8.94 Let $X \sim \mathcal{N}(\mu, \sigma^2)$. Show that, for all $t > 0$,
a) $P(|X - \mu| \le t) = 2\Phi(t/\sigma) - 1$. b) $P(|X - \mu| \ge t) = 2(1 - \Phi(t/\sigma))$.

8.95 Directly verify Equation (8.36) on page 443. That is, show that

$$\frac{1}{\sigma}\phi\left(\frac{x-\mu}{\sigma}\right) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-(x-\mu)^2/2\sigma^2}$$

directly from the definition of ϕ.

8.96 Let $X \sim \mathcal{N}(\mu, \sigma^2)$ and let $z > 0$.
a) Without evaluation, explain why a probability of the form $P(\mu - z\sigma \le X \le \mu + z\sigma)$ doesn't depend on either μ or σ.
b) Determine $P(\mu - z\sigma \le X \le \mu + z\sigma)$ for $z = 1, 2,$ and 3.

8.97 Two normal random variables, X and Y, have the same μ and σ^2 parameters. What can you say about the probability distributions of X and Y? Explain your answer.

8.98 Students in an introductory statistics course at the U.S. Air Force Academy participated in Nabisco's "Chips Ahoy! 1,000 Chips Challenge" by confirming that there were at least 1000 chips in every 18-ounce bag of cookies that they examined. As part of their assignment, they concluded that the number of chips per bag is approximately normally distributed. [Source: Brad Warner and Jim Rutledge, "Checking the Chips Ahoy! Guarantee," *Chance*, 1999, Vol. 12(1), pp. 10–14] Give two reasons why the number of chips in a bag couldn't be exactly normally distributed.

8.99 As reported in *Runner's World* magazine, the times of the finishers in the New York City 10 km run are normally distributed with $\mu = 61$ minutes and $\sigma = 9$ minutes. Let X be the time, in minutes, of a randomly selected finisher. Find
a) $P(X > 75)$. b) $P(X < 50$ or $X > 70)$.

8.100 In 1905, R. Pearl published the article "Biometrical Studies on Man. I. Variation and Correlation in Brain Weight" (*Biometrika*, Vol. 4, pp. 13–104). According to the study, brain weights of Swedish men are normally distributed with $\mu = 1.40$ kg and $\sigma = 0.11$ kg. Obtain the percentage of Swedish men who have brain weights
a) between 1.50 kg and 1.70 kg. b) less than 1.6 kg.

8.101 Refer to Example 8.14 on page 444.
a) What percentage of pregnant women give birth before 300 days?
b) Among those women with a longer than average gestation, what percentage give birth within 300 days? *Note:* As we show in Chapter 10, the parameter μ of a normal random variable is its average (expected) value.
c) In a court case, the prosecuting attorney claims that the defendant is the father of a child who was born on July 6, 2002. The defendant can prove that he was out of town from September 1, 2001, to April 3, 2002. Can the defendant use this information to refute the prosecuting attorney's claim? Explain your answer.

8.102 When you put your money into a soft drink machine at the Student Union, a paper cup comes down, and some cola is put into the cup. You are supposed to get 8 oz of cola. However, the actual amount of cola dispensed is random, having a normal distribution with μ equal to the machine setting and $\sigma = 0.25$ oz. What should the machine setting be so that, in the long run, only 2% of the drinks will contain less than 8 oz?

8.103 A hardware manufacturer produces 10 mm bolts. The manufacturer knows that the diameters of the bolts produced vary somewhat from 10 mm and also from each other. But even if he is willing to accept some variation in bolt diameters, he can't tolerate too much variation—if the variation is too large, too many of the bolts produced will be unusable. The manufacturer has set the tolerance specifications for the 10 mm bolts at ± 0.3 mm; that is, a bolt's diameter is considered satisfactory if it is between 9.7 mm and 10.3 mm. Furthermore, the manufacturer has decided that only 1 in 1000 bolts produced should be defective. Assuming that the diameters of bolts produced are normally distributed with $\mu = 10$ mm, what must σ be to insure that the manufacturer's production criteria are met?

