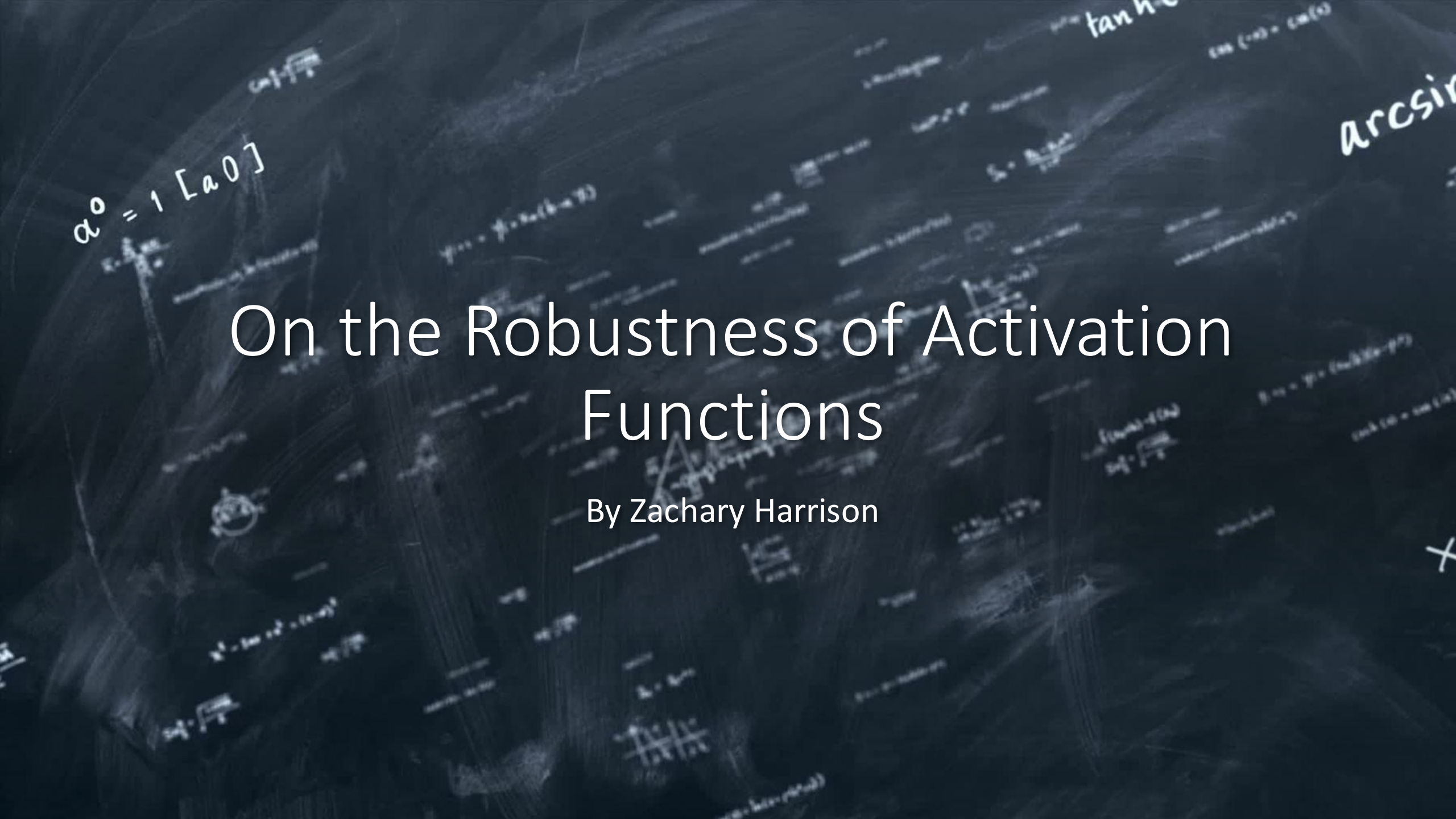
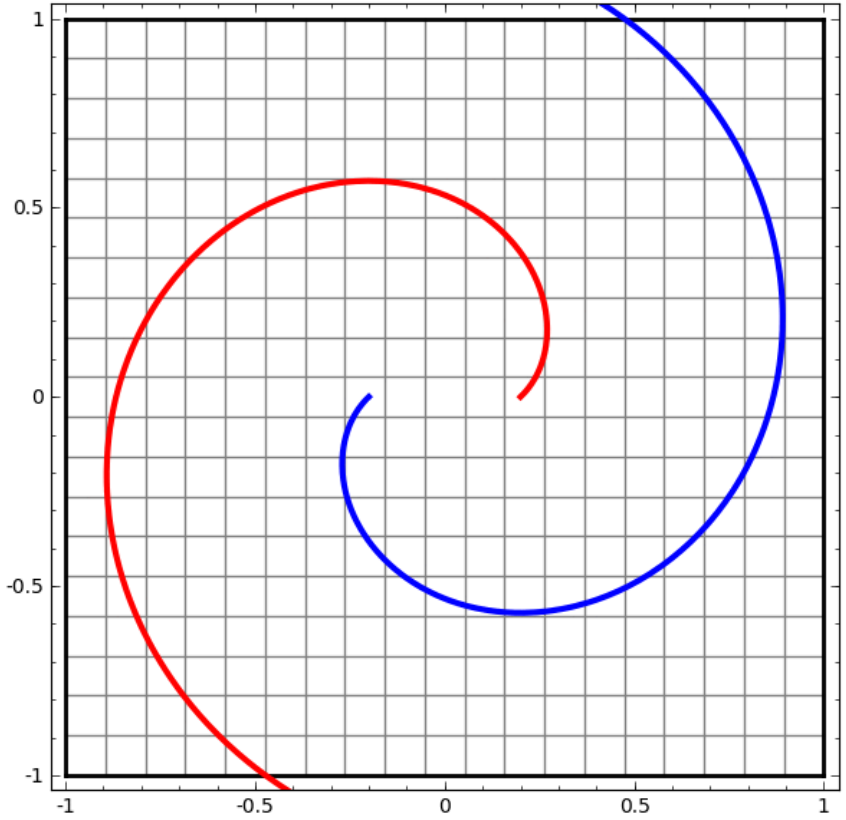
