9.2 Suppose that X and Y are continuous random variables defined on the same sample space. Let $U = X^2$ and $V = Y^2$. Find the joint CDF of U and V in terms of that of X and Y.

9.3 Refer to Example 9.1 on page 487.
a) Use Proposition 9.2 to obtain the marginal CDF of X.
b) Use the result of part (a) to identify the (marginal) probability distribution of X.
c) Repeat parts (a) and (b) for the random variable Y.

9.4 Let X and Y be the x and y coordinates, respectively, of a point selected at random from the upper half of the unit disk—that is, from the set $\{ (x, y) \in \mathcal{R}^2 : x^2 + y^2 < 1,\ y > 0 \}$.
a) Determine the CDF of the random variable X without using the joint CDF of X and Y.
b) Determine the CDF of the random variable Y without using the joint CDF of X and Y.
c) Determine the joint CDF of the random variables X and Y.
d) Use the result of part (c) and Proposition 9.2 to obtain the marginal CDF of X. Compare your answer with that found in part (a).
e) Use the result of part (c) and Proposition 9.2 to obtain the marginal CDF of Y. Compare your answer with that found in part (b).

9.5 Let X and Y be the x and y coordinates, respectively, of a point selected at random from the four vertices of the unit square.
a) Determine the joint CDF of the random variables X and Y.
b) Use the result of part (a) to obtain the marginal CDF of X.
c) Use the result of part (b) to identify the (marginal) probability distribution of X.
d) Repeat parts (b) and (c) for the random variable Y.
e) What is the relationship between the joint CDF of X and Y and the marginal CDFs of X and Y?

9.6 Let X and Y be independent random variables, both having the Bernoulli distribution with parameter p.
a) Find the joint PMF of X and Y.
b) Use part (a) and Definition 9.1 on page 486 to obtain the joint CDF of X and Y.
c) Find the marginal CDFs of X and Y from the joint CDF of X and Y obtained in part (b).
d) Find the marginal CDFs of X and Y without using the joint CDF of X and Y.
e) Use the results of part (d) and the independence of X and Y to determine the joint CDF of X and Y. *Hint:* Apply the definition of independent random variables given in Definition 6.4 on page 291 with the sets $A = (-\infty, x]$ and $B = (-\infty, y]$.
f) Compare your answers in parts (b) and (e).

9.7 Let X and Y be independent exponential random variables with parameters λ and μ, respectively.
a) Determine the marginal CDFs of X and Y without using the joint CDF of X and Y.
b) Use part (a) and the independence of X and Y to obtain the joint CDF of X and Y.

9.8 The function $G: \mathcal{R} \to \mathcal{R}$ defined by $G(x) = 1 - e^{-x}$ for $x \geq 0$, and $G(x) = 0$ otherwise, is a univariate CDF—namely, the CDF of an exponential random variable with parameter $\lambda = 1$. Show that the function $F: \mathcal{R}^2 \to \mathcal{R}$ defined by $F(x, y) = 1 - e^{-(x+y)}$ for $x, y \geq 0$, and $F(x, y) = 0$ otherwise, can't be a joint CDF. *Hint:* Suppose that F is the joint CDF of random variables X and Y and compute an appropriate probability of the form $P(a < X \leq b,\ c < Y \leq d)$.

9.9 For random variables X and Y defined on the same sample space, let $U = \min\{X, Y\}$ and $V = \max\{X, Y\}$.
a) Determine the CDF of V in terms of the joint CDF of X and Y.

b) Determine the CDF of U in terms of the joint and marginal CDFs of X and Y.
c) Determine the CDF of U in terms of the joint CDF of X and Y.

9.10 In this exercise, you are to generalize the results of Exercise 9.9. Let $X_1, \ldots, X_m$ be random variables defined on the same sample space and set $U = \min\{X_1, \ldots, X_m\}$ and $V = \max\{X_1, \ldots, X_m\}$.
a) Determine the CDF of V in terms of the joint CDF of $X_1, \ldots, X_m$.
b) Determine the CDF of U in terms of the joint and marginal CDFs of $X_1, \ldots, X_m$. *Hint:* Use the inclusion–exclusion principle, Proposition 2.10 on page 73.

