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4.45 According to the table: $P(A_2 \cap L_1) = 0.038$.

However, $P(A_2) \cdot P(L_1) = 0.401 \cdot 0.126 \approx 0.051$

$\because P(A_2 \cap L_1) \neq P(A_2) \cdot P(L_1)$ $\boxed{\therefore \text{They are not independent events.}}$

4.47 (a) When $P(A)=0$: $P(A) \cdot P(B) = 0 \cdot P(B) = 0$

$\because (A \cap B) \subseteq A \quad \therefore P(A \cap B) \leq P(A) \Rightarrow P(A \cap B) = 0$

$\because P(A \cap B) = P(A) \cdot P(B) = 0 \quad \therefore$ They're independent.

When $P(A)=1$: $P(A) \cdot P(B) = 1 \cdot P(B) = P(B)$

$\because A, A^c$ forms a partition of $B \quad \therefore P(B) = P(A \cap B) + P(A^c \cap B)$

$\because P(A) = 1 \quad \therefore P(A^c) = 1 - P(A) = 0 \Rightarrow P(A^c \cap B) = 0$

$\therefore P(B) = P(A \cap B) = P(A) \cdot P(B) \quad \therefore A, B$ are independent.

(b) Given that A and A are independent event

$\therefore P(A \cap A) = P(A) \cdot P(A)$

$\because 0 \leq P(A) \leq 1$

$\therefore$ The only values that satifies the above condition are: $P(A) = 0$ or $P(A) = 1$

4.55 (a) Let A, B be 2 arbitrary events. $\because A, B$ are mutually exclusive

$\therefore A \cap B = \phi \Rightarrow P(A \cap B) = 0$

(b) Let A, B be 2 arbitrary events with positive probability and are independent

$\therefore P(A \cap B) = P(A) \cdot P(B)$

$\because P(A) > 0 \wedge P(B) > 0 \quad \therefore P(A \cap B) > 0$

$\because P(A \cap B) \neq 0 \quad \therefore A \cap B \neq \phi \Rightarrow$ They're not disjoint.

(C) Suppose there's an experiment: A person draws 2 balls at a time from a bin with equal number of red and purple balls. After each draw, the balls are permenantly removed from the bin.

$A = \{$ drawing at least 1 red ball $\}$

$B = \{$ Both balls drawn are red $\}$

In this case $A \cap B \neq \phi$; A and B are dependent.

4.68 let $D = \{$ democratic voters $\}$; $R = \{$ Republican voters $\}$; $I = \{$ independent voters $\}$.

$F = \{$ voters favoring increased spending to combat terrorism $\}$

$$P(D \mid F) = \frac{P(D \cap F)}{P(F)}$$

$\because D \cup R \cup I = \Omega$

$$\therefore P(F) = P(D \cap F) + P(R \cap F) + P(I \cap F)$$

$$= (0.4 \cdot 0.6) + (0.32 \cdot 0.8) + (0.28 \cdot 0.3) = 0.58$$

$$P(D \cap F) = 0.4 \cdot 0.6 = 0.24$$

$$\therefore P(D \mid F) = \frac{P(D \cap F)}{P(F)} = \frac{0.24}{0.58} \approx \boxed{0.414}$$

4.72 Let $A = \{$ choose chest A $\}$, $B = \{$ choose chest B $\}$, $C = \{$ choose chest C $\}$, $D = \{$ choose chest D $\}$, $S = \{$ getting silver coin $\}$, $G = \{$ getting a gold coin $\}$.

Box A: S, G

Box B: S, G

Box C: S, S

Box D: G, G

A, B, C, D forms partition of Ω

$$P(C \mid S) = \frac{P(C \cap S)}{P(S)} = \frac{P(C \cap S)}{P(A \cap S) + P(B \cap S) + P(C \cap S) + P(D \cap S)}$$

$$= \frac{P(C) \cdot P(S \mid C)}{P(A) \cdot P(S \mid A) + P(B) \cdot P(S \mid B) + P(C) \cdot P(S \mid C) + P(D) + P(S \mid D)}$$

$P(A) = P(B) = P(C) = P(D) = \frac{1}{4}$

$P(S|A) = \frac{1}{2}$; $P(S|B) = \frac{1}{2}$; $P(S|C) = 1$; $P(S|D) = 0$.

$$\therefore P(C|S) = \frac{\frac{1}{4} \cdot 1}{\frac{1}{4} \cdot \left(\frac{1}{2} + \frac{1}{2} + 1 + 0\right)} = \boxed{\frac{1}{2}}$$

1: 4R, 1W; 2: 1R, 4W

5.7 (a) $\{$Urn I: $\{(R,R), (R,W), (W,R)\}$; Urn II: $\{(W,W), (W,R), (R,W)\}\}$

(b)

	ω	X		ω	X
I	(R, R)	2	II	(W, W)	0
	(R, W)	1		(W, R)	1
	(W, R)	0		(R, W)	1

5.14 (a) Suppose there're n values of $x \in \mathbb{R}$, then we can write: $\sum_{x}^{X} P_X(X=x) = 1$

Divide both sides by n, we get: $\text{Avg}\_P_X(X=x) = \frac{1}{n}$

In other words, even where the pmf is completely evenly distributed, at most n values of x can satify $P_X(X=x) = \frac{1}{n}$. Thus, to achieve the most number of $x \in \mathbb{R}$ that satisfies inequality $P_X(X=x) > \frac{1}{n}$, we must reduce the number of x, n, by 1, which is n-1.

