a) Find the conditional expectation of Y given $X = x$. *Note:* Exercise 6.46(a) on page 288 asks for the conditional PMF of Y given $X = x$.
b) Find the conditional variance of Y given $X = x$.

7.118 Refer to Exercise 7.117 and note that Exercise 6.3 on page 269 asks for the joint PMF of X and Y and the marginal PMF of Y.
a) Use the law of total expectation to determine $\mathcal{E}(Y)$.
b) Determine $\mathcal{E}(Y)$ by using the definition of expected value. Compare your answer to the one obtained in part (a).
c) Use the law of total variance to determine $\mathrm{Var}(Y)$.
d) Determine $\mathrm{Var}(Y)$ by using the definition of variance. Compare your answer to the one obtained in part (c).

7.119 An automobile insurance company divides its policyholders into three groups, which constitute 45.8%, 32.6%, and 21.6% of the policyholders. The average claim amounts for the three groups are \$1457, \$2234, and \$2516, respectively. Obtain the average claim amount among all policyholders.

7.120 The hourly number of customers entering a bank for the purpose of making a deposit has a Poisson distribution with parameter 25.8. Each such customer deposits an average of \$574 with a standard deviation of \$3167. For a 1-hour period,
a) find the expected total deposits by all entering customers.
b) find the standard deviation of deposit amounts by all entering customers.

7.121 Part of a homeowner's insurance policy covers one miscellaneous loss per year, which is known to have a 10% chance of occurring. If there is a miscellaneous loss, the probability is c/x that the loss amount is \$$100x$, for $x = 1, 2, \ldots, 5$, where c is a constant. These are the only loss amounts possible. If the deductible for a miscellaneous loss is \$200, determine the net premium for this part of the policy—that is, the amount that the insurance company must charge to break even.

7.122 An automobile insurance company classifies its policyholders as either good or bad drivers, of which there are 75% and 25%, respectively. The mean claim amounts for good and bad drivers are \$2000 and \$3500, respectively, and the standard deviations of those claim amounts are \$200 and \$400, respectively. Determine the mean and standard deviation of the claim amounts for all policyholders of this company.

7.123 Let $X \sim \mathcal{G}(p)$. Assume as known that $\mathcal{E}(X) = 1/p$, as found in Example 7.25 on page 382. Determine $\mathrm{Var}(X)$ by using each of the following methods.
a) Condition on the outcome of the first trial to obtain the second moment of X.
b) Use the law of total variance.

7.124 Let $X \sim \mathcal{NB}(r, p)$. Assume as known that the mean of a geometric random variable with parameter p is $1/p$, as found in Example 7.25 on page 382. Use mathematical induction on r to show that $\mathcal{E}(X) = r/p$ by conditioning on the outcome of the first trial.

7.125 Let $X \sim \mathcal{NB}(r, p)$. Assume as known that $\mathcal{E}(X) = r/p$, as found in Exercise 7.124, and that the variance of a geometric random variable with parameter p is $(1 - p)/p^2$, as found in Exercise 7.123. Show that $\mathrm{Var}(X) = r(1 - p)/p^2$ by using mathematical induction on r with each of the following methods.
a) Condition on the outcome of the first trial to obtain the second moment of X.
b) Use the law of total variance.

7.126 Let $X_1, \ldots, X_n$ be a random sample from the distribution of a discrete random variable X with finite mean. For $1 \leq k \leq n$, determine $\mathcal{E}(X_k \mid \bar{X}_n = x)$

a) without doing any computations. Explain your reasoning.

b) by using properties of (conditional) expectation. *Hint:* Consider $\mathcal{E}(\bar{X}_n \mid \bar{X}_n = x)$.

Theory Exercises

7.127 Let X and Y be random variables defined on the same sample space.

a) Prove that, if Y has finite expectation, then so does $Y_{|X=x}$ for each possible value x of X.

b) Prove that, if Y has finite variance, then so does $Y_{|X=x}$ for each possible value x of X.