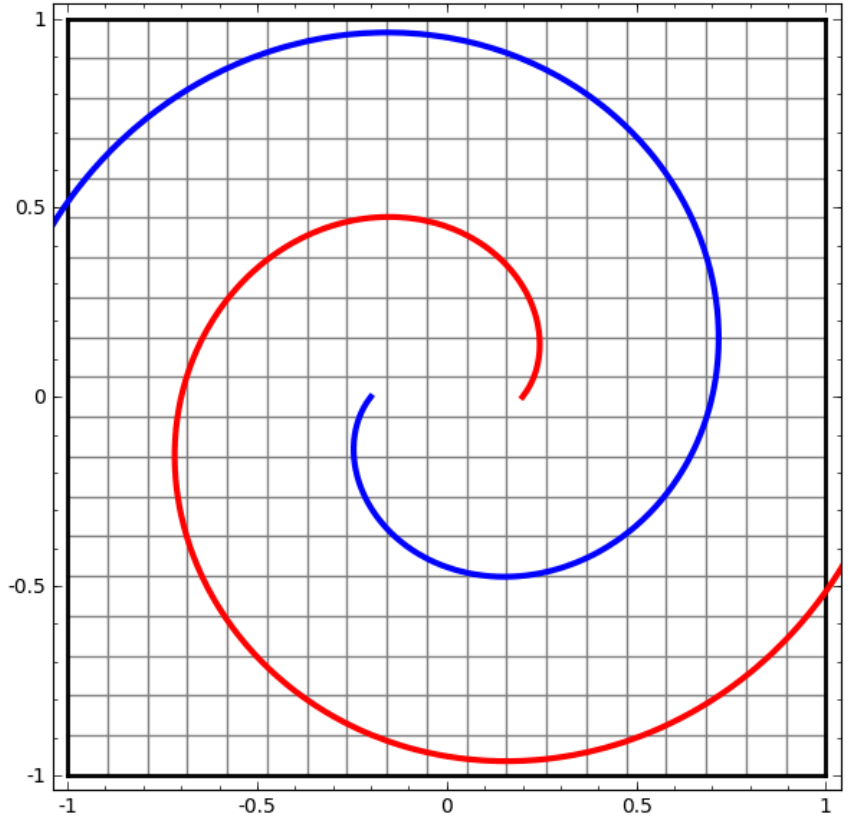