Theory Exercises

9.11 Suppose that X and Y are random variables defined on the same sample space. Prove that $F_{X,Y}$—the joint CDF of X and Y—satisfies the following properties.
a) $F_{X,Y}$ is nondecreasing in each variable separately.
b) $F_{X,Y}$ is everywhere right-continuous in each variable separately.
c) $F_{X,Y}(x, -\infty) \equiv \lim_{y\to-\infty} F_{X,Y}(x, y) = 0$
d) $F_{X,Y}(-\infty, y) \equiv \lim_{x\to-\infty} F_{X,Y}(x, y) = 0$
e) $F_{X,Y}(\infty, \infty) \equiv \lim_{x,y\to\infty} F_{X,Y}(x, y) = 1$.

9.12 Generalize Proposition 9.1 on page 488 to the case of three random variables, X_1, X_2, and X_3, defined on the same sample space. In other words, state and prove a formula for $P(a_1 < X_1 \le b_1,\ a_2 < X_2 \le b_2,\ a_3 < X_3 \le b_3)$ in terms of the joint CDF of X_1, X_2, and X_3, where a_j and b_j are real numbers with $a_j < b_j$ for $j = 1, 2, 3$.

9.13 Prove Proposition 9.2 on page 490, which gives formulas for obtaining marginal CDFs from the joint CDF of two random variables.

Advanced Exercises

9.14 Let X and Y be the x and y coordinates, respectively, of a point selected at random from the diagonal of the unit square—that is, from $\{\,(x, y) \in \mathcal{R}^2 : y = x,\ 0 < x < 1\,\}$.
a) Determine the CDF of the random variable X without using the joint CDF of X and Y.
b) Use the result of part (a) to identify the probability distribution of X.
c) Determine the joint CDF of the random variables X and Y.
d) Use the result of part (c) and Proposition 9.2 on page 490 to obtain the marginal CDF of X. Compare your answer with that found in part (a).
e) Without doing any further computations, determine the marginal CDF of Y and identify the probability distribution of Y.

9.15 Let X and Y be random variables defined on the same sample space.
a) Prove that, if X and Y are independent random variables, then their joint CDF equals the product of their marginal CDFs—that is, $F_{X,Y}(x, y) = F_X(x)F_Y(y)$ for all $x, y \in \mathcal{R}$. *Hint:* Refer to Definition 6.4 on page 291.
b) Prove that, if the joint CDF of X and Y equals the product of their marginal CDFs, then X and Y are independent random variables. *Hint:* Use the fact that the joint CDF of two random variables completely determines their joint probability distribution.

9.16 Generalize Exercise 9.15 to the multivariate case of m random variables defined on the same sample space. *Hint:* Refer to Definition 6.5 on page 296. Also, use the fact that the joint CDF of m random variables completely determines their joint probability distribution.

c) Here we assume a random sample of size n from a uniform distribution on the interval $(0, 1)$. For $0 < t < 1$, we have $F(t) = t$ and $f(t) = 1$; otherwise, $f(t) = 0$. In this case, Equation (9.20) becomes

$$f_{X,Y}(x, y) = n(n-1)(y-x)^{n-2}, \qquad 0 < x < y < 1,$$

and $f_{X,Y}(x, y) = 0$ otherwise. ■

Another Procedure for Finding a Joint PDF

Proposition 9.3 on page 496 provides a method for finding a joint PDF of two continuous random variables. An alternative and more general method is presented in Proposition 9.4, whose proof is left to you as Exercise 9.28.

◆◆◆ **Proposition 9.4 An Equivalent Condition for the Existence of a Joint PDF**

Random variables X and Y defined on the same sample space have a joint PDF if and only if there is a nonnegative function f defined on $\mathcal{R}^2$ such that

$$F_{X,Y}(x, y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f(s, t)\, ds\, dt, \qquad x, y \in \mathcal{R}. \tag{9.21}$$

In this case, f is a joint PDF of X and Y, and

$$f(x, y) = \frac{\partial^2}{\partial x\, \partial y} F_{X,Y}(x, y) \tag{9.22}$$

at all points (x, y) where the mixed partial exists.

Multivariate Joint Probability Density Functions

The definitions and results that we presented for bivariate joint PDFs are easily generalized to the multivariate case of m continuous random variables. We ask you to supply these generalizations in Exercise 9.29.

EXERCISES 9.2 Basic Exercises

Note: Several of the exercises in this section are continuations of exercises presented in Section 9.1.

