Electrical Chaos

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Summary

Chaotic behavior of an electronic circuit is characterized by the circuit producing hundreds of voltage maxima for one frequency. A simple circuit consisting of an inductor, resistor, and a diode was created and driven at different frequencies using Waveforms software in order to find the frequency threshold for chaos. The threshold was discovered to be from 2 to 3.5 MHz. The results were plotted in a bifurcation diagram to indicate the chaotic nature of the circuit.

1 Introduction

Chaotic behavior is behavior that exhibits randomness and unpredictability [1]. In terms of a circuit, the chaotic behavior is when an output is producing a wave with a large amount of random maximum voltage peaks. The number of peaks are determined by how many components the Voltage vs time wave is. For example, an N component wave will have peaks at N values, where N is an integer greater than 0. For a non chaotic output, usually 4 to 5 peak voltages can be observed. However, for a chaotic output there are hundreds of unique random peak voltages causing the input vs output graph to look as if it is filled with noise [3]. A chaotic circuit can actually be simple in its creation. The simplest one consists of a resistor, an inductor, and a diode. The way the circuit produces chaos can be attributed to the way the inductor acts with the diode. A good analogy, taken from Hillborn's paper, is that the diode is like "a water pipe with a flap valve". Where water is analogous to current. As the current travels through the diode the flap of the diode allows it to travel freely though. However, if the current wants to backtrack the "flap" goes down and inhibits the current from going back. The flap requires time to close to block the current, this allows some current to slip through and go back the other way. Another important property of the flap is that when it is held up by current it contains potential energy related to its capacitance.

The inductor creates an electric potential proportional to the rate of change of current, this proportion is the inductance of the inductor. The inductor allows a buffer for the current and potential of the circuit to allow freedom for chaos to occur. The inductor works with the capacitance of the diode to find a frequency to oscillate current and voltage. when the oscillatory period matches the time it takes for the flap to close, electrical chaos can occur [1]. Chaos can happen because the current and voltage change so fast that the closing and opening of flap's natural behavior is disrupted which causes chaos [3]. Although the unpredictability of chaos can seem as though it would be detrimental, recently scientists and engineers have embraced its utility. Studying chaos has led to advancement in regulating artificial heartbeats, telecommunication, and encryption due to the trait that chaotic systems have the capability to switch between behaviors rapidly [2].

2 Methods

2.1 Apparatus

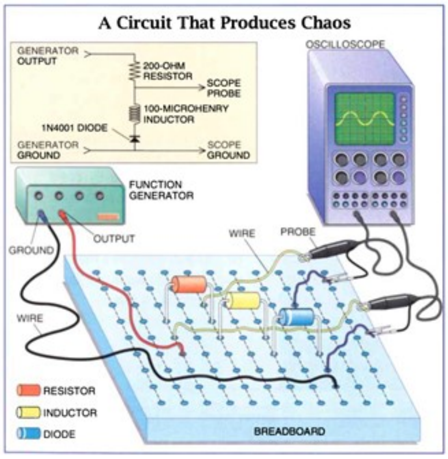

Figure 1: The circuit used to generate the chaotic output used for the bifurcation diagram [3].

The apparatus consisted of three main components, a breadboard circuit, an oscilloscope, and function generator. Both the oscilloscope and the function generator were powered by the Waveforms software. The circuit shown in Fig. 1 consists of a 200 Ohm resistor in series, a 1 microhenry inductor in series with the resistor configuration, and a diode in series with the inductor. The circuit's input came via the function generator in Waveforms. The generator was set to a sine wave input with an input voltage of 3 V and the frequency of the wave varied from 0.5 MHz to 4.6 MHz. The reason the frequency was manipulated was to explore a full cycle of component waves ranging from one component wave to chaotic waves back to one component waves. The resulting output of the circuit was shown on the oscilloscope in Waveforms. The purpose of viewing the oscilloscope was to be able to see at what frequency each component wave occurred, and for every 100 KHz, the data from the oscilloscope was exported and put into a spreadsheet for analysis.

2.2 Procedure

Each data set of output voltage for the corresponding frequency had to be analyzed in a python program using the function *scipy.peaks* to determine the peak voltage. The amount of peaks depends on the type of wave, i.e an n component wave has n peaks and a chaotic wave has hundreds of peaks. However, the circuit was sensitive and any movement resulted in certain component waves having many more peaks than they should have. For example when plotting V_{in} vs V_{out} it is clear to see how many components you are dealing with based on the number of rings (See Fig 2.) We would have clear,two ring plots with over a thousand maxima values. To correct this mistake, a filter was created in python to estimate the voltage peak value. The voltage peaks for each frequency were imported into the program as a list of values and if the wave was a one component wave, we assumed the average value of the list would be the max.