On the Robustness of Activation Functions

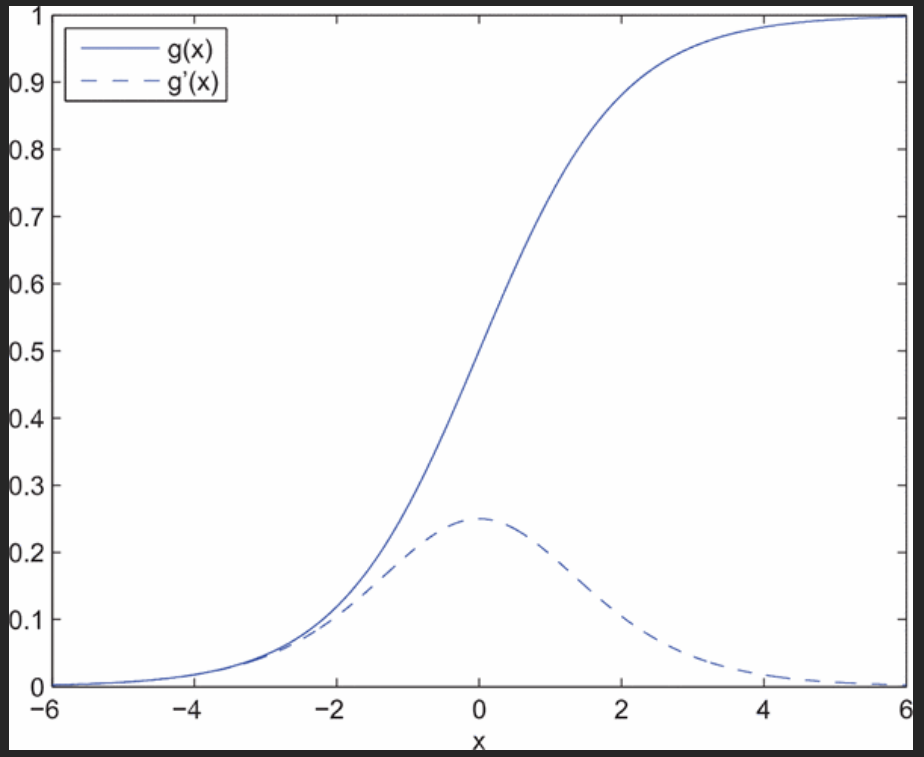
By Zachary Harrison

Visualizing Hidden Layers





Sigmoid

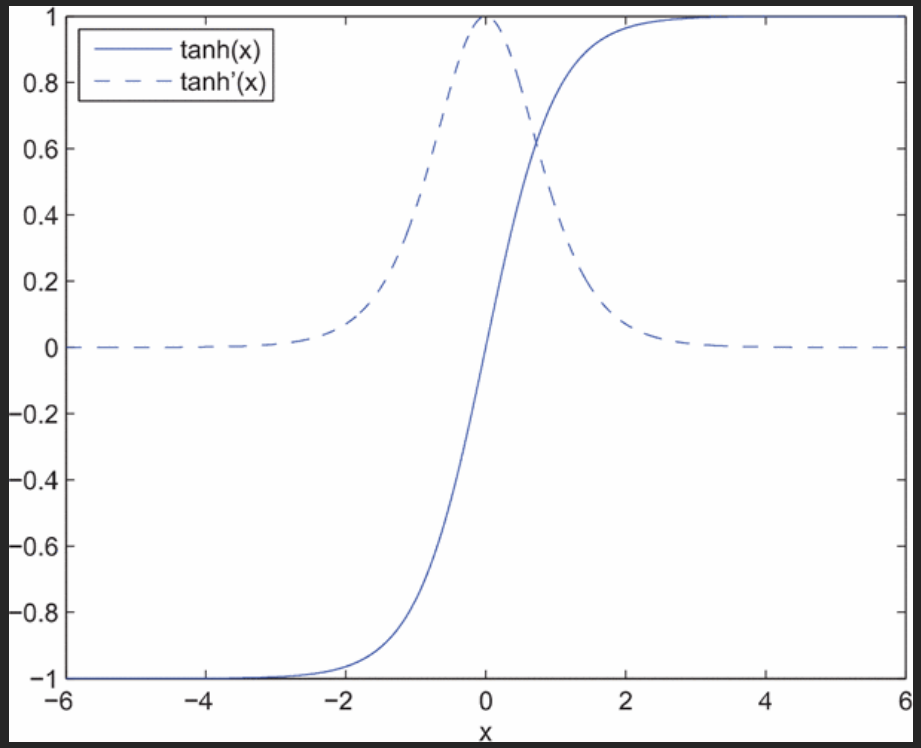


- One of the first activation functions
- Very susceptible to vanishing gradients

$$g(x) = \frac{1}{1 + e^{-x}}$$