7.128 Let X and Y be discrete random variables defined on the same sample space and let x be a possible value of X. Suppose that g is a real-valued function of two real variables defined on the range of (X, Y) and such that $\sum_y |g(x, y)| p_{Y \mid X}(y \mid x) < \infty$.

a) Prove that $g(X, Y)_{|X=x}$ has finite expectation and that

$$\mathcal{E}\big(g(X, Y) \mid X = x\big) = \sum_y g(x, y) p_{Y \mid X}(y \mid x).$$

Hint: Refer to Equation (7.70) on page 388.

b) Let $\xi(x) = \mathcal{E}\big(g(X, Y) \mid X = x\big)$ and define $\mathcal{E}\big(g(X, Y) \mid X\big)$ to be $\xi(X)$. Let k be a function of x alone and let ℓ be a function of both x and y. Prove that

$$\mathcal{E}\big(k(X)\ell(X, Y) \mid X\big) = k(X)\mathcal{E}\big(\ell(X, Y) \mid X\big),$$

provided the expectation on the right makes sense.

c) Prove that $\mathcal{E}\big(g(X, Y)\big) = \mathcal{E}\Big(\mathcal{E}\big(g(X, Y) \mid X\big)\Big)$.

Advanced Exercises

7.129 Refer to Example 6.15 on page 287. Given that $X = 0$, determine the expected number of unreimbursed automobile-accident losses for the family.

7.130 Let $X_1, \ldots, X_m$ have the multinomial distribution with parameters n and $p_1, \ldots, p_m$ and let $k \neq \ell$ be integers between 1 and m, inclusive.

a) Use conditional expectation to determine $\mathcal{E}(X_k X_\ell)$. *Hint:* Refer to Exercise 7.128.

b) Use part (a) to determine $\mathrm{Cov}(X_k, X_\ell)$. Compare your work with that done in Exercise 7.93 on page 375.

7.131 The number of customers who enter a certain store in a day is a random variable X having a Poisson distribution with parameter λ. Each customer makes a purchase with probability p, independent of all other customers. Let Y be the number of purchasing customers in a day. Determine $\rho(X, Y)$. *Hint:* Refer to Exercise 7.128.

7.132 In Exercise 7.75 (page 364) and Exercise 7.104 (page 376), we discussed how to best estimate a random variable Y by a constant c and by a linear function of a random variable X, respectively. In both cases, we use the minimum mean square as the criterion for "best." With that criterion for best, prove that the conditional expectation of Y given X is the best estimate among all functions of X. That is, prove that

$$\mathcal{E}\Big(\big(Y - \mathcal{E}(Y \mid X)\big)^2\Big) = \min_{h \in \mathcal{H}} \mathcal{E}\Big(\big(Y - h(X)\big)^2\Big), \tag{$*$}$$

where $\mathcal{H}$ consists of all real-valued functions h defined on the range of X such that $h(X)$ has finite variance. Refer to Exercise 7.128 and use the following steps. Let $\psi(x) = \mathcal{E}(Y \mid X = x)$.

Describing the Probability Distribution of a Continuous Random Variable

As we mentioned earlier, the probabilities $P(X = x)$ completely determine the probability distribution of a discrete random variable X. However, for a continuous random variable X, the probabilities $P(X = x)$ always equal 0 and hence are totally useless in describing the probability distribution of X.

How then do we describe the probability distribution of a continuous random variable X? The answer is that, instead of considering, for each $x \in \mathcal{R}$, the probability that X equals x, we consider the probability that X is "near" x. First, however, we must examine another function associated with a random variable, the *cumulative distribution function,* which we do in Section 8.2.

EXERCISES 8.1 Basic Exercises

8.1 On page 402, we stated that: "...because a continuous random variable X typically involves measurement, the probability should be 0 that X equals any particular value x." Explain in your own words why this statement makes sense.

8.2 A point is chosen at random from the interior of a sphere of radius r. Let Z denote the distance from the center of the sphere to the point chosen. Use both methods in Example 8.2 on page 403 to show that Z is a continuous random variable.

8.3 Let X denote the tangent of an angle chosen at random from the interval $(-\pi/2, \pi/2)$. Show that X is a continuous random variable.

8.4 Let X be a continuous random variable. Verify that each of the following functions of X is a continuous random variable.
a) $a + bX$, where a and $b \neq 0$ are real numbers
b) X^2 **c)** $\sin X$

8.5 A point is chosen at random from the interior of a triangle with base b and height h. Let Y denote the distance of the point chosen to the base of the triangle. Show that Y is a continuous random variable.

8.6 Let f be a nonnegative real-valued function of a real variable such that $\int_{-\infty}^{\infty} f(x)\,dx = 1$. Suppose that X is a random variable that satisfies $P(X \in A) = \int_A f(x)\,dx$ for all subsets A of $\mathcal{R}$. Show that X is a continuous random variable.

