Formal Languages, Regular Expressions, Automata, Transducers

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Outline

- Formal Languages in the Chomsky Hierarchy
- Regular Expressions
- Finite State Automata
- Finite State Transducers
- Some Sample CL tasks using Regexps
- Concluding Remarks

Formal Language = Set of Strings of Symbols

- A Formal Language Can Model a Phenomenon, e.g., written English
- Examples
  - All Combinations of the letters A and B: ***ABAB, AABB, AAAB***, etc.
  - Any number of As, followed by any number of Bs: ***AB, AABB, AB, AAAAAAAABBB***, etc.
  - Mathematical Equations: ***1 + 2 = 5, 2 + 3 = 4 + 1, 6 = 6***
  - All the sentences of a simplified version of written English, e.g., ***My pet wombat is invisible.***
  - A sequence of musical notation (e.g., the notes in Beethoven's 9th Symphony), e.g., ***A-sharp B-flat C G A-sharp***

What is a Formal Grammar for?

- A formal grammar
  - set of rules
  - matches **<u>all and only</u>** instances of **a formal language**
- A formal grammar defines a formal language
- In Computer Science, Formal grammars are used to **generate** and **recognize** formal languages (e.g., programming languages)
  - Parsing a string of a language involves:
    - Recognizing the string and
    - Recording the analysis showing it is part of the language
  - A compiler translates from language X to language Y, e.g.,
    - This may include parsing language X and generating language Y
  - If all natural languages were formal languages, then Machine Translation systems would just be compilers

A Formal Grammar Consists of:

- **N**: a Finite set of nonterminal symbols
  - Symbols that can be replaced by other symbols
- T: a Finite set of terminal symbols
  - Symbols that cannot be replaced by other symbols
- R: a set of rewrite rules, e.g., XYZ → abXzY
  - Replace the symbol sequence XYZ with abXzY
- S: A special nonterminal that is the start symbol