8.104 Blood alcohol concentration (BAC) is the amount of alcohol in the bloodstream, measured in percentages. In many states, a driver is considered legally intoxicated if his or her BAC is 0.10% (i.e., 1 part alcohol per 1000 parts blood in the body) or higher. When a suspected DUI driver is stopped, police request that the person take a breathalyzer test to determine his or her BAC. Such tests are imperfect and exhibit a certain amount of measurement error. Suppose that the measured BAC is a normal random variable with μ equal to the person's actual BAC and $\sigma = 0.005\%$.

a) What is the probability that a driver with a BAC of 0.11% will pass the breathalyzer test?
b) What is the probability that a driver with a BAC of 0.095% will incorrectly be determined to be DUI?

8.105 Let $X \sim \mathcal{N}(0, 1/\alpha^2)$, where α is a positive real number. Determine the PDF of the random variable $1/X^2$. *Note:* The distribution of $1/X^2$ is called a *one-sided stable distribution of index 1/2*.

8.106 The diameters of ball bearings made by the Acme Ball Bearing Company are normally distributed with $\mu = 1.4$ cm and $\sigma = 0.025$ cm. The bearings are fully inspected and those that have diameters either less than 1.35 cm or greater than 1.48 cm are discarded. Determine the PDF of the diameters of the remaining ball bearings.

8.107 At a bottling plant, two machines are used for filling 16 oz bottles of soda. Machine I has an average fill (μ) of 16.21 oz with $\sigma = 0.14$ oz; Machine II has an average fill (μ) of 16.12 oz with $\sigma = 0.07$ oz. Both fill amounts are normally distributed. Machine I fills twice as many bottles per day as Machine II. What percentage of bottles that contain less than 15.96 oz of soda are filled by Machine I?

8.108 As reported by a spokesperson for Southwest Airlines, the no-show rate for reservations is 16%—that is, the probability is 0.16 that a person making a reservation will not take the flight. For a certain flight, 42 people have reservations. For each part, determine and compare the exact probability by using the appropriate binomial PMF and an approximate probability by using the normal approximation to the binomial as given in Proposition 8.12 on page 445. The probability that the number of people who don't take the flight is
a) exactly 5. **b)** between 9 and 12, inclusive. **c)** at least 1. **d)** at most 2.

8.109 According to *USA TODAY*, Anchorage, Alaska, is the city with the highest rate of cell phone ownership, with 56% of the residents owning cell phones. Of 500 randomly selected (without replacement) Anchorage residents, let X denote the number who own a cell phone.

a) Identify the exact probability distribution of the random variable X.
b) Identify the binomial distribution that should be used to approximate the probability distribution of X.
c) Identify the normal distribution that should be used to approximate the probability distribution of X.

Use the local De Moivre–Laplace theorem to approximate the probability that, of 500 randomly selected Anchorage residents, the number who own a cell phone is
d) exactly 280. **e)** between 278 and 280, inclusive.

Solution Let X denote the time, relative to noon, that the person arrives at the specified place. By assumption, $X \sim \mathcal{T}(-5, 5)$. From Equation (8.57), we can express the PDF of X as

$$f_X(x) = \frac{1}{5}\left(1 - \frac{|x|}{5}\right), \qquad -5 < x < 5, \tag{8.58}$$

and $f_X(x) = 0$ otherwise.

The problem is to determine $P(|X| \le 2)$. Applying the FPF, Equation (8.58), and the symmetry of the PDF of X, we get

$$P(|X| \le 2) = \int_{|x|\le 2} f_X(x)\,dx = \int_{-2}^{2} \frac{1}{5}\left(1 - \frac{|x|}{5}\right) dx = \frac{2}{5}\int_0^2 \left(1 - \frac{x}{5}\right) dx = 0.64.$$

Chances are 64% that the person will arrive within 2 minutes of noon. ■

Other Continuous Random Variables

In addition to the families of continuous random variables that we have discussed so far, many others occur frequently in practice. They include the *Weibull, Pareto, lognormal,* and *Cauchy* families. We discuss some of them in the exercises for this section and others in Section 8.7. Later in the book, we encounter still other important families of continuous random variables.