$$g'(x) = \frac{e^{-x}}{(1 + e^{-x})^2}$$

Tanh

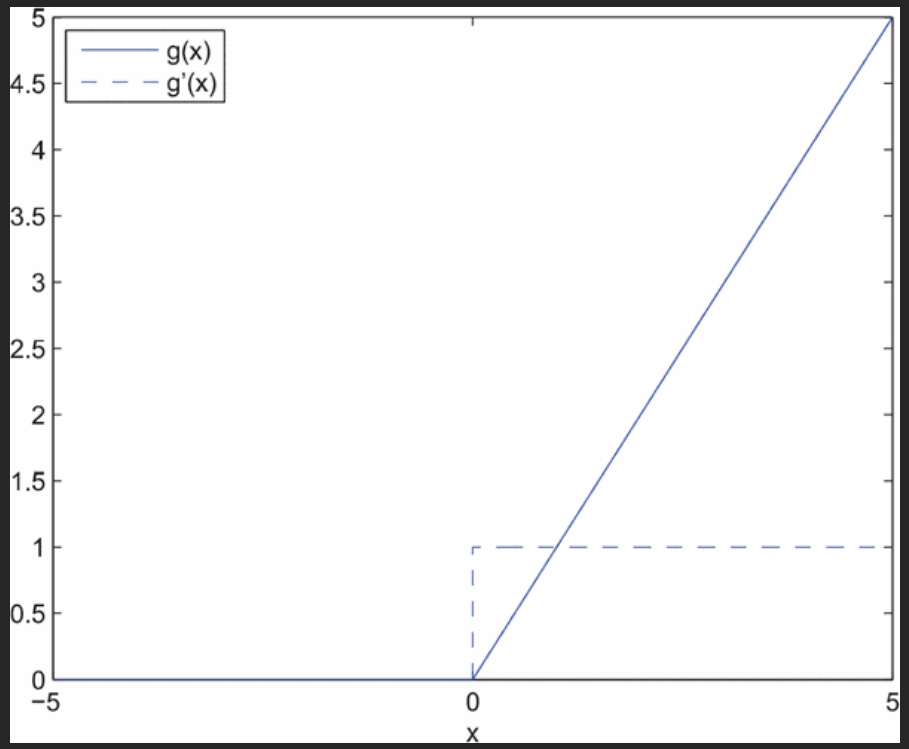


- Similar to Sigmoid
- Also susceptible to vanishing gradients

$$tanh(x) = \frac{sinh(x)}{cosh(x)} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$$tanh'(x) = 2sigmoid'(2x) - 1 = \frac{4e^{-2x}}{(1 + e^{-2x})^2}$$