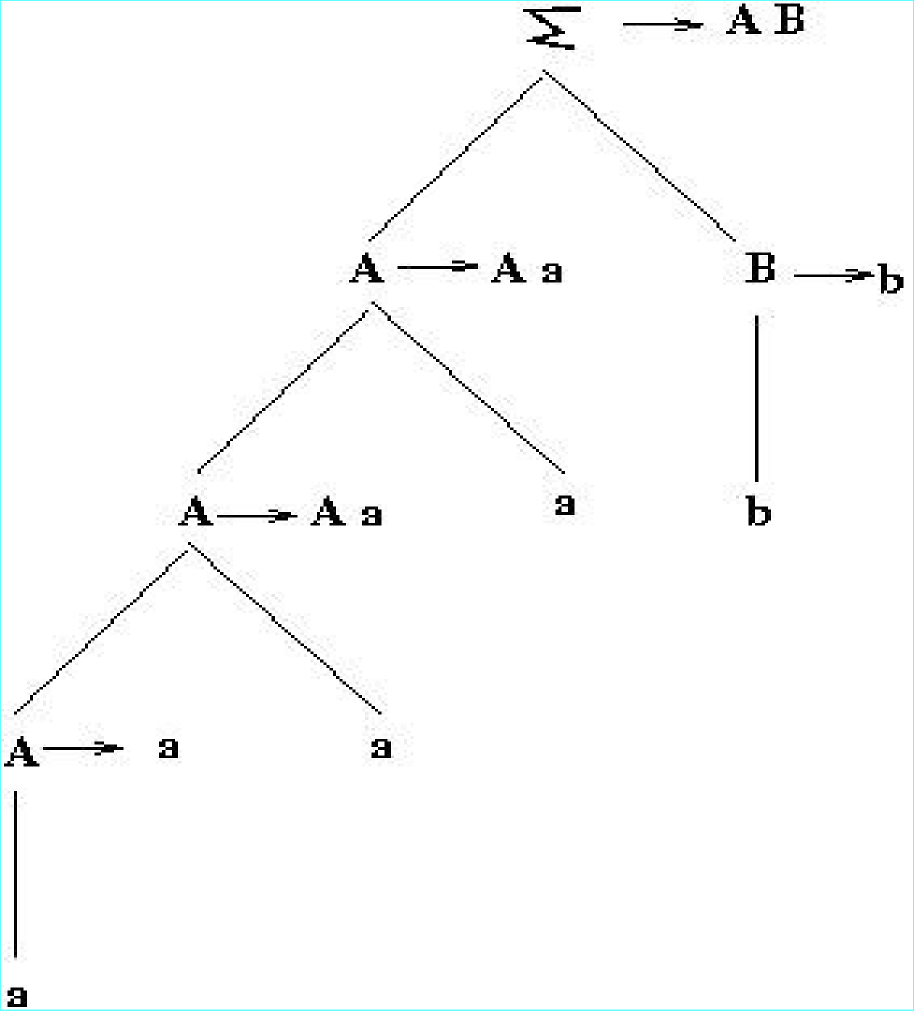
A Very Simple Formal Grammar

- **Language_AB = 1 or more a, followed by 1 or more b, e.g., ab, aab, abb, aaaaaaabb, etc.**
- **N = {A,B}**
- **T={a,b}**
- **S=Σ**
- **R={A→a, A→Aa, B→b B→Bb, Σ→AB}**

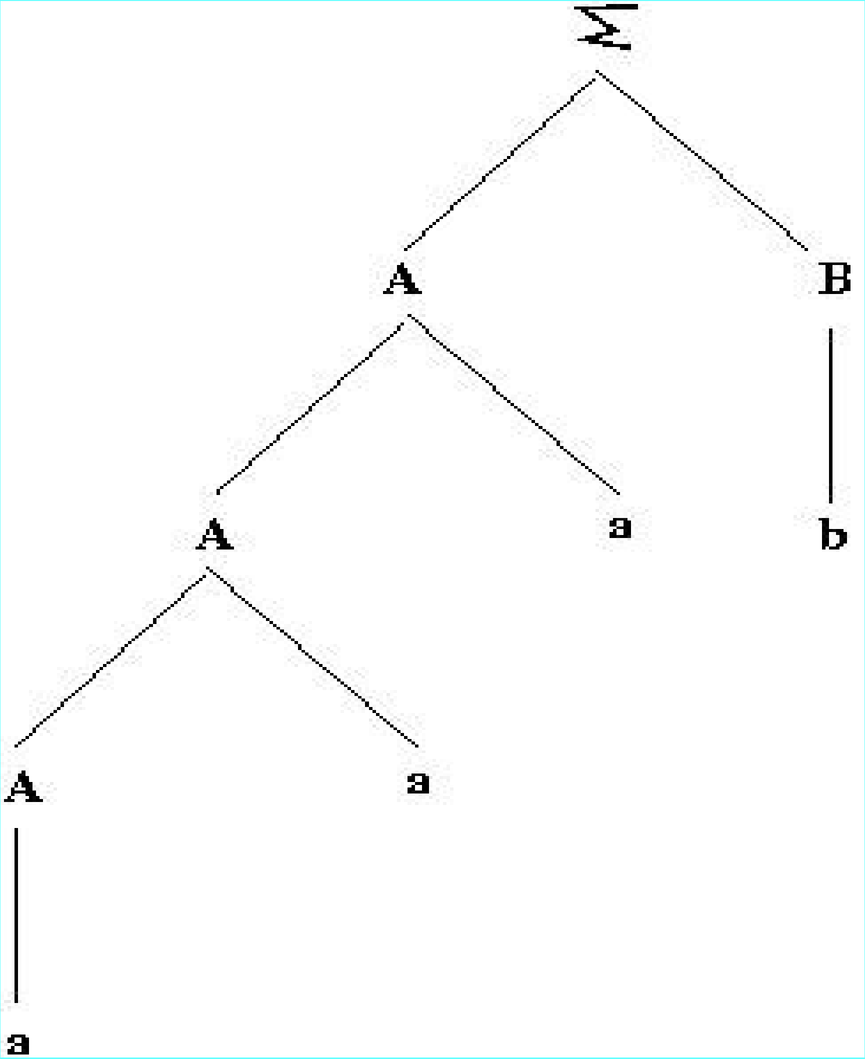
Generating a Sample String

- Start with **Σ**
- Apply **Σ→AB**, Generate A B
- Apply **A→Aa**, Generate A a B
- Apply **A→Aa**, Generate A a a B
- Apply **A→a**, Generate a a a B
- Apply **B→b**, Generate a a a b