9.19 On page 493, we stated that "For continuous random variables X and Y, the probabilities $P(X = x, Y = y)$ are all 0 ...." Verify that statement.

9.20 Let X and Y be random variables defined on the same sample space. Set $U = a + bX$ and $V = c + dY$, where a, b, c, and d are real numbers with $b > 0$ and $d > 0$. Assuming that X and Y have a joint PDF, determine a joint PDF of U and V in terms of that of X and Y. *Note:* Exercise 9.1 on page 490 asks for the joint CDF of U and V in terms of that of X and Y.

9.21 Suppose that X and Y are continuous random variables with a joint PDF. Let $U = X^2$ and $V = Y^2$. Find a joint PDF of U and V in terms of that of X and Y. *Note:* Exercise 9.2 on page 491 asks for the joint CDF of U and V in terms of that of X and Y.

9.22 Let X and Y be the x and y coordinates of a point selected at random from the unit square. Determine $P(1/2 \le X \le 3/4,\ 1/4 \le Y \le 3/4)$ by using

a) Equation (9.4) on page 487.

b) the joint CDF of X and Y, obtained in Example 9.1 on page 487.

c) the joint PDF of X and Y, obtained in Example 9.3 on page 496.

d) Repeat parts (a)–(c) for $P(X > 0.6 \text{ or } Y < 0.2)$.

9.23 Let X and Y be the x and y coordinates, respectively, of a point selected at random from the upper half of the unit disk—that is, from the set $\{ (x, y) \in \mathcal{R}^2 : x^2 + y^2 < 1,\ y > 0 \}$.

a) Determine a joint PDF of X and Y. *Note:* Exercise 9.4(c) on page 491 asks for the joint CDF of X and Y.

b) Explain why the joint PDF obtained in part (a) is a nonzero constant on the upper half of the unit disk and is zero elsewhere.

9.24 Let X and Y be independent exponential random variables with parameters λ and μ, respectively.

a) Obtain a joint PDF of X and Y. *Note:* Exercise 9.7(b) on page 491 asks for the joint CDF of X and Y.

b) What is the relationship between the joint PDF of X and Y found in part (a) and the (standard) individual PDFs of X and Y?

c) Does the relationship in part (b) surprise you? Explain your answer.

9.25 Let X and Y be the minimum and maximum, respectively, of a random sample of size n taken from an exponential distribution with parameter λ. Determine a joint PDF of X and Y.

9.26 Let X and Y have joint PDF given by $f(x, y) = x + y$ for $0 < x < 1$ and $0 < y < 1$, and $f(x, y) = 0$ otherwise.

a) Use the joint PDF to determine $P(1/4 < X < 3/4,\ 1/2 < Y < 1)$.

b) Determine the joint CDF of the random variables X and Y.

c) Use the joint CDF to determine $P(1/4 < X < 3/4,\ 1/2 < Y < 1)$. Compare your answer with that obtained in part (a).

Theory Exercises

9.27 Prove Proposition 9.3 on page 496 when the partial derivatives of $F_{X,Y}$, up to and including those of the second order, exist and are continuous everywhere.

9.28 Prove Proposition 9.4 on page 499, which provides an equivalent condition for the existence of a bivariate joint PDF.

9.29 Let $X_1, \ldots, X_m$ be random variables defined on the same sample space.

a) Define *joint probability density function* for these random variables. *Hint:* Refer to Definition 9.2 on page 495.

b) State the m-variate analogue of Proposition 9.3 on page 496.

c) State the m-variate analogue of Proposition 9.4 on page 499.

Advanced Exercises

9.30 Refer to Exercise 9.14 on page 492, where X and Y are the x and y coordinates, respectively, of a point selected at random from the diagonal of the unit square—that is, from $\{ (x, y) \in \mathcal{R}^2 : y = x,\ 0 < x < 1 \}$.

a) Show that X and Y are continuous random variables.

b) Show that X and Y can't possibly have a joint PDF.

b) Compare the work entailed in part (a) to that in Example 9.3 on page 496 (which also depended on the work done in Example 9.1 on page 487).
c) Find the probability that the magnitude of the difference of the x and y coordinates of the point obtained is at most 1/4 by using the joint PDF of X and Y; by using Exercise 9.40(b).

