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3.32 # of possible outcomes = $\binom{53}{5} \cdot 42$

$= \boxed{120,526,770}$

3.63 $P = \frac{3!\,(5!\cdot 4!\cdot 3!)}{12!} \approx \boxed{2.165 \times 10^{-4}}$

3.64 $P(k) = \boxed{\frac{\binom{k}{k}\binom{N-k}{n-k}}{\binom{N}{n}}}$

4.9 (a) $\because$ The cards were dealt randomly $\wedge$ there're 13 spades in 52 card set.

$\therefore$ P (1st Spade) $= \frac{13}{52} = \boxed{\frac{1}{4}}$

(b) Total # of face card $= 3 \cdot 4 = 12$

$\therefore$ P(face card) $= \frac{12}{52} = \boxed{\frac{3}{13}}$

4.11 (a) let A = first child is a boy

B = both children are boys

$\therefore P(B|A) = \frac{P(B\cap A)}{P(A)}$;

Assume having boys and girls are equally likely, $P(A) = \frac{1}{2}$

$\because B \subseteq A \quad \therefore P(B \cap A) = P(B) = \frac{1}{2}\cdot\frac{1}{2} = \frac{1}{4}$

$\therefore P(B|A) = \frac{\frac{1}{4}}{\frac{1}{2}} = \boxed{\frac{1}{2}}$

Problem 4.11 cont'd

(b) Let A = at least one child is a boy.

C = both children are boys ; B = boy , G = girl

$\therefore P(C|A) = \frac{P(C\cap A)}{P(A)}$

$\because \Omega = \{(BB), (BG), (GB), (GG)\}$

$\therefore P(A) = \frac{3}{4}$; $P(C\cap A) = P(C) = \frac{1}{4}$

$P(C|A) = \frac{1/4}{3/4} = \boxed{\frac{1}{3}}$

4.12 $\because$ among the first 7 tosses, #6 did not occur

$\therefore \Omega = \{1, 2, 3, 4, 5\} \Rightarrow P(\text{getting a } 4) = \frac{1}{5}$

$P(\text{Getting } 4 = 2) = \binom{7}{2}\left(\frac{1}{5}\right)^2 \cdot \left(1-\frac{1}{5}\right)^5$

$= 21 \cdot \frac{1}{25} \cdot \frac{1024}{3125}$

$= \frac{21504}{78125}$

$\boxed{\approx 0.275}$

4.19 $LHS = P(C \mid A\cap B)$

$= \frac{P(C\cap (A\cap B))}{P(A\cap B)} = \frac{P(C\cap A\cap B)}{P(B|A)\cdot P(A)}$

$= \frac{P(B\cap C \mid A)\cdot P(A)}{P(B|A)\cdot P(A)} = \frac{P(B\cap C \mid A)}{P(B|A)} = RHS$, Q.E.D

4.23 (a) Let A = the person has access to the internet

B = the person is a regular user.

C = the person's social contacts is reduced by the internet.

According to the prompt, We know the following:

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = 0.36$$

$$P(C|B) = \frac{P(C \cap B)}{P(B)} = 0.25$$

We're looking for: $P(C \cap B \mid A)$

$$P(C \cap B | A) = \frac{P(C \cap B \cap A)}{P(A)}$$

$\because \; C \subseteq B \subseteq A$

$\therefore \; P(C \cap B) = 0.25 \cdot P(B) = P(C) \Rightarrow P(C) = 0.25 \cdot P(B)$

$P(B \cap A) = P(B) = 0.36 \cdot P(A) \Rightarrow P(A) = \frac{P(B)}{0.36}$

$\therefore \; P(C \cap B|A) = P\frac{(C)}{(A)} = 0.25\, P(B) \cdot \frac{0.36}{P(B)} = 0.25 \cdot 0.36 = \boxed{0.09}$

(b) If a ramdomly selected Americon person has access to the internet, and if they're also a regular user, the chance that they feel that the Web has reduced their social contacts is: $0.09 \times 100 = 9\%$

4.31 let i = number on of the die

A = rolling the die

B = obtaining exactly 2 heads.

$$\therefore P(B) = \sum_{i=1}^{6} P(B \cap A_i) = \sum_{i=1}^{6} P(A_i) \cdot P(B|A_i)$$

$\because$ Each side of the die is equally likely to appear

$\therefore \; P(A_1) = P(A_2) = P(A_3) = P(A_4) = P(A_5) = P(A_6) = \frac{1}{6}$

$P(B|A_1) = 0$, since the die is only rolled once.

$$P(B|A_2) = \binom{2}{2} \cdot \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^0 = 1 \cdot \frac{1}{4} \cdot 1 = \frac{1}{4}$$

$$P(B|A_3) = \binom{3}{2} \cdot \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^1 = 3 \cdot \frac{1}{4} \cdot \frac{1}{2} = \frac{3}{8}$$

$$P(B|A_4) = \binom{4}{2} \cdot \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^2 = 6 \cdot \frac{1}{4} \cdot \frac{1}{4} = \frac{3}{8}$$

$P(B \mid A_5) = \binom{5}{2} \cdot \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^3 = 10 \cdot \frac{1}{4} \cdot \frac{1}{8} = \frac{5}{16}$

$P(B \mid A_6) = \binom{6}{2} \cdot \left(\frac{1}{2}\right)^2 \cdot \left(\frac{1}{2}\right)^4 = 15 \cdot \frac{1}{4} \cdot \frac{1}{16} = \frac{15}{64}$

$\therefore P(B) = \frac{1}{6} \cdot \left(0 + \frac{1}{4} + \frac{3}{8} + \frac{3}{8} + \frac{5}{16} + \frac{15}{64}\right) = \boxed{\frac{33}{128}}$

4.44 (a) $P(C_1) = \frac{9.3}{61.4} = \frac{93}{614} \approx 0.151$

$P(S_2) = \frac{25.8}{61.4} = \frac{129}{307} \approx 0.42$

$P(C_1 \cap S_2) = \frac{1.3}{61.4} = \frac{13}{614} \approx 0.021$

$$P(C_1 \mid S_2) = \frac{P(C_1 \cap S_2)}{P(S_2)} = \frac{\frac{13}{614}}{\frac{129}{307}} = \frac{13}{258} \approx 0.05$$

$\because P(C_1) \neq P(C_1 \mid S_2)$ $\therefore$ They are not independent

(b) $P(S_1) = \frac{35.6}{61.4} = \frac{178}{307} \approx 0.58$

$P(C_2) = \frac{21.4}{61.4} = \frac{107}{307} \approx 0.35$

$P(S_1 \cap C_2) = \frac{9.8}{61.4} = \frac{49}{307} \approx 0.16$

$$P(S_1 \mid C_2) = \frac{P(S_1 \cap C_2)}{P(C_2)} = \frac{\frac{49}{307}}{\frac{107}{307}} \approx 0.46$$

$\because P(S_1) \neq P(S_1 \mid C_2)$ $\therefore$ S_1 and C_2 aren't independent