in front, we have:

$$\frac{dy}{du} = 0,5e^{u} \tag{1}$$

## 2.3 Step 2: Derivative of the Inner Function

Next, we differentiate the inner function $u = -x^2$ with respect to $x$ using the basic power rule differentiation:

$$\frac{du}{dx} = \frac{d}{dx}\left(-x^2\right) = -2x \tag{2}$$

## 2.4 Step 3: Applying the Chain Rule

Now, we apply the chain rule by multiplying the results from Step 1 and Step 2:

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 0,5e^{-x^2} \cdot (-2x) \tag{3}$$

And by simplifying this expression we get:

$$\frac{dy}{dx} = -x \cdot e^{-x^2} \tag{4}$$

Thus we have the derivative of $y$.

### 2.5 Finding the Slope at $x = 0,8$

Now that we have the general expression for the derivative, we can calculate the slope of the curve at the point $x$. Substituting $x = 0,8$ into the derivative as well as simplifying:

$$\begin{aligned} y'(0,8) &= -0,8 \cdot e^{-(0,8)^2} \\ &= -0,8 \cdot e^{-(0,64)} \\ &\approx -0,8 \cdot 0,5272 \approx -0,42 \end{aligned} \tag{5}$$

So the answer is that the slope of the tangent line at $x = 0,8$ is approximately $-0,42$.

## 3 Second derivative

Here our task is:

Find an equation where the angle is the steepest using this function:

$$y = 0,5e^{-ax^2}$$

Where $a$ is a constant and $x$ is in the range $0 \leq x \leq 2,5$. The second derivative will tell us how the slope of the curve is changing and will let us analyze the concavity of the function. But first, we need to calculate the first derivative $y'$.

### 3.1 First Derivative

Just like earlier we will make use of the **chain rule** for this, so, again:

- The outer function is $f(u) = 0.5e^u$, where $u = -ax^2$
- The inner function is $g(x) = -ax^2$.

The outer function's derivative with respect to $u$ is, once again, $e^u$, so we have:

$$\frac{dy}{du} = 0,5e^u \tag{6}$$

And the inner function's derivative with respect to $x$ using the power rule, we get:

$$\begin{aligned} \frac{du}{dx} &= \frac{d}{dx}\left(-ax^2\right) \\ &= -2ax \end{aligned} \tag{7}$$

Now we can apply the chain rule:

$$\begin{aligned} \frac{dy}{dx} &= \frac{dy}{du} \cdot \frac{du}{dx} \\ &= 0,5e^{-ax^2} \cdot (-2ax) \end{aligned} \tag{8}$$

Now that we have the first derivative, we can simplify it:

$$\begin{aligned} y' &= 0,5e^{-ax^2} \cdot (-2ax) \\ &= -ax \cdot e^{-ax^2} \end{aligned}$$

### 3.2 Finding the Second Derivative

Finally we can work toward finding the Second Derivative $\frac{d^2y}{dx^2}$. To do this we can use the **product rule** and, once again, the **chain rule**.

### The Product Rule

The product rule states:

$$\frac{d}{dx}\left(u\left(x\right)\cdot v\left(x\right)\right)=u'\left(x\right)\cdot v\left(x\right)+u\left(x\right)\cdot v'\left(x\right)$$

And in our case:

- $u\left(x\right)=-ax$
- $v\left(x\right)=e^{-ax^2}$

Of which we need to find the derivative for both:

$$\begin{aligned} u'\left(x\right)&=-as\\ v'\left(x\right)&=e^{-ax^2}\cdot\left(-2ax\right)\\ &=-2axe^{-ax^2} \end{aligned} \tag{9}$$

Now we need to apply the product rule:

$$\begin{aligned} \frac{d^2y}{dx^2}&=\frac{d}{dx}\left(u\left(x\right)\cdot v\left(x\right)\right)\\ &=u'\left(x\right)\cdot v\left(x\right)+u\left(x\right)\cdot v'\left(x\right)\\ &=\left(-a\right)\cdot e^{-ax2}+\left(-ax\right)\cdot\left(-2axe^{-ax^2}\right) \end{aligned} \tag{10}$$

## 4 Och ännu nästa (del-) uppgift...

## 5 Diskussion [och slutsatser]

Sammanfatta vad som avhandlats i rapporten, vad du kommit fram till, och sätt det i sitt sammanhang.

## Referenser

[1] Michel Goossens, Frank Mittelbach, and Alexander Samarin. *The LaTeX Companion*. Addison-Wesley, Reading, Massachusetts, 1993.

[2] Albert Einstein. *Zur Elektrodynamik bewegter Körper*. (German) [*On the electrodynamics of moving bodies*]. Annalen der Physik, 322(10):891–921, 1905.