Derivation of a a a b



Phrase Structure Tree for a a a b



The Chomsky Hierarchy: Type 0 and 1

- Type 0: No restrictions on rules
  - Equivalent to Turing Machine
    - General System capable of Simulating any Algorithm
- Type 1: Context-sensitive rules
  - $\alpha A \beta \rightarrow \alpha \gamma \beta$
    - Greek letters = 0 or more nonterms/terms
    - A = nonterminal
    - Rule means: replace A with γ, when A is between α and β
  - For example,
    - DUCK DUCK DUCK→ DUCK DUCK GOOSE
    - Means convert DUCK to a GOOSE, if preceded by 2 DUCKS

Chomsky Hierarchy Type 2

- Context-free rules
- $A \rightarrow \gamma$
- Like context-sensitive, except left-hand side can only contain exactly one nonterminal
- Example Rule from linguistics:
  - NP → POSSP n PP
  - NP → Det n
  - NP → n
  - POSSP → NP 's
  - PP → p NP
  - [NP [POSSP [NP [Det ***The***] [*n* ***group***]] 's]

    [n ***discussion***]

    [PP [p ***about***][NP [n ***food***]]]]
    - ***The group's discussion about food***

Chomsky Hierarchy Type 3

- Regular (finite state) grammars
  - A → βa or A → ϵ (left regular)
  - A → aβ, or A → ϵ (right regular)
- Like Type 2, except right hand side is constrained
  - Non-terminals precede (but don't follow) terminals in left regular grammar
  - Non-terminals follow (but don't precede) terminals in right regular grammar
  - null string is allowed
- Example left regular rules from linguistics:
  - NP → POSSP n
  - NP → n
  - NP → det n
  - POSSP → NP 's
- [NP [POSSP [NP [det ***The***] [n ***group***]] 's]

  [n ***discussion***]]
  - ***The group's discussion***

Chomsky Hierarchy

- $Type0 \supseteq Type1 \supseteq Type2 \supseteq Type3$
- Type 3 grammars
  - Least expressive, Most efficient processors
- Processors for Type 0 grammars
  - Most expressive, Least efficient processors
- Complexity of recognizer for languages:
  - Type 0 = exponential; Type 1 = polynomial; Type 2 = $O(n^3)$; Type 3 = O(n log n)

CL mainly features Type 2 & 3 Grammars

- Type 3 grammars
  - Include regular expressions and finite state automata (aka, finite state machines)
  - The focal point of the rest of this talk
  - Also see Nooj platform for NLP:
    - http://www.nooj-association.org/
    - might work best in Windows
- Type 2 grammars
  - Commonly used for natural language parsers
  - Used to model syntactic structure in many linguistic theories (often supplemented by other mechanisms)
  - Important for later talks on constituent structure & parsing

Type 1.5 Grammars

- Human Language believed to be “mildly context sensitive”
  - Less expressive than type 1 (context sensitive)
  - More expressive than type 2 (context free)
- Some complex dependencies cannot be expressed in context free rules, e.g., see https://dash.harvard.edu/bitstream/handle/1/2026618/Shieber_EvidenceAgainst.pdf?sequence=2
- Tree Adjoining Grammars
  - https://repository.upenn.edu/cgi/viewcontent.cgi?article=1706&context=cis_reports
  - https://www.aclweb.org/anthology/H86-1020.pdf
  - Formalism by A. Joshi & others
  - May be able to handle these cases

Regular Expressions

- The language of ***regular expressions*** (regexps)
  - A standardized way of representing search strings
  - Kleene (1956)
- Computer Languages with regexp facilities:
  - Python, JAVA, Perl, Ruby, most scripting languages, …
  - If not officially supported, a library still may exist
- UNIX (linux, Apple, etc.) utilities and text editors
  - grep (grep -E regexp file)
    - different versions: -E,-F,-G,-P
  - emacs, vi, ex, …
- Other
  - Mysql, Microsoft Office, Open Office, ...

