Chapter 5 Will Zang

5.45 (a) Assuming that there're only 2 outcomes: hit / miss

Also assuming that the player's probability for hit stays constant.

In addition, successive hits do not affect each other (independent)

(b) $P(X=1) = \binom{4}{1}(0.260)^1 \cdot (1-0.260)^{4-1} \approx 0.4214$

$P(X=0) = \binom{4}{0}(0.260)^0 \cdot (1-0.260)^4 \approx 0.2999$

$\therefore P(X \geq 2) = 1 - [P(X=1) + P(X=0)] \approx 0.279$

5.49 Let head $(X=1)$, tail $(X=0)$

$P(\text{only 1 head}) = P(X=1) = \binom{n}{1} p^1 (1-p)^{n-1}$

$P(\text{only 1 tail}) = P(X=0) = \binom{n}{1} p^{(n-1)} (1-p)^1$

$\therefore P(\text{odd}) = P(\text{only 1 head}) + P(\text{only 1 tail})$

$= \binom{n}{1} p(1-p)^{n-1} + \binom{n}{1} p^{n-1}(1-p)$

5.62 $$P(X=X) = \frac{\binom{N \cdot \frac{X}{n}}{n} \cdot \binom{N \cdot (1-\frac{X}{n})}{n-X}}{\binom{N}{n}}$$

5.65 Let A: Bin 1 is chosen ; B = Bin 2 is chosen

C = 2 of the 3 parts are defective

Look for: $P(A \mid C) = \frac{P(A \cap C)}{P(C)}$

$P(C) = P(C \mid A) \cdot P(A) + P(C \mid B) \cdot P(B)$

$$= \frac{\binom{20 \cdot (\frac{5}{20})}{2} \cdot \binom{20 \cdot (1-\frac{5}{20})}{1}}{\binom{20}{3}} \cdot \left(\frac{1}{2}\right) + \frac{\binom{15 \cdot \frac{4}{15}}{2} \cdot \binom{15 \cdot (1-\frac{4}{15})}{1}}{\binom{15}{3}} \cdot \left(\frac{1}{2}\right)$$

$$= \frac{10 \cdot 15}{1140} \cdot \frac{1}{2} + \frac{6 \cdot 11}{455} \cdot \frac{1}{2} \approx 0.1383$$

$P(A \cap C) = P(C|A) \cdot P(A) = \frac{10 \cdot 15}{1140} \cdot \frac{1}{2} \approx 0.06579$

$\therefore P(A|C) = \frac{P(A \cap C)}{P(C)} \approx \frac{0.06579}{0.13831} \approx 0.476.$

5.70 $\lim_{N \to \infty} \frac{\binom{NP}{x}\binom{N(1-P)}{n-x}}{\binom{N}{n}} = \binom{n}{x} P^x (1-P)^{n-x}$ $\binom{n}{x} = \frac{n}{x!(n-x)!}$

$$LHS = \frac{\frac{(NP)!}{x!(NP-x)!} \cdot \frac{(N-NP)!}{(n-x)!(N-NP-n+x)!}}{\frac{N!}{n!(N-n)!}}$$

$$= \frac{(NP)! \cdot (N-NP)! \cdot n!(N-n)!}{x!(NP-x)! \cdot (n-x)!(N-NP-n+x)! \cdot N!}$$?

$$= \frac{n!}{x!(n-x)!} \cdot \frac{(NP)! \cdot (N-NP)! \cdot (N-n)!}{(NP-x)! \cdot (N-NP-N+x)! \cdot N!} = \binom{n}{x} \cdot \frac{(NP)! \cdot (N-NP)! \cdot (N-n)!}{(NP-x)! \cdot (N-NP-N+x)! \cdot N!}$$

5.80 $n = 10,000$, $P = \frac{1}{1500}$

$\therefore \lambda = nP = 10,000 \cdot \frac{1}{1500} = \frac{20}{3} \Rightarrow P(X=x) = \frac{e^{-\frac{20}{3}} \cdot (\frac{20}{3})^x}{x!}$

(a) $P(X \leq 7) = \sum_{x=0}^{7} \frac{e^{-\frac{20}{3}} \cdot (\frac{20}{3})^x}{x!} \approx 0.648$

$\therefore P(X > 7) = 1 - P(X \leq 7) \approx 0.352$

(b) $P(X \leq 10) = \sum_{x=0}^{10} \frac{e^{-\frac{20}{3}} \cdot (\frac{20}{3})^x}{x!} \approx 0.9234$

(c) Using binomial formula:

$P(X > 7) = 1 - P(X \leq 7)$

$= 1 - \sum_{x=0}^{7} \binom{10,000}{x} \cdot (\frac{1}{1500})^x (1 - \frac{1}{1500})^{10000-x} \approx 0.352$

$P(X \leq 10) = \sum_{x=0}^{10} \binom{10000}{x} \cdot (\frac{1}{1500})^x (1 - \frac{1}{1500})^{10000-x} \approx 0.923$

$\therefore$ They're basically the same.

5.84 Use poissoin approximation:

$e^{-\lambda} \cdot \frac{\lambda^k}{k!} = 0.9$, where $\lambda = n \cdot p = 0.005 \cdot n$, $k=1$

$e^{-0.005n} \cdot -0.005n = 0.9$

$e^{-0.005n} = -180 \cdot \frac{0.9}{n}$

$-0.005n = \log(180 \cdot \frac{0.9}{n})$?

5.96 Definition of lack of memory property:

$P(X = n+k \mid X > n) = P(X=k)$

in this case: $k=4$, $n=6$

$\therefore$ Equation 1 must hold

5.97 (a) $P(X=k \mid X>n) = \frac{P(X=k, X>n)}{P(X>n)} = \frac{P(X=k)}{P(X>n)}$

$= \frac{P(1-P)^{k-1}}{(1-P)^n} = P \cdot (1-P)^{k-n-1}$

(b) use lack of memory property: $P(X=n+k \mid X>n) = P(X=k)$

$P(X=k \mid X>n)$

in this case: "k" $= k-n$

$\therefore P(X=k \mid X>n) = P(X=k-n) = P(1-P)^{(k-n)-1}$

5.116 (a) $\because$ It's # of bets until win $\therefore X \sim G(p=0.67)$

$\therefore P(k) = (1-P)^{k-1} \cdot P$, $k = 1, 2, 3 \dots$, $P = 0.67$

(b) Exactly 4: $P(X=4)$

$P(X=4) = (1-0.67)^{4-1} \cdot 0.67 \approx 0.0241$

At most 4: $P(X \leq 4)$

$p(x=3) = (1-0.67)^2 \cdot 0.67 \approx 0.07296$

$p(x=2) = (1-0.67)^1 \cdot 0.67 \approx 0.2211$

$p(x=1) = (1-0.67)^0 \cdot 0.67 = 0.67$

$\therefore\ p(x \leq 4) \approx 0.67 + 0.2211 + 0.07296 + 0.0241 \approx 0.988$

At least 4: $p(x \geqslant 4)$

$p(x \geqslant 4) = 1 - (p(x=3) + p(x=2) + p(x=1))$

≈ 0.036