ReLU

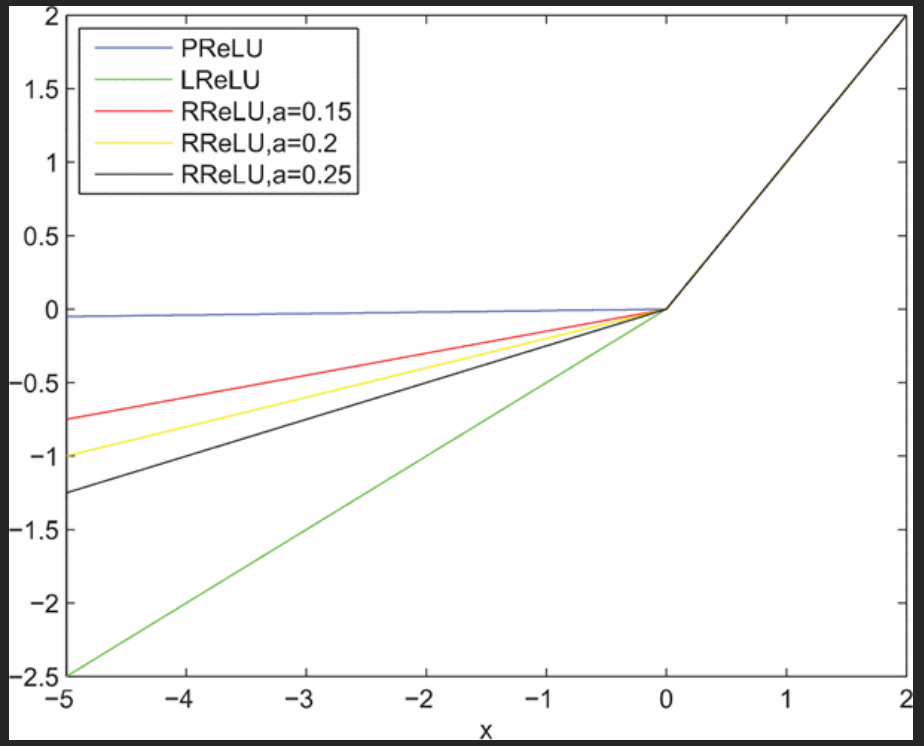


- **Extremely** fast computation
- Much less susceptible to vanishing gradients

$$g(x) = max(0, x) = \begin{cases} x & if\ x \geq 0 \\ 0 & if\ x < 0 \end{cases}$$