9.42 Let X and Y be the x and y coordinates, respectively, of a point selected at random from the upper half of the unit disk—that is, from the set $\{ (x, y) \in \mathcal{R}^2 : x^2 + y^2 < 1,\ y > 0 \}$.
a) Use Exercise 9.40(a) to obtain a joint PDF of X and Y.
b) Compare the work entailed in part (a) to that in Exercise 9.23 on page 500 (which also depended on the work done in Exercise 9.4(c) on page 491).
c) Find the probability that the point obtained lies in the triangle with vertices $(-1, 0)$, $(0, 1)$, and $(1, 0)$ by using the joint PDF of X and Y; by using Exercise 9.40(b).
d) Determine the probability that the x coordinate of the point obtained is at least 1/2 unit from the origin.

9.43 Multivariate uniform random variables: Let S be a subset of $\mathcal{R}^m$ with finite nonzero m-dimensional volume. Suppose that a point is selected at random (i.e., uniformly) from S and, for $1 \leq j \leq m$, let X_j denote the x_j coordinate of the point obtained.
a) Determine a joint PDF of $X_1, \ldots, X_m$.
b) Determine a simple formula for $P\left((X_1, \ldots, X_m) \in A\right)$, where $A \subset \mathcal{R}^m$.

9.44 A point is selected a random from the unit cube, $\{ (x, y, z) \in \mathcal{R}^3 : 0 < x, y, z < 1 \}$. Let X, Y, and Z be the x, y, and z coordinates, respectively, of the point obtained.
a) Use the result of Exercise 9.43(a) to determine a joint PDF of X, Y and Z.
b) Use the result of part (a) to find the probability that $Z = \max\{X, Y, Z\}$—that is, that Z is the largest among X, Y and Z.
c) Use a symmetry argument to solve part (b).
d) Find the probability that the point obtained lies in the sphere of radius 1/4 centered at the point $(1/2, 1/2, 1/2)$.
e) Find the probability that, for the point obtained, the sum of the x and y coordinates exceeds the z coordinate.

9.45 A company is reviewing tornado damage claims under a farm insurance policy. Let X be the portion of a claim representing damage to the house and let Y be the portion of the same claim representing damage to the rest of the property. A joint density function of X and Y is $f_{X,Y}(x, y) = 6(1 - x - y)$ in the triangle with vertices $(0, 0)$, $(1, 0)$, and $(0, 1)$, and $f_{X,Y}(x, y) = 0$ otherwise.
a) Determine the probability that the portion of a claim representing damage to the house exceeds the portion of the same claim representing damage to the rest of the property.
b) Determine the probability that the portion of a claim representing damage to the house is less than 0.2.

9.46 In Example 9.5 on page 503, we considered two electrical components, A and B, with respective lifetimes X and Y whose joint PDF is $f_{X,Y}(x, y) = \lambda\mu e^{-(\lambda x + \mu y)}$ for $x > 0$ and $y > 0$, and $f_{X,Y}(x, y) = 0$ otherwise. Suppose that components A and B constitute an electrical unit.
a) Find a PDF of this electrical unit's lifetime if it's a parallel system—that is, if it functions when at least one of the components is working. *Hint:* First obtain the CDF of the electrical unit's lifetime by using the FPF.
b) Find a PDF of this electrical unit's lifetime if it's a series system—that is, if it functions only when both components are working. Identify the lifetime distribution in this case.
c) Determine the probability that exactly one of the two components is working at time t.

However, when considering more than two continuous random variables, there are many more marginal PDFs.

For instance, suppose that X, Y, and Z are three continuous random variables with a joint PDF. In this case, there are $\binom{3}{1} = 3$ univariate marginal PDFs—namely, f_X (the PDF of X), f_Y (the PDF of Y), and f_Z (the PDF of Z). Additionally, there are $\binom{3}{2} = 3$ bivariate marginal PDFs—namely, $f_{X,Y}$ (the joint PDF of X and Y), $f_{X,Z}$ (the joint PDF of X and Z), and $f_{Y,Z}$ (the joint PDF of Y and Z). Each marginal PDF is obtained by integrating the joint PDF of X, Y, and Z on the unwanted variable(s). To illustrate, the univariate marginal PDF of Y and the bivariate marginal PDF of Y and Z are obtained as follows:

$$f_Y(y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{X,Y,Z}(x, y, z)\,dx\,dz$$

and

$$f_{Y,Z}(y, z) = \int_{-\infty}^{\infty} f_{X,Y,Z}(x, y, z)\,dx.$$

In general, if $X_1, \ldots, X_m$ are m continuous random variables with a joint PDF, there are $\binom{m}{j}$ j-variate marginal PDFs ($1 \le j \le m-1$). Each such j-variate marginal PDF is obtained by integrating the joint PDF of $X_1, \ldots, X_m$ on the other $m - j$ variables.