My T-Shirt

- My T-Shirt says: **/(BB|[^B]{2})/**
  - The “/”, “(“ and “)” can be ignored for now
  - B represents the string “B”
  - “|” represents the operator 'inclusive or'
  - “^” represents the negative operator
  - [] represents a single character
  - {N} represents N repetitions of preceding item
- What famous quote could this represent?
- What details are different from the quote?

Regexp = formula specifying set of strings

- Regexp = $\varnothing$
  - The empty set
- Regexp = ε
  - The empty string
- Regexp = sequence of one or more characters
  - *X*
  - *Y*
  - *This sentence contains characters like &T^**%P*
- Regexp = Disjunction, concatenation or repetition of regexps

Concatenation, Disjunction, Repetition

- Concatenation
  - If X is a regexp and Y is a regexp, then XY is a regexp
  - Examples
    - If ***ABC*** and ***DEF*** are regexps, then ***ABCDEF*** is a regexp
    - If ***AB**** and ***BC**** are regexps, then ***AB*BC**** is a regexp
      - Note: Kleene * is explained below
- Disjunction
  - If X is a regexp and Y is a regexp, then X | Y is a regexp
  - Example: ***ABC|DEF*** will match either ***ABC*** or ***DEF***
- Repetition
  - If X is a regexp than a repetition of X will also be a regexp
    - The Kleene Star: ***A**** means 0 or more instances of ***A***
    - Regexp{number}: ***A{2}*** means exactly 2 instances of ***A***

Regexp Notation Slide 2

- Disjunction of characters
  - ***[ABC]*** – means the same thing as ***A* | *B* | *C***
  - ***[a-zA-Z0-9]*** – character ranges are equivalent to lists: a|b|c|...|A|B|...|0|1|...|9|
- Negation of character lists/sequences
  - ^ inside bracket means complement of disjunction, e.g., ***[^a-z]*** means a character that is neither ***a*** nor ***b*** nor ***c*** … nor ***z***
  - Question: **Why is character negation equivalent to a disjunction?**
- Parentheses
  - Disambiguate scope of operators
    - ***(ABC)|(DEF)*** means ABC or ADEF
    - Otherwise defaults apply, e.g., ***ABC|D*** means ***ABC*** or ***ABD***
- ***?*** signifies optionality
  - ***ABC?*** is equivalent to ***(ABC)|(AB)***
- **+** indiates 1 or more
  - ***A(BC)**** is equivalent to A|(A(BC)+)

Regexp Notation Slide 3

- Special Symbols:
  - Period means any character, e.g., ***A.*B*** – matches A and B and any characters between
  - Carrot (^) means the beginning of a line, e.g., **^*ABC*** matches ABC at the beginning of a line [*Note dual usage of ^ as negation operator]
  - Dollar sign (***$***) means the end of a line, .e.g., ***[\.?!] *$*** matches final punctuation, zero or more spaces and the end of a line
- Python's Regexp Module
  - Searching
    - Groups and Group Numbers
  - Compiling
  - Substitution
- Similar Modules for: Java, Perl, etc.

Other Details

- See various manuals, e.g., https://docs.python.org/3/library/re.html
- The info above should be enough for most regexps, but there is more
- Sets of characters:
  - \w = [A-Za-z0-9_]
  - \W = [^A-Za-z0-9_]
  - etc.
- All repetition modifiers are greedy, but there are non-greedy versions – Usually, unnecessary if you use appropriate parentheses
- Etc.