$$g'(x) = \begin{cases} 1 & if\ x \geq 0 \\ 0 & if\ x < 0 \end{cases}$$

ReLU's Wacky Family

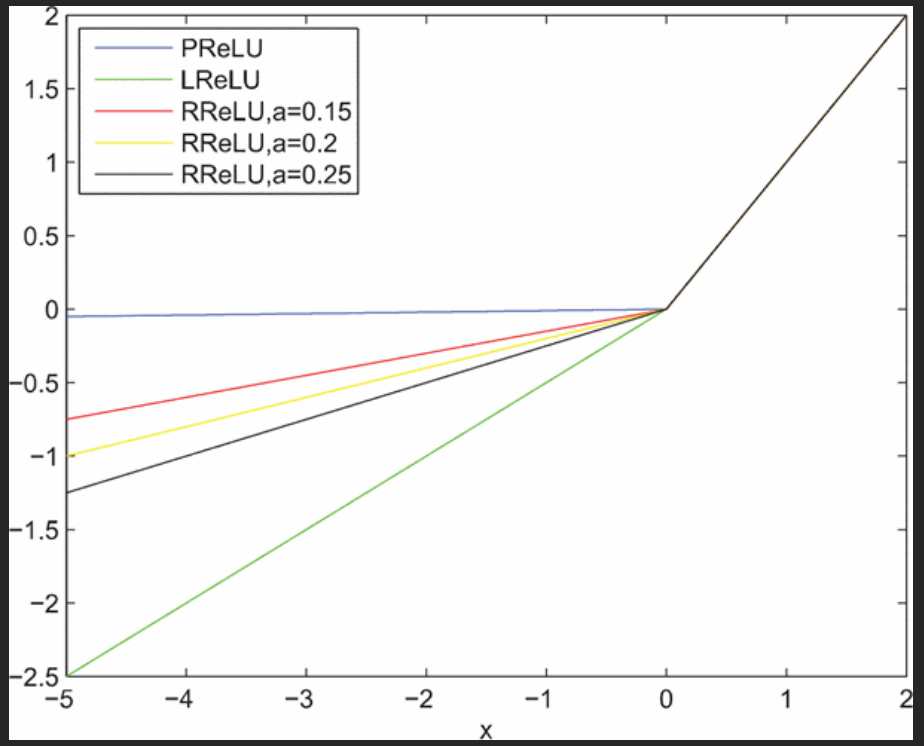


- LReLU or Leaky ReLU allows for small negative values
- PReLU is like Leaky ReLU, but has a learnable parameter ***a*** instead of a constant 0.01.

$$g(x) = \begin{cases} x & if \ x \geq 0 \\ ax & if \ x < 0 \end{cases}$$

$$g'(x) = \frac{e^{-x}}{(1+e^{-x})^2}$$

ReLU's Wacky Family



- LReLU allows for small negative values.
- PReLU is like Leaky ReLU, but has a learnable parameter ***a*** instead of a constant 0.01.
- In RReLU, the slopes are randomized during training and fixed during testing.

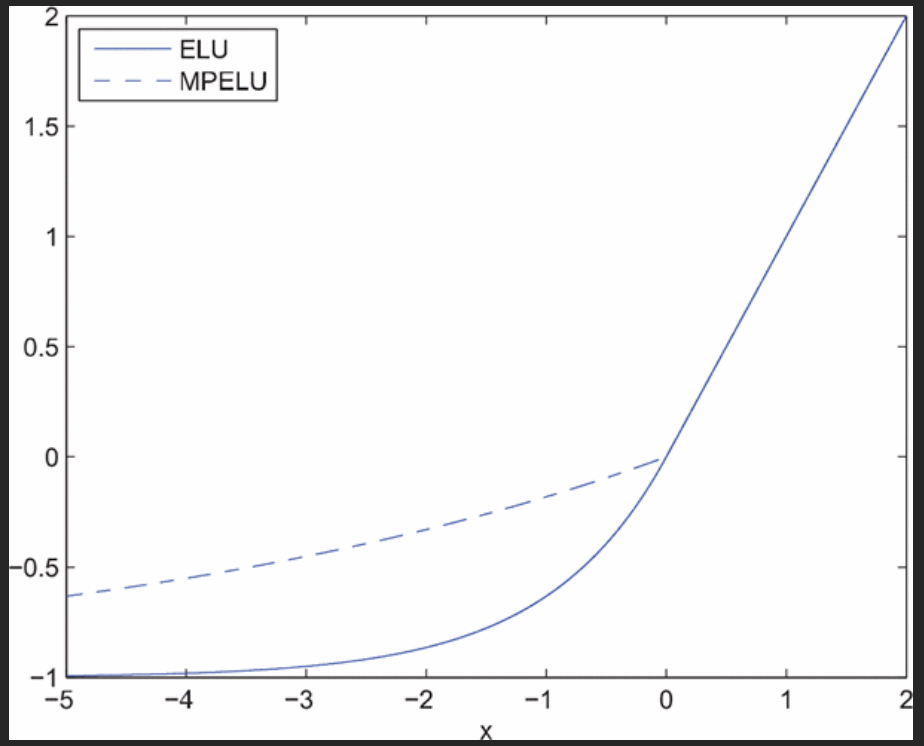
LReLU (Leaky ReLU)