Theory Exercises

8.7 Let X be a continuous random variable.
a) Show that $P(X \in K) = 0$ for any countable set K.
b) Use the result of part (a) to conclude that a continuous random variable can't be a discrete random variable. *Hint:* Refer to Definition 5.2 on page 178.

Advanced Exercises

8.8 Refer to Example 8.1 on page 402, where X denotes a randomly chosen number from the interval (0, 1). Determine the probability that the number chosen is a rational number.

8.9 Let X be a continuous random variable and let p be a polynomial of positive degree. Show that $p(X)$ is a continuous random variable.

From Equation (8.7), we can also conclude that

$$P(X = x) = F_X(x) - F_X(x-), \qquad x \in \mathcal{R}. \tag{8.8}$$

Geometrically, this relation means that the probability that X takes the value x equals the jump (if any) of the CDF of X at x. To show why Equation (8.8) is true, we first observe that $\{X \le x\} = \{X < x\} \cup \{X = x\}$. Noting that the two events on the right of this equation are mutually exclusive and referring to Equation (8.7), we have

$$F_X(x) = P(X \le x) = P(X < x) + P(X = x) = F_X(x-) + P(X = x).$$

Hence Equation (8.8) holds. From Equation (8.8) and Proposition 8.1(b), we deduce the important fact presented in Proposition 8.3.

◆◆◆ **Proposition 8.3 Continuous Random Variables and Continuous CDFs**

A random variable is continuous if and only if its CDF is an everywhere continuous function.

Proof We first recall from Proposition 8.1(b) that the CDF of any random variable is everywhere right-continuous. Thus a CDF is continuous at a point if and only if it is left-continuous at that point.

Suppose that X is a continuous random variable. Then, by definition, $P(X = x) = 0$ for all $x \in \mathcal{R}$. This fact and Equation (8.8) imply that F_X is left-continuous and hence continuous for all $x \in \mathcal{R}$. Conversely, suppose that the CDF of a random variable X is everywhere continuous. Then, in particular, it is everywhere left-continuous. From Equation (8.8), $P(X = x) = 0$ for all $x \in \mathcal{R}$. So X is a continuous random variable. ◆

Note: Proposition 8.3 shows that, for a continuous random variable X, the right sides of all four equations in Proposition 8.2 equal $F_X(b) - F_X(a)$.

Proposition 8.3 provides another justification for using the adjective "continuous" for a random variable X that satisfies $P(X = x) = 0$ for all $x \in \mathcal{R}$—namely, the CDF of such a random variable is everywhere continuous.

EXERCISES 8.2 Basic Exercises

In Exercises 8.15–8.21, do the following.

a) Obtain and graph the CDF of the specified random variable.
b) Use the results of part (a) to verify both mathematically and graphically that the CDF of the specified random variable satisfies properties (a)–(d) of Proposition 8.1 on page 411.
c) Use Proposition 8.3 and the results of part (a) to decide whether the specified random variable is a continuous random variable.

8.15 A point is chosen at random from the interior of a sphere of radius r. Let Z denote the distance from the center of the sphere to the point chosen.

8.16 Let X denote the tangent of an angle chosen at random from the interval $(-\pi/2, \pi/2)$.

8.17 A point is chosen at random from the interior of a triangle with base b and height h. Let Y denote the distance from the point chosen to the base of the triangle.

8.18 Let X be the number of siblings of a randomly selected student from one of Professor Weiss's classes, as discussed in Example 5.1 on page 176. *Note:* Table 5.5 on page 186 gives the PMF of the random variable X.

8.19 Let Y be the indicator random variable of an event E.

8.20 According to *Vital Statistics of the United States*, published by the U.S. National Center for Health Statistics, chances are 80% that a person aged 20 will be alive at age 65. Of three people aged 20 selected at random, let X denote the number who live to be at least age 65.

8.21 Six men and five women apply for a job at Alpha, Inc. Three of the applicants are selected for interviews. Let X denote the number of women in the interview pool.

8.22 Let X be a discrete random variable.
a) Express F_X in terms of p_X. **b)** Express p_X in terms of F_X.

8.23 Let X have the discrete uniform distribution on the set of the first N positive integers.
a) Without using the PMF of X, obtain and graph the CDF of X.
b) Use part (a) and Equation (8.8) on page 413 to obtain the PMF of X.