Regexp in NLTK's Chatbot

- Running eliza
  - import nltk
  - from nltk.chat.eliza import *
  - eliza_chat()
- NLTK's chatbots:
  - See util.py and eliza.py
  - In your nltk/chat/ directory
  - Full path depends on how you install nltk
- How it works
  - It creates a Chat object (defined in util.py) that includes a substitute method
  - The settings for this chat object are in eliza.py
  - For each pair in pairs, the 1st item is matched against the input string, to produce an answer listed as the 2nd item. The use of %1 indicates repeated parts of the strings.
  - In util.py – note that the matching pattern for the 1st item is created with ***re.compile***, a method that turns a regular expression into a match-able pattern, although in the current examples (.*), a very simple (and general) regexp.

Regexps in Python

- import re imports regexp package
- Example re functions
  - re.search(regexp,input_string) creates a search object
  - re.sub (regexp,repl,string)
- search_object methods
  - start() and end() -- respectively output start and end position in the string
  - group(0) – outputs whole match
  - group(N) – outputs the nth group (item in parentheses)
- Patterns can be compiled
  - Pattern1 = re.compile(r'[Aa]Bc')
  - Methods takes additional parameters (e.g., starting position)
    - Pattern1.search('ABcaBc',2)
      - starts search at position 2

Regexp with Unix tools

- grep -E '\$[0-9\.,]+' all-OANC |less
  – Different flavors of regexp used by grep
    - -P and -E seem to work pretty well (P = Perl and E = Extended)
- In the program less
  – /\$[0-9.,]+
    - Highlights numeric instances
    - Note some of the problems with this regexp for characterizing money strings
    - Your HW will include an expanded version of this problem (finding dollar amounts in text)

RegExp to Search for Common Types of Numeric Strings

- An XML (or html) tag
  - <[^>]+>
- Money
  - $[0-9\.,]+
  - Would this match the string '$,,,,,,'?
    - Maybe that doesn't matter?
  - How might we handle cases like “$4 million”?
  - What might be a better regexp for money? (Part 1 of homework)
- Others
  - Dates, Roman Numerals, Social Security, Telephone Numbers, Zip Codes, Library Call Numbers, etc.
- Time of Day – Let's Do this one as a joint exercise

Time of Day

- Let's agree on the components of a time of day as printed
- For 5 minutes, Everyone should attempt to write such an expression independently. You can test your regexp with Python or grep.
- Let's look at some of the proposed answers, test them and possibly combine aspects.

A “good” regexp?

- It should match most sample cases of the target type of string
- It should not match many “incorrect” strings
- Sample “correct” and “incorrect” strings can be used to tune regexps
- So can running on a large set of sample data (like the all-OANC.txt file)
- You should have some confidence that the regexp will “generalize” well.
  - It should correctly match (and not match) cases that are not in your input data.
- **Midterm question regexps are expected to correctly match and not match examples that are not provided as part of the test.**

NLTK's Regexp Language for Chunking

- sentence = '''The big grey dog with three heads was on my lap'''
- tokens = nltk.word_tokenize(sentence)
- pos_tagged_items = nltk.pos_tag(tokens)
- chunk_grammar = nltk.RegexpParser(r"""

```
NG: {(<CD|DT|JJ|NN|PRP\$>)*(<NN|NNS>)}
VG: {<MD|VB|VBD|VBN|VBZ|VBP|VBG>*<VB|VBD|VBN|VBZ|VBP|VBG><RP>?}
"""
```

- chunk_grammar.parse(pos_tagged_items)
- Structure:
  - 1 rule per line
  - Nonterminal: Regexp
  - Regexp = terminals, nonterminals & operators (*+?{}…)
- See sample_chunks.py

Chunking Rules On the right side, Nonterminals precede terminals

- chunks2 = r"""

```
DTP: {<PDT><DT|CD>}
NG: {(<CD|DT|JJ|NN|DTP|PRP\$|DTP>)*(<NN|NNS>)}
VG:{<MD|VB|VBD|VBN|VBZ|VBP|VBG>*<VB|VBD|VBN|VBZ|VBP|VBG><RP>?}
PG:{<RB><IN|TO>}
VP: {<VG> <NG|PG>*}
"""
```

