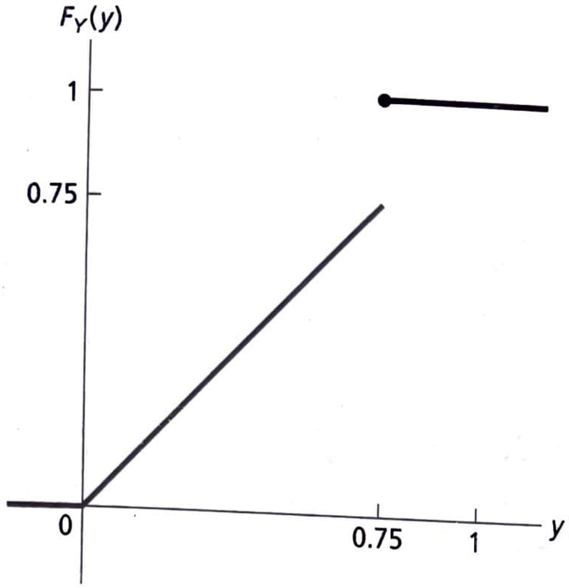
Figure 8.6 CDF of the random variable Y



Basic Properties of a CDF

Figures 8.3–8.6 show that all four CDFs are nondecreasing and right-continuous. In fact, every CDF satisfies the four basic properties presented in Proposition 8.1.

◆◆◆ **Proposition 8.1 Basic Properties of a CDF**

The cumulative distribution function F_X of a random variable X satisfies the following four properties.

a) *F_X is nondecreasing.*

b) *F_X is everywhere right-continuous.*

c) $F_X(-\infty) \equiv \lim_{x\to-\infty} F_X(x) = 0$.

d) $F_X(\infty) \equiv \lim_{x\to\infty} F_X(x) = 1$.

Proof a) Suppose that $x_1 < x_2$. Then we have $\{X \le x_1\} \subset \{X \le x_2\}$ and, so, by the domination principle (Proposition 2.6 on page 64),

$$F_X(x_1) = P(X \le x_1) \le P(X \le x_2) = F_X(x_2).$$

Thus F_X is nondecreasing.

b) Suppose that $x \in \mathcal{R}$ and let $\{x_n\}_{n=1}^{\infty}$ be any decreasing sequence of real numbers that converges to x. For each $n \in \mathcal{N}$, set $A_n = \{X \le x_n\}$. Then $A_1 \supset A_2 \supset \cdots$ and $\bigcap_{n=1}^{\infty} A_n = \{X \le x\}$. Applying the continuity property of probability (Proposition 2.11 on page 74) gives

$$\begin{aligned}\lim_{n\to\infty} F_X(x_n) &= \lim_{n\to\infty} P(X \le x_n) = \lim_{n\to\infty} P(A_n) \\ &= P\left(\bigcap_{n=1}^{\infty} A_n\right) = P(X \le x) = F_X(x).\end{aligned}$$

Hence F_X is everywhere right-continuous.