EXERCISES 8.6 Basic Exercises

8.115 Apply calculus techniques to obtain a graph of the gamma function, as defined in Equation (8.42) on page 451. *Note:* You may assume that it's permissible to take the derivative of the gamma function by differentiating under the integral sign with respect to t.

8.116 Let λ be a positive real number and let r be a positive integer. Show that the function $F: \mathcal{R} \to \mathcal{R}$ defined by

$$F(x) = 1 - e^{-\lambda x}\sum_{j=0}^{r-1}\frac{(\lambda x)^j}{j!}, \qquad x \ge 0,$$

and $F(x) = 0$ otherwise, is the CDF of a continuous random variable. *Note:* You must show that F is everywhere continuous and satisfies properties (a)–(d) of Proposition 8.1 on page 411.

8.117 Determine a PDF of each of the following distributions.

a) Erlang distribution with parameters r and λ

b) Chi-square distribution with ν degrees of freedom

8.118 Let $X \sim \mathcal{N}(0, \sigma^2)$. Determine and identify the probability distribution of each of the following random variables.

a) X^2 **b)** X^2/σ^2

8.119 Suppose that X has the beta distribution with parameters m and n, both positive integers. Show that, for $0 < x < 1$, the identity $P(X \le x) = P(Y \ge m)$ holds, where Y has the binomial distribution with parameters $m + n - 1$ and x.

8.120 Let X have the beta distribution with parameters α and β. Determine a PDF of the random variable $X^{-1} - 1$.

8.121 Verify the properties of beta distributions that are given in the bulleted list on page 457.

8.122 Let X have the uniform distribution on the interval $(0, 1)$ and let α be a positive real number. Obtain and identify the probability distribution of the random variable $Y = X^{1/\alpha}$.

8.123 For a certain manufactured item, the proportion that require service during the first 5 years of use has the beta distribution with parameters $\alpha = 2$ and $\beta = 3$. Determine the probability that the percentage of these manufactured items that require service during the first 5 years of use is

a) at most 30%.

b) between 10% and 20%.

8.124 For this exercise, you'll need access to statistical software. In his article "A Juiced Analysis" (*Chance*, 2002, Vol. 15, No. 4, pp. 50–53), Scott M. Berry modeled the probability of a Barry Bonds home run by a beta distribution with parameters $\alpha = 51.3$ and $\beta = 539.75$.

a) Graph and interpret a PDF for the probability of a Barry Bonds home run.

b) Determine the probability that the probability of a Barry Bonds home run is between 0.07 and 0.10.

8.125 Show that any triangular distribution can be obtained from a particular triangular distribution, say, $\mathcal{T}(-1, 1)$, by a location and scale change.

8.126 Obtain and graph the CDF of a random variable with a triangular distribution on the interval (a, b).

8.127 According to the paper "Table for the Likelihood Solutions of Gamma Distribution and Its Medical Applications" (*Reports of Statistical Application Research (JUSE)*, 1952, Vol. 1, pp. 18–23) by M. Masuyama and Y. Kuroiwa, during the 30th week of pregnancy the sedimentation rate, X, has (approximately) the gamma distribution with parameters $\alpha = 5$ and $\lambda = 0.1$.

a) Specify the PDF and CDF of X.

b) Determine the probability that, during the 30th week of pregnancy, the sedimentation rate is at most 60.

c) Determine the probability that, during the 30th week of pregnancy, the sedimentation rate is between 40 and 50.