- Rules assume Penn Treebank POS tags on next slide

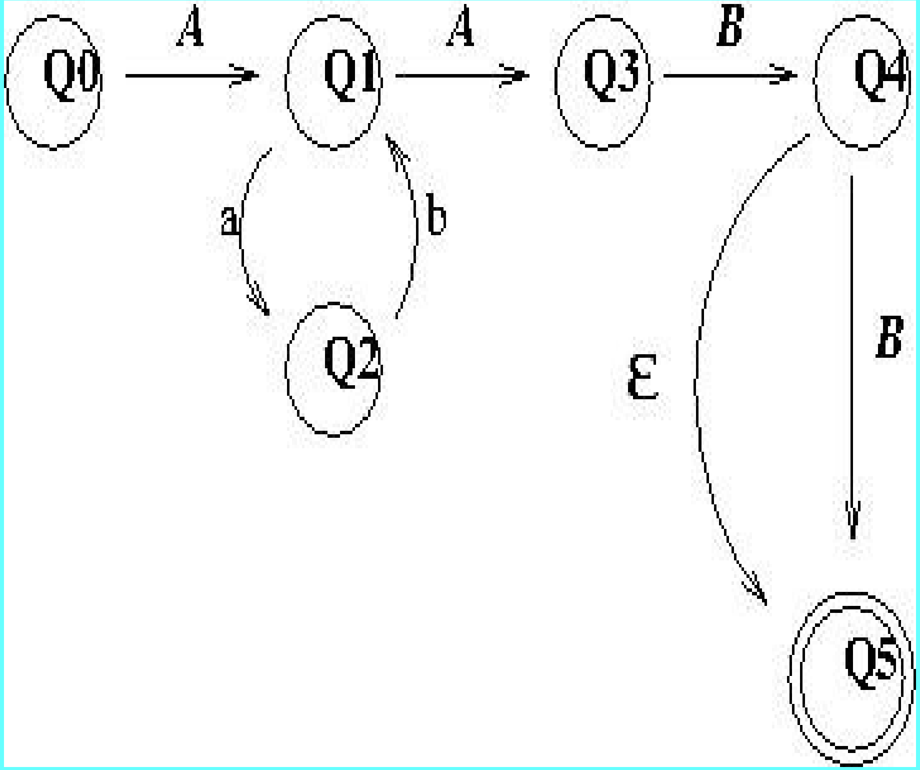
The Penn Treebank II POS tagset

- Verbs: VB, VBP, VBZ, VBD, VBG, VBN
  - base, present-non-3rd, present-3rd, past, -ing, -en
- Nouns: NNP, NNPS, NN, NNS
  - proper/common, singular/plural (singular includes mass + generic)
- Adjectives: JJ, JJR, JJS (base, comparative, superlative)
- Adverbs: RB, RBR, RBS, RP (base, comparative, superlative, particle)
- Pronouns: PRP, PP$ (personal, possessive)
- Interogatives: WP, WP$, WDT, WRB (compare to: PRP, PP$, DT, RB)
- Other Closed Class: CC, CD, DT, PDT, IN, MD
- Punctuation: **# $. , : ( ) “ ” '' ' `**
- Weird Cases: FW(***deja vu***), SYM (***@***), LS (***1, 2, a, b***), TO (***to***), POS(***'s, '***), UH (***no, OK, well***), EX (***it/there***)
- Newer tags: HYPH, PU

Finite State Automata

- Devices for recognizing finite state grammars (include regexps)
- Two types
  - Deterministic Finite State Automata (DFSA)
    - Rules are unambiguous
  - NonDeterministic FSA (NDFSA)
    - Rules are ambiguous
      - Sometimes more than one sequence of rules must be attempted to determine if a string matches the grammar
        - » Backtracking
        - » Parallel Processing
        - » Look Ahead
  - Any NDFSA can be mapped into an equivalent (but larger) DFSA

DFSA for Regexp: *A(ab)*ABB?*



State	Input				
	A	B	a	b	ε
Q0	Q1				
Q1	Q3		Q2		
Q2				Q1	
Q3		Q4			
Q4		Q5			Q5
Q5					