Thus, n-1 is the largest number of x for which $P_X(X=x) > \frac{1}{n}$ holds.

(b) Since $n \in \mathbb{N}$, n-1 must be countable. According to (a), there're at most (n-1) $x \in \mathbb{R}$ such that $P_X(X=x) > \frac{1}{n}$, and $\frac{1}{n} \neq 0$. Thus, $\{\bigcup_{0}^{n} x \in \mathbb{R}\}$, where x satisfies $P_X(X=x) > \frac{1}{n}$, must also be countable. Then we can infer that $\{x \in \mathbb{R} : P(X=x) \neq 0\}$ is countable.

(c) If the probability for any specific input x for a PMF $P_X(X=x)$ does not equal to zero, then the set of input X is a countable set.

5.24 (a): $|\Omega| = 6^2 = 36$, Let E_i be events where the sum is i.

$E_2 = \{(1,1)\}$

$E_3 = \{(1,2), (2,1)\}$

$E_4 = \{(1,3), (3,1), (2,2)\}$

$E_5 = \{(1,4), (4,1), (2,3), (3,2)\}$

$E_6 = \{(1,5), (5,1), (2,4), (4,2), (3,3)\}$

$E_7 = \{(1,6), (6,1), (2,5), (5,2), (3,4), (4,3)\}$

$E_8 = \{(2,6), (6,2), (3,5), (5,3), (4,4)\}$

$E_9 = \{(3,6), (6,3), (4,5), (5,4)\}$

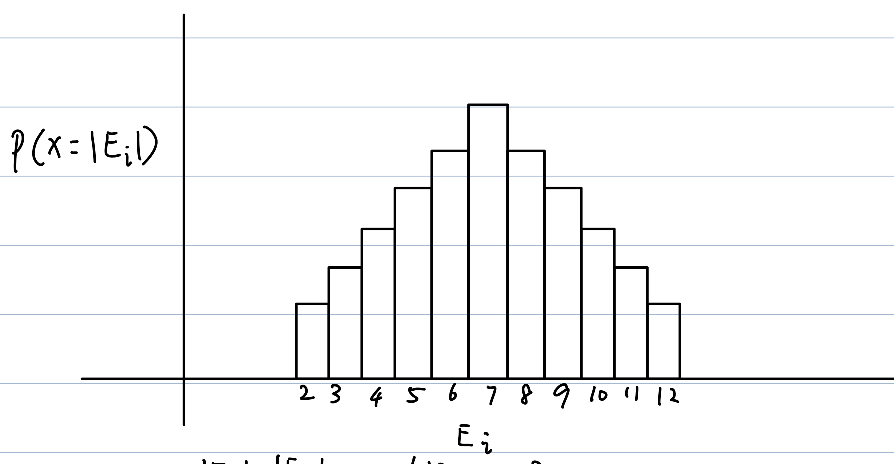
$E_{10} = \{(4,6), (6,4), (5,5)\}$

$E_{11} = \{(5,6), (6,5)\}$

$E_{12} = \{(6,6)\}$

$$P_X(x) = \begin{cases} p(X=2) = \frac{|E_2|}{\Omega} = \frac{1}{36} \\ p(X=3) = \frac{|E_3|}{\Omega} = \frac{2}{36} \\ p(X=4) = \frac{3}{36} \\ p(X=5) = \frac{4}{36} \\ p(X=6) = \frac{5}{36} \\ p(X=7) = \frac{6}{36} \\ p(X=8) = \frac{5}{36} \\ p(X=9) = \frac{4}{36} \\ p(X=10) = \frac{3}{36} \\ p(X=11) = \frac{2}{36} \\ p(X=12) = \frac{1}{36} \end{cases}$$

(b)



(c) $p(\text{win}) = \frac{|E_7| + |E_{11}|}{\Omega} = \frac{6+2}{36} = \frac{2}{9}$

(d) $p(\text{loss}) = \frac{|E_2| + |E_3| + |E_{12}|}{\Omega} = \frac{1+2+1}{36} = \frac{1}{9}$

(e) A is the set of all of the sums. i.e., the set of 36 permutations.