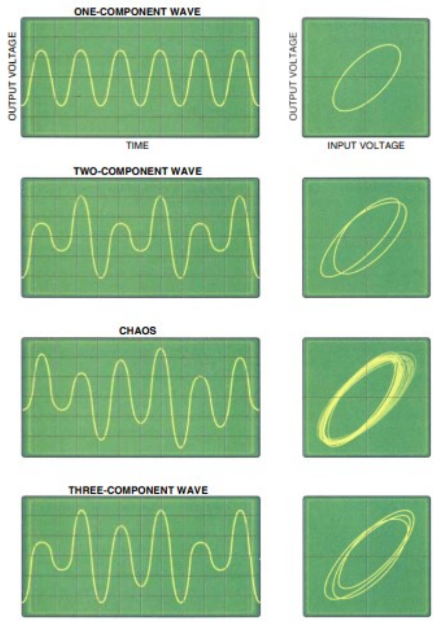

Figure 2: Showing how an N component wave translates into an input vs output graph where the circle has N rings [3].

For a two component wave the average value of the list was taken and the values below the average were again averaged out to find the lower value maximum and the voltage values above the average were averaged out and that value was used as the higher value maximum. For 3 and 4 component waves, we wrote an if else statement filtering out data between 3 or 4 ranges based on a split in the data. The new data was appended to 3 or 4 lists and averaged them out to find the respected maxima for the wave.

From there we created two lists of data to plot the bifurcation diagram describing the maximum Voltage(s) versus Frequency. Fig. 3 shows the distribution of peak voltage values for the whole sweep via a histogram.

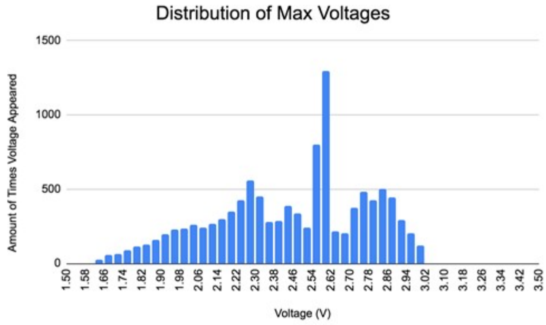

Figure 3: A histogram showing the distribution of voltage maxima. Note, where a large peak is seen, it indicates the voltages fall within a 0.4 V range of the labeled value and that most peaks were different by a factor of 0.01 to 0.00001 for chaotic frequencies (2MHz - 3.5MHz)

3 Results

The results are shown in Fig. 4. The bifurcation diagram shows that when the circuit was driven with a frequency of 2 to 3.5 MHz it exhibits chaotic behavior. This is because there are several maxima voltage values at one frequency, a key trait of a chaotic circuit. The average peak voltage value of the frequency sweep is 2.45 ± 0.3199 V with a range of 1.62 V to 3.02 V.

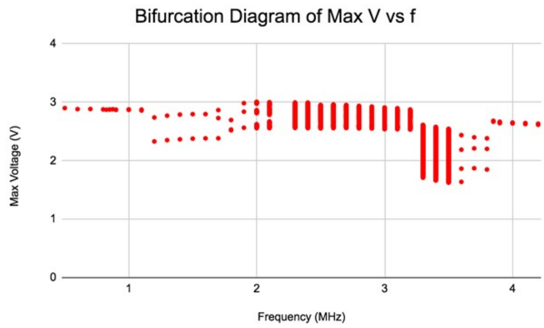

Figure 4: Bifurcation Diagram showing the Max Voltage vs Frequency. The vertical lines show a cluster of maxima at that frequency, the main indication that the circuit produces chaos.

4 Discussion

4.1 Uncertainty

The main uncertainty came from filtering out the maxima when a low n component wave had several hundred peak voltages. We estimated the uncertainty in the maxima calculation to be ± 0.1 volts because when we had multiple maxima, all max voltage values were in a range of less than 1 volt from each other. There are several factors that could have contributed to the incorrect number of maxima being recorded by the software such as not having the electrical components all the way in the breadboard, the breadboard also has some capacitance, and the usb wire importing the data to waveforms was very sensitive on my computer and actually lost signal a couple of times during the data taking experiment. In addition, whenever the circuit moved its output changed, which is a testament to the sensitivity of the circuit. The average peak voltage value of the frequency sweep is 2.45 ± 0.3199 V with a range of 1.62 V to 3.02 V which means that all our values greater than the mean fall within 2σ of the mean and all the values less than the mean fall within 3σ. The distribution of voltages varies due to the fact that the voltage output is dependent on the frequency. In addition, the values less than the mean fall within 3σ instead of 2σ because we took more data from the frequencies preceding chaotic behavior compared to the amount we took after the chaotic behavior which was an experimental error.

4.2 Conclusion

Based on the bifurcation diagram and our output vs input voltage plot, it is clear that the circuit created chaos. Chaos happened at the 2.0 MHz threshold and ended at the 3.5 MHz threshold. The bifurcation diagram, which plotted max amplitude vs frequency demonstrates chaotic behavior at the aforementioned frequencies due to the fact there are several max voltages for one specific frequency, resulting in a vertical line. The average peak voltage value of the frequency sweep is 2.45 ± 0.3199 V and all values of voltage fall within 3σ of the mean.

References

[1] Robert C. Hillborn. *Chaos and Non-Linear Dynamics*. Oxford University Press, 1994, pp. 1–18.

[2] Louis M. Pecora. *Mastering Chaos*. Vol. 269. Scientific American, 1993.

[3] Douglas Smith. “How to Generate Chaos at Home”. In: *Scientific American* 266.1 (1992), pp. 144–146.