8.24 Let X have the geometric distribution with parameter p.
a) Use the FPF to obtain the CDF of X.
b) Use tail probabilities to obtain the CDF of X.
c) Graph the CDF of X.
d) Use the CDF of X and Equation (8.8) on page 413 to obtain the PMF of X.

8.25 Refer to Example 8.3 on page 407. Use the CDF obtained in that example to determine $P(a < W \leq b)$, $P(a < W < b)$, $P(a \leq W < b)$, and $P(a \leq W \leq b)$ for the specified values of a and b.
a) $a = 1, b = 2$ **b)** $a = 0.5, b = 2$ **c)** $a = 1, b = 2.75$ **d)** $a = 0.5, b = 2.75$
e) Repeat parts (a)–(d) by using the FPF and the PMF of W (Table 8.1 on page 407).

8.26 Refer to Example 8.4 on page 408. Use the CDF obtained in that example to determine $P(a < X \leq b)$, $P(a < X < b)$, $P(a \leq X < b)$, and $P(a \leq X \leq b)$ for the specified values of a and b.
a) $a = 0.2, b = 0.8$ **b)** $a = 0, b = 0.8$ **c)** $a = 0.2, b = 1.5$
d) $a = -1, b = 1.5$ **e)** $a = -2, b = -1$ **f)** $a = 1, b = 2$
g) For each of parts (a)–(f), why are the four probabilities identical?

8.27 Refer to Example 8.5 on page 409. Use the CDF obtained in that example to determine $P(a < Z \leq b)$, $P(a < Z < b)$, $P(a \leq Z < b)$, and $P(a \leq Z \leq b)$ for the specified values of a and b.
a) $a = 0.2, b = 0.8$ **b)** $a = 0, b = 0.8$ **c)** $a = 0.2, b = 1.5$
d) $a = -1, b = 1.5$ **e)** $a = -2, b = -1$ **f)** $a = 1, b = 2$
g) For each of parts (a)–(f), why are the four probabilities identical?

8.28 Refer to Example 8.6 on page 410. Use the CDF obtained in that example to determine $P(a < Y \leq b)$, $P(a < Y < b)$, $P(a \leq Y < b)$, and $P(a \leq Y \leq b)$ for the specified values of a and b.
a) $a = 0.2, b = 0.8$ **b)** $a = 0.2, b = 0.75$ **c)** $a = -1, b = 1.5$
d) $a = -1, b = 0.75$ **e)** $a = -2, b = -1$ **f)** $a = 0.75, b = 2$

8.29 Let f be a nonnegative real-valued function such that $\int_{-\infty}^{\infty} f(x)\,dx = 1$. Suppose that X is a random variable that satisfies $P(X \in A) = \int_A f(x)\,dx$ for all subsets A of $\mathcal{R}$.
a) Obtain F_X in terms of f.
b) What is the relationship between F_X' and f?

c) Let a and b be real numbers with $a < b$. Determine $P(a < X \leq b)$, $P(a < X < b)$, $P(a \leq X < b)$, and $P(a \leq X \leq b)$ in terms of f.
d) Why are all four answers in part (c) the same?

8.30 For each function f, let X be as in Exercise 8.29. Graph f and determine and graph F_X.
a) $f(x) = \lambda e^{-\lambda x}$ if $x > 0$ and $f(x) = 0$ otherwise, where λ is a positive real number.
b) $f(x) = 1/(b-a)$ if $a < x < b$ and $f(x) = 0$ otherwise, where a and b are real numbers with $a < b$.
c) $f(x) = b^{-1}(1 - |x|/b)$ if $-b < x < b$ and $f(x) = 0$ otherwise, where b is a positive real number.