8.128 **Pareto random variable:** Let X have the exponential distribution with parameter α and let β be a positive real number.

a) Determine the CDF of the random variable $Y = \beta e^X$.

b) Determine a PDF of Y. *Note:* A random variable with this PDF is called a *Pareto random variable* and is said to have the *Pareto distribution with parameters α and β.*

8.129 Refer to Exercise 8.128. An actuary models the loss amounts of a certain policy by the Pareto distribution with parameters $\alpha = 2$ and $\beta = 3$, where loss amounts are measured in thousands of dollars.

a) Graph the PDF and CDF of the loss amounts.

b) Determine the probability that a loss amount exceeds \$8000.

c) Determine the probability that a loss amount is between \$4000 and \$5000.

8.130 **Weibull random variable:** Let X have the exponential distribution with parameter α and let β be a positive real number.

a) Determine the CDF of the random variable $Y = X^{1/\beta}$.

b) Determine a PDF of Y. *Note:* A random variable with this PDF is called a *Weibull random variable* and is said to have the *Weibull distribution with parameters α and β.*

c) Graph the PDF and CDF of a Weibull random variable if $\beta < 1$; $\beta = 1$; $\beta > 1$.

RCISES 8.7 Basic Exercises

In Exercises 8.141–8.149, we cite exercises that appeared earlier in this chapter. Basically, their solutions involved the application of the CDF method, Procedure 8.2 on page 465. In each case, decide whether the transformation method, Procedure 8.3 on page 467, is appropriate; if it is, apply it.

8.141 Exercise 8.67(b) on page 435.

8.142 Exercise 8.72(a) on page 435.

8.143 Exercise 8.73(a) on page 435.

8.144 Exercise 8.105 on page 449.

8.145 Exercise 8.118 on page 461.

8.146 Exercise 8.120 on page 461.

8.147 Exercise 8.122 on page 462.

8.148 Exercise 8.128(b) on page 462.

8.149 Exercise 8.130(b) on page 462.

8.150 Solve Example 8.20 on page 468 by using the CDF method, Procedure 8.2 on page 465.

8.151 Use the transformation method to verify Equation (8.64): If X is a continuous random variable with a PDF and a and $b \neq 0$ are real numbers, then $f_{a+bX}(y) = |b|^{-1} f_X\big((y-a)/b\big)$.

8.152 An actuary modeled the lifetime of a device by the random variable $Y = 10X^{0.8}$, where X is exponentially distributed with parameter 1. Find a PDF of Y.

8.153 Let $X \sim C(\eta, \theta)$ and let ψ denote the standard Cauchy PDF.
a) Verify Equation (8.66) on page 471: $f_X(x) = \theta^{-1}\psi\big((x-\eta)/\theta\big)$.
b) Use part (a) to conclude that $(X - \eta)/\theta$ has the standard Cauchy distribution.

8.154 Lognormal random variable: Let $X \sim \mathcal{N}(\mu, \sigma^2)$.
a) Determine a PDF of the random variable $Y = e^X$. *Note:* A random variable with this PDF is called a *lognormal random variable* and is said to have the *lognormal distribution with parameters μ and σ^2.*
b) Explain why the term "lognormal" is used for a random variable with a PDF as found in part (a).

8.155 Find a PDF of the random variable $Y = \sin X$ when
a) $X \sim \mathcal{U}(-\pi/2, \pi/2)$. b) $X \sim \mathcal{U}(-\pi, \pi)$.

8.156 A point is selected at random from the unit circle (i.e., boundary of the unit disk).
a) Determine a PDF of the x coordinate of the point chosen.
b) Without doing any calculations, determine a PDF of the y coordinate of the point chosen. Explain your reasoning.
c) Determine a PDF of the distance from the point chosen to the point $(1, 0)$.

8.157 Let X have the beta distribution with parameters α and β and set $Y = a + (b-a)X$, where a and b are real numbers with $a < b$.
a) Identify the range of Y. b) Obtain a PDF of Y.

8.158 Show that, if X has the standard Cauchy distribution, so does $1/X$.