We can also define conditional PDFs in the general multivariate case. The formulas for these conditional PDFs are analogous to those in the bivariate case. For instance, if X, Y, and Z are continuous random variables with a joint PDF, then

$$f_{Y,Z\,|\,X}(y, z\,|\,x) = \frac{f_{X,Y,Z}(x, y, z)}{f_X(x)}$$

and

$$f_{Z\,|\,X,Y}(z\,|\,x, y) = \frac{f_{X,Y,Z}(x, y, z)}{f_{X,Y}(x, y)}.$$

From the previous formula and the general multiplication rule for the joint PDF of two continuous random variables—Equation (9.34) on page 517—we easily obtain the general multiplication rule in the trivariate case:

$$f_{X,Y,Z}(x, y, z) = f_X(x)\,f_{Y\,|\,X}(y\,|\,x)\,f_{Z\,|\,X,Y}(z\,|\,x, y). \tag{9.36}$$

Exercise 9.80 asks you to state the general multiplication rule for the joint PDF of m continuous random variables.

EXERCISES 9.4 Basic Exercises

9.63 Let X and Y denote the x and y coordinates, respectively, of a point selected at random from the unit square. In Example 9.7 on page 510, we obtained marginal PDFs of X and Y.

a) Obtain and identify all conditional PDFs.

b) Compare the conditional PDFs of Y given $X = x$ to each other and to the marginal PDF of Y. Interpret the results.

9.64 For each part, determine $P(X > 0.9\,|\,Y = 0.8)$ for the specified joint PDF of a random point (X, Y) in the unit square.

a) $f_{X,Y}(x, y) = 1$ **b)** $f_{X,Y}(x, y) = x + y$ **c)** $f_{X,Y}(x, y) = \frac{3}{2}(x^2 + y^2)$

Multivariate Independent Continuous Random Variables

As in the bivariate continuous case, a necessary and sufficient condition for several continuous random variables with a joint PDF to be independent is that their joint PDF equals the product of their marginal PDFs. We leave the precise statement of this result and its proof to you as Exercise 9.108.

ES 9.5 Basic Exercises

9.88 In the petri-dish illustration of Example 9.8 on page 511, let X and Y denote the x and y coordinates, respectively, of the center of the first spot (visible bacteria colony) to appear. We showed, in Example 9.14 on page 524, that the random variables X and Y aren't independent. Here you are asked to provide three other arguments to establish that result.

a) Argue heuristically that X and Y aren't independent by considering the possible values of Y among different specified values of X.

b) Use the results of Examples 9.8(b) and 9.10 (pages 512 and 515, respectively) and Proposition 9.10 (page 527) to show that X and Y aren't independent.

c) Use Proposition 9.4 on page 499—specifically, Equation (9.22)—to show that X and Y aren't independent. *Hint:* Assume that the product of the marginals of X and Y is a joint PDF of X and Y. Obtain a contradiction.

9.89 Provide another verification that the random variables X and Y in Example 9.15 on page 525 are independent. *Hint:* Refer to Example 9.11 on page 516.

9.90 Solve Example 9.16 on page 526 if the arrival times are independent uniform random variables on the interval $(-5, 5)$, where time is measured in minutes relative to noon.

9.91 Let X and Y be the x and y coordinates, respectively, of a point selected at random from the unit square. Determine whether X and Y are independent random variables.

9.92 Let X and Y be the x and y coordinates, respectively, of a point selected at random from the upper half of the unit disk, that is, from the set $\{(x, y) \in \mathcal{R}^2 : x^2 + y^2 < 1,\ y > 0\}$. Determine whether X and Y are independent random variables. *Note:* Exercise 9.65 on page 520 provides a joint PDF of X and Y and asks for marginal and conditional PDFs.