8.31 Decide whether each function F is the CDF of a random variable by checking properties (a)–(d) of Proposition 8.1 on page 411. For each F that is the CDF of a random variable, classify the random variable as discrete, continuous, or mixed.
a) $F(x) = 0$ if $x < 0$ and $F(x) = 1$ if $x \geq 0$.
b) $F(x) = 0$ if $x < 0$, $F(x) = 1 - p$ if $0 \leq x < 1$, and $F(x) = 1$ if $x \geq 1$. Here p is a real number with $0 < p < 1$.
c) $F(x) = 0$ if $x < 0$ and $F(x) = \lfloor x \rfloor$ if $x \geq 0$.
d) $F(x) = 0$ if $x < 0$ and $F(x) = \sum_{n=0}^{\lfloor x \rfloor} a_n$ for $x \geq 0$, where $\{a_n\}_{n=0}^{\infty}$ is a sequence of nonnegative real numbers whose sum is 1.
e) $F(x) = 0$ if $x < 0$ and $F(x) = 1 - e^{-\lambda x} \sum_{j=0}^{r-1} (\lambda x)^j / j!$ if $x \geq 0$. Here λ is a positive real number and r is a positive integer.
f) $F(x) = 0$ if $x < 0$ and $F(x) = x$ if $x \geq 0$.
g) $F(x) = 0$ if $x < -1$, $F(x) = \frac{1}{2} + \frac{3}{8}x$ if $-1 \leq x < 1$, and $F(x) = 1$ if $x \geq 1$.

8.32 Let X be a random variable and let m be a real number. Determine the CDF of each of the following random variables in terms of the CDF of X.
a) $Y = \max\{X, m\}$ b) $Z = \min\{X, m\}$

8.33 Let $X_1, \ldots, X_m$ be independent random variables, each having the same probability distribution as a random variable X. Determine the CDF of each of the following random variables in terms of the CDF of X.
a) $Y = \max\{X_1, \ldots, X_m\}$ b) $Z = \min\{X_1, \ldots, X_m\}$

8.34 A function F can be the CDF of a continuous random variable X and still have "flat spots." What is the probabilistic meaning of $F(x) = c$ for all $x \in [a, b]$, where c is a constant and a and b are real numbers with $a < b$?

Theory Exercises

8.35 Prove Equation (8.7) on page 412: If X is a random variable, then $F_X(x-) = P(X < x)$ for all $x \in \mathcal{R}$.

8.36 Prove parts (a), (b), and (d) of Proposition 8.2 on page 412.

Advanced Exercises

8.37 Let X be a random variable. Prove that the CDF of X has a countable number of discontinuities. *Hint:* For each $n \in \mathcal{N}$, consider the set $D_n = \{\, x \in \mathcal{R} : F_X(x) - F_X(x-) \geq 1/n \,\}$.

8.38 For each $n \in \mathcal{N}$, let the random variable X_n have the discrete uniform distribution on the set $\{0, 1/n, \ldots, (n-1)/n\}$.
a) Determine the CDF of X_n.

Solution Let Z denote the distance of the center of the first spot from the center of the petri dish. In Example 8.8, we obtained a PDF of Z:

$$f_Z(z) = \begin{cases} 2z, & \text{if } 0 \le z < 1; \\ 0, & \text{otherwise.} \end{cases}$$

Using this PDF and the FPF, we can obtain the required probabilities.

a) We want to determine $P(Z \le 1/2)$. Applying the FPF, we get

$$P(Z \le 1/2) = \int_{-\infty}^{1/2} f_Z(z)\,dz = \int_{-\infty}^{0} 0\,dz + \int_{0}^{1/2} 2z\,dz = 1/4.$$

The probability is 0.25 that the center of the first spot to appear will be within 1/2 unit of the center of the petri dish.

b) We need to find $P(Z < 1/4 \text{ or } Z > 3/4)$. Applying the FPF, we get

$$P(Z < 1/4 \text{ or } Z > 3/4) = \int_{-\infty}^{1/4} f_Z(z)\,dz + \int_{3/4}^{\infty} f_Z(z)\,dz$$
$$= \int_{0}^{1/4} 2z\,dz + \int_{3/4}^{1} 2z\,dz = 1/16 + 7/16 = 0.5.$$

The probability is 0.5 that the center of the first spot to appear will be either less than 1/4 unit or more than 3/4 unit from the center of the petri dish. ■

EXERCISES 8.3 Basic Exercises

8.41 A point is chosen at random from the interior of a sphere of radius r. Let Z denote the distance from the center of the sphere to the point chosen.

a) Determine a PDF of the random variable Z. *Note:* Exercise 8.15 on page 413 asks for the CDF of Z.

b) Interpret the PDF of Z obtained in part (a) with regard to which values of Z are more likely than others.

c) Use the PDF of Z obtained in part (a) to find the probability that the distance from the center of the sphere to the point chosen is at most $r/2$; at least $r/4$; between $r/4$ and $r/2$.

d) Use the CDF of Z obtained in Exercise 8.15 to determine the three probabilities required in part (c).