$$g(x) = \begin{cases} x & if\ x \geq 0 \\ 0.01x & if\ x < 0 \end{cases}$$

MPELU

$$g(x) = \begin{cases} x & if\ \ x > 0 \\ \alpha(e^{\beta x} - 1) & if\ \ x \leq 0 \end{cases}$$

ELU and MPELU



- ELU is more robust to the input perturbation or noise due to its convergence property as x approaches negative infinity.
- Because MPELU can become ReLU, LReLU, PReLU, or ELU through training, it is usually considered better.

ELU

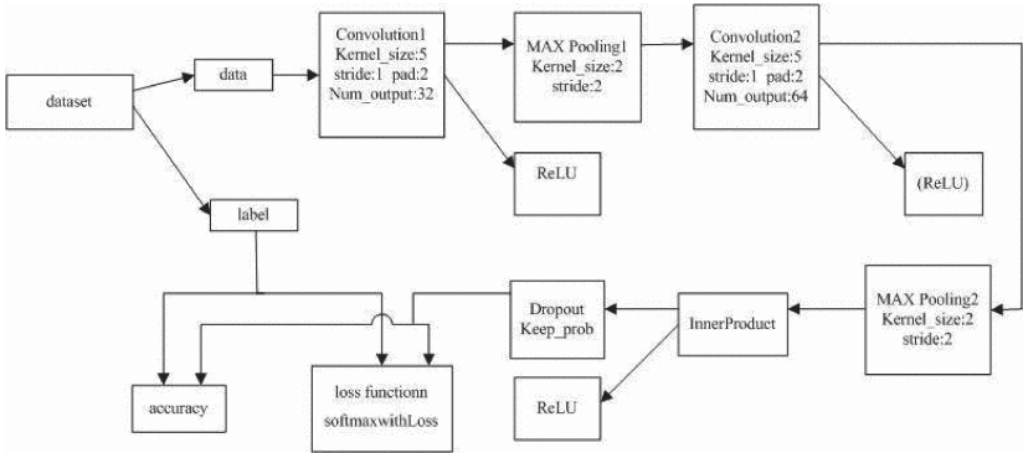
$$g(x) = \begin{cases} x & if \ x > 0 \\ \alpha(e^x - 1) & if \ x \leq 0 \end{cases}$$

MPELU

$$g(x) = \begin{cases} x & if \ x > 0 \\ \alpha(e^{\beta x} - 1) & if \ x \leq 0 \end{cases}$$

Results from DCNN

- Using the model architecture defined in the top right, each activation type's performance was recorded.
- It's important to note that models only had 20,000 iterations on their training set of 1,000 28x28 grayscale images.



Activation function	Parameter	Error (%)
Sigmoid	-	1.15
Tanh	-	1.12
ReLU	-	0.8
RReLU	$a = 0.5$	0.99
ELU	$\alpha = 1$	1.1

Sigmoid revisited

- A huge innovation that ELU brought was this convergence property as x approaches negative infinity. The reason this increases robustness is because it reduces outliers' ability to change the model during the training process.
- Both Sigmoid and Tanh have this property, but they are widely considered inferior to all ReLU's family of activation functions. Why is that?
- That's what I'm going to research!

Thank you for listening!

References

1. Activation functions and their characteristics in deep neural networks
2. The Power of Approximating: a Comparison of Activation Functions
3. How to Choose an Activation Function
4. Small nonlinearities in activation functions create bad local minima in neural networks
5. Neural Networks, Manifolds, and Topology