DFSA algorithm

- D-Recognize(tape, machine)

  pointer ← beginning of tape

  current state ← initial state Q0

  repeat until the end of the input is reached

  look up (current state,input symbol) in transition table

  if found: set current state as per table look up

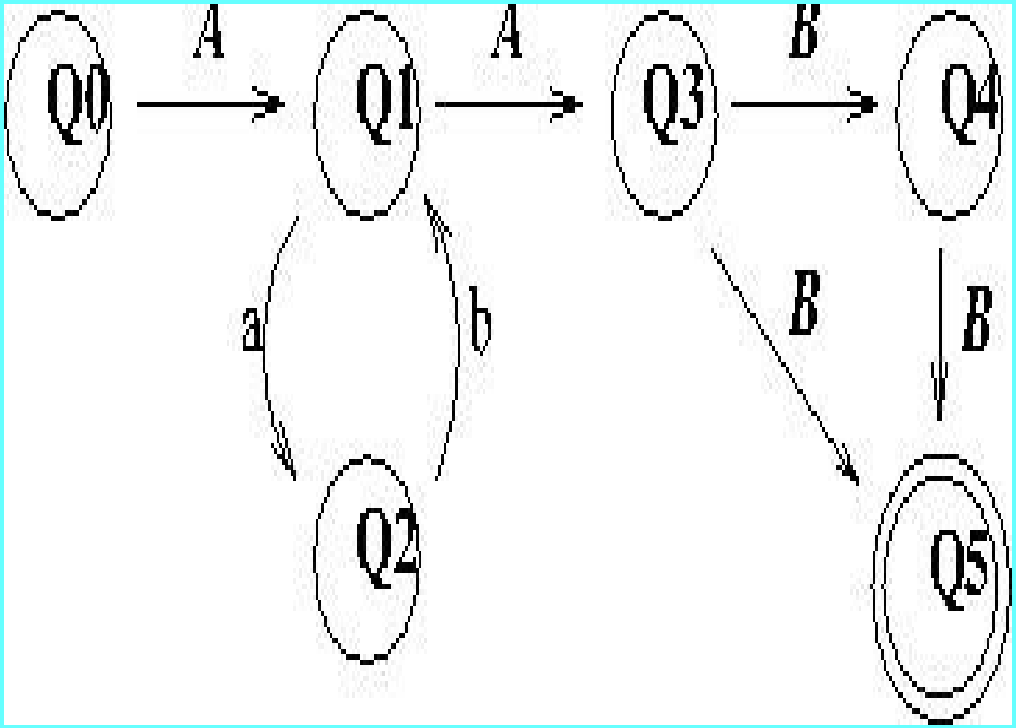
  advance pointer to next position on tape

  else: reject string and exit function

  if current state is a final state: accept the string

  else: reject the string

NDFSA for Regexp: *A(ab)*ABB?*



State	Input			
	A	B	a	b
Q0	Q1			
Q1	Q3		Q2	
Q2				Q1
Q3		Q4 Q5		
Q4		Q5		

NDFSA algorithm

- ND-Recognize(tape, machine)

  agenda ← {(initial state, start of tape)}

  current state ← next(agenda)

  repeat until accept(current state) or agenda is empty

    agenda ← Union(agenda,look_up_in_table(current state,next_symbol))

    current state ← **next**(agenda)

  if accept(current state): return(True)

  else: false

- Accept if at the end of the tape and current state is a final state
- **Next** defined differently for different types of search
  - Choose most recently added state first (depth first)
  - Chose least recently added state first (breadth first)
  - Etc.

A Right Regular Grammar Equivalent to: *A(ab)*ABB?*
(Red = Terminal, Black = Nonterminal)

- Q→ARS
- R→ϵ
- R→abR
- S→ABB
- S→AB

Readings and Homework

- Readings
  - Chapters 2 and 3 in Jurafsky and Martin
  - Chapters 2 and 3 in NLTK
- Homework
  - http://cs.nyu.edu/courses/fall22/CSCI-UA.0480-057/homework2.html