8.42 By referring to parts (c) and (d) of Exercise 8.41, answer the following question for a continuous random variable with a PDF: If you know (in closed form) the CDF of the random variable, is there any need to apply the FPF to the PDF to obtain the probability that the random variable takes a value in any specified interval? Explain your answer.

8.43 Let X denote the tangent of an angle chosen at random from the interval $(-\pi/2, \pi/2)$.

a) Determine a PDF of the random variable X. *Note:* Exercise 8.16 on page 413 asks for the CDF of X.

b) Interpret the PDF of X obtained in part (a) with regard to which values of X are more likely than others.

c) Apply the FPF to the PDF of X obtained in part (a) to find the probability that the tangent of the angle chosen is at most 1.

d) Find the probability required in part (c) by using the CDF of X obtained in Exercise 8.16.

8.67 Let $X \sim \mathcal{U}(0, 1)$ and let c and $d > 0$ be real numbers.

a) Without doing any calculations, make an educated guess about the distribution of the random variable $Y = c + dX$.

b) Use Procedure 8.1 on page 419 to determine and identify a PDF of $Y = c + dX$.

c) Suppose that a and b be real numbers with $a < b$. Use part (b) to find c and d such that $c + dX \sim \mathcal{U}(a, b)$.

8.68 Simulation: This exercise requires access to a computer or graphing calculator.

a) Use the result of Exercise 8.67(c) to simulate 10,000 values of a $\mathcal{U}(-2, 3)$ random variable by employing a basic random number generator—that is, a random number generator that simulates the selection of a number at random from the interval $(0, 1)$.

b) Roughly, what would you expect a histogram of the 10,000 values obtained in part (a) to look like?

c) Obtain a histogram of the 10,000 values obtained in part (a) and compare it to your expectation in part (b).

8.69 Define $F: \mathcal{R} \to \mathcal{R}$ by $F(x) = 1 - e^{-\lambda x}$ for $x \geq 0$ and $F(x) = 0$ otherwise. Show that F is the CDF of a continuous random variable—that is, that F is everywhere continuous and satisfies properties (a)–(d) of Proposition 8.1 on page 411.

8.70 Let X be exponentially distributed with parameter λ.

a) Use the PDF of X and the FPF to show that the CDF of X is given by

$$F_X(x) = \begin{cases} 0, & \text{if } x < 0; \\ 1 - e^{-\lambda x}, & \text{if } x \geq 0. \end{cases}$$

b) Graph the CDF of X.

8.71 Solve Example 8.12 on page 433 by using the CDF of X. *Note:* Refer to Exercise 8.70.

8.72 Let $X \sim \mathcal{E}(1)$ and let b be a positive real number.

a) Use Procedure 8.1 on page 419 to determine and identify a PDF of $Y = bX$.

b) Let λ be a positive real number. Use part (a) to find b such that $bX \sim \mathcal{E}(\lambda)$.

8.73 Let $X \sim \mathcal{U}(0, 1)$ and let λ be a positive real number.

a) Use Procedure 8.1 on page 419 to show that $-\ln X \sim \mathcal{E}(1)$.

b) Use part (a) and Exercise 8.72(b) to conclude that $-\lambda^{-1} \ln X \sim \mathcal{E}(\lambda)$.

8.74 Simulation: This exercise requires access to a computer or graphing calculator.

a) Use the result of Exercise 8.73(b) to simulate 10,000 values of an $\mathcal{E}(6.9)$ random variable by employing a basic random number generator, that is, a random number generator that simulates the selection of a number at random from the interval $(0, 1)$.

b) Roughly, what would you expect a histogram of the 10,000 values obtained in part (a) to look like?

c) Obtain a histogram of the 10,000 values obtained in part (a) and compare it to your expectation in part (b).

8.75 According to the text *Rhythms of Dialogue* by J. Jaffee and S. Feldstein (New York: Academic Press, 1970), the duration, in seconds, of a pause during a monologue has an exponential distribution with parameter 1.4. Determine the probability that a pause

a) lasts between 0.5 second and 1 second.

b) exceeds 1 second.

c) exceeds 3 seconds.

d) exceeds 3 seconds, given that it exceeds 2 seconds.