9.93 Refer to the regression analysis illustration of Example 9.13 on page 517. Use Proposition 9.10 on page 527 to determine necessary and sufficient conditions for X and Y to be independent random variables.

9.94 Let X and Y be independent random variables, each uniform on the interval $(-1, 1)$. Find the probability that the roots of the random quadratic equation $x^2 + Xx + Y = 0$ are real.

9.95 Let X and Y be continuous random variables with a joint PDF. Suppose that there are nonnegative functions g and h defined on $\mathcal{R}$ such that $f_{X,Y}(x, y) = g(x)h(y)$ for all $x, y \in \mathcal{R}$. Show that X and Y are independent by proceeding as follows.

a) Obtain a marginal PDF of X in terms of g and h.

b) Obtain a marginal PDF of Y in terms of g and h.

c) Explain why $\left(\int_{-\infty}^{\infty} g(x)\,dx\right)\left(\int_{-\infty}^{\infty} h(y)\,dy\right) = 1$.

d) Verify that X and Y are independent random variables.

9.96 In Exercise 9.95, is it necessarily true that g is a marginal PDF of X and h is a marginal PDF of Y? If not, find conditions when that is the case.

EXERCISES 9.6 Basic Exercises

9.115 This exercise considers the two-dimensional analogue of the Maxwell distribution discussed in Examples 9.18 and 9.21. Suppose that X and Y are independent normal random variables, both having parameters 0 and σ^2. Show that the random variable $R = \sqrt{X^2 + Y^2}$ has the Rayleigh distribution—introduced in Exercise 8.159 on page 475—by using an argument similar to that in

a) Example 9.18 on page 533.
b) Example 9.21 on page 537.

9.116 Midrange of a random sample: Let $X_1, \ldots, X_n$ be a random sample from a continuous distribution with CDF F and PDF f. The *midrange* of the random sample is defined to be $M = (X + Y)/2$, where $X = \min\{X_1, \ldots, X_n\}$ and $Y = \max\{X_1, \ldots, X_n\}$.

a) Show that a PDF of the random variable M is given by

$$f_M(m) = 2n(n-1) \int_{-\infty}^{m} f(x) f(2m - x) \left(F(2m - x) - F(x)\right)^{n-2} dx,$$

for $-\infty < m < \infty$. *Note:* A joint PDF of X and Y can be found in Equation (9.20) on page 498.

b) Apply the result of part (a) to the special case of a random sample of size n from a uniform distribution on the interval (0, 1).

9.117 Mechanical or electrical units often consist of several components, each of which is subject to failure. A unit is said to be a *parallel system* if it functions when at least one of the components is working. Consider a parallel system of n components, $C_1, \ldots, C_n$, with respective lifetimes $X_1, \ldots, X_n$, where $X_1, \ldots, X_n$ are independent exponential random variables with parameters $\lambda_1, \ldots, \lambda_n$, respectively. Determine the probability distribution of the lifetime of this parallel system.

9.118 A critical component in a complex system has a built-in spare. When the original component fails, the spare comes on immediately and the system continues to function. When the spare fails, the system goes down. Assuming that the lifetimes of both the original component and the spare are exponentially distributed with parameter λ, find and identify the CDF of the proportion of time until system failure that the original component is working.

9.119 Bilateral exponential random variable: Let X and Y be independent random variables both having the exponential distribution with parameter λ. Determine and graph a PDF of $Z = X - Y$. *Note:* Any random variable with this PDF is called a *bilateral exponential random variable* and is said to have the *bilateral exponential distribution with parameter* λ.

9.120 Let X and Y be random variables with joint PDF given by $f_{X,Y}(x, y) = e^{-(x+y)}$ for $x > 0$ and $y > 0$, and $f_{X,Y}(x, y) = 0$ otherwise. An insurance policy is written to reimburse $X + Y$. Determine the probability that the reimbursement is at most 1 by using

a) the FPF.
b) Proposition 9.13 on page 537 and Equation (8.49) on page 453.

9.121 An insurance company sells two types of auto insurance policies: basic and deluxe. The time, in days, until the next basic policy claim is an exponential random variable with parameter 1/2. The time, in days, until the next deluxe policy claim is an independent exponential random variable with parameter 1/3. Determine the probability that the next claim will be a deluxe policy claim by using

a) the FPF. **b)** Proposition 9.11 on page 532.